

Measuring the leading hadronic contribution to the muon g-2 via μe scattering.

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The precision measurement of the anomalous magnetic moment g-2 of the muon, presently exhibits a 3.5σ deviation between theory and experiments. In the next few years the anomalous magnetic moment will be measured with higher precision at Fermilab and J-PARC. The theoretical prediction from the other side is mainly limited by the uncertainty on the leading hadronic correction a_μ^{HLO} to the g-2. We have proposed a novel approach to determine a_μ^{HLO} with space-like data, by means of precise measurement of the hadronic shift of the effective electromagnetic coupling α exploiting the elastic scattering of 150 GeV muons (currently available at CERN North area) on atomic electrons of a low-Z target. The direct measurement of a_μ^{HLO} in the space-like region will provide an independent determination competitive with the time-like dispersive approach, and will consolidate the theoretical prediction of the muon g-2 in the Standard Model. It will allow therefore a firmer interpretation of the measurements of the future muon g-2 experiments at Fermilab and J-PARC

1 Introduction

The discrepancy between the experimental value of the muon anomalous magnetic moment and the Standard Model (SM) prediction, $\Delta a_\mu \sim (28 \pm 8) \times 10^{-10}$ ^{1,2}, is an intriguing indication of possible physics beyond the SM. The error achieved by the BNL E821 experiment³, corresponding to 0.54 ppm, is dominated by the available statistics. New experiments at Fermilab and J-PARC, aiming at measuring the muon g-2 to a precision of 1.6×10^{-10} (0.14 ppm), are in preparation^{4,5}. The current accuracy of the SM prediction, 5×10^{-10} , is limited by strong interaction effects, which cannot be computed perturbatively at low energies. The leading-order hadronic contribution to the muon g-2, a_μ^{HLO} , can be computed using time-like data via a dispersion integral of the hadron production cross section in e^+e^- annihilation at low-energy. Our proposal is to determine a_μ^{HLO} from a measurement of the effective electromagnetic coupling in the space-like region, where the vacuum polarization is a smooth function of the squared momentum transfer. This method has been originally proposed⁶ to be used with e^+e^- Bhabha scattering data. It has been then realized that the hadronic contribution to the running of α can be effectively determined through the t -channel μe elastic scattering process, from which a_μ^{HLO} can be obtained directly⁷.

2 Measuring the Hadronic Leading Contribution.

The master formula for the calculation of hadronic leading contribution a_μ^{HLO} using space-like data is the following⁶:

$$a_\mu^{\text{HLO}} = \frac{\alpha}{\pi} \int_0^1 dx (1-x) \Delta\alpha_{\text{had}}[t(x)], \quad (1)$$

where $\Delta\alpha_{\text{had}}(t)$ is the hadronic contribution to the running of the fine-structure constant, evaluated at

$$t(x) = \frac{x^2 m_\mu^2}{x-1} < 0, \quad (2)$$

corresponding to a space-like (negative) squared four-momentum transfer.

The integrand in Eq. (1), because of the negative q^2 , is a smooth function and free of resonances. Fig. 1 (left) shows the expected shifts $\Delta\alpha_{\text{had}}$ and, for comparison, $\Delta\alpha_{\text{lep}}$, as a function of the variables integration variable x and the Mandelstam t . The range $x \in (0, 1)$ corresponds to $t \in (-\infty, 0)$, with $x = 0$ for $t = 0$. The expected function integrand of Eq. (1) is plotted in Fig. 1 (right). The peak of the integrand occurs at $x_{\text{peak}} \simeq 0.914$ (corresponding to $t_{\text{peak}} \simeq -0.108 \text{ GeV}^2$) with $\Delta\alpha_{\text{had}}(t_{\text{peak}}) \simeq 7.86 \times 10^{-4}$ (see Fig. 1 (right)).

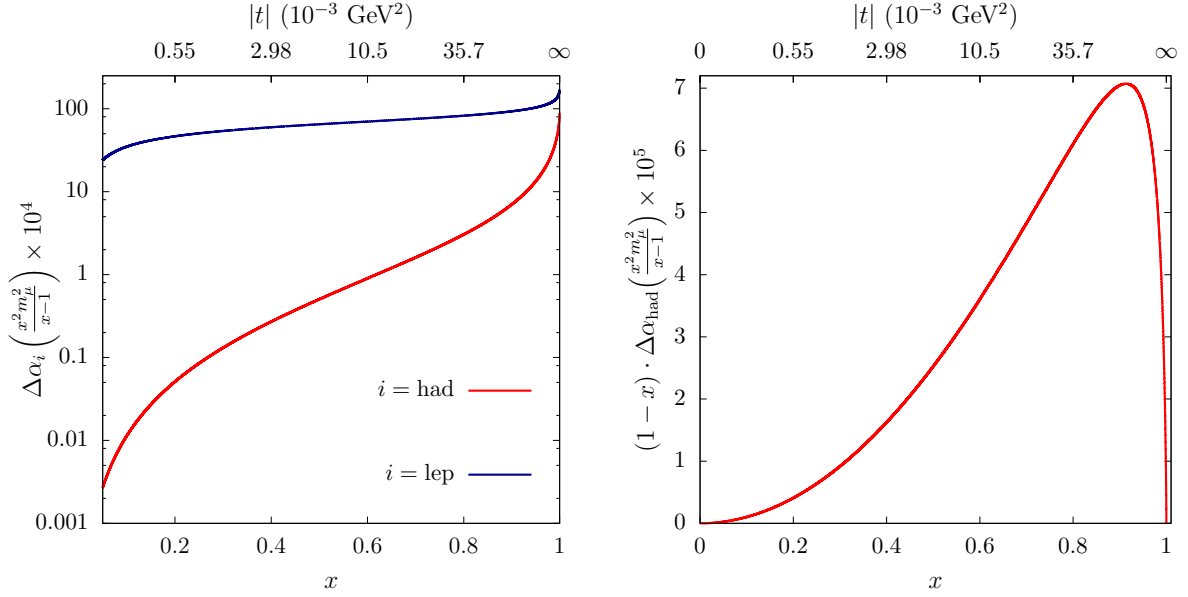


Figure 1 – Left: $\Delta\alpha_{\text{had}}[t(x)] \times 10^4$ (red) and, for comparison, $\Delta\alpha_{\text{lep}}[t(x)] \times 10^4$ (blue), as a function of x and t (upper scale). Right: the integrand $(1-x)\Delta\alpha_{\text{had}}[t(x)] \times 10^5$ as a function of x and t . The peak value is at $x_{\text{peak}} \simeq 0.914$, corresponding to $t_{\text{peak}} \simeq -0.108 \text{ GeV}^2$.

We propose to measure the running of $\alpha(t)$ by using a muon beam of $E_\mu = 150 \text{ GeV}$ colliding on a fixed electron target. This technique is similar to the one used for the measurement of the space-like pion form factor⁸. It is very appealing for the following reasons:

- It is a t -channel process, making the dependence on t of the differential cross section proportional to $|\alpha(t)/\alpha(0)|^2$. It is understood that for a high precision measurement also higher-order radiative corrections must be included^{9,10}.
- Given the incoming muon energy E_μ^i the t variable is related to scattered electron angle θ_e^f or to the energy E_e^f . The angle θ_e^f spans the range (0–31.85) mrad for the electron energy E_e^f in the range (1–139.8) GeV.
- For $E_\mu^i = 150 \text{ GeV}$ the region of x extends up to 0.93, covering 87% of the integral, while the peak of the integrand function of Eq. (1) is at $x_{\text{peak}} = 0.914$, corresponding to an electron scattering angle of 1.5 mrad, as visible in Fig. 1 (right).
- The angles of the scattered electron and muon are correlated as shown in Fig. 2 (drawn for incoming muon energy of 150 GeV). This constraint is extremely important to select elastic scattering events, rejecting background events from radiative or inelastic processes and to minimize systematic effects in the determination of t .

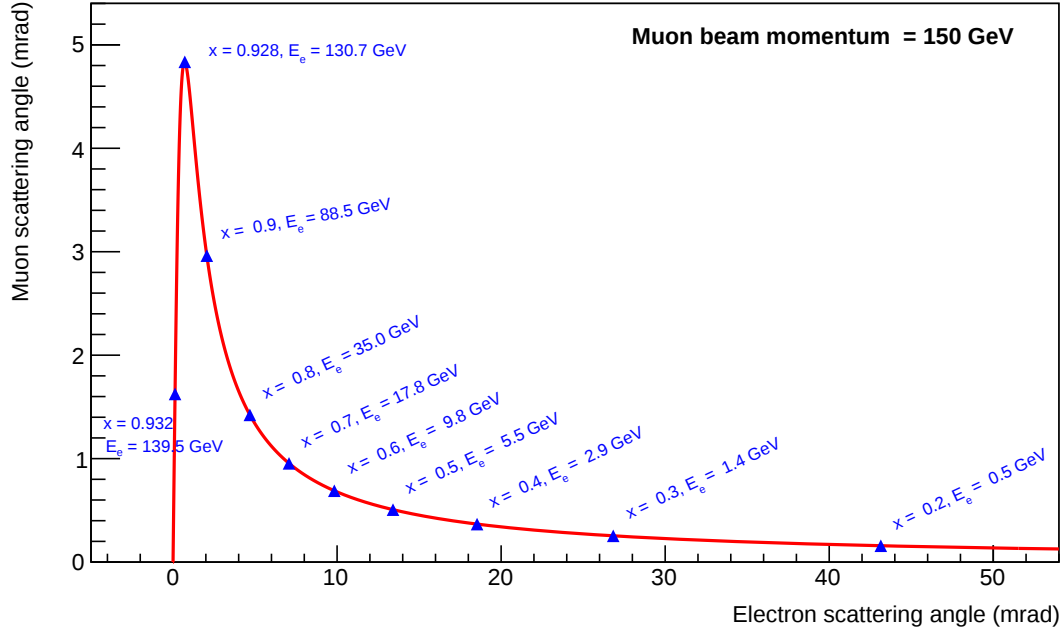


Figure 2 – The relation between the muon and electron scattering angles for 150 GeV incident muon beam momentum. Blue triangles indicate reference values of the Feynman's x and electron energy.

- The boosted kinematics allows the same detector to cover the whole acceptance. Many systematic errors, *e.g.* on the efficiency, will cancel out (at least at first order) in the relative ratios of event counts in the high and low q^2 regions (signal and normalization regions).

Assuming to use the CERN muon beam available at the CERN North Area, of energy of 150 GeV and average intensity $\sim 1.3 \times 10^7$ muon/s, and assuming a running time of 2×10^7 s/yr, we estimate the statistical sensitivity of this experiment on the value of a_μ^{HLO} to be $\sim 0.3\%$ (using 30 experimental points in x supplemented with large $|t|$ contributions that can be derived from pQCD).

3 Detection technique

The CERN muon beam M2 has the ideal characteristics for the measurement. The beam intensity, of more than 10^7 muon/s, can provide the required event yield. The time structure allows to tag incident muons. The target must be of low- Z in order to minimize multiple scattering and have high radiation length. The whole thickness must be of the order of 60 cm, to get enough electron scattering centres. To cope with the effects of multiple scattering the target must be segmented in thin layers, to be distributed in 20 identical modules. Each module will have the size of one meter, and consists of the thin layer of Be (or C) and Si planes for precise tracking (no magnetic field being applied). Fig. 3 shows the basic layout. As downstream particle identifiers we plan to use a calorimeter. It is needed to resolve the muon-electron ambiguity for electron scattering angles around (2–3) mrad (*cf.* Fig. 2). Preliminary studies of the apparatus indicate an angular resolution for tracks of ~ 0.02 mrad. The detector acceptance of $10 \text{ cm} \times 10 \text{ cm}$ in transverse plane, covers both the signal, with the electron emitted at extremely forward angles and high energies, and the normalization region, where the electron has much lower energy (around 1 GeV) and an emission angle of some tens of mrad.

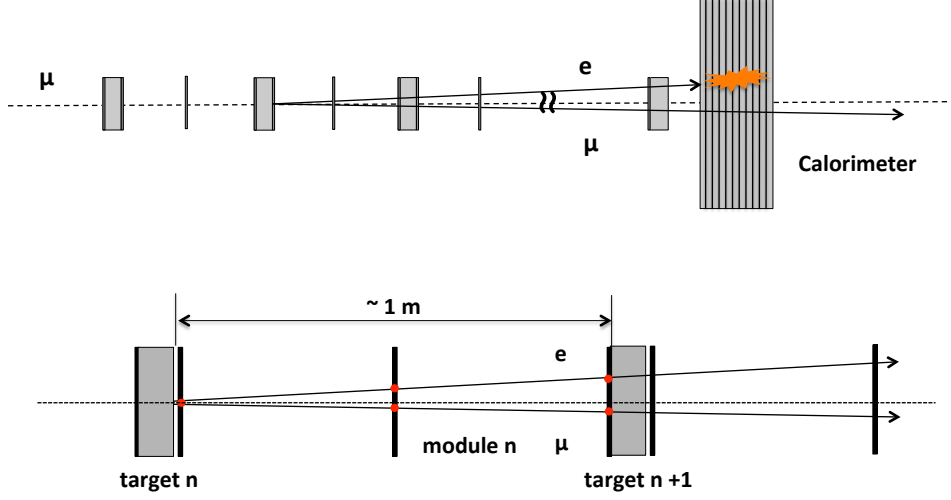


Figure 3 – Scheme of a possible detector layout. The detector is a modular system. Each module consists of a low- Z target (Be or C) and two silicon tracking stations located at a distance of one meter. To perform the μ/e discrimination in the case of small scattering angles (both θ_μ and θ_e below 5 mrad) the detector is equipped with an electromagnetic calorimeter.

4 Considerations on systematic uncertainties

Significant contributions of the hadronic vacuum polarization to the $\mu e \rightarrow \mu e$ differential cross section are essentially restricted to electron scattering angles below 10 mrad, corresponding to electron energies above 10 GeV. The net effect of these contributions is to increase the cross section by a few per mille. A precise determination of a_μ^{HLO} requires therefore not only high statistics, but also a high systematic accuracy, as the final goal of the experiment is equivalent to a determination of the differential cross section with ~ 10 ppm systematic uncertainty at the peak of the integrand function (*cf.* Fig. 1).

Such an accuracy can be achieved if the efficiency is kept highly uniform over the entire q^2 range, including the normalization region, and over all the detector components. This motivates the choice of a purely angular measurement: an acceptance of tens of mrad can be covered with a single sensor of modern silicon detectors, positioned at a distance of about one meter from the target. It has to be stressed that particle identification (electromagnetic calorimeter) is necessary to solve the electron-muon ambiguity in the region below a few mrad.

An important effect is the multiple scattering, as the electron energy is as low as 1 GeV. Multiple scattering breaks the muon-electron two-body angular correlation, moving events out of the kinematic line in the 2D plot of Fig. 2. In addition, multiple scattering in general causes acoplanarity, while two-body events are planar, within the resolution. These facts allow effects to be modeled and measured by using data. This possibility will be studied in detail with simulation. In high-precision experiments several systematic effects can be explored within the experiment itself. In this respect the proposed modularity of the apparatus will help since a test with a single module could provide a proof-of-concept of the proposed method.

5 Conclusions

The experiment is primarily based on the precise measurement of the scattering angles of the outgoing particles, as the q^2 of the muon-electron interaction can be directly determined by the electron (or muon) scattering angle. An advantage of the muon beam is the possibility of employing a modular apparatus, with the target subdivided in subsequent layers. A low- Z solid target is preferred in order to provide the required event rate, limiting at the same time the effect of multiple scattering as well as of other types of muon interactions (pair production, bremsstrahlung and nuclear interactions). The normalization of the cross section is provided by the very same $\mu e \rightarrow \mu e$ process in the low- q^2 region, where the effect of the hadronic corrections on $\alpha(t)$ is negligible. Such a simple and robust technique has the potential to keep systematic effects under control, aiming to reach a systematic uncertainty of the same order as the statistical one. For this purpose a preliminary detector layout has been described. By considering a beam of 150 GeV muons with an average intensity of $\sim 1.3 \times 10^7$ muon/s, currently available at the CERN North Area, a statistical uncertainty of $\sim 0.3\%$ can be achieved on a_μ^{HLO} in two years of data taking.

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