COMPLETE DESCRIPTION OF NON-LINEAR COMPTON AND BREIT–WHEELER PROCESSES* **

D.Yu. Ivanov, G.L. Kotkin, V.G. Serbo

Novosibirsk State University
Pirogova 2, Novosibirsk, 6300090, Russia

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We consider emission of a photon by an electron in the field of a strong laser wave and production of e^+e^- pair by a high-energy photon in the field of a strong laser wave. A probability of these processes for circularly or linearly polarized laser photons and for arbitrary polarization of all other particles is calculated. We obtain the complete set of functions which describes such a probability in a compact invariant form. Besides, we discuss the polarization effects in the kinematics relevant to the problem of $e \to \gamma$ conversion at $\gamma\gamma$ and γe colliders.

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1. Introduction

The analysis of polarization effects in the Compton scattering

$$e(p) + \gamma(k) \to e(p') + \gamma(k') \tag{1}$$

is now included in text-books (see, for example, [1], §87). Nevertheless, the complete results for the cross sections with both initial and final particles polarized has been obtained only recently (see [2–4] and the literature therein). One interesting application of the process (1) is the collision of an ultra-relativistic electron with a beam of polarized laser photons. In this case the Compton effect is the basic process for obtaining of high-energy photons for contemporary experiments in nuclear physics and for future $\gamma\gamma$ and γe colliders [5]. The importance of the particle polarization is clearly

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seen from the fact that in comparison with the unpolarized case the number of final photons with maximum energy is nearly doubled when the helicities of the initial electron and photon are opposite [3].

With the growth of the laser field intensity, an electron starts to interact coherently with n laser photons,

$$e(q) + n\,\gamma_L(k) \to e(q') + \gamma(k')\,, \tag{2}$$

thus the Compton scattering becomes non-linear.

Besides the non-linear Compton scattering, in the conversion region the non-linear Breit–Wheeler process takes place:

$$\gamma(k_1) + n\,\gamma_{\rm L}(k_2) \to e^+(q_+) + e^-(q_-)\,. \tag{3}$$

The non-linear Breit–Wheeler process is a crossing channel for the non-linear Compton scattering. The main results for the process (3) can be obtained by usual rules of the crossing relations from the corresponding results for the non-linear Compton scattering. Therefore, we concentrate in this report on the basic process — the non-linear Compton scattering.

The non-linear processes (2) and (3) were observed in the recent experiment at SLAC [6]. The polarization properties of these processes are important for future $\gamma\gamma$ and γe colliders (see [7, 8] and the literature therein). Such reactions must be taken into account in simulations of the processes in a conversion region of these colliders. The method of calculation for such cross sections was developed by Nikishov and Ritus [9]. It is based on the exact solution of the Dirac equation in the field of the external electromagnetic plane wave. Some particular polarization properties of these processes were considered and have already been included in the existing simulation codes (see references in [10] and [11]). Five (four) from sixteen (sixteen) functions for the circularly (linearly) polarized laser photons were found out, however not always in an exact form.

We obtained exact results for all 16 functions for the non-linear Compton scattering in paper [10] and for the non-linear Breit–Wheeler processes in paper [11]. We presented the complete description of both these processes for the case of circularly or linearly polarized laser photons and arbitrary polarization of all other particles. Besides, we derived *(i)* the approximate formulae relevant for the problem of $e \to \gamma$ conversion; *(ii)* the polarization of the final particles averaged over their azimuthal angles; *(iii)* the limiting cases of the small and large energies of the final particles; *(iv)* some numerical results obtained for the range of parameters close to those considered for the TESLA project [12]. Here we present the short review of the results obtained in [10] and [11]. Below we use the system of units in which $c = 1$, $\hbar = 1$.

2. Kinematics

Let us consider the interaction of an electron with a monochromatic plane wave. The corresponding electric and magnetic fields are $\boldsymbol{E}$ and $\boldsymbol{B}$, a frequency is ω, and let F be the root-mean-squared field strength, $F^2 = \langle \boldsymbol{B}^2 \rangle = \langle \boldsymbol{E}^2 \rangle$ and n_{L} be the density of photons in the laser wave. The parameter describing the intensity of the laser field (the parameter of non-linearity) is defined as

$$\xi^2 = \left(\frac{eF}{m\omega}\right)^2 = \frac{4\pi\alpha}{m^2\,\omega}\, n_{\mathrm{L}}\,, \tag{4}$$

where e and m are the electron charge and the mass, $\alpha \approx 1/137$.

It is convenient to use the same invariant variables as for the linear Compton scattering:

$$x = \frac{2pk}{m^2} \approx \frac{4E\omega}{m^2}\,, \quad y = \frac{kk'}{pk} \approx \frac{\omega'}{E} \le y_n = \frac{nx}{1 + nx + \xi^2}\,. \tag{5}$$

The invariant description of the polarization properties of both the initial and the final photons can be performed in the standard way (see [1], §87) using notations ξ_j and ξ'_j for the Stokes parameters of the initial and final photons. For the initial and final electrons we use the invariant polarization parameters ζ_j and ζ'_j, which are the projection of the electron-spin four-vectors a and a' on the convenient orts e_j and e'_j (for detail see [10]).

The standard notion of the cross section is not applicable for the reaction (2) and usually its description is given in terms of the probability of the process per second $\dot{W}^{(n)}$. However, in the procedure of simulation in the conversion region as well as for the simple comparison with the linear case, it is useful to introduce the "effective cross section" defined as

$$d\sigma^{(n)} = \frac{d\dot{W}^{(n)}}{j}\,, \tag{6}$$

where j is the flux density of colliding particles. Contrary to the standard cross section, this effective cross section does depend on the laser beam intensity, *i.e.* on the parameter ξ^2. The total effective cross section is defined as the sum over harmonics, corresponding to the reaction (2) with a given number n of the absorbed laser photons:

$$d\sigma = \sum_n d\sigma^{(n)}\,. \tag{7}$$

The effective differential cross section can be presented in the following invariant form:

$$d\sigma(\boldsymbol{\zeta},\, \boldsymbol{\xi},\, \boldsymbol{\zeta}',\, \boldsymbol{\xi}') = \frac{r_e^2}{4x}\, \sum_n F^{(n)}\, d\Gamma_n\,, \quad d\Gamma_n = dy\, d\varphi\,, \tag{8}$$

where $r_e = \alpha/m$, φ is the azimuthal angle of the final photon, and

$$F^{(n)} = F_0^{(n)} + \sum_{j=1}^{3} \left(F_j^{(n)} \xi_j' \; + \; G_j^{(n)} \zeta_j' \right) + \sum_{i,j=1}^{3} H_{ij}^{(n)} \, \zeta_i' \, \xi_j' \, . \tag{9}$$

The function $F_0^{(n)}$ describes the total cross section for a given harmonic n, summed over spin states of the final particles:

$$\sigma^{(n)}(\boldsymbol{\zeta}, \, \boldsymbol{\xi}) = \frac{r_e^2}{x} \int F_0^{(n)} \, d\Gamma_n \, . \tag{10}$$

The terms $F_j^{(n)} \xi_j'$ and $G_j^{(n)} \zeta_j'$ in (9) describe the polarization of the final photons and the final electrons, respectively. The last terms $H_{ij}^{(n)} \zeta_i' \, \xi_j'$ stand for the correlation of the final particles' polarizations.

3. Some results

We have calculated the coefficients $F_j^{(n)}$, $G_j^{(n)}$ and $H_{ij}^{(n)}$ using the standard technique presented in [1], §101. The necessary traces have been found using the package MATHEMATICA. As an example, we present here the functions $F_{0,1,2,3}$ for the circularly polarized laser photons. In the considered case of the 100 % circularly polarized ($P_c = \pm 1$) laser beam, almost all dependence on the non-linearity parameter ξ^2 accumulates in three functions:

$$\begin{aligned} f_n \; &\equiv \; f_n(z_n) = J_{n-1}^2(z_n) + J_{n+1}^2(z_n) - 2 J_n^2(z_n) \, , \\ g_n \; &\equiv \; g_n(z_n) = \frac{4n^2 J_n^2(z_n)}{z_n^2} \, , \\ h_n \; &\equiv \; h_n(z_n) = J_{n-1}^2(z_n) - J_{n+1}^2(z_n) \, , \end{aligned} \tag{11}$$

where $J_n(z)$ is the Bessel function. The functions (11) depend on x, y and ξ^2 via the single argument

$$z_n = \frac{\xi}{\sqrt{1+\xi^2}} \, n \, s_n \, , \tag{12}$$

where

$$s_n = 2\sqrt{r_n(1-r_n)}, \quad c_n = 1 - 2 r_n \, , \quad r_n = \frac{y \, (1+\xi^2)}{(1-y) \, n x} \, . \tag{13}$$

The results of our calculations are the following. The function $F_0^{(n)}$, related to the total cross section (10), reads:

$$F_0^{(n)} = \left(\frac{1}{1-y} + 1 - y \right) f_n - \frac{s_n^2}{1+\xi^2} g_n - \left[\frac{y s_n}{\sqrt{1+\xi^2}} \zeta_2 - \frac{y(2-y)}{1-y} c_n \zeta_3 \right] h_n P_c \, .$$

The polarization of the final photons is related to the functions

$$
\begin{aligned}
F_1^{(n)} &= \frac{y}{1-y}\frac{s_n}{\sqrt{1+\xi^2}}h_n P_c \zeta_1\,, \\
F_2^{(n)} &= \left(\frac{1}{1-y}+1-y\right) c_n h_n P_c - \frac{y s_n c_n}{\sqrt{1+\xi^2}} g_n \zeta_2 \\
&\quad + y\left(\frac{2-y}{1-y} f_n - \frac{s_n^2}{1+\xi^2} g_n\right)\zeta_3\,, \\
F_3^{(n)} &= 2(f_n - g_n) + s_n^2\left(1+\frac{\xi^2}{1+\xi^2}\right) g_n - \frac{y}{1-y}\frac{s_n}{\sqrt{1+\xi^2}} h_n P_c \zeta_2\,.
\end{aligned}
$$

Up to now only $F_0^{(n)}$ and $F_2^{(n)}$ at $\zeta_1 = \zeta_2 = 0$ were known. The detailed discussion of the results and numerous figures for spectra and polarizations of the final particles can be found in [10] and [11].

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