Symmetry in nature: discreteness, three-dimensional extension, motion, and minimum potential energy

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Abstract. At every scale of Nature symmetry manifests in many ways, under many guises, and apparently disjoint descriptions, from Platonic solids and the Fibonacci sequence, to modern charge, parity, and time invariance. It is posited here that discreteness, three-dimensional extension, motion, and minimization of potential energy may be common traits and principles leading to observed symmetries. In 1986 this author found that, when driven by an inverse potential field, N discrete punctual particles array on the surface and inside a sphere in highly symmetrical configurations exhibiting tangential equilibrium and low potential energy. We add here new constraints and boundary conditions arising from three-dimensional extension and motion of the discrete corpuscles within the symmetrical N-particle arrays ($2 \le N \le 20$).

1. Introduction: a unified fluid and field (UFF) theory of Nature

The three-dimensional wave equation is listed by Ian Stewart as one of the seventeen equations that changed the world, and its connections to Fourier transforms, Maxwell equations, and Schrödinger equation were duly noted [1]. Around 1994 present writer discovered novel solutions for the threedimensional classical wave equation (3D-CWE) in spherical coordinates (r, θ, ϕ) [2, 3, 4]. Skipping the formal mathematical problem of uniqueness, our solutions are new in the pragmatic and intuitive sense that the independent variables time t and distance r, appear in our solutions as the ratio q = Ct/r, rather than in the usual additive structure. As such, our 24-year-old solutions might be a suitable candidate for the Navier-Stokes Millenia Problem posed by the Clay Mathematics Institute.

The new solutions for 3D-CWE are a main input to our unified fluid and field (UFF) theory of Nature [5-12], that may fulfill Einstein, De Broglie, and 'tHooft dreams. Let us briefly elaborate:

- 1) Einstein, Infeld and Hoffmann [13] demonstrated that the equations of motion of discrete Newtonian particles may be obtained from general relativity adding to the gravitational field equations Dirac delta-functions to handle singularities (i.e. discreteness of particles). In that context Jammer wrote that "if it were possible to work out a unified field theory that subjects electromagnetic and possibly also nuclear forces to a similar treatment as gravitation, then it would lead us to a final stage in the history of the concept of force. While the modern treatment of classical mechanics still admitted, tolerantly, so to say, the concept of force as a methodological intermediate, the theory of fields would have to banish it even from this humble position", underlining added [14].
- 2) Louis de Broglie always advocated a realistic and Lorentz invariant quantum mechanical fluid [15-18]: "the wave function Ψ certainly is not a physical reality... it only is a representation of probabilities dependent upon the state of our knowledge" (our literal translation from French, [16]

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page 266). De Broglie's ideas eventually led to the stochastic interpretation of quantum mechanics [19], associated to an underlying physical fluid, often identified with Dirac's ether [20], and here with a primordial fluid (PF).

3) Gerard 'tHooft recently suggested that causality may underlie both quantum mechanics, and a theory of matter [21]. Present paper is our first building block in that direction.

UFF theory postulates the existence of a primordial fluid (PF) obeying the laws of the classical theory of fluids [5-7]. Sagions are the elementary constituents of PF, described as indestructible discrete 3D-extended and massless atoms of energy in permanent translational and spinning motion. Force, mass, and charge are not primitive notions in UFF theory; they are derived concepts. Electromagnetic force is easily derived from UFF theory [8], preliminary applications to planetary structure and to other gravitational problems appear in [9], and to nuclear problems in [10], possible qualitative connections to general relativity are described in [5-7, 11].

In our view the fundamental traits of macroscopic everyday Nature are individuality, discreteness, 3D-extension, and motion [12]. The principle of universality that underlies classical philosophy and physics requires that properties of Nature should be the same at all scales, from the cosmos, passing through the human scale, down to the smallest bits of *matter* (as opposed to energy); the latter are highly symmetrical arrays of a few sagions in permanent tangential motion. Present paper describes geometrical structure of the most elementary pieces of matter formed by N sagions ($2 \le N \le 20$).

2. Minimum potential energy in static N-particle spherical arrays

In the related but different context that the manifest discreteness exhibited by Nature requires discrete mathematics, in 1986 present writer investigated [22] properties of *static* arrays of N discrete punctual charges e on the surface and inside a macroscopic sphere of radius R, subject to the electrostatic Coulomb potential in spherical coordinates (r, θ, φ) , where r is polar distance, θ is altitude angle relative to the X-Y plane, and φ is azimuth angle on the X-Y plane. Of course, those results are also applicable to arrays under an inverse Newtonian gravity potential.

An N-particle array is in tangential equilibrium on the surface of a sphere if, for each particle, there are no net components of field intensity along direction (θ , φ); the net component of field intensity along radial distance r may be non-zero. A simple numerical method was used to find arrays in rotational equilibrium. Potential energy U for each equilibrium array was calculated in units of e^2/R (or Gm^2/R in the Newtonian case). Table 1 summarizes main results relevant for present paper, comments are in column 9, several rows are added to include a new nested structure for N = 8, and some other aspects addressed here for the first time.

A convex polygon with *n* vertices is denoted as n-gon; thus, a two-gon (2-gon) is a line, a three-gon (3-gon) is a triangle, and a four-gon (4-gon) is a rhombus (rectangle and square are special cases). Second column in table 1 is potential energy U(2D) associated with two dimensional planar arrays occupying an equatorial plane of the sphere. For $2 \le N \le 11$, regular N-vertex polygons exhibit the lowest potential energy in planar configurations, but for $12 \le N \le 20$, lowest U(2D) contains a central charge and/or a central nested coplanar dipole (see footnotes in table 1). From merely geometric considerations, we chose in 1986 a system of Cartesian coordinates with origin at the geometric center of the regular N-gon containing the planar equatorial array in the X-Y plane. This choice turns out to be quite appropriate when the N-array rotates around the Z-axis, which thus becomes an intrinsic direction for spin and orbital angular momentum in the N-body array, as in present paper.

Third column in table 1 describes three-dimensional N-arrays on the surface and inside the sphere as a superposition of several simpler sub-arrays: (a) An equatorial n-gon on the XY-plane. (b) A rectangular n-prism nP with n-gon bases parallel to X-Y plane, the limiting case is 1P (i.e., n = 1), pictured here as a dipole (DP) aligned along the Z-axis (ZDP). (c) A twisted prism (nTP) for n > 1 obtains by rotating one of the bases of nP around the Z-axis through angle π/n radians. (d) Nested arrays of several twisted prisms along the Z-axis, having same number n of vertices in each base, but different size: one twisted prism, 1x(nTP); two twisted prisms, 2x(nTP); three twisted prisms, 3x(nTP), and so on.

N	U(2D)	3D-arrays	vertex array	d_1	d_2	shift	U(3D)	comments
2	0.50	2-gon = DS	0/2/0	0	_	_	0.500	equatorial dipole
3	1.73	3-gon = TS	0/3/0	0	_	_	1.732	equatorial triangle
4	3.83	4-gon = square	0/4/0	0	_	_	3.828	equatorial square
4		2-gon+ZDP	1/2/1	0	1	_	3.828	unstable square
4		1x(2TP)	2/0/2	0.577	_	$\pi/4$	3.674	tetrahedron
5	6.88	3-gon+ZDP	1/3/1	0	1	_	6.475	unstable array
6	10.96	1x(3TP)	3/0/3	0.577	_	π/3	9.985	octahedron-1
6		2-gon+1x(2P)	2/2/2	0	0.707	$\pi/2$	9.985	octahedron-2
7	16.13	5-gon+ZDP	1/5/1	0	1	_	14.453	unstable array
7		3-gon+1x(2TP)	2/3/2	0				to be calculated
8	22.44	2x(2TP)	2,2/0/2,2	0.323	0.792	$\pi/2$	18.356	missed in 1986
8		1x(4TP)	4/0/4	0.56	_	$\pi/4$	19.675	reported in 1986
8		1x(4P)	4/0/4	0.577	_	0	19.741	cube-1
8		3TP+ZDP	1,3/0/3,1	0.333	1	$\pi/3$	19.741	cube-2
9	29.92	3-gon+3P	3/3/3	0	0.70	$\pi/3$	25.760	
10	38.62	4TP+ZDP	1,4/0/4,1	0.42	1	$\pi/2$	32.717	unstable array
11	48.68	3-gon+3P+ZDP	1,3/3/3,1	0	0.53	$\pi/3$	40.622	unstable array
12	59.58^{a}	2x(3TP)	3,3/0/3,3	0.188	0.795	π/3	49.165	icosahedron-1
12		4-gon+ $2x(2P)$	2,2/4/2,2	0.526	0.851	$\pi/2$	49.165	icosahedron-2
12		5TP+ZDP	1,5/0/5,1	0.447	1	π/5	49.165	icosahedron-3
12		3x(2TP)	2,2,2/0/2,2,2			$\pi/4?$		to be calculated
13	71.81^{a}	5-gon+4TP	4/5/4	0	0.75	$\pi/40$	59.058	tangent unstable
14	85.35 ^a	6TP+ZDP	1,6/0/6,1	0.46	1	π/6	69.306	unstable array
15	100.22^{a}	7-gon+4TP	4/7/4	0	0.78	π/56	80.759	tangent unstable
16	116.45 ^a	2x(4TP)	4,4/0/4,4	0.20	0.78	π/4	92.920	two nested 4TP
16		4x(2TP)	2,2,2,2/0/2,2,2,2			$\pi/4?$		to be calculated
17	134.06 ^a	5-gon+5P+ZDP	1,5/5/5,1	0	0.61	π/5	106.050	unstable array
18	152.49 ^b	2x(4TP)+ZDP	1,4,4/0/4,4,1	0.20	0.68	π/4	120.084	unstable array
18		3x(3TP)	3,3,3/0/3,3,3			$\pi/6?$		to be calculated
19	173.40 ^c	7-gon+5TP+ZDP	1,5/7/5,1	0	0.67	$\pi/70$	135.161	tangent unstable
20	194.02 ^d	6P+3TP+ZDP	1,3,6/0/6,3,1	?	0.62	π/6	150.908	unstable orientation
20		3x(3TP)+ZDP	1,3,3,3/0/3,3,3,1			$\pi/6?$		to be calculated
20		2x(5TP)	5,5/0/5,5	0.188	0.795	π/5	151.799	dodecahedron

Table 1. Potential energy of N-sagion arrays in tangential equilibrium (partially adapted from [22]).

^a(N-1)-gon+1 central sagion. ^b(N-2)-gon+1 nested disagion (inner radius, shift = 0.304, $\pi/16$). ^c(N-3)gon+1 nested disagion (inn radius, shift = 0.49, $\pi/16$). ^d(N-2)-gon+1 nested disagion (inn radius, shift = 0.293, $\pi/18$).

Note that a given N-array (say, the square, the cube, the octahedron, ...) may be described in several ways, depending upon its orientation relative to the inherent Z-axis. Some important physical implications are noted in sections 4 and 5.

Fourth column describes the arrays according to the distribution of vertices, starting from the south: number of vertices on each southern plane separated by commas/equatorial vertices/number of vertices on each northern plane separated by commas. For the range under consideration (N \leq 20), there are up to nine planes parallel to the X-Y plane. If N is even, the number of planes is also even, distributed in symmetrical pairs above/below the X-Y plane, the first pair has the upper/lower planes at distance d₁ above/below the X-Y plane; the second pair at distance d_2 , and so on. The limiting case is a single particle at each the north/south pole of the sphere (d=1), arrangement called a bi-capped configuration in [22], and renamed here as a Z-axis dipole (ZDP); when the array is in rotation there are significant physical implications (see section 5). If N is odd, the number of parallel planes is also odd; the additional plane is the XY-plane itself. Distances d_1 and d_2 appear in columns 5 and 6, where values obtained from the numerical optimization are given to only two significant digits. It is noteworthy that for N = 12, 16, 20 the ratio d_1/d_2 is related to Phidias number and to the ubiquitous golden ratio, where d_1 and d_2 are sqrt{[sqrt(5)±2]/[3sqrt(5)]}[22]. Since our procedure was a brute force numerical optimization, present writer missed the golden ratio connection in 1986; further links of our arrays to Fibonnacci sequence and to living beings [23] will be explored elsewhere. Relative angular orientation between planar substructures is in column 7 of table 1. For the particular case of two nested n-twisted prisms, the relative angular shift between the two polygonal bases (n-gons) always is π/n rad.

It is known that configurations with low potential energy tend to have triangular lateral faces (see [22] for references); in that sense the cube N = 8 (square faces) and the dodecahedron N = 20 (pentagonals faces) differ from the remaining Platonic solids. Also, they are sub-optimal, as shown in table 1, where potential energy associated with the twisted cube (square bases and triangular lateral faces) is lower than potential energy of the cube.

Back in 1986 three members in the family of two nested nTP were identified: (a) 2x(3TP) for the icosahedron N = 12, (b) 2x(4TP) for N = 16, and (c) 2x(5TP) for the dodecahedron N = 20. We missed the first member of the family: 2x(2TP) for N = 8. It has just been calculated and, indeed, all faces in this novel array are triangular, and its potential energy is a minimum (see table 1).

Previous findings suggest that the beautiful symmetrical shapes that manifest everywhere in Nature [23], are not driven by an abstract "principle of symmetry" or by some unidentified "principle of beauty", but rather by the mundane and ubiquitous principle of minimization of potential energy of the N-body discrete array —principle that often appears in physics and engineering.

Thirty years ago Good reported [24] that masses of all elementary particles seemed to be integer multiples of some sub-structures containing 2, 3, and possibly 5 simpler components. In the context of present paper it is as if the disagion, the trisagion, and possibly the pentasagion were the basic constituents in all fundamental particles. The unavoidable implication is that arrays in table 1 containing sub-structures with 4, 6, 7 sagions should be simplified. In that spirit, table 1 shows new N-configurations formed by three and four nested sub-arrays of twisted prisms, that may lead to novel arrays complying with Good's hypothesis, but having lower potential energy. Those results will be reported elsewhere.

3. Peaceful coexistence of time-independent and time-dependent potentials

Physics is, of course, a creation of the human mind. We side with Einstein in believing that hard sciences should describe an outside reality, the Universe, which is always there, even when we do not look at it, and that such real Universe will continue existing after earth and humankind eventually dissapear, say in a collision with a large body. The rest of Universe would hardly notice our absence!

What is the exact meaning of the word "static" in the arrays that were considered in previous section? Does it mean that the agent driving the system, say a physical potential Π , is constant over time in the sense that it never changes? Or, is it constant in the sense that, over time, there is an inflow of something that is cancelled by an equal outflow? Without entering in details, mathematically, both

cases are handled by some null first derivative. A pragmatic answer may be that "static" is the array registered in a single photograph taken with a very-high-speed camera. The physical fact is that beautiful mineral crystals do exist in Nature; they can be touched, over and over again, there is 3D-immanency beyond a single 2D-high-speed photograph.

From the viewpoint of logic, the human observer tends to consider that time-dependence and timeindependence constitute a dichotomy. That is, at a given observation time, the system may behave as time-independent <u>or</u> as time-dependent. However, the general solution of time-dependent differential equations, as the classical wave equation, destroys the dichotomy through the back-door by including the time-independent solution (i.e. Laplace equation) as part of the general solution. In such case the "or" in the dichotomy may become "and".

Our general solution to the 3D-HWE takes a further step and explicitly introduces the timeindependent solution through the front door as a permanent potential field I(r) that is always there [2,3,4]; the notation I(.) was consciously chosen to stress independency of time. In this way the dichotomy smoothly disappears, and there is peaceful coexistence between two apparently contradictory notions, as presciently foreseen long ago by Heraclitus of Ephesus.

As reported at the 71^{st} Meeting of the Division of Fluid Dynamics of the American Institute of Physics (Atlanta, 18-20 November 2018), we now interpret the background force resulting from I(r) as Newtonian gravity with a small correction.

4. Three-dimensional extension and occupation of 3D-space

Configurations in section 2 were obtained for static arrays of a small number N of *punctual* objects $2 \le N \le 20$. The center of mass theorem of Newtonian mechanics [25] allows treating discrete macroscopic objects of mass M as if they were mathematical points carrying said mass M. Of course, the potential energy of our solar system may be calculated in first approximation by treating sun and planets as punctual, but for the analysis of solar and lunar tides the macroscopic size of earth must be taken explicitly into account; for additional considerations see [7,12].



Figure 1. Arrays formed by extended sagions of radius \mathcal{R} for N = 2, 3, 4. Each case shows two spheres: (1) innner sphere of radius RN_{CM} circumscribed to CMs of individual sagions in four arrays (line, triangle, square, and tetrahedron); (2) outer sphere of radius RN_{OUT} = \mathcal{R} + RN_{CM}.

In UFF theory, sagions are treated as homogeneous 3D-extended spherical dots of radius \mathcal{R} , so that the center of mass (CM) is at center of the sagion. Figure 1 shows that the optimal punctual arrays discussed in section 2 are directly valid if the arrays are interpreted as formed by the CM of sagions. However, since sagions have extension, some punctual arrays are physically impossible. Some examples: (1) Outer radius of sphere circumscribed to a given N-array cannot decrease indefinitely; for disagion the minimum outer radius is $2\mathcal{R}$. (2) There always is a finite inner void region of radius ε_N .

All examples in figure 1 correspond to closest packing, where two or more sagions are in (almost) contact. For the disagion minimum(ε_2) \rightarrow 0, but for all other arrays minimum(ε_N) > 0. (3) Minimum distance between the two bases in a prism is $2\mathcal{R}$. (4) Constraints arise in the case of nested substructures, consider, for instance, an (N–2)-gon on the XY-plane plus an inner nested disagion (see planar configurations for N = 18-20 in column 2 of table 1). It is necessary to check if inner space may accommodate the extended disagion of radius $2\mathcal{R}$.

A corollary of utmost physical importance is the following. If the inner void region inside an array is $\varepsilon_N < \mathcal{R}$, then it cannot accommodate one sagion, and cannot be even traversed by a free sagion, that region is *permanently* empty. There is *absolute vacuum*, and the array is extremely stable because the pressure difference is large, even in intergalactic regions where number density of sagions is low.

5. Motion, 3D-extension, and voids in the occupation of 3D-space

Consider now free sagions carrying linear momentum \mathcal{P} , total energy $\mathcal{E} = \mathcal{P}\mathcal{C}$, and spin \mathcal{S} — positive if counterclockwise CCW, negative if clockwise CW. Sagions move with constant speed \mathcal{C} relative to a geometrical absolute 3D-space Σ , that according to light bending during eclipses may have positive curvature. For numerical values and further details see proceedings of Vigier IX at Baltimore, USA, 2014 [6], and Vigier X at PortoNovo, Italy, 2016 [7]. Sagions may exist in two states: free, and coalesced, the latter being building blocks for quarks and all fundamental particles, including photons.

5.1. Toroidal 3D-space occupied by the kinematic disagion

When in collinear motion, punctual particles can only have frontal collisions, but extended sagions, as all macroscopic bodies, may also enter into slanting collisions [6,7,26]. Two sagions in a slanting encounter may be locked into mutual orbital motion to form a disagion, which is the simplest form of coalescence. The process conserves total energy, and angular and linear momentum, which becomes tangential motion in a planar orbit. The physical mechanism that keeps sagions locked is the difference in pressure between the fluid outside the array, and the fluid inside the 3D-array. The pressure inside the array tends to zero when the two sagions are (almost) in contact, i.e. when $\varepsilon_2 \rightarrow 0$. At macroscopic scale, this "force" mechanism is easily observed in the Magdeburg hemispheres; a microscopic scale example is Casimir force.



Figure 2. Toroidal region occupied by disagion (left) and trisagion (right) in permanent orbital motion. CM of each sagion moves along a circular path, radius = R2, R3 respectively.

Angular momentum before a slanting sagion-sagion collision is $\mathbf{L} = \pm 2\mathcal{PR}$, sign is positive/negative according to CCW/CW orbital motion of sagions; panels in figure 2 depict CCW motion. Kinetic energy before collision becomes internal energy of disagion after collision, hidden as tangential motion of individual sagions. Immediately after its coalescence, the CM of each sagion moves with

tangential speed \mathcal{C} along a circle of radius R2 = \mathcal{R} , thus generating a torus in 3D-space, as in figure 2. The CM of the disagion does not stay at rest and gets linear motion from sagion-disagion and disagion-disagion elastic collisions [6,7].

At planetary scale, the toroidal orbit of our Earth may be traversed by other objects, and may be also occupied by Trojan companions. The toroidal orbits of disagion and trisagion are similar —but at the most fundamental scale. At arbitrary time, the whole 3D-space inside the torus is not completely filled by the pair of sagions, rather space is occupied in the same *average* sense that Earth occupies her toroidal orbit around the sun. In passing, it is stressed that there is nothing random in the position of Earth around the Sun, or a sagion along its orbit. Physical probability is causal [27], probabilistic and frequentist calculations simply reflect lack of knowledge and/or lack of experimental control.

The disagion is the simplest coalesced state, so that a single free sagion (i.e. in rectilinear motion) cannot be part of a coalesced array, possibly at rest. Hence, any array in table 1 of section 2 containing a single particle, for instance at the center of the equatorial plane for N > 12, is not physically viable.

5.2. Toroidal 3D-space occupied by the kinematic trisagion

Capture of a free sagion by a disagion produces a trisagion; conservation of angular momentum leads to a symmetrical arrangement with the three CMs of individual sagions placed at vertices of a moving equilateral triangle [6,7]. The array keeps its shape during the orbital motion as indicated in figure 2.

As in planetary orbits, at any arbitray time, large regions of the toroidal region of space occupied by disagion and trisagion are empty, so that the 3D-space occupied by the torus may be traversed by free sagions and by sagion arrays, provided that the usual occupants of the orbit do not coincide with the crossing objects at same place over the crossing interval of time. It also means that if the radius is right, a torus may accomodate several additional sagions in a given orbit, for instance, a fourth sagion may fit in the trisagion orbit of figure 2.

The sense of orbital motion and the associated angular momentun L of the disagion/trisagion define an internal or *inherent direction* for the coalesced sagion array. Simply choose the plane of orbital motion of CMs of individual sagions as the XY plane for the coordinate system, and select the direction of the Z-axis according to the right-hand rule as sketched in figure 2.

The triangular array in figure 2 is an "instant photograph" of a periodic orbit, which implies that the word "static" in section 2 means steady state.

Note that the toroidal orbits of disagion/trisagion are topological objects with one hole, subject receiving high attention since 2016, when the physics Nobel Prize was awarded to "exotic matter" [28,29]. In contrast to other theories, our toroidal orbits in figure 2 are not deformations of 3D-space, but rather straightforward average occupation of some 3D-regions during each orbital period. Note also that the occupied and the void regions of the tori are not at rest in absolute space; they move along with the CM of the disagion/trisagion, similarly to a human being moving around in a room, carrying along the internal void regions of his body.

5.3. Three-dimensional space occupied by higher N-arrays

Disagions and trisagions are 2D-planar structures in the sense that CMs of individual sagions are disposed in plane arrays. Additional collisions with free sagions lead to formation of other sagion arrays for N = 4,5,... [6,7]. Further inelastic collisions between the arrays and the primordial fluid drives sagion arrays towards minimum potential energy, which in nuclear language is the binding energy of the array.

Arrays derived from 2D-planar sagion arrays with N > 3 are listed firstly, moving next to more complex arrays.

5.3.1. Equatorial polygonal arrays = one torus with N sagions. Any regular polygon with N sagions at the vertices and occupying the XY-plane defines an inherent direction along the Z-axis; an example is the square in figure 1 with vertex distribution 0/4/0. Column 2 in table 1 refers to equatorial arrays only. Other possible choices for the Z-axis are considered next.

5.3.2. Stacked disagion arrays = several tangential tori. For even N, the Z-axis is defined by the middle point of two opposite edges in a regular polygon. Simplest case is the vertical square N = 4 in figure 1C, formed by two stacked disagions, just touching each other: A-B disagion parallel and below the XY plane, C-D disagion parallel and above the XY plane. The distribution of vertices is 2/0/2, and angular momentum is the same as in the equatorial case. Orbital motion leads to occupation of 3D-space in the shape of two tangential tori.

Next array in this class is the vertical hexagon N = 6, formed by equatorial disagion C-F, plus A-B disagion parallel and below the XY plane, plus D-E disagion parallel and above the XY plane; this array may be also viewed as rectangle ABDE occupying the inner void region in equatorial dipole CF. Vertex distribution is 2/2/2, and orbital sagion motion leads to three tangential tori as in figure 3A.



Figure 3. Planar hexagon in vertical orientations. (A) Z-axis through two opposite sides. (B) Z-axis through two opposite vertices; note that the 2D-hexagon only exists twice in a rotation.

5.3.3. Arrays with a transversal Z-disagion.

Vigier

For even N, the Z-axis is defined by two opposite vertices in an N-gon, as in figure 3B for the hexagon N = 6, with a vertex distribution 1,2/0/2/1. Rotation of the two horizontal disagions B-F and C-E generates two tangential tori with a finite inner void region whose minimum radius is ε . The Z-disagion A-D has radius R6, rotates on a plane orthogonal to the XY-plane, and generates a torus with inner void radius R6– \mathcal{R} . Left side of figure 3B shows a planar hexagonal configuration that only appears twice in every rotation period. The simplest case in this family of arrays is the square N = 4 (figure 1C), with vertex distribution 1/2/1, which corresponds to a B-D disagion on the equatorial XY-plane (say, in CCW motion), and one sagion each at the south (sagion A) and north (sagion C) poles of the sphere circumscribed to the array. As in the hexagon, the Z-disagion A-C, also called Z-dipole, has radius R4, rotates on a plane orthogonal to the XY-plane, and generates a torus with inner void radius $\varepsilon = R4-\mathcal{R}$.

As illustrated in figure 3B, net angular momentum is not parallel to the Z-axis: it is the vector addition of a Z-component associated with horizontal disagions plus the horizontal angular momentum associated with the dipole along the Z-axis. This class of arrays may be stable or unstable. This is due to interception of orthogonal torus A-D upon the two tangential tori. Arrays are stable if the timing of rotation, and distances of horizontal tori from the XY-plane allows polar sagions to pass through the horizontal orbits without colliding. In the case of the square 1/2/1, there is a collision on the XY-plane at the first quarter of period. The array is thus destroyed in the collision. This serendipitous finding provides a realist microscopic and classical mechanism for radioactivity!

Several arrays in table 1 contain one particle at each the north and south pole. The stability of each array must be individually determined. Stable arrays may exhibit topologies with several holes.

5.3.4. Stacked or prism arrays = two tangential tori.

A stacked prism is a generalization of the stacked disagions in 5.3.2 to the case of higher n-polygonal

bases (triangle, square, pentagon, etc.) formed by n-gons with n = N/2 > 2, as illustrated in figure 4A for N = 4, 6, 8. Prisms are denoted here as (N/2)P, for instance the triangular prism is 3P. The cube is the special case of 4P when all edges are equal. Sagion rotation in each base leads to two tangential tori, each one with N/2 sagions. Note that potential energy associated with the cube N = 8 is non-optimal; in the stacked orientation the distribution of vertices in the cube is 4/0/4.

Potential energy is, of course, independent of the orientation of the array, but angular momentum depends upon the manner that the array rotates. Two additional orientations of the cube are: (a) Z-axis through the middle of two opposite edges, leading to disagion AB parallel and above the XY plane, disagion GH parallel and below the XY plane, and equatorial square CDEF, for a vertex distribution 2/4/2. Sagion rotation generates three stacked and tangential tori. (b) Z-axis through two opposite vertices A and G, leading to a Z-dipole plus a twisted prism with triangular bases BDE and CFH; vertex distribution 1,3/0/3,1 (this array is cube-2 in table 1, section 2). Sagion rotation generates a torus with number "8" cross section (see next paragraph) and an orthogonally intersecting torus generated by disagion AG.



Figure 4. Prism and twisted prism arrays. Panels (B) and (D) are projections upon the XY-plane.

5.3.5. Twisted prism arrays = two intersecting tori.

The two adjacent orbits in the two tangential tori in a prism array may be converted into another array with a more efficient use of 3D-space by partially sharing the average space they occupy during one period of rotation; additionally, the process leads to lower potential energy arrays. Consider the square of figure 1C in the vertical orientation analysed in 5.3.2, and turn one of the bases, say disagion C-D, through 90° (i.e. $\pi/2$ radians), as in figure 4B. This rotation immediately unfolds the 2D-square into a 3D-pyramid with isosceles triangles in the four faces. Distance 2d between the bases of the prism may be decreased to obtain a tetrahedron with equilateral triangles in all four faces (see figure 1D). The microscopic physical mechanism is provided by collisions of primordial fluid against the pyramidal array, process that involves minimization of potential energy. In the square the two tangential tori were separated by distance $2d=2\mathcal{R}$, when the value of d is decreased to d₁ < \mathcal{R} (see also figure 1D), the tangential tori mutually intersect to form a single torus with number eight "8" cross-section.

Likewise, as shown in figures 4A and 4B for N = 4,6,8, any prism array *n*P may be converted into a companion twisted prism array (*n*TP) by rotating one of the bases through π/n radians. The potential energy U(*n*TP) for the twisted array is consistently lower than the energy U(*n*P) associated with the stacked array. Moreover, the potential energy of two platonic solids —the tetrahedron (2TP) and the octahedron (3TP) — are absolute mimima for N = 4 and 6 respectively. For N = 8 it also holds that

U(twisted cube) = U(4TP) < U(4P) = U(cube) (see table 1), but not all faces of the twisted cube are triangular, so that it may be expected that there exist arrays with lower energy. Indeed, that is the case as discussed in next subsection 5.3.6.

For completeness, two additional rotational orientations of the octahedron are: (a) Z-axis through the middle of two opposite edges, leading to disagion AB parallel and above the XY plane, disagion EF parallel and below the XY plane, and equatorial disagion CD, for a vertex distribution 2/2/2, shown as octahedron-2 in table 1. Sagion rotation generates three stacked and tangential tori; this spatial occupation is similar to the cube. (b) Z-axis through two opposite vertices A and E, leading to a Z-dipole plus an equatorial square BCDF; vertex distribution 1/4/1 (this is a popular representation of the octahedron as two pyramids joined at the base). The stability of this orientation has not been calculated as yet.

Summarizing, a given array is characterized by potential energy, but angular momentum depends on the axis of rotation. All arrays described as *n*TP occupy 3D-space as a torus with number "8" cross section, which is the intersection of two parallel orbits, each with N/2 sagions.

5.3.6. Nested prisms and twisted prisms arrays = several intersecting tori.

As N grows, optimal arrays tend to be nested structures of three basic objects already described in the foregoing: equatorial n-gons, prisms with *n*-gons at the base (nP), and twisted prisms with *n*-gons at the base (nTP), see sketches in figures 4C and 4D. Additionally, there are increasing levels of nesting with two, three, and more nested arrays.

Good's analysis of the empirical evidence related to fundamental particles [24] indicates that the value of *n* seems to be restricted to 2 and 3, with 5 occassionaly appearing. The optimization process in 1986 selected arrays with information available at that time. For instance, no effort was made to look for structures *without* squares (n = 4). This 4-gon appeared in the optimal arrays for N =8 (the twisted cube), and N = 16, a nested structure with two twisted prisms with square bases shown in figure 4D. The new information leads to reconsideration of the optimization processes. For instance, an array containing a square may be unfolded as two nested prisms (or, nested twisted prisms) with base n = 2. This new array has potential energy lower than the twisted cube, and all lateral faces are triangular. The vertex distribution is 2,2/0/2,2 corresponding to two nested 2TP shown in table 1. It is likely that this novel array has the absolute minimum energy for N = 8.

Platonic solids for N = 12 (icosahedron) and N = 20 (dodecahedron) may be viewed as two nested twisted prisms with triangular and pentagonal bases respectively (see table 1). Since all faces in the icosahedron are triangular there is a good chance that this array is the absolute optimal for N = 12. On the contrary, the dodecahedron N = 20 only has pentagonal faces, so that it is quite likely that there exist arrays with lower potential energy. An example was found in 1986, but the new information may lead to better arrays, for instance, a five nested 2P (or 2TP) with vertex distribution (2,2,2,2,2/0/2,2,2,2,2) may be an optimal array, another candidate is listed in table 1.

6. Prolegomena for a classical kinematic theory of matter

Heeding t'Hooft suggestions [21], this writer is developing a causal classical theory of matter based on 3D-extended objects, and linked to the broader context of our unified fluid and field (UFF) theory of Nature. To choose the basic content for a credible theory of matter several extremely deep preliminary issues must be addressed beforehand. In the style of Newton's Optics, some queries are posed next.

6.1. What is the origin of matter?

For some people this is not even a relevant question. Matter is just assumed to be there. Mass conservation is an old law in chemistry and fluid theory. Exothermic and endothermic reactions indicated that heat was related to internal energy of matter. So, the total macroscopic balance includes mass, heat and internal energy, the latter often related to motion, as in molecular oscillations. By the end of XIX century there were several suggestions that mass and energy might be related in a very simple way, quite similar to Einstein's $E = mc^2$, expression that he axiomatically derived. This mass-

energy equivalence is widely used today to generate electricity in nuclear power plants. Thus, mass, which is a property or at least a manifestation of matter, seems to be intimately linked to energy.

But, is there any *direct* evidence of matter-energy interconversion? Yes. At any nuclear physics laboratory it is easy to observe direct microscopic evidence of electron-positron annihilation, and its inverse phenomenon, creation of electron-positron pairs from photons with energy above 1.022 MeV [30]. Similar reactions for protons and other massive particles are routinely observed at CERN, and at large national laboratories in technologically advanced countries.

A subsidiary question arises: which is more fundamental, matter or energy? Some processes in Nature exhibit traits quite different from those usually assigned to matter. Examples are life, mind, and even electromagnetic radiation. Overall, it seems that energy is a broader notion than matter. Many physicists believe that material particles may be created as energy fluctuations of zero-point energy, or from physical vacuum, but the scanty physical details are hidden behind mathematical formalisms, as the creation-annihilation operator.

The principle of universality underlies classical thinking, including Newton's Principia [25]. In that vein our UFF theory postulates an energy-like primordial fluid (PF) [6,7], that obeys same physical laws at all scales from the cosmos down to the PF itself. At the most fundamental level, sagion-sagion coalescence is conversion of energy from PF into matter, while liberation of sagions in the destruction of an N-sagion array runs in the opposite direction. Final arbiter is, of course, experimental test and consistency with all empirical evidence.

6.2. Is matter of electromagnetic origin?

The reversible electron-positron annihilation process opens new questions: "*If it were not for the different nature of the states on the left and right side, this reaction would, if taken alone, seem to suggest that the leptons and photons are in fact different states of the same object*" [31]. Another issue is causality in the propagation of electrical field associated with electron/positron [32], and hints that electric charge might be hidden somehow inside photons [33]. By the way, nobody knows what a photon is.

Some people answer the question above affirmatively, thus assuming that electromagnetic (EM) force is more fundamental than other interactions or forces, and leaves open the question about the origin of EM force. Our answer is negative: structure of both, electrons and photons, may be explained by another more fundamental theory. This is simpler and more economic.

For the sake of this discussion assume that matter is indeed of EM origin. Is that Maxwell's equations? Or, is it another competing EM theory? Some of them claiming a better consistency with empirical observations, for instance Weber's theory [34].

But even within the realm of Maxwell equations there are open questions. Dirac asked long ago why Maxwell equations were not completely symmetric regarding the sources, and conjectured a magnetic monopole. Several decade long search for such object yields negative results [35]. In the mid-1990s present writer offered a different solution: by using vector algebra only, and *without* any physical assumptions [36], Maxwell equations were rewrriten in a symmetrical form for electrical sources $\pm \rho$ associated with a new pair of vectors (**P**,**N**). Our claim is that the symmetrized equations for (**P**,**N**) are tautologically equivalent to the usual Maxwell equations for (**E**,**B**). In other words, (**E**,**B**) and (**P**,**N**) are two views of the same problem that complement each other (see figure 5). Absence of a source for **B** is a mere artifact of the choice of fields —(**E**,**B**) instead of (**P**,**N**)— rather than an intrinsic deficiency in Maxwell equations. No wonder that the mythical monopole has not been found!

Symmetrized fields (\mathbf{P}, \mathbf{N}) hide additional surprises. The usual derivation of wave equations from Maxwell equations requires Coulomb and Lorentz gauges as additional constraints [37]. On the contrary, wave equations for (\mathbf{P}, \mathbf{N}) are obtained *without* additional assumptions. This implies that Coulomb and Lorentz gauges play no fundamental role in EM theory, they are unnecessary if one uses pair (\mathbf{P}, \mathbf{N}) instead of (\mathbf{E}, \mathbf{B}) . Figure 5 puts in evidence another surprise. Propagation of EM waves in Hertz oscillating dipole may be viewed as transversal (\mathbf{E}, \mathbf{B}) waves [38] at a given time, or as helical

(**P**,**N**) waves [39,40] evolving over space at different times; T is the dipole period. Hence, as already claimed elsewhere [41,42], transversality is not an intrinsic trait of EM radiation!



Figure 5. Complementary representations of Hertz dipole. (A) Usual stationary transversal (E,B) [38] waves at t=1T. (B) Spatial evolution of helical (P,N) waves [36] (from figures 1.11 and 3.20b in [39]).

Let us return to our negative answer and assume that fluid theory is the fundamental notion underlying the structure of leptons, baryons and photons. It is easy to obtain Maxwell equations and forces from the theory of fluids [5,8], without introducing an explicit definition for charge, which only appears embedded in dimensional constants. The implication for a theory of matter is that charge is not a fundamental property of sagions.

6.3. Is gravitational force an inherent property of matter?

The popular answer is positive —this includes many physicists and engineers. As explicitly stated in a letter to Bentley dated January 17, 1692/3, for Newton the answer was negative: "You sometimes speak of gravity as essential and inherent to matter. Pray do not ascribe that notion to me, for the cause of gravity is what I do not pretend to know and therefore would take more time to consider of it" [43]. A possible mechanism for the cause of gravity and its propagation considered by Newton at that time was the pushing gravity idea suggested by the Swiss Fatio de Duiller [44]. However, Newton never took a definite position and in the Principia [25] he stated his famous dictum: hypotheses non fingo. Newton's pragmatism eventually led to the widespread (but incorrect) view that he was the creator of the action-at-a-distance (AAAD) paradigm.

In UFF theory the focus is on a single force, that at Newton's time was epithomized by gravity. We accept Newton's private opinion, and do not ascribe gravitational force to matter. On this regard we fully agree with the Cartesian view that AAAD is an unacceptable notion from the view point of logic. This immediately leads to next query.

6.4. Is there room for a primordial fluid?

In a letter to chemist Robert Boyle on February 28, 1678/9, Newton described ether as a gas formed by discrete corpuscles [43]: "I suppose, that there is diffused through all places an etherial substance, capable of contraction and dilatation, strongly elastic, and, in a word, much <u>like air in all respects</u>, <u>but far more subtle</u>. I suppose this ether pervades all gross bodies, but yet so as to stand rarer in their pores than in free spaces, and so much the rarer, as their pores are less" (underlining added). In his Optics and in private letters Newton was a strong advocate for aether as a medium required for the propagation of gravity. In the third letter to Bentley dated 25 February 1692/3 he argued that [43]: "It is inconceivable that inanimate brute matter should, <u>without the mediation of something else which is not material</u>, operate upon and affect other matter without mutual contact, as it must be if gravitation, in the sense of Epicurus, be essential and inherent in it. And this is one reason why I desired you

would not ascribe innate gravity to me. That gravity should be innate, inherent, and essential to matter, so that one body may act upon another at a distance through a vacuum, without the mediation of anything else, by and through which their action and force may be conveyed from one to another, is to me so great an absurdity that I believe no man who has in philosophical matters a competent faculty of thinking can ever fall into it. <u>Gravity must be caused by an agent acting constantly</u> according to certain laws, but whether this agent is material or immaterial I have left to the consideration of the readers" (underlinings added).

In the second half of the XIX century the notion that transversal EM waves could not propagate in a fluid aether led to the appearance of internally contradictory models of aether; for a brief recount see [45]. After studying some of those absurd ideas, Einstein deprecated ether in 1905. It seems that Einstein was unaware of the dynamical ether proposed by Hertz around 1891, published in 1894 in the lengthy introduction to Hertz's posthumous book on mechanics in a new form [46, 47]. Unfortunately, *"Hertz himself never found a disciple with the understanding of his Mechanics to advance his work further"* [47]. A similar remark applies to his Galilean invariant electromagnetic theory, where Hertz used total derivatives (as in fluid theory) instead of Maxwell's partial derivatives. Such fundamental approach recently received some attention [48]. It is a pity that one of the top-ten physicists of all time passed away before his prime. In Mulligan's opinion: *"had Hertz lived, he would have been one of the leaders in the development of modern physics"* [49]. Indeed!

Thus, it appears that a fluid aether may serve as a fundamental basis for both gravity and electrodynamics. A subsidiary question arises: is there any empirical evidence *against* aether?

The conventional answer is positive. The main evidence is the negative interpretation of the 1887 Michelson-Morley experiment (MMX) [50], which was a follow up to Michelson's 1881 work [51] at Berlin and Postdam when he was a student under Helmholtz. Both Lorentz and Poincaré mentioned Michelson as empirical source for their theories: Lorentz contraction and principle of relativity, respectively.

Present writer became interested in MMX in the late 1980s, and indentified some faults in the analysis of data [52] and a *significant flaw* in Michelson's initial conceptual design [51] based on an extremely small solar velocity. Calculation of expected fringe-shifts with modern values of solar velocity [53] demonstrated that the MM apparatus surely produced variations in amplitude of *several* fringe-widths, while in the actual experiments [50,51] only the *fractional* part of the fringe-shift was recorded. This explains the small value of velocity obtained in all classical experiments [54], including Miller's work during almost 30 years [55]. So, this writer opted for repeating MMX in a stationary mode, automatically recording almost continuously during two years (2003-2005) [56,57,58]. After correcting for environmental effects (pressure, humidity, and temperature), the residuals exhibit daily variations dependent upon terrestrial motion relative to a frame attached to the fixed stars [57,58], thus supporting Miller's final interpretation of his massive data [55]. Hence, there is no empirical evidence against the existence of a primordial fluid.

6.5. Disagions and trisagions: the building blocks of fundamental particles

Our research is close to Helmhotz and Hertz programme: reduce all phenomena to mechanics [49], that we tackle via a classical fluid theory for 3D-extended sagions. We strictly adhere to Leibiniz principle of continuity, that includes, as a special case, the dictum *Natura non facit saltus*. In our approach, quantum mechanical discontinuities are related to discreteness of 3D-objects and processes [12], without conflicts with Leibniz principle. Formation of discrete N-sagion arrays is guided by minimization of potential energy, which is in the same conceptual spirit of Helmholtz least-action principle, and Hertz straightest paths.

According to empirical evidence, the majority of fundamental particles contain an integer number of substructures with 2 and 3 elements [24]. Thus, disagions and trisagions are identified here as the main building blocks of quarks, photons, and the rest of fundamental particles. Disagions are closely related to leptons, trisagions to baryons.

As shown in fig 2, the 3D-space occupied by disagions and trisagions is toroidal in an average sense, but this has nothing to do with QM probabilities, rather it is similar to our planetary system. Both in theoretical and in phenomenological models, the electron is often described as toroidal [31, 59-61].

In the kinematical theory of matter under development, the fractional charge of quarks is associated with spin of the sagion; the sign is positive when spin is "up" or CCW, and negative when "down" or CW. The disagion in fig 2 is charge neutral, while the trisagion has one positive fractional charge. The Hertzian electrical dipole is similar to the disagion in figure 2, and the corresponding (E,B) and (P,N) fields shown in figure 5 may be easily calculated from fluid theory using Hertz method [38,39].

Different sagions in an N-sagion array may have their individual spins aligned differently, thus generating an apparent helical motion on the surface of the torus, that may be pictorially described as the trajectory of the tip of the arrow representing the vector superposition of the N-spins in the array.

Figure 6 shows 12 permutations of two disagions in CCW orbital motion, say, particles. Another 12 permutations arise for the antiparticles represented by the opposite CW orbital motion of sagions. Arrangements in second and fourth rows on right side of figure 6 are physically indistinguishable, and correspond to the concept of multiplicity, thus increasing the probability (or, more accurately) the frequency with which that combination is observed. That is, the number of potentially observable combinations reduces to 9 for each direction of orbital motion (particles and antiparticles).



Figure 6. Spin orientation of each sagion relative to XYZ coordinates, with Z-axis fixed by orbital angular momentum L of disagion/trisagion. Tentative assignments compatible with standard theory [35] are: hypercharge Y = X-axis, charm C = Y-axis, isospin Iz = Z-axis. Arrangements in second and fourth lines may be physically indistinguishable.

Also, directions on the XY plane may be physically indistinguishable. Distinction between X and Y orientations on the XY-plane requires a physical feature to define which direction is X. This may be an internal feature of the array, or an external beacon, for instance a magnetic or electric field used in the process of measurement. If X and Y directions are indistinguishable, they merge as a single "horizontal" direction and the number of distinguishable combinations reduces to 6 for each direction of orbital motion.

If attention is restricted to arrays with parallel/antiparallel spin only, the number of configurations reduces to three for each direction of orbital motion (particles and antiparticles): (a) positive charge: both sagions up, (b) neutral charge: one sagion up and one sagion down, (c) negative charge: both

sagions down. In this case, the disagion has three different states, which is reminiscent of the remark noted above by Williamson and van der Mark [31].

Permutations similar to figure 6 also exist for the three sagions in a trisagion; reported elsewhere.

7. Concluding remarks: towards a revision of preconceived mathematical tenets

As a concluding summary, present writer discovered novel nonperiodic and quantized solutions for the classical 3D-wave equation [2,3,4,6,7]. It is postulated that the 3D-CWE represents a primordial fluid (PF) that populates the entire universe, which exists in an absolute curved 3D-space Σ . The basic component of PF it a 3D-extended atom of energy called sagion, which is the smallest object in Nature. Sagion radius is $R_1 = \mathcal{R}$, and next object in size and complexity is the disagion: a dipole permanently rotating on a given plane, thus defining an internal XY-plane and its orthogonal direction Z for that array. Orbital rotation occupies a torus in 3D-space, inscribed in a sphere of radius $R_2 = 2\mathcal{R} + 2\mathcal{R}$ ε , where ε is the inner radius of the torus. In closest packing geometry sagions are in (almost) contact, so that $\varepsilon = 0$, and $R_2 = 2\mathcal{R}$. The latter is the smallest radius of a *material* object, and constitutes the lowest limit for the size of any composite array. Arrays are kept together by differences in fluid pressure. Absolute vacuum appears in regions with $\varepsilon < \mathcal{R}$, that cannot be traversed without disrupting the array. Configurations containing absolute vacuum are very stable, and difference in fluid pressure is at the highest local value. In contrast to conventional arrays formed by "punctual" particles, 3Dextension establishes lower size limits. Of course, sagion and Σ are mental constructs. Our claim is that from those very few metaphysical assumptions one obtains a self-consistent unified fluid and field (UFF) theory applicable to all interactions in Nature ("forces" in Newtonian language). The economy preconized in Ockham's principle is on our side!

To end this paper in a ruminative mood, let us list some mathematical preconceptions that may need revision to reach an internally consistent description of nature.

7.1. Continuous versus discrete mathematics

For Plato, geometry was a prior requisite for philosophizing, and two thousand years later, Galileo considered that mathematics, in the sense of geometry, was the appropriate language to express ideas in natural philosophy. In the 1780s and early XIX century, Coulomb, Laplace and Poisson used summations to describe the field generated by discrete charges [34]. Somewhere in the XIX century the idea shifted, and by early XX century continuous differential equations became the "exact" representation of nature. By the mid-XX century discreteness explicitly re-appeared as fractal geometry. Our 1986 paper used the same discrete Coulomb formula [22] to demonstrate that for few discrete charges the calculation of electric field using the "exact" continuous mathematics differs from the discrete calculation. The shift towards discreteness goes on, and in the words of a recipient of the 2016 Nobel Physics Award: "in many ways a physics based on integers and rational numbers is more natural than a continuum theory, and it was a shock to Greek mathematicians to discover that the ratio of the diagonal of a square to its side could not be expressed as a ratio of integers" [62]. In our view, irrational numbers should be also explicitly included in the description of small N systems, thus acknowledging the existence of discrete 3D-extended atoms in Nature; for further comments see [12]. Needless to say, systems containing a very large number of particles, for instance larger than Avogadro's number, may be appropriately represented by continuous equations.

7.2. Linear versus non-linear mathematics

Development of computers in past century allowed formerly impossible non-linear calculations. For operational convenience, linear mathematical models were widely used in the XIX-century; but it does not follow that Nature is linear. In the first decades of XX century John von Neumann proposed several logically and intuitively appealing axioms, and derived quantum mechanics (QM) therefrom. However, one of von Neumann's axioms is a linear addition in vector space, thus ignoring all non-

linear operations. It follows a fortiori that quantum mechanics, as known today, is an incomplete theory. This simple remark, clearly and definitively, vindicates Einstein in his controversy with Bohr.

In developing QM theory Schrödinger made a similar choice. According to a well-known textbook [63], Schrödinger's initial candidate for his theory was "the most familiar one-dimensional wave equation, that which describes the motion of transverse waves on a string or plane sound waves in a gas". From linear superposition considerations, Schrödinger discarded the 3D-CWE, which contains time as a second-order partial time-derivative, and selected his well known equation with only a first-order partial time-derivative.

Thus, a right step in the direction towards unification of QM and Einstein's general relativity would be to reformulate QM in terms of the intrinsically Lorentz-invariant 3D-CWE (which is equivalent to the Klein-Gordon equation). Such approach was suggested by De Broglie long ago [15-18].

7.3. Periodic versus non-periodic solutions for the wave equations

Pioneers and creators of most theories often make simplifying assumptions, that in some cases become a fundamental part of the theory. An example are the periodic solutions of Schrödinger equation usually written in spherical coordinates as proportional to $D(\theta, \varphi)exp\{\pm i(kr - \omega t)\}$. The periodicity implicit in sines and cosines is an appropriate boundary condition in many cases, *but not always*. A similar case is the solution for the scalar potential Π in the 3D-CWE, often written as periodical functions for separation constants ℓ and *m* restricted as in equation (1) [64], which also imposes periodicity upon azimuth angle φ :

$$\Box \Pi = \left(\frac{\partial^2}{\partial w^2} - \nabla^2\right) \Pi = 0, \ \Pi(w, r, \theta, \varphi) = R(r) D(\theta, \varphi) \exp\{-ikw\}, \ w \equiv Ct,$$

$$D(\theta, \varphi) = Y(\theta, \varphi; \ell, m), \ \text{for } \lambda = -\ell(\ell+1), \ \lambda_1 = -m^2, \ \ell \ge m, \ \ell, m = 0, 1, 2, \dots$$

$$D(\theta, \varphi) = H(\theta, \varphi), \ \text{for } \lambda = -\ell(\ell+1), \ \ell = 0, 1, 2, \dots, \ \lambda_1 = +m^2 = -\mu^2, \ \mu = \pm im = \pm i\sqrt{\lambda_1}$$
(1)
$$D(\theta, \varphi) = H(\theta, \varphi), \ \text{for } \lambda = -\ell(\ell+1), \ \ell = 0, 1, 2, \dots, \ \lambda_1 = +m^2 = -\mu^2, \ \mu = \pm im = \pm i\sqrt{\lambda_1}$$

Equation (2) allows positive values for separation constant λ_1 , thus relaxing the periodicity constraint imposed upon φ by equation (1). It leads to helical potentials $H(\theta, \varphi)$ evolving on the surface of cones of quantized half-vertex $\theta_1, \theta_2, \dots$ [2-4,6]. Vortices are studied by various other methods in fluid theory [65].

A completely new set of solutions obtains for time and distance handled as a pair of mixed independent variables (r,q) in a mixed potential M(r,q) defined in equations (3) and (4) [2-4,6]:

$$\Pi(w,r,\theta,\varphi) = M(r,q)D(\theta,\varphi) = I(r,\ell,\eta)D(\theta,\varphi) + Q(q,\ell,\eta)D(\theta,\varphi), \ q \equiv w/r$$
(3)

The time-independent potential $I(r; \ell, \eta)$ is similar to the standard solution of homogeneous Laplace equation [64,65], usually arising for: (a) time-independent problems, and (b) steady state potentials. Here, it appears as an intrinsic part of the general time-dependent mixed potential M(r,q), thus bringing in an additional background potential $f(r; \ell, \eta)$, where η is a new (dimensional) separation constant. The <u>quantized isomorph neo-Ga</u>lilean and Lorentz quingal function $Q(q; \ell, \eta)$ is formed by three families of polynomials (Q_1, Q_2, Q_3), each containing a finite number of terms according to quantum number ℓ [2-4,6,7]:

$$I(r; \ell, \eta) = Ar^{\ell} + \frac{B}{r^{\ell+1}} + f(r; \ell, \eta); \ Q(q; \ell, \eta) = A_1 Q_1(q; \ell) + A_2 Q_2(q; \ell) + \eta Q_3(q; \ell)$$

$$f(r; 0, \eta) = \eta \ln(r) \text{ for } \ell = 0; \ f(r; \ell, \eta) = -\frac{\eta}{\ell(\ell+1)} \text{ for } \ell = 1, 2, \dots$$
(4)

In the language of general relativity, the separation of variables in equation (3) amounts to the definition of a new mixed metric $(r,q,\theta,\varphi) = (r,\theta,\varphi)+(q,\theta,\varphi)$ as a superposition of two metrics of lower order on the surfaces of 3-spheres, thus joining the small family of traditional metrics: Minkowski (w,x,y,z) and Schwarzchild (w,r,θ,φ) [66]. The time-independent component of the metric (r,θ,φ) represents a background permanent field that *always is there*, thus providing an unexpected solution to the nagging question of action-at-a-distance (AAAD) attributed to Newtonian gravity, namely: the dichotomy AAAD <u>or</u> delayed propagation does not exist, rather both time-independent (r,θ,φ) and time-dependent (q,θ,φ) potentials are always there; this coincides with some recent remarks in [67].

It is well known that "Birkhoff's theorem is the statement that the Schwarzchild metric is the <u>unique</u> vacuum solution with spherical symmetry (and in particular, that there are no time-dependent solutions of this form)", emphasis in the original [66]. Of course, Birkhoff's theorem applies to the component (r, θ, φ) of our mixed metric (r, q, θ, φ) , but the component (q, θ, φ) constitutes a novel set of quantized non-periodic and time-dependent solutions in vacuum to which Birkhoff's theorem is not applicable (in particular, equations (5.37) and (5.38) in [66] are not valid for our (q, θ, φ) , whose explicit generic solutions are the quingal functions.

The remarkable property of isomorphism exhibited by our quingal functions [6,7,11] means that, when observed from moving frames, the value of the potential is independent of the transformation used for the calculations: Lorentz, Poincaré, Einstein, or the classical Galilean-Doppler, described by Voigt in 1887 [68]. In other words, quingal functions are universal in the sense that they are independent of the relativistic transformation. In that context, our Q-functions may be a concrete realization of the concept of absolute relativity recently introduced by Williamson [69,70].

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References

Vigier

- [1] Stewart I 2012 Seventeen Equations that Changed the World (Joat Enterprises)
- Spanish translation by L Sánchez-Fernández 2013 (Barcelona, Spain: Crítica S.L. and Editorial Planeta)
- [2] Múnera H A, Buriticá D, Guzmán O and Vallejo J 1995 Non-conventional solutions for the travelling wave equation (in Spanish) *Revista Colombiana de Física* 27 (1) 215-218
- [3] Múnera H A and Guzmán O 1997 New explicit nonperiodic solutions of the homogeneous wave equation *Foundations of Physics Letters* **10** (1) 31-41
- [4] Múnera H A 2000 New closed solutions in spherical coordinates for the three-dimensional homogenous wave equation (in Spanish) *Momento, Journal of Physics Department, National University of Colombia, Bogota* 20 1-30

Available at: http://www.revistas.unal.edu.co/index.php/momento/article/view/35307.

- [5] Múnera H A 2000 A realistic four-dimensional hydrodynamic aether interpreted as a unified field equation *Proceedings of International Workshop Lorentz Group, CPT and Neutrinos* (*Zacatecas, Mexico, 23-26 June 1999*) ed A E Chubykalo, V V Dvoeglazov, D J Ernst, V G Kadyshevsky and Y S Kim (Singapore: World Scientific) 425–33
- [6] Múnera H A 2014 Interconnection of all forces of nature via the energy and momentum equations for a fluid aether Proceedings of the IX Vigier Symposium, Unified Field Mechanics: Natural Science Beyond the Veil of Spacetime (Morgan State University, Baltimore, MD, USA, 16-19 Nov. 2014) ed R L Amoroso, L H Kauffman and P Rowlands (Singapore: World Scientific) 247-267
- [7] Múnera H A 2016 Neo-Cartesian unified fluid theory: from the classical wave equation to De

Broglie's Lorentzian quantized mechanics and quantized gravity *Proceedings of Unified Field Mechanics II -- X Vigier Symposium* (Porto Novo, Ancona, Italy, July 24-26, 2016) ed R L Amoroso Published in present *IOP Conference Proceedings*

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- [8] Múnera H A 2000 An electromagnetic force containing two new terms: derivation from a 4Daether *Apeiron* 7 (1-2) 67-75, available at http://redshift.vif.com.
- [9] Múnera H A 2015 A novel approach to gravitation from fluid theory: Titius-Bode structures, flat rotation rate of galaxies, and other predictions. Poster FM18 (p. 22) at XIXth General Assembly of the International Astronomical Union (IAU) (3-14 August, Honolulu, Hawaii, USA, 2015). Abstract in Astronomy in Focus XIXB ed P Benvenuti (Cambridge, UK: Cambridge University Press 2016)
- [10] Múnera H A 2016 Unified field theory from the classical wave equation: preliminary application to atomic and nuclear structure *Proceedings of 11th Latin American Symposium* on Nuclear Physics and Applications LASNPA XI (University of Antioquia, Medellín, Colombia, Nov. 30 - Dec. 4, 2015) ed F Cristancho, AIP Conference Proceedings 1753 030015-1/8 DOI = http://dx.doi.org/10.1063/1.4955356
- [11] Múnera H A 2018 General relativity as a unified fluid and field theory Proceedings of the PIRT-2017 Conference on Physical Interpretations of Relativity Theory (Bauman State University, Moscow, Russia, July 03 - 06, 2017) ed V Gladishev *IOP Conf. Series: Journal of Physics: Conf. Series* **1051** (2018) 012024 doi:10.1088/1742-6596/1051/1/012024
- [12] Múnera H A 2018 Unified fluid and field theory solves long standing controversies (Athens Institute for Education and Research, 20-22 May 2018, Athens, Greece) ATINER's Conference Paper Series EDU2018-23XX
- [13] Einstein A, Infeld L, and Hoffmann B 1938 The gravitational equations and the problem of motion *Annals of Mathematics* **39** (1) 65-100
- [14] Jammer M 1999 Concepts of Force: A Study in the Foundations of Dynamics (Mineola, NY, USA: Dover) 264
- [15] De Broglie L 1927 The five-dimensional universe and wave mechanics (in French) *Le Journal de Physique et le Radium* series VI **8** (2) 65-73, 242-4
- [16] De Broglie L 1952 On the possibility of a causal and objective interpretation of wave mechanics (in French) Comptes rendus Académie des Sciences (Paris) 234 265-8
- [17] De Broglie L 1962 Remarks on the interpretation of the duality of waves and corpuscles (in French) Cahiers de Physique 16 (147) 425-45
- [18] De Broglie L 1970 The reinterpretation of wave mechanics Foundations of Physics 1 (1) 5-15
- [19] Bohm D and Vigier J P 1954 Model of the causal interpretation of quantum theory in terms of a fluid with irregular fluctuations *Physical Review* **96** (1) 208-17
- [20] Cufaro Petroni N and Vigier J P 1983 Dirac's aether in relativistic quantum mechanics Foundations of Physics 13 (2) 253-85
- [21] 't Hooft G 2002 Determinism beneath quantum mechanics Preprint arXiv:0212095v1
- [22] Múnera H A 1986 Properties of discrete electrostatic systems Nature 320 (6063) 597-600
- [23] Ghyka M C 1983 *Esthétique des Proportions dans la Nature et dans les Arts* (in French) Translated into Spanish by J Bosch-Bousquet (Barcelona, Spain: Editorial Poseidon)
- [24] Good I J 1988 What are the masses of elementary particles? Nature 332 (6164) 495-6
- [25] Newton I 1726 The Principia Mathematical Principles of Natural Philosophy Third edition Cohen I B and Whitman A 1999 A New Translation (Berkeley, CA, USA: University of California Press) 371-974 Cohen I B 1999 Guide to Newton's Principia (Berkeley, CA, USA: University of California
- Press) 1-370
 [26] Múnera H A 2011 A Le Sagian atomic-type model for propagation and generation of gravity Should the Laws of Gravitation be Reconsidered? The Scientific Legacy of Maurice Allais H A Múnera ed (Montreal, Quebec, Canada: Apeiron) 385-422
- [27] Múnera H A 1992 A deterministic event tree approach to uncertainty, randomness and

probability in individual chance processes Theory and Decision 32 (1) 21-55

- [28] The Royal Swedish Academy of Sciences 2016 Strange phenomena in matter's flatlands *Popular Science Background The Nobel Prize in Physics 2016* http://kva.se
- [29] The Royal Swedish Academy of Sciences 2016 Topological phase transitions and topological phases of matter *Scientific Background on the Nobel Prize in Physics 2016* http://kva.se
- [30] Segrè E 1977 Nuclei and Particles—An Introduction to Nuclear and Subnuclear Physics 2nd edition (Reading, Mass, USA: W. A. Benjamin)
- [31] Williamson J G and van der Mark M B 1997 Is the electron a photon with toroidal topology? Annales de la Fondation Louis de Broglie 22 (2) 133-58
- [32] Chubykalo A E, Múnera H A and Smirnov-Rueda R 1998 Is the free electromagnetic field a consequence of Maxwell's equations or a postulate? *Found. Phys. Lett.* **11** (6) 573-84
- [33] Múnera H A 2002 The photon as a charge-neutral and mass-neutral composite particle Gravitation and Cosmology: From the Hubble Radius to the Planck Scale R L Amoroso, G Hunter, M Kafatos and J P Vigier eds (Dordrecht, The Netherlands: Kluwer Academic Publishers) 469-476
- [34] Assis A K T 1994 Weber's Electrodynamics (Dordrecht, The Netherlands: Kluwer Academic Publishers) 26-7, 223-37
- [35] PDG Particle Data Group 2016 Particle Physics Booklet (Berkeley, California: Regents of the University of California) 188, 228-30
- [36] Múnera H A and Guzmán O 1997 A symmetric formulation of Maxwell's equations Modern Physics Letters A 12 (28) 2089-101
- [37] Jackson J D 1966 *Classical Electrodynamics* (New York, NY, USA: John Wiley) Spanish translation by G Marsal Aleixandre (Madrid, Spain: Editorial Alhambra) 176-8
- [38] Hertz H R 1892 Electric waves: being researches on the propagation of electric action with finite velocity through space (in German) (Leipzig, Germany: Johann Ambrosius Barth)
 Spanish translation by M García-Doncel and X Roqué 1990 Las Ondas Electromagnéticas (Bellaterra, Barcelona, Spain: Universitat Autónoma de Barcelona) 107-122, 199-217
- [39] Garzón-Barragán I 2005 Representation of the electromagnetic field of an electrical oscillating dipole by symmetrical fields and potentials: Hertzian fields (P,N) and potentials (Π_e, Π_m) (M.Sc. thesis in Spanish) (Bogota, Colombia: Department of Physics, National University)
- [40] Múnera H A and Garzón I 2007 Unfolding the Maxwellian helical fields of the vacuum International Conference on Fundamental and Applied Research in Physics FARPHYS2007 (Iasi, Romania: Physics Department, Alexander I. Cuza University, 25-28 October 2007)
- [41] Múnera H A and Guzmán O 1998 An explicit example of a family of non-planar free-space electromagnetic waves containing magnetic scalar potentials *Progress in Electromagnetics Research Symposium PIERS98* (Nantes, France: Academie de l'Electromagnetisme, 13-17 July, 1998)
- [42] Múnera H A and Guzmán O 2000 Explicit examples of free-space non-planar electromagnetic waves Apeiron 7 (1-2) 59-66
- [43] Newton I 1953 Newton's Philosophy of Nature—Selections from his Writings H S Thayer ed (New York, NY, USA: Hafner Publishing) 53-4, 112-3
- [44] Edwards M R (ed) 2002 Pushing Gravity New Perspectives on Le Sage's Theory of Gravitation (Montreal, Quebec, Canada: Apeiron) 41-64
- [45] Múnera H A 2017 From the classical ethers of Descartes and Newton to cosmons and sagions The Galileo of Palomar: Essays in Memory of Halton Arp C C Fulton and M Kokus eds (Montreal, Canada: Apeiron) 243-292 Also published in 2015 at Apeiron 20 (2) 1-67 available at http://redshift.vif.com
- [46] Hertz H R 1894 Principles of Mechanics Presented in a New Form volume 3 Collected Works (in German) P. Lennard ed (Leipzig, Germany: J. A. Barth) English translation D E Jones and J T Walley 1899 (London: MacMillan)
- [47] Mulligan J E (ed) 1994 Heinrich Rudolf Hertz (1857-1894) A Collection of Articles and

IOP Publishing

Addresses (New York and London: Garland Publishing) 72-3, 305-77

- [48] Phipps T E Jr 2006 Old Physics for New (Montreal, Canada: Apeiron) 17-25
- [49] Mulligan J E 1987 The influence of Hermann von Helmholtz on Heinrich Hertz's contributions to physics American Journal of Physics 55 (8) 711-9
- [50] Michelson A A and Morley E W 1887 On the relative motion of the earth and the luminiferous ether *American Journal of Science* series 3 **134** (Nov.) 333-45
- [51] Michelson A A 1881 The relative motion of the earth and the luminiferous aether *American* Journal of Science series 3 22 120-9
- [52] Múnera H A 1998 Michelson-Morley experiments revisited: systematic errors, consistency among different experiments, and compatibility with absolute space *Apeiron* **5** (1-2) 37-54
- [53] Múnera H A 2002 The effect of solar motion upon the fringe-shifts in a Michelson-Morley interferometer à la Miller *Annales de la Fondation Louis de Broglie (Paris)* 27 (2) 463-84
- [54] Múnera H A 2006 The evidence for length contraction at the turn of the 20th century: nonexistent *Einstein and Poincaré — The Physical Vacuum* Valeriy V. Dvoeglazov ed (Montreal, Canada: Apeiron) 87-102
- [55] Miller D C 1933 The ether-drift experiment and the determination of the absolute motion of the earth *Revs. Mod. Phys.* 5 203-42
- [56] Múnera H A, Hernández-Deckers D, Arenas G and Alfonso E 2005 Observation during 2004 of periodic fringe-shifts in an adialeiptometric stationary Michelson-Morley experiment *Electromagnetic Phenomena (Institute for Electromagnetic Research Kharkov, Ukraine)* 6 1(16) 70-92
- [57] Múnera H A, Hernández-Deckers D, Arenas G and Alfonso E 2007 Observation of a significant influence of earth's motion on the velocity of photons in our terrestrial laboratory *The Nature* of Light: What Are Photons? C Roychoudhuri, A F Kracklauer and K Creath eds Proceedings of International Society for Optical Engineering, SPIE 6664 66640K
- [58] Múnera H A, Hernández-Deckers D, Arenas G, Alfonso E and López I 2009 Observation of a non-conventional influence of earth's motion on the velocity of photons, and calculation of the velocity of our galaxy *Progress in Electromagnetics Research Symposium PIERS-2009* (Beijing, China, 23-27 March 2009) volume 1, 98-104 ISSN:1559-9450 www.piers.org
- [59] Bergman D L and Wesley J P 1990 Spinning charged ring model of electron yielding anomalous magnetic moment *Galilean Electrodynamics* **2** 63-7
- [60] Lucas C W 2013 *The Universal Force Volume 1* (Charleston, SC, USA: www.commonsense science.org) 138-40
- [61] Consa O 2018 Helical solenoid model of the electron Progress in Physics 14 (2) 80-9
- [62] Thouless D J 1998 Topological Quantum Numbers in Nonrelativistic Physics (Singapore: World Scientific Publishing, reprinted 2016) 1
- [63] Schiff L I 1968 Quantum Mechanics, 3rd edition (Singapore: McGraw-Hill International) 21-3
- [64] Butkov E 1968 Mathematical Physics (Reading, Mass, USA: Addison-Wesley) 332-7, 372-81
- [65] Milne-Thomson L M 1938 Theoretical Hydrodynamics (Mineola, NY, USA: Dover) 363, 572-4
- [66] Carroll S 2004 Spacetime and Geometry (Addison-Wesley Longman) 197-204
- [67] Espinoza A, Chubykalo A and Pérez-Carlos D 2016 Gauge invariance of gravitodynamical potentials in the Jefimenko's generalized theory of gravitation *Journal of Modern Physics* 7 (September 6) 1617-26 http://dx.doi.org/10.4236/jmp.2016.713146
- [68] Voigt W 1887 On the Doppler's principle (in German) Nachrichten von der Königlichen Gesellschaft der Wissenschaften und der Georg-Augusts Universität zu Göttingen 2 41–51
- [69] Williamson J G 2018 The nature of "dark matter" in new theory of light and matter *Monthly Notices Royal Astronomical Society* preprint 14 June 2018
- [70] Williamson J G 2018 Root-energy space-time dynamics (XI Vigier Symposium, Liege, Belgium August 6-9, 2018) Present IOP Conference proceedings