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On the time-dependent Aharonov–Bohm effect

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ABSTRACT

The Aharonov–Bohm effect in the background of a time-dependent vector potential is re-examined for both non-relativistic and relativistic cases. Based on the solutions to the Schrodinger and Dirac equations which contain the time-dependent magnetic vector potential, we find that contrary to the conclusions in a recent paper (Singleton and Vagenas 2013 [4]), the interference pattern will be altered with respect to time because of the time-dependent vector potential.

The Aharonov–Bohm (AB) effect plays an important role in modern quantum theories [1]. There are two kinds of AB effect which are named vector and scalar AB effects respectively. The vector AB effect predicts that compared with the conventional double-slit experiment in quantum mechanics, the interference pattern will be shifted if a long–thin flux-carried solenoid is located between these two slits [1]. This prediction has been confirmed by several experiments [2,3].

Quantum mechanically, the dynamics of electrons moving in a region with the vector potential is governed by the Schrodinger equation (we study the non-relativistic case firstly and consider the relativistic case at the end of this paper)

$$i\hbar\frac{\partial\psi}{\partial t} = H\psi \tag{1}$$

where H is the Hamiltonian

$$H = \frac{1}{2m}(\mathbf{p} - e\mathbf{A})^2 \tag{2}$$

with e, m being the charge and mass of the electron, $\mathbf{A}$ and $\mathbf{p} = -i\hbar\nabla$ being the magnetic potential and the canonical momentum. For the sake of simplicity, we set $c = 1$. It is well-known that the solution to equation (1) can be gotten from the free Schrodinger equation

$$i\hbar\frac{\partial\psi_0}{\partial t} = H_0\psi_0, \quad H_0 = \frac{\mathbf{p}^2}{2m} \tag{3}$$

by multiplying a phase factor (Dirac factor) $e^{i\frac{e}{\hbar}\int_{\mathbf{r_0}}^{\mathbf{r}}\mathbf{A}(\mathbf{r}')\cdot d\mathbf{r}'}$ to the solution of the free one

$$\psi(\mathbf{r}, t) = \psi_0(\mathbf{r}, t)e^{i\frac{e}{\hbar}\int_{\mathbf{r_0}}^{\mathbf{r}}\mathbf{A}(\mathbf{r}')\cdot d\mathbf{r}'} \tag{4}$$

provided the vector potential is time-independent, i.e., $\frac{\partial\mathbf{A}}{\partial t} = 0$.

It means that if two electrons start from the same point and arrive at the screen via two different paths which enclose the long–thin flux-carried solenoid will acquire a relative phase difference although there are no forces acting on them. This phase difference can be obtained directly from the solution (4). It is

$$\delta\alpha_B = \frac{e}{\hbar}\oint\mathbf{A}(\mathbf{r})\cdot d\mathbf{r} = \frac{e}{\hbar}\int\mathbf{B}(\mathbf{r})\cdot d\mathbf{S} = \frac{e\Phi}{\hbar} \tag{5}$$

where the integral is performed along a closed loop which is formed by one of the paths and reversing the other, Φ is the magnetic flux inside the long–thin solenoid. The phase difference (5) can also be written in the form [4]

$$\delta(\text{phase}) \propto (\text{field}) \times (\text{area}) \tag{6}$$

by considering the second expression of equation (5). The vector AB effect indicates that the vector potential $\mathbf{A}$, which was introduced as an auxiliary in classical theory, is in fact observable in quantum theory.

The scalar AB effect predicts that the interference pattern in the conventional two-slit interference experiment will be shifted if two electrons travel in two regions with different scalar potentials when they are recombined in the screen. In contrast with the vector AB effect, the observation of scalar AB effect in experiments is rather later [6]. The scalar AB effect can be analyzed from the

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dynamics of electrons in the background of a scalar potential. The Hamiltonian is given by

$$H = \frac{\mathbf{p}^2}{2m} + e\phi \tag{7}$$

where ϕ is the scalar potential. It can be verified that the solution to the Schrodinger equation with the Hamiltonian (7) can be achieved from the solution to the free Schrodinger equation ψ_0 by multiplying a factor $e^{-\frac{iq}{\hbar}\int_{t_0}^{t}\phi\, dt}$, i.e.,

$$\psi(\mathbf{r}, t) = \psi_0(\mathbf{r}, t) e^{-\frac{ie}{\hbar}\int_{t_0}^{t}\phi(t')\, dt'} \tag{8}$$

provided $\nabla\phi = 0$.

Therefore, two electrons pass through two regions with different scalar potentials will acquire a phase difference although there is no electric field. The phase difference is determined by the solution (8). It is

$$\delta\alpha_E = \frac{e}{\hbar}\int_{t_1}^{t_2} \Delta\phi\, dt \tag{9}$$

where $\Delta\phi$ is the scalar potential difference between two regions in which two electrons pass through and $t_2 - t_1$ is the time the electrons spend in the regions. In order to write the phase difference (9) in the same form as (6), one can introduce the electric field $\mathbf{E}$ and rewrite $\Delta\phi$ in (9) as $\Delta\phi = \int \mathbf{E}\cdot d\mathbf{r}$. Thus, the phase difference in scalar AB effect can also be written formally as [4]

$$\delta\alpha_E = \frac{e}{\hbar}\int_{t_1}^{t_2}\int \mathbf{E}\cdot d\mathbf{r}dt. \tag{10}$$

The scalar AB effect indicates that contrary to the classical theory in which the scalar potential is taken as an auxiliary, it has observable effect in quantum theory.

The phase difference in vector and scalar AB effects can be written in a unified covariant formula [4,5],

$$\delta\alpha_{BE} = \frac{e}{\hbar}\oint A_\mu dx^\mu = \frac{e}{\hbar}\left(\int_{t_1}^{t_2}\Delta\phi dt - \oint \mathbf{A}\cdot d\mathbf{r}\right) \tag{11}$$

if

$$\frac{\partial\mathbf{A}}{\partial t} = \nabla\phi = 0 \tag{12}$$

is satisfied. The exact meaning of the closed loop integral in the first expression is explained in Ref. [4].

By expanding the Faraday two-form $F = -\frac{1}{2}F_{\mu\nu}dx^\mu \wedge dx^\nu$ explicitly as $F = (E_x dx + E_y dy + E_z dz)\wedge dt + B_x dy \wedge dz + B_y dz \wedge dx + B_z dx \wedge dy$, the authors of Ref. [4] show that the expression (11) can also be written in an equivalent covariant form

$$\delta\alpha_{BE} = -\frac{e}{2\hbar}\int F_{\mu\nu}\, dx^\mu \wedge dx^\nu = \frac{e}{\hbar}\int F. \tag{13}$$

Both the expressions (11) and (13) are the starting points of analyzing the time-dependent vector AB effect in Ref. [4]

In fact, the unified covariant expression (11) is based on the solution to the Schrodinger equation. To this end, let us consider the Hamiltonian of an electron moving in the background of electric and magnetic potentials. It is

$$H = \frac{1}{2m}(\mathbf{p} - e\mathbf{A})^2 + e\phi. \tag{14}$$

The solution to the Schrodinger equation with the above Hamiltonian is

$$\psi(\mathbf{r}, t) = \psi_0(\mathbf{r}, t) e^{i\frac{e}{\hbar}\left(\int_{\mathbf{r_0}}^{\mathbf{r}}\mathbf{A}(\mathbf{r}')\cdot d\mathbf{r}' - \int_{t_0}^{t}\phi(t')dt'\right)} \tag{15}$$

provided $\psi_0(\mathbf{r}, t)$ is the solution of the free Schrodinger equation and the condition (12) is satisfied. The phase difference (11) can be obtained naturally from the solution (15) if the paths of two electrons enclose a long–thin flux-carried solenoid or the two electrons pass through two regions with different scalar potentials.

Both vector and scalar AB effects can not be understood from classical theories since there are neither electric nor magnetic forces acting on the electrons locally. Therefore, the AB effects not only indicate that the electric–magnetic potentials $(\phi, \mathbf{A})$ which were introduced as auxiliaries in classical theories are observable in quantum theories, but also reveal the non-locality of the phase in quantum theories.

Up to now, the time-independent vector AB effect, which is also named as type I AB effect because of the electrons moving in a field-free region, is well studied and confirmed experimentally. However, there are some controversies on the time-dependent vector AB effect (the scalar potential is set to zero, i.e., $\phi = 0$). Obviously, the time-dependent AB effect differs the time-independent one greatly since electrons move in a region where the field and the force are not zero in the former case. Therefore, one must take into account both the AB phase shift and the phase shift coming from the direct electric or magnetic forces in studying the time-dependent AB effect. Because of it, the time-dependent AB effect is also named type II AB effect.[1] The distinction between type I and type II AB effects is not commonly discussed until a recent paper [8].

There are some theoretical studies on the time-dependent AB effect. Ref. [9] presents a general discussion on the vector potential of the time-dependent AB effect. The author finds that if the magnetic field vanishes at the outer region of solenoid then the vector potential can only depend on time linearly. The authors of [10] investigates the Hamiltonian of both time-independent and time-dependent AB effects. In [11], the author concludes that there is no time-dependent shifting of the interference pattern in time-dependent AB effect since the contributions from the electric and the magnetic parts cancel exactly. The authors of [12] finds that the electric and magnetic contributions to the phase shift will be canceled partly since the magnetic phase shift is bigger than the electric one. However, this article only considers impulsive changes of the flux inside the solenoid. The path integral formulation and holonomies are applied to investigate time-dependent AB effect in [13]. In Refs. [14], the authors generalize the results of the time-independent vector AB phase difference to the time-dependent one by substituting $\mathbf{A}(\mathbf{r})$ by $\mathbf{A}(\mathbf{r}, t)$ or $\mathbf{B}(\mathbf{r})$ by $\mathbf{B}(\mathbf{r}, t)$ on the right hand side of (5). Therefore, the conclusion in Ref. [14] is that the interference pattern will be altered with respect to time if the vector potential is time-dependent.

The time-dependent AB effect is also studied from experimental aspect. Following the suggestions of [14], the authors of [15] carried out an experiment. The results of this experiment show that there are no time-dependent interference pattern. The authors of [16] give an analysis for the reasons why one did not observe the time-dependent interference pattern in the experiment [15]. In fact, there was an much earlier accidental experiments on the time-dependent AB effect in [17]. In this experiment, an interferometric experiment was carried out. Later, it was realized that

[1] In fact, the Aharonov–Casher [7] effect can be regarded as a kind of type II AB effect.

the region where the electron beams passed through was 'contaminated' by a 60 Hz magnetic field. However, this experiment only observed a stationary interference pattern. An explanation to the results of [17] was given in [18]. In this paper, the authors suggested that the combination of the time-independent AB effect plus an additional shift coming from the force $\sim \mathbf{r} \times \mathbf{B}$ exactly cancel and lead to the stationary interference pattern. Recently, an analysis of this experiment is given in [19]. In this paper, the authors pointed out that the results of experiment [17] can also be explained by combining the regular AB phase effect and the phase shift due to the induced electric field.

In a more recent paper [4], the authors get a similar conclusion with [11] that there is no net phase shift difference coming from the time-dependent vector potential. They find that the vector and the scalar AB phase shifts will be canceled exactly both from expression (11) and (13). This work is further illustrated and generalized in Refs. [20–24]. It seems that the conclusions of [11,4] coincide with the observations in experiments [15,17].

In Ref. [4], the authors start their analysis from the covariant expression (13).[2] Expanding the Faraday two-form $F = -\frac{1}{2}F_{\mu\nu}dx^{\mu} \wedge dx^{\nu}$ explicitly, one finds that the contribution from three magnetic terms is $\int(B_x dy \wedge dz + B_y dz \wedge dx + B_z dx \wedge dy) = \int \mathbf{B}(\mathbf{r},t) \cdot d\mathbf{S}$. While the electric terms contribute $\int(E_x dx + E_y dy + E_z dz) \wedge dt$. According to Faraday's law, the varied magnetic field will induce an electric field, i.e., $\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t}$, the authors of Ref. [4] conclude that the contribution from electric terms equals to $-\oint \mathbf{A}(\mathbf{r},t) \cdot d\mathbf{r} = -\int \mathbf{B}(\mathbf{r},t) \cdot d\mathbf{S}$. It seems that the contributions from magnetic and electric parts cancel exactly.

Then, the authors of Ref. [4] analyze this problem from the semi-classical point of view. Based on the phase difference expression (11), the authors consider the case of the magnetic vector depending on time linearly. For the sake of clarity, we present a schematic drawing of the configuration in Fig. 1 which is similar with the one in Ref. [14]. A beam of electrons are split into two beams at the point A. Then these two beams of electrons travel along two half circles and recombine in the screen B. The long–thin solenoid with the linearly increased magnetic flux is located at the center of the circle. We assume that the direction of the vector potential $\mathbf{A}(\mathbf{r},t)$ is clockwise and with an increasing magnitude. According to Faraday's law, the direction of the induced electric field is anti-clock. As a result, the upper path electrons will be accelerated while the lower path electrons will be decelerated by the induced electric field.

The vector potential outside the solenoid is

$$\mathbf{A} = \frac{kI(t)}{r}\hat{\theta}, \tag{16}$$

where k is a constant, $I(t)$ is the time-dependent current, $\hat{\theta}$ is a unit vector in the angular direction. Accordingly, the induced electric field is given by

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} = -\frac{k}{r}\frac{dI(t)}{dt}\hat{\theta}. \tag{17}$$

As is shown in Ref. [4], the contribution from the time-dependent vector to the phase difference in an infinitesimal arc can be calculated by using the expression (11). It is

$$\delta\alpha_{A(t)} = \frac{ek\Delta t\Delta\theta}{2\hbar}\frac{dI(t)}{dt}, \tag{18}$$

where Δt is the time the electron spends in the infinitesimal arc and $\Delta\theta$ is the angular displacement during the time Δt.

[2] We note that there are some queries on the covariance of the expression (13) in Ref. [25].

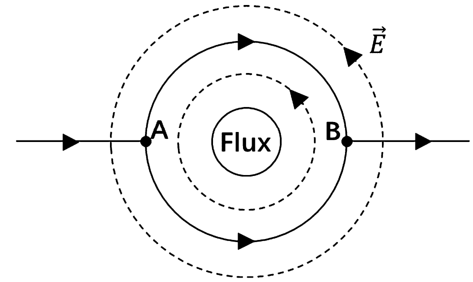

Fig. 1. A schematic drawing of the configuration. A beam of electrons are split into two at point A. These two beams of electrons travel along two half circles and recombine in the screen B. The thin-long solenoid is located at the center of the circle. We assume that the direction of vector potential is clockwise and the magnetic flux inside the solenoid is increased linearly. Therefore, the direction of the induced electric field is anti-clock. The directions of forces the induced electric field act on the upper and the lower electrons are opposite.

The contribution from the electric part to the phase difference is calculated by applying the semi-classical method in Ref. [4]. Because of the induced electric field, the electron in the upper path will be accelerated. The acceleration is

$$\mathbf{a} = \frac{e\mathbf{E}}{m} = -\frac{ek}{mr}\frac{dI(t)}{dt}\hat{\theta}, \tag{19}$$

and the change in distance due to the acceleration is

$$\Delta d = \frac{1}{2}a(\Delta t)^2 = -\frac{ek\Delta\theta\Delta t}{2mv}\frac{dI(t)}{dt} \tag{20}$$

where v is the velocity of the electron. Due to this change in distance, the effect of the induced electric field on the phase difference within the infinitesimal arc is given by

$$\delta\alpha_{E\text{-}field} = \frac{2\pi\,\Delta d}{\lambda} = -\frac{ek\Delta t\Delta\theta}{2\hbar}\frac{dI(t)}{dt} \tag{21}$$

in which λ is the de Broglie wavelength, i.e., $\lambda = \frac{h}{mv}$. Thus it seems that in an arbitrary infinitesimal arc in the upper path, the AB phase shift due to the time variation of the potential (18) exactly cancels the phase shift due to the effect of the induced electric field (21).

We shall point out that the analysis and the conclusions in Ref. [4] in fact, are invalid. Firstly, the expressions of phase difference (11), or, equivalently (13) are only valid in the case that there are no electric or magnetic forces acting on the electrons in the whole process, i.e., the type I AB effect. Obviously, the time-dependent vector potential will induce an electric field, which exerts on the electrons in the process of the traveling. Therefore, it is inappropriate to apply the expression (11) and (13) to the present case although it seems natural from the expression $\delta\alpha = \frac{e}{\hbar}\int F$ that the magnetic contribution $\int(B_x dy \wedge dz + B_y dz \wedge dx + B_z dx \wedge dy) = \int \mathbf{B}(\mathbf{r},t) \cdot d\mathbf{S}$ cancels exactly the electric contribution $\int(E_x dx + E_y dy + E_z dz) \wedge dt = -\int \mathbf{B} \cdot d\mathbf{S}$.

Secondly, there are also errors in applying the semi-classical method to analyze this problem in Ref. [4]. The analysis in Ref. [4] are only valid in the upper path of electrons in Fig. 1. For the lower path of electrons, the contribution from the varied magnetic vector is the same as the upper one, but the induced electric field will decelerate the electrons, therefore, two contributions can not cancel as in the upper case. In fact, one can image an extreme case in which the vector potential varies rapidly so that the induced electric field is strong enough to prevent the electrons in the lower path to reach the screen. As a result, there will be no interference pattern in the screen. It is obvious different from the time-independent case.

According to the principles of quantum mechanics [26], wave functions must be the solutions to the dynamical equations (Schrodinger or Dirac equations in the present study). However, it is easy to verify that the wave function (15) on which the results of phase difference (11), (13) based, is not the solution to Schrodinger equation with a time-dependent vector potential. Or, in another word, the wave function (15) is the solution to the Schrodinger equation for the type I AB effect rather than the type II one. It is the key point of the invalidity of [4]. To see it clearly, we remind that the Hamiltonian of an electron in the background of a time-dependent vector potential can not be obtained from (2) simply by replacing $\mathbf{A}(\mathbf{r})$ by $\mathbf{A}(\mathbf{r},t)$ since there will be an induced electric field $\mathbf{E}=-\frac{\partial \mathbf{A}(\mathbf{r},t)}{\partial t}$. Different from the electrostatic field, this induced electric field is not curl-free, i.e., $\nabla\times\mathbf{E}\neq 0$. Therefore, it is impossible to introduce the concept of 'potential' with respect to this induced electric field. However, this induced electric field will act on the electrons by accelerating or decelerating them. Thus, the Hamiltonian of an electron interacting with a time-dependent vector potential should include a term which can describe the effect of the induced electric field. The appropriate form should be

$$H=\frac{1}{2m}[\mathbf{p}-e\mathbf{A}(\mathbf{r},t)]^2-e\int_{\mathbf{r}_0}^{\mathbf{r}}\frac{\partial \mathbf{A}}{\partial t}\cdot d\mathbf{r}, \tag{22}$$

where the integral in the last term is path-dependent. A Hamiltonian which is similar with (22) was obtained in Ref. [14]. In that paper, the authors argued that the last term can be dropped for some specific situations.

The solution to the Schrodinger equation with Hamiltonian (22) can be solved. It is

$$\psi(\mathbf{r},t)=\psi_0(\mathbf{r},t)e^{i\frac{e}{\hbar}\int_{\mathbf{r}_0}^{\mathbf{r}}\mathbf{A}(\mathbf{r}',t)\cdot d\mathbf{r}'} \tag{23}$$

where $\psi_0(\mathbf{r},t)$ is the solution to the free Schrodinger equation. Therefore, for the specific configuration Fig. 1, the phase difference of two electrons traveling in the upper and lower paths will be

$$\delta\alpha=\oint\mathbf{A}(\mathbf{r},t)\cdot d\mathbf{r}=\int\mathbf{B}(\mathbf{r},t)\cdot d\mathbf{S}=\Phi(t). \tag{24}$$

Clearly, the interference pattern will be altered with respect to time according to our result.

At the end of this paper, we would like to mention the time-dependent vector AB effect in relativistic quantum mechanics. The relativistic motion of electrons in the background of a magnetic potential is described by the Dirac equation,

$$i\hbar\frac{\partial\psi}{\partial t}=H\psi,\quad H=c\boldsymbol{\alpha}\cdot(\mathbf{p}-e\mathbf{A})+m\beta, \tag{25}$$

in which $\boldsymbol{\alpha}=\begin{pmatrix}0&\boldsymbol{\sigma}\\ \boldsymbol{\sigma}&0\end{pmatrix}$, $\beta=\begin{pmatrix}\mathbf{I}&0\\0&-\mathbf{I}\end{pmatrix}$ are 4×4 matrices with $\boldsymbol{\sigma}$ and $\mathbf{I}$ being the Pauli matrices and 2×2 identity matrix, $\psi(\mathbf{r},t)$ is a four-component spinor.

If the vector potential $\mathbf{A}$ is time-independent, one can verify directly that similar with the non-relativistic case, the solution to the Dirac equation (25) is

$$\psi(\mathbf{r},t)=\psi_0(\mathbf{r},t)e^{i\frac{e}{\hbar}\int_{\mathbf{r}_0}^{\mathbf{r}}\mathbf{A}(\mathbf{r}')\cdot d\mathbf{r}'} \tag{26}$$

if $\psi_0(\mathbf{r},t)$ is the solution to the free Dirac equation, i.e., $i\hbar\frac{\partial\psi_0}{\partial t}=H_0\psi$, $H_0=c\boldsymbol{\alpha}\cdot\mathbf{p}+m\beta$.

However, if the vector potential depends on time explicitly, i.e., $\mathbf{A}=\mathbf{A}(\mathbf{r},t)$, an induced electric field will appear. Thus, the Hamiltonian (25) is insufficient to describe the present situation. One must add a term in Hamiltonian (25) so as to reflect the fact that the induced electric field will accelerate or decelerate the electrons. This term is exactly the same as the one in non-relativistic case. Therefore, the Hamiltonian with the time-dependent vector potential should be

$$H=c\boldsymbol{\alpha}\cdot[\mathbf{p}-e\mathbf{A}(\mathbf{r},t)]+m\beta-e\int_{\mathbf{r}_0}^{\mathbf{r}}\frac{\partial \mathbf{A}}{\partial t}\cdot d\mathbf{r}. \tag{27}$$

As in the non-relativistic case, the integral in the last term is also path-dependent.

The solution to the Dirac equation with the Hamiltonian (27) is

$$\psi(\mathbf{r},t)=\psi_0(\mathbf{r},t)e^{i\frac{e}{\hbar}\int_{\mathbf{r}_0}^{\mathbf{r}}\mathbf{A}(\mathbf{r}',t)\cdot d\mathbf{r}'} \tag{28}$$

if $\psi_0(\mathbf{r},t)$ is the solution to the free Dirac equation.

As a result, there will be a phase difference between the electron traveling from the upper and lower paths. The phase difference is the same as the non-relativistic one (24). Therefore, the interference pattern will also be altered in the relativistic case.

To summarize, based on one of the principles of quantum mechanics that wave functions must satisfy the dynamical equations (Schrodinger equation for non-relativistic and Dirac equation for the relativistic cases in the present study), we find that the analysis and conclusions in Ref. [4] are invalid. The interference pattern, in our conclusion, should be altered with respect with time. Of course, both the predictions in Ref. [4] and ours need to be confirmed by careful and accurate experiments.

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