ORDER OF THE PHASE TRANSITION RELATED TO THE DEGENERACY AND INTERACTION WITH PROPOSAL FOR AN EXACT PROBLEM

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The system of many identical interacting units with certain degenerate states is investigated[1]. The state of a unit system is represented by a spin-like variable σ_i $(i=1,2,\cdots,N)$, which takes the values $-s,-s+1,\cdots,s-1,s$ with integral or half-integral s. The Hamiltonian of the system is expressed as a sum of pair interaction energies, $\sum_{(i,j)} V(\sigma_i,\sigma_j)$. The state designated by σ_i is assumed to be degenerate and its degeneracy is denoted by $D(\sigma_i)$. The importance of symmetry properties of $D(\sigma_i)$ and $V(\sigma_i,\sigma_j)$ are stressed, where the symmetry means that $D(\sigma_i)$ and $V(\sigma_i,\sigma_j)$ are even functions of σ_i and σ_j : i.e. $D(\sigma_i)=D(-\sigma_i)$, $V(\sigma_i,\sigma_j)=V(-\sigma_i,-\sigma_j)$. The system of magnetic spins with usual exchange interactions belongs to the symmetric category of constant degeneracy. The thermostatistical average of any σ_i which is denoted by σ represents the order parameter of the system. Dependences of σ upon temperature T and a certain applied field H causing a perturbing energy $-H\sum_i \sigma_i$ are investigated. For the present the pair interaction is confined to such a simple form as $V(\sigma_i,\sigma_j)=-U\sigma_i\sigma_j-W(\sigma_i+\sigma_j)$ with the use of two constants U and W. The partition function of such a system is expressed as

$$Z(T,H)=\sum_{\sigma_1}\cdots\sum_{\sigma_N} D(\sigma_1)\cdots D(\sigma_N)\exp(\beta V\sum_{(i,j)}\sigma_i\sigma_j+\beta H\sum_i\sigma_i) ,$$

where β denotes the inverse of the Boltzmann constant times temperature. The properties of $Z(T,H)$ in relation to T and H are the object of the present investigation.

The symmetric system with constant degeneracy is equivalent to the magnetic spin system with exchange interactions and exhibits the phase transition of second order. The Lee-Yang theorem[2] is valid for such systems, where the singular line of the partition function on the H-T plane is such as shown by the thick line in Fig.1. In the simplest case of these, $s=1/2$ and the degeneracy $D(1/2)=D(-1/2)$ is constant. We have therefore necessarily a second-order transition. In case $s=1$, there exists such symmetric systems that have a non-constant $D(\sigma_i)$. If a certain condition is satistfied by $D(\sigma_i)$, the phase transition of first order occurs, where the singular line is given by such a thick curve as shown in Fig.2. Otherwise we have the second-order transition similar to the symmtric case of s=1/2. The mean field theory leads to the condition expressed as the inequality $D(1)=D(-1)< D(0)/4$ for the first-order transition to occur in the case s=1. There occurs also an induced transition by the applied field H between the temperatures T_c and T_c' such as shown in Fig.2.

For the asymmetric system[3)4)] we get the singular lines such as shown in Figs.3 and 4 corresponding to Figs.1 and 2, respectively, where several branches represent the lines for various typical cases (not exhaustively) of the relevant parameters. The intersection and contact points of the singular line with the temperature axis correspond to the transition temperatures of first-and second-order transitions, respectively. The induced transition by the field can be also read from Figs.3 and 4 as well as Fig.2.

Finally the following notes are to be added.

(1) The symmetric system on plane lattice with s=1 and $D(1)=D(-1)< cD(0)$,c being a certain constant,is proposed as a simplest model which might be exactly proved to exhibit the first-order phase transition of usual sort with such a singular line as shown in Fig.2. Accordingly the Lee-Yang theorem corresponding to Fig.1 should be replaced for such a system by the theorem corresponding to Fig.2. The mean field approximation leads to the value 1/4 for the above critical constant c. In fact the asymmetric system with s=1/2 on plane lattice has been already proved exactly to exhibit the first order transition[4)],which is, however,of a peculiar type (Fig.3).

(2) The calculation in the present paper can be applied to see the occurence of the transition between different phases of dissipative process in the theory such as investigated recently[5)].

(3) By generalizing the state variable σ_i to a continuous one, some more specific features may be found,although the main feature will not essentially change. Such a generalization is adequate in particular for the problem mentioned above in (2).

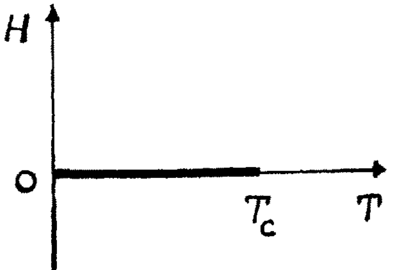

Fig.1

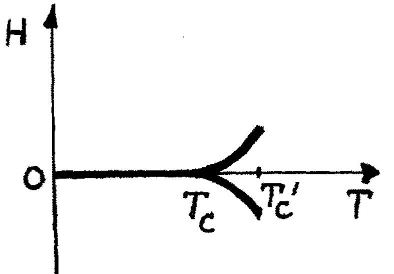

Fig.2

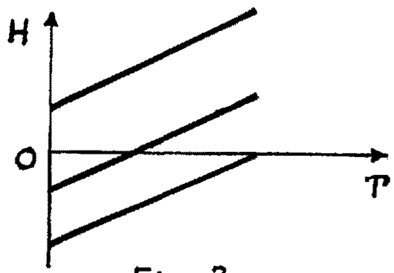

Fig. 3

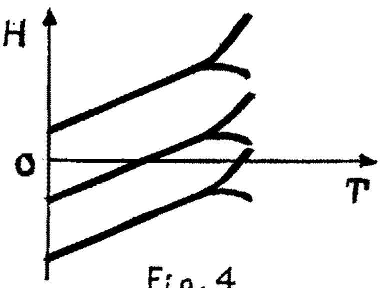

Fig. 4

Peferences

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3) S.Strassler and C.Kittel, Phys. Rev. 139 (1965),A758.

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Discussions

Lobowitz and Kubo asked about the relation to the Lee-Yang theorem. Further Lebowitz questioned whether the singularity shown by Fig.2 is valid for the system with s=1/2. Nakano denied it and clarified that such a singularity can appear under a certain condition on the degeneracy function D($_i$) only in the case s is equal to or more than the unity. M.Suzuki expected his work on the Lee Yang theorem for the Ising ferromagnets of general spin with degeneracy (J. Math. Phys. 14 (1973) 1088) might be useful. Nakano hoped to make use of it to decide the limit of that theorem.