IBs in normal deformed odd-A nuclei

Pardeep Singh,* Harjeet Kaur, and Sham S. Malik
Department of Physics, Guru Nanak Dev University, Amritsar-143005

Introduction

Rotational bands of nuclei are known to display several puzzling features [1], and many of them are still very poorly understood. We expect the band structure of each nucleus to be unique. It is, therefore, very unusual that two nuclei should have identical bands (IBs). Many examples of IBs in odd-A normal deformed nuclei were pointed out for the first time by A. K. Jain [2]. Later on, many examples of IBs in the rare-earth region [3] and the actinide region [4] were also pointed out. An excellent review of the IBs and the various explanations may be found in [7]. IBs in high-K 3qp rotational bands based on same band head spin have been found and analyzed on the basis of TAC model [5].

A number of criteria have been used in the literature for the selection of IBs. The systematic studies would be very beneficial in the investigation of the origin of IBs because of the known spin, configuration, and excitation energy of normal deformed bands. Mostly equality of γ-ray energies, and dynamic moment of inertia $\Im^{(2)}$ of two bands or several other quantities indirectly related to these parameters have been chosen as criteria for selection of IBs [9].

Results and discussion

In the present study of IBs in normal deformed odd-A nuclei, based on different band head spin, we found the fractional change in dynamic moment of inertia $F.C.(\Im^{(2)})$, which is defined as

$$F.C.(\Im^{(2)}) = \frac{\Im_A^{(2)} - \Im_B^{(2)}}{\Im_A^{(2)}}$$

where $A > B$ and dynamic moment of inertia ($\Im^{(2)}$) is the ratio of the variations of angular momentum and rotational frequency.

We also applied energy factor method [6, 8, 9] to generalize the definition of IBs. The method is explained as

$E_\gamma(I_A)$ and $E_\gamma(I_B)$ are the transition energies of bands A and B of nuclei having IBs. By introducing energy factor x, $E_\gamma(I_A)$ is defined as

$$E_\gamma(I_A) = xE_\gamma(I_A) + (1-x)E_\gamma(I_A+2) \quad (1)$$

here $I_A = I_A^0 + 2k, I_B = I_B^0 + 2k, k = 0, 1, 2, ..., N-1$

In this equation, x is a parameter restricted to $[0, 1]$, I_A^0 and I_B^0 are the angular momentum of the band head for band A and band B, respectively. The band head energies satisfies the relation $E_\gamma(I_A^0) \leqslant E_\gamma(I_B^0) \leqslant E_\gamma(I_A^0 + 2)$. the factor x describes the relationship of the transition energies of band A and band B, and thus we may call it energy factor x. The factor x can be determined by minimizing the quantity s, with respect to the factor x,

$$s = \sum_{k=0}^{N-1} [E_\gamma(I_A) - E_\gamma(I_B)]^2 \quad (2)$$

and the result is obtained from $ds/dx = 0$ as

$$x = \frac{\sum_k [E_\gamma(I_A+2) - E_\gamma(I_A)][E_\gamma(I_A+2) - E_\gamma(I_B)]}{\sum_k [E_\gamma(I_A+2) - E_\gamma(I_A)]^2} \quad (3)$$

For appropriate N and δ, if $\Delta E_\gamma(k) = |E_\gamma(I_A) - E_\gamma(I_B)| \leqslant \delta$ is fulfilled for all k and a fixed x in the region of $[0, 1]$, we regard for the two bands as IBs.

For a pair of IBs, the incremental alignment Δi is defined as

$$\Delta i = 2\frac{E_{\gamma near} - E_\gamma(I_B)}{E_\gamma(I_A+2) - E_\gamma(I_A)} \quad (4)$$

*Electronic address: pardeepphy@gmail.com

TABLE I: γ -ray energies for different pairs of nuclei.

^{171}Hf		^{173}Hf		^{163}Er		^{175}Ta	
I^π	E_γ	I^π	E_γ	I^π	E_γ	I^π	E_γ
$25/2^-$	177	$25/2^-$	163.3	$21/2^+$	162.0	$23/2^+$	165.7
$27/2^-$	211	$27/2^-$	209.1	$23/2^+$	187.5	$25/2^+$	191.0
$29/2^-$	239.4	$29/2^-$	242.0	$25/2^+$	210.3	$27/2^+$	212.4
$31/2^-$	265.6	$31/2^-$	269.2	$27/2^+$	230.5	$29/2^+$	232.3
$33/2^-$	289.3	$33/2^-$	293.8	$29/2^+$	249.4	$31/2^+$	251.3
$35/2^-$	311	$35/2^-$	315.5	$31/2^+$	266.5	$33/2^+$	269.5
$37/2^-$	331.0	$37/2^-$	336.3	$33/2^+$	281.9	$35/2^+$	286.6
$39/2^-$	349.0	$39/2^-$	355.0				
^{163}Er		^{181}Ir		^{177}Lu		^{179}Ta	
I^π	E_γ	I^π	E_γ	I^π	E_γ	I^π	E_γ
$21/2^+$	193.7	$23/2^-$	195.6	$25/2^-$	272.8	$27/2^+$	273.8
$23/2^+$	210.3	$25/2^-$	206.8	$27/2^-$	293.5	$29/2^+$	293.7
$25/2^+$	225.4	$27/2^-$	220.5	$29/2^-$	314.1	$31/2^+$	313.5
$27/2^+$	238.3	$29/2^-$	234.5	$31/2^-$	334.4	$33/2^+$	332.5
$29/2^+$	249.4	$31/2^-$	247.2	$33/2^-$	353.8	$35/2^+$	351.3
$31/2^+$	258.4	$33/2^-$	256.7	$35/2^-$	372.8	$37/2^+$	369.5
$33/2^+$	263.7	$35/2^-$	259.8	$37/2^-$	392.2	$39/2^+$	386.6
^{171}Hf		^{177}Re		^{177}W		^{177}Re	
I^π	E_γ	I^π	E_γ	I^π	E_γ	I^π	E_γ
$21/2^+$	149.4	$23/2^+$	150.7	$21/2^+$	144.3	$23/2^+$	150.7
$23/2^+$	183	$25/2^+$	180.2	$23/2^+$	187.5	$25/2^+$	180.2
$25/2^+$	212	$27/2^+$	211.3	$25/2^+$	217.5	$27/2^+$	211.3
$27/2^+$	236.9	$29/2^+$	237.7	$27/2^+$	241.4	$29/2^+$	237.7
$29/2^+$	259.4	$31/2^+$	260.2	$29/2^+$	261.7	$31/2^+$	260.2
$31/2^+$	280.8	$33/2^+$	279.8	$31/2^+$	276.0	$33/2^+$	279.8

where $E_{\gamma near}$ is the transition energy in band A which is the closest to $E_\gamma(I_B)$ and band A is supposed to be a reference band. From Eqs. (1), (3),(4) and an assumption that the $\delta E_\gamma = E_\gamma(I_A+2) - E_\gamma(I_A)$ is almost a constant value for rotational band, we could find the relation between x and $\overline{\Delta i}$:

$$\overline{\Delta i} \approx 2x \tag{5}$$

if $E_{\gamma near} = E_\gamma(I_A + 2)$ and

$$\overline{\Delta i} \approx 2(x-1) \tag{6}$$

if $E_{\gamma near} = E_\gamma(I_A)$

From TABLE I, we can notice the similarity in the γ-ray energies for six pairs of nuclei. In TABLE II, we have calculated x (energy factor), Δi (incremental alignment), $\overline{\Delta i}$ (average value of incremental alignment), $|\Delta i - \overline{\Delta i}|$. The Δi and its average $\overline{\Delta i}$ are restricted in region [-1,1] according to Eq. (4). The difference between them is very small and less than 0.02. We have also tabulated the calculated value of $F.C.\Im^{(2)}$ in TABLE II. One may notice that these values are also very small and less than 0.2. These results suggest that, the nuclei pairs, given in TABLE I and TABLE II, exhibit IBs.

TABLE II: Calculated values of x, Δi, $\overline{\Delta i}$, $|\Delta i - \overline{\Delta i}|$ and $F.C.(\Im^{(2)})$ for different pairs of normal deformed odd-A nuclei exhibiting IBs.

Band (A)	Band (B)	x	Δi	$\overline{\Delta i}$	$\lvert\Delta i - \overline{\Delta i}\rvert$	$F.C.\Im^{(2)}$
^{171}Hf	^{173}Hf	0.916	-0.179	-0.168	0.011	0.103
^{163}Er	^{175}Ta	0.929	-0.135	-0.141	0.006	0.016
^{181}Ir	^{163}Er	0.840	-0.328	-0.319	0.009	0.044
^{179}Ta	^{177}Lu	0.981	-0.043	-0.038	0.005	0.056
^{177}Re	^{171}Hf	0.992	-0.011	-0.015	0.004	0.015
^{177}Re	^{177}W	0.882	-0.228	-0.235	0.007	0.188

References

[1] S. Singh, S. S. Malik, A. K. Jain and B. Singh, At. Data and Nucl. Data Tables, **92** 1, 2006.

[2] K. Jain and A. K. Jain, Phys. Rev. C, **30** 2050, 1984.

[3] A. K. Jain, R. K. Sheline, P. C. Sood and K Jain, Rev. Mod. Phys., **62** 393, 1990.

[4] K Jain K and A. K. Jain, At. Data and Nucl. Data Tables, **50** 269, 1992.

[5] H. Kaur, P. Singh and S. S. Malik, J. Phys. G, **42** 025105, 2015.

[6] C. Shen, W. Luo, and Y. S. Chen, Phys. Rev. C, **55**, 1762, 1997.

[7] C. Baktash, B. Haas and W. Nazarewicz, Annu. Rev. Nucl. Part. Sci., **45** 485, 1995.

[8] F. S. Stephens, M. A. Deleplanque, J. E. Draper, R. M. Diamond et al., Phys. Rev. Lett., **64** 2623, 1990.

[9] J. Y. Zhang and L. L. Riedinger, Phys. Rev. Lett., **69** 3448, 1992.