SLAC–PUB–6245
August 1993
(T)

# HADRONIZATION THROUGH PARTON-MESON FLUCTUATIONS⋆

Thomas Hyer

*Stanford Linear Accelerator Center,*

*Stanford University, Stanford, California 94309*

## ABSTRACT

We examine the hadronization process in QCD, modeling it as a series of independent parton-meson scatterings. In the limit of rapid scatterings, obtained by neglecting mass terms, we find a simple description of the hadronizing system. The consequences of this description, and the effects of heavy quarks, are discussed.

Submitted to *Physical Review D*

⋆ Work supported by Department of Energy contract DE–AC03–76SF00515.

## I. INTRODUCTION

The mechanism by which quarks hadronize into color-singlet bound states is perhaps the least understood of all processes in Quantum Chromodynamics (QCD). This is due to the confining nature of the theory; as we attempt to follow the hadronization process perturbatively to smaller and smaller values of the factorization scale $Q$ above which perturbative QCD (pQCD) is deemed valid, we find that the perturbation expansion becomes unreliable due to the growth of the running coupling constant at values of $Q$ that are still too large to permit detailed study of the formation of hadrons.

In this paper, we study the same process from another viewpoint, in which nonperturbative physics is absorbed into the wavefunctions of hadrons [1]. We work from the assumption that scattering into bound states, as opposed to scattering of free partons, becomes important at some momentum scale $Q_0$ (as must be the case, if hadronization is to proceed at all). The 'elementary' subprocesses of our model are $qg \to qH$, $q\bar{q} \to Hg$, and $gg \to Hg$, where $H$ represents some meson. We will show how consideration of these processes leads to a picture of the hadronization process which, in the massless limit, possesses remarkably few free parameters.

The paper is organized as follows: Section 2 outlines the important processes, and explains how they proceed in pQCD. Section 3 introduces the limit from which our results are obtained, discusses the validity of that limit, and extracts information about the behavior of the parton-meson ensemble. Section 4 presents the resulting final state, and draws comparisons with experimental information. Finally, Section 5 presents our assessment of the method and conclusions.

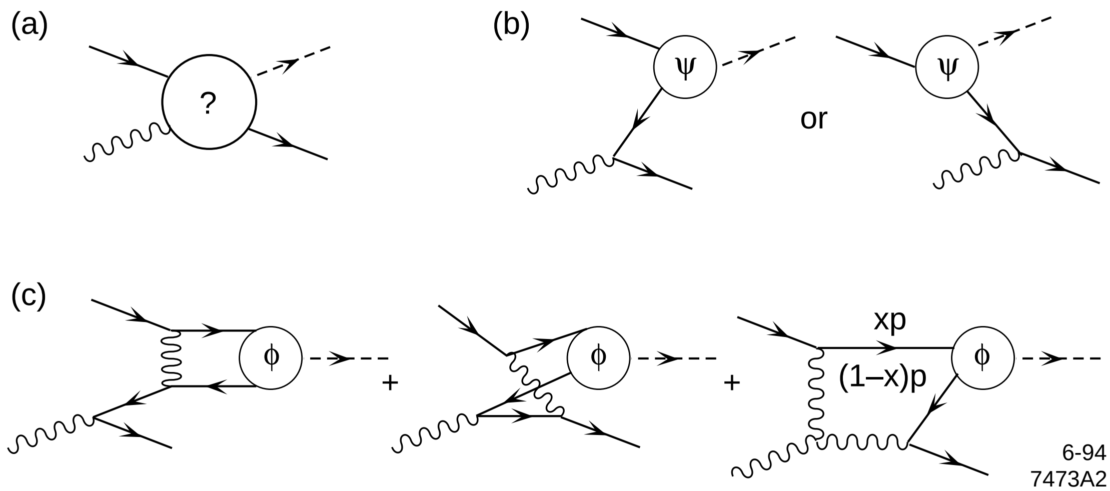


Figure 1. Three ways of looking at the one-meson amplitude: (a) represents the full, nonperturbative amplitude, whose calculation will require extensive use of numerical methods; (b) shows the two LCPTh graphs contributing to the amplitude to form the meson in its valence ($q\bar{q}$) Fock state; and (c) shows the Feynman graphs that contribute in the high-energy limit.

## FLUCTUATION SUBPROCESSES

We need to consider as generally as possible the generic process $pp \leftrightarrow Hp$, where $H$ is any meson and $p = q, \bar{q}, g$ any parton. In perturbative QCD, such processes can be written in the language of light-cone quantization (LCQ) as convolutions of scattering amplitudes in light-cone perturbation theory (LCPTh), with nonperturbative light-cone wavefunctions of the mesons involved [1]; see Fig. 1. Though we are interested in the region in which perturbation theory is inaccurate, it is nonetheless instructive to examine briefly the conceptual framework provided by LCQ.

A theory quantized on the light cone shares many features with 'old-fashioned' time-ordered perturbation theory; in particular, the 'energy' $P^- \equiv P^0 - P^z$ is not conserved in intermediate states, while all internal particles propagate on their mass shell (defined by the requirement $p^2 \equiv p^+p^- - p_\perp^2 = m^2$).

One notable difference is that the LCQ vacuum is very simple: since $P^+ \equiv P^0 + P^z$ is both positive and conserved, no vacuum fluctuations can occur [2].

In this framework, the suppression of amplitudes in which particles propagate far from the mass shell is ensured by 'energy denominators' $[\sum_i k_i^- - P^-]^{-1}$ from each intermediate state containing momenta $k_i$. If we carry out a calculation in pQCD down to scale $Q$, each internal particle (in the Feynman language) will be on-shell to within an amount $\sim Q$; in light-cone language, each intermediate state will provide an energy denominator $\sim Q$. Thus processes involving the transfer of momenta larger than $Q$ are suppressed their energy denominators, while soft processes proceed with increasing facility as $Q$ decreases.

We assume that, at some momentum scale $Q_0$, the familiar parton interactions of QCD may be switched off and replaced by the (multiplicity-conserving) interactions of the form $pp \leftrightarrow Hp$. While extreme in appearance, this assumption is not as odd as it may seem in the region of momentum transfer applicable to the formation of hadrons.

First, note that interactions of the type $p \leftrightarrow pp$ that involve sizable momentum transfer will push the intermediate states far off-shell; thus, they will be suppressed relative to near-collinear or four-point interactions. The former will not spoil the applicability of our results because collinear splittings, which serve to 'dress' partons at large momentum transfer, will not greatly affect the intrinsically soft process of combination into bound states; a near-collinear $qg$ system will interact in much the same way as a bare quark.

Both the $pp \to pp$ interactions of pQCD and the $pp \leftrightarrow Hp$ interactions with which we are concerned require the re-absorption of an intermediate-state parton into a final-state parton or meson. Since we expect the color-singlet states of QCD to be much more closely bound than nonsinglets, we expect that this recombination will proceed more readily in the parton-meson interaction, so that it will dominate the four-parton scattering at sufficiently low momentum transfer. Thus our *ansatz* is a plausible one, at least in the region of interest [3].

### Parton-meson fluctuation processes

In perturbative QCD, a scattering $pp \leftrightarrow Hp$ includes a sum over all Fock states of the wavefunction $\psi_H$, convolved with hard-scattering amplitudes over all possible values of the transverse momenta $k_{\perp,i}$ and longitudinal momentum fractions $x_i$ carried by the Fock-state constituents. Such a calculation rapidly becomes overwhelmingly arduous, and introduces dependence on the separate projections of the wavefunction into each Fock state. attempt any such computation, we will content ourselves with commenting on the general behavior of such amplitudes.

We begin by examining the scattering amplitude in the high-energy region, where it is comparatively well understood. Here the dependence of the hard-scattering amplitude on the internal transverse momentum may be ignored, leading to the simpler form [1]

$$\mathcal{M} \ = \ \int [\mathrm{d}x] \ T_H(x;Q^2) \ \phi(x;Q^2) \ ,$$

where $\phi(x;Q^2) \equiv \int^Q \psi(x,k_\perp)\mathrm{d}^2k_\perp/16\pi^2$ is the hadron distribution amplitude and $T_H$ is the pQCD amplitude to produce a state in which the hadron is replaced by collinear quarks with momenta $p_i = x_i p_H$; see Fig. 1(c). The Appendix contains some amplitudes computed in this limit, and comments on their behavior.

The point we wish to make is that the perturbative amplitude $T_H$ is strongly divergent in the forward-scattering limit; to demonstrate the finiteness of the full amplitude, one must take into account intrinsic transverse momenta or Sudakov effects [4]. The physics of individual scatterings is highly sensitive to the implementation of such a cutoff, which greatly hampers the reliable extraction of information about hadronic wavefunctions.

## MULTIPLE-FLUCTUATION LIMIT

We now model the parton-meson subamplitudes at low momentum transfer in a way that incorporates the salient features discussed above. Rather than attempt to compute these subamplitudes with some cutoff procedure, we will instead attempt to work with quantities that remain finite even in the absence of any cutoff. While the resulting treatment cannot be entirely accurate (the full physical amplitudes are indeed finite), it yields a simple picture of the hadronizing system.

As we examine the amplitude at smaller and smaller scattering angles, the rate at which the system oscillates between $pp$ and $Hp$ states diverges. However, since the divergence is only in the forward direction, the quantity

$$\frac{\mathrm{d}}{\mathrm{d}t}\left\langle\theta^2\right\rangle \;=\; \left\langle\theta^2,\mathcal{H}\right\rangle = \left\langle\theta^2 V\right\rangle + O\left(\left\langle\theta^2\right\rangle\right)\;, \tag{1}$$

where $V$ is the interaction part of the Hamiltonian $\mathcal{H}$, remains finite: the limit of increasingly frequent scatterings through diminishing angles is a random walk in the angular variables $\mathrm{d}\Omega$, which can be parametrized by the 'diffusion' constant

$$D \;\equiv\; \lim_{\theta\to 0}\; \theta^2\, V \;.$$

By failing to impose any cutoff, we have given up knowledge of the composition of the system, which can now oscillate freely between states; however, we have retained information about its orientation in a very simple way.

This loss of information can be described heuristically by envisioning a very densely populated QCD vacuum. A $qg$ system, for example, need only pull a soft $\bar{q}q$ pair from this vacuum; the resulting $(q\bar{q})(gq)$ system now must be treated as an $hq$ system, since the pairs in parentheses are highly collinear. Thus the flavor quantum numbers of any state are subject to change without notice, while its momentum is altered only gradually.

The above discussion neglects the divergent backward-scattering peaks that are present in $qg \to Hq$ scattering. However, the effect of backward scattering can be duplicated by a chain of forward scatterings: for example, $qg \to qH$ with $p_g \simeq p_H$ (near the backward-scattering pole) can proceed through the forward scatterings $q_{p_1}\bar{q}_{p_2}g_{p_3} \to H'_{p_1}g_{p_2}g_{p_3} \to H'_{p_1}g_{p_2}H_{p_3} \to q_{p_1}\bar{q}_{p_2}H_{p_3}$. Here the subscripts indicate the momentum carried by each particle; comparing the initial and final states, we see that this is indeed a backward scattering.

Since the backward-scattering amplitude diverges more slowly at small angles than the forward amplitude, it is numerically smaller in the region in question and gives no contribution to the diffusion constant $D$. Hence no new effects arise from the consideration of backward scattering, so that we may safely neglect it.

The angular diffusion constant $D$ is proportional to the meson wavefunction $\psi(x, k_\perp)$; hence, by dimensional analysis, $D \propto E^0$ where $E$ is the center-of-mass energy of the $pp$ system, with $2E^2 = p_1 \cdot p_2$. The rate of diffusion in momentum space is related to $D$ by $\mathrm{d}\left\langle p^2 \right\rangle / \mathrm{d}t = E^2 D$. Since each parton may scatter independently with any other, we sum the diffusion constants to obtain

$$\frac{\mathrm{d}}{\mathrm{d}t}\left\langle \big(p_i(t) - p_i(0)\big)^2 \right\rangle = \sum_{\text{partons } j\neq i} \frac{(p_i \cdot p_j)}{2}\, D_{ij} \ . \tag{2}$$

This expression can be further simplified. Since we have allowed the rate of forward scattering to diverge, color, flavor and spin will flow freely through the graph. Thus any initial information about the partons is lost: the particle properties can only be predicted statistically over the entire ensemble, not associated with a given line. As a result, the diffusion constants $D_{ij}$ are all identical, and we can perform the sum over partons to obtain

$$\frac{\mathrm{d}}{\mathrm{d}t} \left\langle \big(p_i(t) - p_i(0)\big)^2 \right\rangle \;=\; \frac{1}{2}\, D(p_i \cdot P) \;=\; \left(\frac{1}{2}\, D E_{\mathrm{cm}}\right) E_i \tag{3}$$

where the energy is understood to be evaluated in the center-of-momentum frame of the hadronizing system.

Equation (3) has a very simple interpretation: the momenta are spread out through independent, identically-distributed random walks in the rapidity space whose coordinates are $(\ln E, \Omega)$; see Fig. 2. When does this process terminate?

## THE FINAL STATE

The multiple-fluctuation process described above presupposes that the initial $pp$ state is in a color triplet or octet. Thus, though the number of nonsinglet partons will fluctuate rapidly, it will never decrease to less than two unless some other mechanism intervenes. The first candidate is the process $q\bar{q} \to HH'$ (or $gg \to HH'$).

This 'two-meson' process will not serve to terminate the oscillation process, for two reasons. First, the time-reversed process $HH' \to q\bar{q}$ occurs with equal facility, restarting the fluctuation process; thus the two-meson mechanism is not sufficient to halt the oscillations and produce a final hadronic state.

In fact, the two-meson process will not be important in hadronization. The initial $q\bar{Q}$ (or $gg$) state has a substantial chromoelectric dipole moment in the latter stages of the hadronization process, as the system is spread out to a length scale on the order of $\Lambda_{\mathrm{QCD}}^{-1}$; however, the final $HH'$ state can have only a small dipole

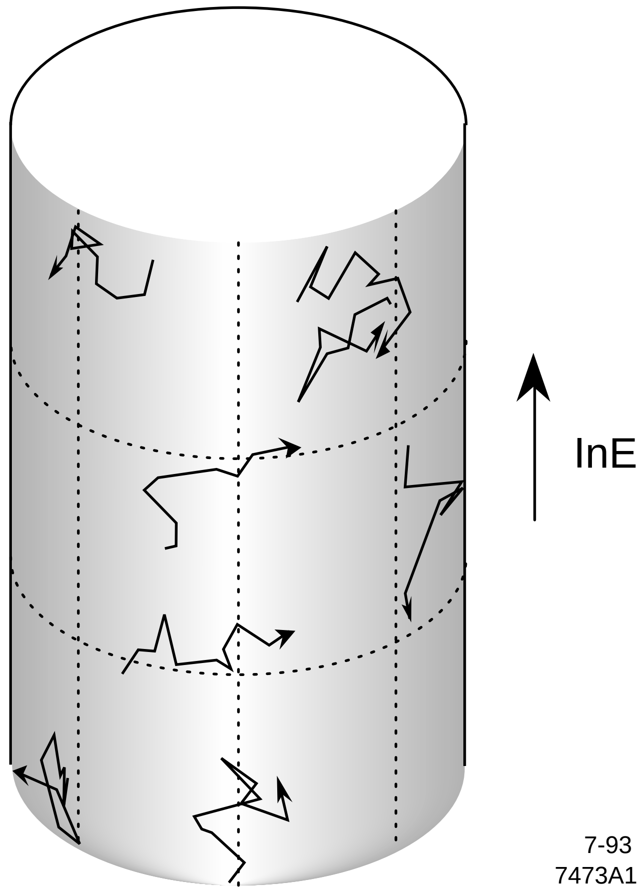


Figure 2. Random walks in two-dimensional rapidity space $(\ln E, \phi)$.

moment, internal to the color-singlet mesons. Thus we expect that the amplitude will disappear more rapidly as the system expands than will the other processes under consideration, since the transition requires the creation at a point of a $q\bar{q}$ pair that cannot readily join with the existing partons to form mesons. For this reason, we ignore its potential both to stop and to restart the fluctuation process.

The remaining mechanism by which we may reach a purely hadronic final state is the process $q\bar{q} \to H$ (or $gg \to H$), which will occur when two partons become sufficiently collinear to combine into a single meson (with lifetime longer than the timescale of hadronization, or equivalently width $\lesssim \Lambda_{\rm QCD}$). Since flavor and color cannot be associated with individual partons, this will occur as soon as any two quanta wander near each other. The time for this to occur is clearly proportional to $D^{-1}$; thus the dependence of the final state on the diffusion parameter $D$ drops out, and our model predicts a statistical distribution of mesons, with momenta obtained by proper smearing of the parton QCD parton momenta.

What is the 'proper' smearing? We have seen that each particle wanders through rapidity space independently (subject to conservation of total momentum) with the same diffusion constant. Thus we parametrize the total smearing by the quantity $\Delta$, the rms total diffusion in rapidity space:

$$\Delta^2 \equiv \left\langle \ln^2 \frac{E}{E_0} + \frac{2p \cdot p_0}{EE_0} \right\rangle ,$$

where $p$ is the final-state hadron momentum corresponding to initial parton momentum $p_0$. It should be emphasized that $\Delta$ is not a parameter of our model, but rather a random variable whose distribution depends only on the distribution of parton momenta produced in pQCD.

Since the partons are not produced independently, we must attempt an estimate based on the parameters of the parton distribution (*i.e.*, jet parameters). Since $\Delta$ is a measure of how far diffusion must proceed before two partons are sufficiently collinear to produce a single meson, we expect a result of the form

$$\Delta^{-2} \propto \sum_{\substack{\text{jets} \\ j}} \ln \left( \frac{n_j(n_j-1)}{M_j^2} \sum_{\substack{\text{mesons} \\ H}} f_H^2 \right) ,$$

where $M_j$ is the jet invariant mass, $n_j$ is the number of particles within the jet, and $f_H$ is the meson decay constant that indicates the amplitude of the wavefunction at the origin. This result is somewhat dependent on the jet definitions; however, most sensible choices of jets will serve to nearly minimize the quantity $\Delta$.

Final-state populations

One set of parameters of the model remains; namely, the expected fraction $\langle n_H/N \rangle$ of the total final-state multiplicity that will be comprised by each meson species $H$. The only prediction of our model is that these fractions should be universal, which is not surprising; theoretical extraction of the population fractions would require a thorough numerical calculation of the mixing rate in lattice QCD or Discretized Light-Cone Quantization (DLCQ).

What *prima facie* implications of this model can we obtain? The most obvious is that all flavor information from the initial parton distribution (except for heavy quarks) is destroyed; the final-state distribution of light mesons is solely determined by the set of parton momenta produced in pQCD. However, we cannot immediately deduce that, for example, jets from $u$ quarks and from $\bar{u}$ quarks should be indistinguishable.

One complicating effect is the fact that there is a very large (8/9) probability that the $q\bar{Q}$ pair in a $q\bar{q}Q\bar{Q}$ system produced in $e^+e^-$ annihilation will be in a color singlet [5]. Thus the two jets may become divorced into separate color singlet states before hadronization occurs, preserving the flavor information they carry. In addition, individual leading mesons may be produced directly [6] in the pQCD process.

In actuality, charge assignment probabilities up to 66% (*i.e.*, primary-quark tagging efficiencies up to 32%) in two-jet events can be obtained by forming a weighted sum of final-state hadronic charges [7]. It remains to be seen whether the mechanisms we have mentioned suffice to explain the observed success of such a method.

A clearer implication is that the ratio of the number of a given type of hadron to the total multiplicity should be independent of the multiplicity of light hadrons, and of the presence of any heavy quarks. This will result in a slight negative correlation between the fraction of any stable hadron (*e.g.*, $n_{\pi^+}/N$) and the observed multiplicity $N$, due to decays of unstable mesons after hadronization (in good agreement with results from the LUND Monte Carlo) [8].

### Heavy quarks

The assertion above that all information on the nature of the pQCD partons is lost in hadronization applies only in the limit of vanishing quark mass. In the opposite limit, a heavy quark $\mathbf{q}$ will combine with light quarks to form mesons, thus exchanging momentum through the same process of fluctuations, but never losing its identity. The diffusion constant $D$ will be marginally decreased (due to the absence of the $u$-channel pole for $\mathbf{q}\bar{q} \to \mathbf{H}g$) for the heavy system, but will remain unchanged for the remaining quarks as long as the total multiplicity $N \gg n_{\mathbf{q}}$.

The presence of heavy quarks will not affect the hadronization of the light-quark system in any other way, so that the conclusions obtained above (including the estimate of $\Delta$) are valid in the presence of heavy quarks, as well as for light-quark systems.

## CONCLUSIONS

We have made two assumptions about the behavior of hadronizing systems; that the parton-meson interactions of our model will dominate the fundamental interactions of pQCD at small momenta, and that the fluctuations between states which these interactions induce are rapid compared with the timescale of hadronization. We have used these limiting properties to draw a simple and very general picture of the hadronization process. Qualitative conclusions of this analysis are found to be in rough agreement with experiment, which is certainly to be expected of any sensible model. What application does it have?

First, it provides us with a new way of looking at the process itself, which may partake more of the physical reality than will models that attempt to calculate the scattering into hadronic states directly. This is accomplished by emphasizing the universality of the light-quark hadronization process, resulting in the parameter-free predictions of the previous section.

Second, it presents an intuitively appealing way to deal with the endpoint divergences of semiexclusive amplitudes [6]. The pQCD amplitude for the process $e^+e^- \to K^-\bar{s}u$ diverges when $p_u \parallel p_K$; this divergence does not appear in the rate for the inclusive process $e^+e^- \to q\bar{q}Q\bar{Q}$, so it is not addressed by the usual renormalization procedures. Instead, the divergence is contained by resummation of the fluctuations $sg \leftrightarrow Ku$, in the manner outlined in this paper.

The greatest drawback of this model is the existence of the $s$ quark, which cannot safely be classified as either 'heavy' or 'light' at the energy scales of interest. The model's predictions of universal light-quark symmetry in hadronization are not in good agreement with experiment if the $s$ is considered light, but we should be very hesitant to treat $s$ quarks created in pQCD as heavy particles throughout the hadronization process.

In sum, we have presented an intuitively attractive model of the hadronization process for light quarks, whose only parameters are the mean population fractions for each meson flavor. In the process, we have shown how certain apparent divergences are naturally regulated by the same process of rapid oscillations from which they arise.

## APPENDIX

## PARTON–MESON SCATTERING PROCESSES

In this Appendix, we comment on the parton-meson scattering processes discussed in this paper, using pQCD calculations at leading twist as our starting point.

We turn first to the process $qg \to HQ$. The perturbative hard-scattering term is the amplitude for the process $qg \to (q\bar{Q})Q$ with the final-state quark momenta $p_q = xp_H$, $p_{\bar{Q}} = (1-x)p_H$. There are three minimally connected graphs; each contains an internal gluon with momentum $(p_q - xp_H)$, leading to a factor of $\left[x(1-\cos\theta_{\rm cm})\right)]^{-1}$ in the amplitude. (This divergence appears only when the $Q$ and $g$ helicities are the same; it is cancelled by numerator factors when they are opposite.) The factor of $1/x$ is cancelled by a factor of $x$ in the distribution amplitude, leaving only the $\theta^{-2}$ divergence in the forward direction.

The second diagram of Fig. 1(c) diverges in the 'backward' direction $p_H \to p_g$. This divergence is proportional to $1/\cos(\theta/2) \sim (\pi-\theta)^{-1}$; thus, it does not contribute to the diffusion constant, which picks out only the quadratically divergent terms (see Fig. 1).

Graphs like $q\bar{Q} \to Hg$ diverge equally in both the forward and backward direction, as $\theta^{-2}$ or $(\pi-\theta)^{-2}$; thus, the fluctuations between partonic and mesonic states destroy information about the quark flavor associated with each momentum. Both graphs contribute to the diffusion constant $D$.

Finally, consider the two-meson graph in Fig. 3. This diverges as $\theta^{-3}$ in the forward limit as two internal gluon propagators approach the mass shell; what justification can there be for ignoring it?

Dimensional counting, or explicit calculation, tells us that the full amplitude for the two-meson process is proportional to $\theta^{-3}(\mu/E)^2$ where $\mu$ is a 'soft' scale

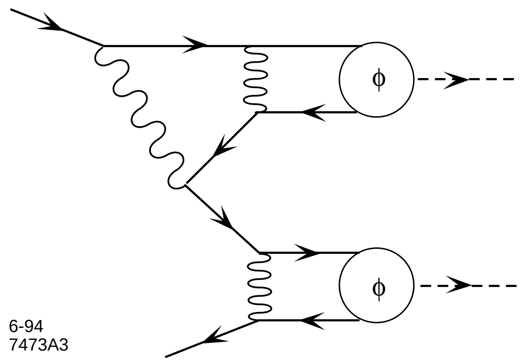


Figure 3. One of the fourteen Feynman diagrams contributing to the process $q\bar{Q} \to HH'$ in the high-energy limit.

intrinsic to the hadron, as opposed to $\theta^{-2}\mu/E$ for the one-meson processes we consider. Mass and transverse momentum terms both serve to cut off the growth of the amplitude beyond some point $\theta \sim \mu/E$, which is precisely where the two amplitudes become comparable. Thus the faster apparent divergence is spurious, and the one-meson amplitude remains dominant. Since the two-meson amplitude is more sharply peaked, its propensity to cause diffusion can also be ignored. The only remaining effect is the threat that the time-reversed $HH' \to q\bar{q}$ scattering will restart the hadronization process from a purely mesonic state. As explained in the text, this process becomes unimportant as the system expands, so that in the final state it can safely be ignored.

**REFERENCES**


1. G. P. Lepage and S. J. Brodsky, *Phys. Rev.* **D22**, 2157 (1980).
2. Actually, in some cases the vacuum need not be trivial due to the existence of nondynamical 'zero modes', which correspond to fluctuations with $P^+ = 0$ in the initial-state boundary conditions. For a review see, for example, S. J. Brodsky et al., SLAC–PUB–5811.
3. We can phrase the same assumption somewhat differently in the language of the Kinoshita-Lee-Nauenberg theorem. To wit: we assume that the actual eigenstates of the Hamiltonian that governs hadronization, while they freely mingle parton and hadron states, have a well-defined total particle multiplicity $N$.
4. A.H. Mueller, *Phys. Rep.* **73**, 237 (1981); J. Botts and G. Sterman, *Nucl. Phys.* **B325**, 62 (1989); H. Li and G. Sterman, *Nucl. Phys.* **B381**, 129 (1992); T. Hyer, *Phys. Rev.* **D47**, 3875 (1993).
5. J. D. Bjorken, S. J. Brodsky and H. Lu, *Phys. Lett.* **B286**, 153 (1992).
6. T. Hyer, *Phys. Rev.* **D48**, 147 (1993).
7. TOPAZ Collaboration, *Phys. Lett.* **B255**, 613 (1991).
8. The correlation coefficient between $n_\pi/N$ and $N$ is about $-5\%$ for center-of-mass energies up to 100 GeV.