PROTON COMPTON SCATTERING

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(presented by B. Stech)

In previous calculations [1-4] of the proton Compton scattering with dispersion relation techniques, simple fits for π-photoproduction matrix elements entering the dispersion integrals have been used. These fits give relatively simple expressions for the imaginary parts of the Compton scattering amplitude but are not in good agreement with the observed angular distributions in π-photoproduction [5].

In this paper differential cross sections for elastic γ-ray scattering by protons have been calculated using fixed momentum-transfer dispersion relations. No kinematical approximations have been made. The approach is based on a more theoretical description of π-photoproduction.

It is assumed that this process is dominated by the resonant $(M1)_{3/2}$- transition and that all non-resonant partial-waves are described by lowest order renormalized perturbation theory. This method has the advantage of not containing parameters to be fitted. It predicts proton Compton scattering in a certain energy region from π-photoproduction which in turn is determined mainly by πN-scattering.

We start with some kinematical preliminaries.

$$\gamma + N \rightarrow \gamma' + N'$$

$$k + p = k' + p'$$

$$k^2 = k'^2 = 0 \qquad p^2 = p'^2 = M^2$$

$$(ke) = (k'e') = 0 \qquad e^2 = e'^2 = -1$$

Let us define

$$P = \tfrac{1}{2}(p + p')$$

$$Q = \tfrac{1}{2}(p - p')$$

$$K = \tfrac{1}{2}(k + k')$$

In the special gauge (radiation gauge in the CMS)

$$(K+P)e = 0 \qquad (K+P)e' = 0$$

the invariant decomposition of the s-matrix element has the form

$$\langle \gamma' N' | S - 1 | \gamma N \rangle = 2\pi i \delta(k + p - k' - p') T$$

$$T = \frac{1}{(2\pi)^3} \sum_{j=1}^{6} K^j H^j(s,t)$$

with

$$K^1 = \frac{1}{M}(e'e)\, \bar{u}(N')\, u(N)$$

$$K^2 = \frac{1}{M^2}(e'e)\, \bar{u}(N')\, (\gamma K)\, u(N)$$

$$K^3 = \frac{i}{M} \bar{u}(N')\, e'^{\mu} \sigma_{\mu\nu} e^{\nu} u(N)$$

$$K^4 = \frac{1}{M^3}(e'Q)\, (Qe)\, \bar{u}(N')\, u(N)$$

$$K^5 = \frac{1}{M^4}(e'Q)\, (Qe)\, \bar{u}(N')\, (\gamma K)\, u(N)$$

$$K^6 = \frac{1}{M^2} \bar{u}(N') \left\{ (\gamma e')\, (Qe) - (e'Q)\, (\gamma e) \right\} u(N)$$

The amplitudes $H^i(s,t)$ are linear combinations of invariant functions $h^i(s,t)$ for which dispersion relations are valid.

$$H^i(s,t) = \sum_{l=1}^{6} V_{jl}(s,t)\, h^l(s,t)$$

This matrix $V(s_1 t)$ is a consequence of the special gauge chosen and has real elements. In the dispersion relations, 2 subtractions are performed, one at infinity to include the Low amplitude, the other at the pole $s = M^2$.

$h^i(s,t) =$

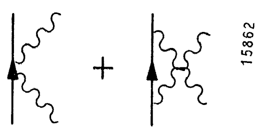
$$\rho^i(t)\left\{\frac{1}{s-M^2}+\frac{\varepsilon^i}{r-M^2}\left(\frac{s-M^2}{-t}\right)\right\}+c^j(t)+(s-M^2)\Lambda^j(t)+\frac{1}{\pi}\int\limits_{(M+m)^2}^{\infty} \mathrm{d}s' a^j(s',t)\left\{\frac{1}{s'-s}\left(\frac{s-M^2}{s'-M^2}\right)+\frac{\varepsilon^i}{s'-r}\left(\frac{s-M^2}{M^2-t-s'}\right)\right\}$$

Hence, the invariant functions $h^j(s,t)$ have the form

$$h^j(s,t) = \frac{\rho^j(t)}{s-M^2}+c^j(t)+f^j(s-M_1^2t)$$

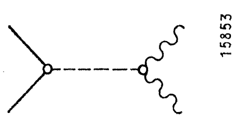
with $f^j(0,t) = 0$

The subtraction constants c^j have been determined from perturbation theory. This is suggested by the low-energy-theorem for Compton scattering [6] which states that the lowest order perturbation theory with anomalous magnetic moments provides the correct scattering amplitude up to the term linear in the photon frequency ω at fixed CMS-angle. We assume therefore that the constants c^j in the Laurent-series of the invariant functions are determined by:

$e_1\mu_a$  $\hat{=}\frac{1}{(2\pi)^3}\sum_{j=1}^{6} K^j B_N^j$

$$c^i = \lim_{s-M^2}\left\{(V^{-1}B_N)^i-\frac{\rho^i}{s-M^2}\right\}$$

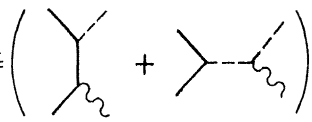
The subtraction constants Λ^i introduce the Low amplitude.

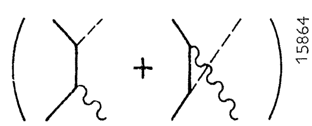
 $\hat{=}\frac{1}{(2\pi)^3}\sum_{j=1}^{6} K^j H_L^j$

$$h_L^j = (V^{-1}H_L)^j$$

$$\Lambda^j = \lim_{s-\infty}\frac{h_L^j(s,t)}{s}$$

The absorptive parts $a^j(s,t)$ are calculated in standard manner from π-photoproduction only. For $E_\gamma < 350$ MeV this should be a good approximation. As regards the π-photoproduction matrix elements we remember that this process is dominated by the resonant $(M1)_{3/2}$-transition. We assume now that the other partial waves are described by lowest order perturbation theory for a proton without anomalous magnetic moment [7]. The resonant $(M1)_{3/2}$ amplitude was calculated by CGLN. This means:

$\langle\pi^+\underline{N}|S|\gamma N\rangle \hat{=}$  $+(M1)^+_{3/2}$ without $j = 3/2$

$\langle\pi^0\underline{N}|S|\gamma N\rangle \hat{=}$  $+(M1)^0_{3/2}$ without $j = 3/2$

In order to facilitate the numerical evaluation of the dispersion integrals, we approximated the crossed graph in the π^0-production by its E_{0+} and M_{1-} projection. This is reasonable because of its smallness compared with the resonant $(M1)_{3/2}$ amplitude.

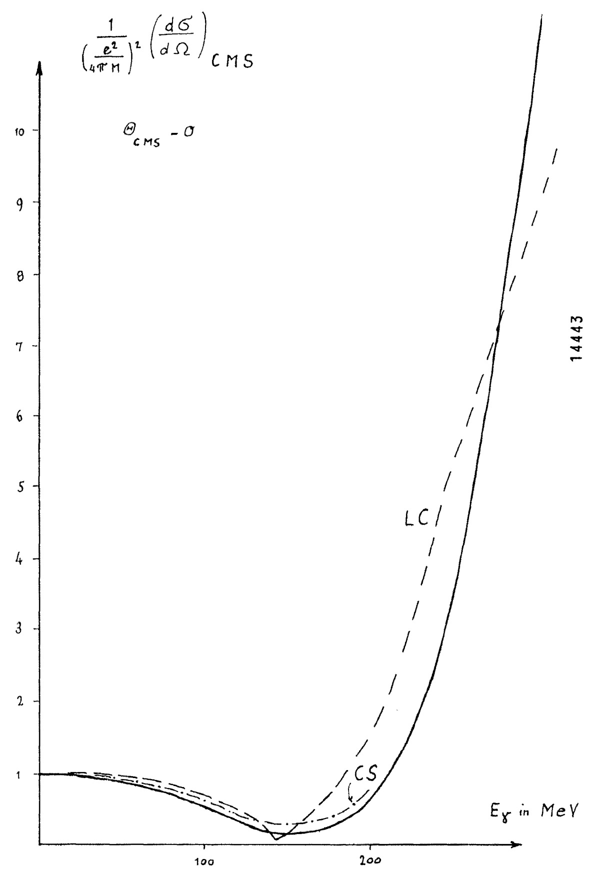


Fig. 1

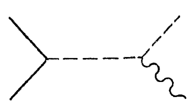
Numerical results of the calculations *) are presented in Figs. 1-5 with

$$\left[\frac{e^2}{4\pi M}\right]^2 = 2.35\cdot 10^{-32}\ \mathrm{cm}^2$$

Fig. 1 shows the forward scattering cross section obtained here. It is compared with the calculations of Cini, Stroffolini [8] (CS) and of Lapidus, Chou Kuang-chao [9] (LC).

Fig. 2 contains the calculated cross section for $\theta_{CMS} = 75°$. Only 2 experimental points are available.

In Fig. 3 our results are compared with the experimental points measured by the Cornell- and Illinois-group. In addition the curve WRT (without retardation term) is plotted in order to demonstrate the influence of the retardation term



in the π-photoproduction matrix elements. This curve is calculated from simplified matrix elements for π-photoproduction, where the π^+-production is described by the resonant $(M1)_{3/2}$-transition plus the nucleon-pole diagram $((E1)_{1/2})$, but π^0-production by the resonant $(M1)_{3/2}$-transition only. This is clearly insufficient.

The results for CMS-angle 135° are plotted in Fig. 4. Here again the calculation with the simplified matrix elements (curve WRT) is shown and also the curve from Mathews [1].

Fig. 5 at least contains angular distributions at various LAB-energies.

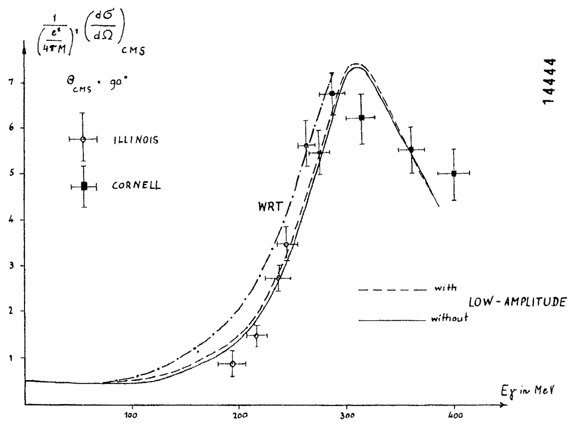


Fig. 3

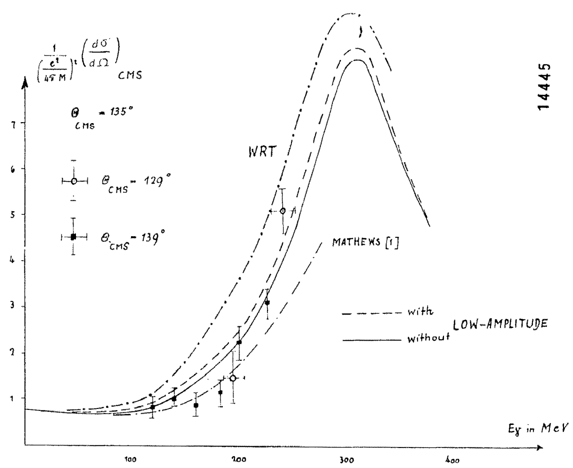


Fig. 4

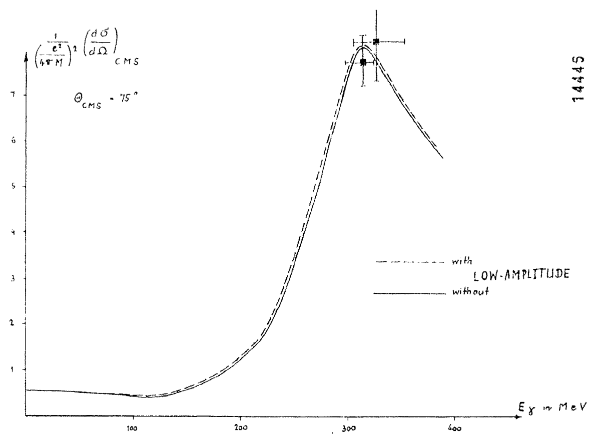


Fig. 2

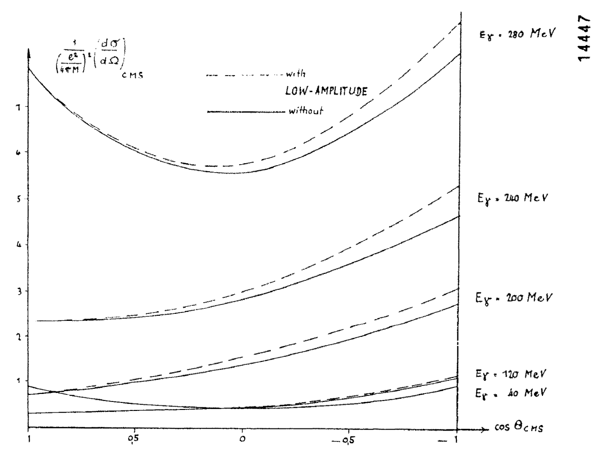


Fig. 5

(*) For the πN phase-shift α_{33} fits have been used, below and in the resonance region the Chew-Low plot of Barnes *et al.*[10], and above the resonance the fit of Höhler [10].

LIST OF REFERENCES

1. J. Mathews; Thesis Cal. Inst. Techn. 1957 (unpublished).
2. M. Jacob, J. Mathews; Phys. Rev. *117*, 854 (1960).
3. L. I. Lapidus, Chou Kuang-chao; Dubna preprint D 740 (1961).
4. A. P. Contogouris; Phys. Rev. *124*, 912 (1961).
5. G. Höhler, private communication.
6. F. E. Low; Phys. Rev. *96*, 1428 (1954); M. Gell-Mann, M. L. Goldberger; Phys. Rev. *96*, 1433. (1954)
7. K. Dietz, G. Höhler, A. Müllensiefen; Zeitschr. f. Physik *159*, 77 (1960).
8. M Cini, R. Stroffolini; Nucl. Phys. *5*, 684 (1958).
9. L. I. Lapidus, Chou Kuang-chao; Soviet Physics, JETP *14*, 210 (1962).
10. S. W. Barnes *et al.* Phys. Rev. *117*, 235 (1960); G. Höhler, Nuovo Cimento, *16*, 585 (1960).

DISCUSSION

LEADER: It is interesting to see from Dr. Stech's slides of the differential cross-section vs. energy at various centre-of-mass angles, that the discrepancy between Müller's calculation and the experiment increases with increasing angle. This just coincides with the fact, as shown in the work of Hearn and myself, that as the angle increases the multi-pion cuts approach the physical region and become increasingly more important.

FUBINI: I want to ask Prof. Stech: there is one point that I do not understand about the subtraction constants. If I remember rightly Gell-Mann, Goldberger and Low, have proved a low energy theorem, at 0 energy, but in the physical region which actually means 0 energy and $t = 0$. So I would think that your determination of the subtraction constants rely on the general theorem of Gell-Mann and Goldberger, and Low, only for $t = 0$. For $t \neq 0$, I believe that you normalize your constants to Born approximation. Is this correct?

STECH: Yes. In order to see the connection with the low energy theorem one has to multiply the invariant functions h^j with somewhat complicated co-variants which themselves are functions of s and t. The subtraction constants depend therefore on the special choice of co-variants. A different choice may remove the t-dependence for most if not all of the co-efficients ϱ^j and c^j.

DISPERSION RELATIONS FOR COMPTON-EFFECT ON A PROTON

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The aim of the present paper is to obtain the one-dimensional dispersion relations (with subtractions) from the double dispersion relation [1] and to use them for the analysis of the experimental data.

Let the four momenta of the photons and nucleons be denoted by $k_1(\mathbf{k}_1, \omega_1)$, $k_2(\mathbf{k}_2, \omega_2)$, $p_1(\mathbf{p}_1, E_1)$, $p_2(\mathbf{p}_2, E_2)$ and consider the following processes:

$$\gamma_1 + N_1 \rightarrow \gamma_2 + N_2 \qquad (I)$$
$$\gamma_2 + N_1 \rightarrow \gamma_1 + N_2 \qquad (II)$$
$$N_1 + \overline{N}_2 \rightarrow \gamma_1 + \gamma_2 \qquad (III).$$

Let us introduce the invariant values [*]:

$$s = (p_1 + k_1)^2, \quad u = (p_1 - k_2)^2, \quad t = (k_2 - k_1)^2$$

If we choose the following system of the orthogonal basic vectors

$$K = \tfrac{1}{2}(k_1 + k_2), \quad Q = \tfrac{1}{2}(k_2 - k_1),$$
$$P' = P - \frac{PK}{K^2}K, \quad N_\mu = \epsilon_{\mu\nu\lambda\sigma} P'_\nu K_\lambda Q_\sigma, \qquad (2)$$

(*) We use the following metrics: $ab = a_0 b_0 - \mathbf{ab}$.