ESTIMATION OF BARYON ASYMMETRY OF THE UNIVERSE FROM NEUTRINO PHYSICS

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I would like to dedicate this thesis to my parents . . .

Declaration

I hereby declare that this thesis entitled "Estimation of Baryon Asymmetry of the Universe from Neutrino Physics" is the result of my own research work which has been carried out under the guidance of Prof N. Nimai Singh of Gauhati University. I further declare that this thesis as a whole or any part thereof has not been submitted to any other university or institute for the award of any degree or diploma.

Manorama Bora

Manorama Bora

4th August 2017

Certificate

This is to certify that the thesis entitled "Estimation of Baryon Asymmetry of the Universe from Neutrino Physics" is the result of research work of Ms.Manorama Bora, carried out under my supervision, submitted to Gauhati University for the award of the degree of Doctor of Philosophy in Physics. The research work reported in this thesis is original and candidates own work. She has fulfilled all the requirements under the Ph.D regulations of Gauhati University.

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Prof. N. Nimai Singh, Supervisor

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Abstract

The discovery of neutrino masses and mixing in neutrino oscillation experiments in 1998 has greatly increased the interest in a mechanism of baryogenesis through leptogenesis, a model of baryogenesis which is a cosmological consequence. The most popular way to explain why neutrinos are massive but at the same time much lighter than all other fermions, is the see-saw mechanism.

Thus, leptogenesis realises a highly non-trivial link between two completely independent experimental observations: the absence of antimatter in the observable universe and the observation of neutrino mixings and masses. Therefore, leptogenesis has a built-in double sided nature.. The discovery of Higgs boson of mass 125GeV having properties consistent with the SM, further supports the leptogenesis mechanism.

In this thesis we present a brief sketch on the phenomenological status of Standard Model (SM) and its extension to GUT with or without SUSY. Then we review on neutrino oscillation and its implication with latest experiments. We discuss baryogenesis via leptogenesis through the decay of heavy Majorana neutrinos. We also discuss formulation of thermal leptogenesis. At last we try to explore the possibilities for the discrimination of the six kinds of Quasi-degenerate neutrino(QDN)mass models in the light of baryogenesis via leptogenesis. We have seen that all the six QDN mass models are relevant in the context of flavoured leptogenesis. But if the leptogenesis is unflvoured or single flavoured,the scenario is little different,where

we see that only QD-NH-IA and QD-IH-IA are dominant. Here type IA means CP Parity patterns in the three absolute neutrino masses i.e $m_{LL}^{d} = diag(+m_1, -m_2, +m_3)$. In order to get specific results, the choice of Dirac neutrino mass matrix as down-quark type is found to be most favourable.

Table of contents

List of figures

List of tables

1

Introduction and Scope

1.1 Introduction

In this introductory chapter we present a brief review of the main features of the phenomenological status of Standard Model(SM) and its extension to grand unified theories (GUT) with or without supersymmetry a necessary ingredients for the following chapters of the thesis. At the end in section 1.3, we present the scope of the thesis and organisation of the chapters.

1.2 Standard Model (SM) and beyond

In nature we have four fundamental interactions(or forces) namely, gravitational, electromagnetic, strong and weak. Three eminent scientists namely Sheldon Glashow[1], Abdus Salam[2] and Steven Weinberg[3] unify the electromagnetic and weak interactions which is known as standard electroweak theory.
The electroweak part of SM is based on the symmetry group $SU(2)_L \times U(1)_Y$, where L stands for left chiral and Y denotes the weak hypercharge, was started by Glashow with $W^{\pm}$ boson mass term put by hand. Salam and Ward [4] also discussed this subsequently. Weinberg[3] and Salam[2] independently presented for the case of leptons, which was used for generations of $W^{\pm}$, and Z^0 bosons masses with Higgs-Kibble mechanism for Spontaneous symmetry Breaking (SSB). The inclusion of hadron in terms of quark into the theory was done by following the suggestions of Glashow, Illiopoulos and Maiani (known as GIM mechanism) [5]. Such inclusion of charm-quark before its discovery, helped to suppress the "Strangeness Changing Neutral Current"in agreement with experiments. The renormalizabilty of the gauge symmetry $SU(2)_L \times U(1)_Y$ which was not spoiled by SSB, was completed by t'Hooft [6]in Feynman gauge using path integral techniques. In this way, Electro-weak gauge theory was developed and popularly known as GWS theory. Around the same period, a non-abelian gauge theory of strong interaction popularly known as Quantum Chromo-dynamics (QCD), based on colour group $SU(3)_C$, was developed, and the resulting theory has eight massless vector bosons known as gluons which glue the quarks together leading to the phenomenon of "quark confinement" at low energies. The combination of strong interaction with electroweak theory leads to $SU(2)_L \times U(1)_Y \times SU(3)_C \equiv G_{213}$, popularly known as the Standard Model (SM).
The use of "Renormalization Group Equation"(RGEs) for the evolution of gauge couplings $\alpha_i (= \frac{g_i^2}{4\pi})$, $1 = 1, 2, 3 = Y, 2L, 3C$, with energy Q^2 was then carried out by Georgi, Quinn and Weinberg [7]. At high energies (a phenomenon known as Asymptotic Freedom),the

strong interaction is not so strong and its coupling constant $\alpha_{3c}(Q^2)$ decreases with the increase of energy,whereas the electromagnetic coupling constant $\alpha_{em} = \alpha(Q^2)$ increases logarithmically with energy. This opposite picture gives the possibility of the meeting of coupling constants of electroweak group with α_{3c} at higher energy scale.
A brief sketch of the particle representations in $SU(2)_L \times U(1)_Y \times SU(3)_C$, is given below, where the transformation properties under G_{213} have been shown [8]

Gauge bosons (before symmetry breaking) :

$$W_\mu^{1,2,3}(3,0,1) + B_\mu(1,0,1) + G^i_{\mu j}(1,0,8)(i,j = 1,.......8) \tag{1.1}$$

where the first, second and third correspond to the gauge bosons of $SU(2)_L, U(1)_Y$ and $SU(3)_C$,respectively.

Fermions Representation

Todays standard model of particle physics described three replicated families of quarks and leptons. The first family consist of up and down-quarks $(u_L, d_L$ and (u_R, d_R) (L and R stand for left and right chirality of spin 1/2 particles). Each quark comes in three colours : red(r), yellow(y) and blue(b). In addition, there are three colourless leptons $(\overline{e}_L, \nu_{eL})$ and $\overline{e}_R$. Thus this family has 12 quarks and 3 leptons (altogether 15 two-component objects) with 30 = 2x15 degrees of freedom:

1st generation : (with transformation properties under G_{213})

$$\begin{pmatrix} u \\ d \end{pmatrix}_{\alpha L}, (2, \frac{1}{3}, 3), \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L, (2, -1, 1)$$

$$u_{\alpha R}, (1, \frac{4}{3}, 3), \quad \bar{e}_R(1, -2, 1) \tag{1.2}$$

$$d_{\alpha R}, (1, -\frac{2}{3}, 3), \quad \alpha = 1, 2, 3 = r, y, b$$

The second family has charm and strange quarks (c,s)(replacing the (u,d) quarks) while the electron and its neutrino are replaced by the muon(μ) and its (ν_μ). Like the first family, there are 15 two component objects in the 2nd generation :

$$\begin{pmatrix} c \\ s \end{pmatrix}_{\alpha L}, (2, \frac{1}{3}, 3), \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L, (2, -1, 1)$$

$$c_{\alpha R}, (1, \frac{4}{3}, 3), \quad \overline{\mu}_R(1, -2, 1) \tag{1.3}$$

$$s_{\alpha R}, (1, -\frac{2}{3}, 3), \quad \alpha = 1, 2, 3 = r, y, b$$

The third family likewise consists of top and bottom(t,b) quarks plus the tauon(τ) and its neutrino (ν)

$$\begin{pmatrix} t \\ b \end{pmatrix}_{\alpha L} (2, \frac{1}{3}, 3), \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L, (2, -1, 1)$$

$$t_{\alpha R}(1, \frac{4}{3}, 3), \quad \overline{\tau}_R(1, -2, 1) \tag{1.4}$$

$$b_{\alpha R}(1,-\frac{2}{3},3), \quad \alpha = 1,2,3 = r,y,b$$

Neither the structure of multiplets in a generation nor the mass spectrum and the number of generations are explained by the SM. The relation $Q = T_3 + \frac{Y}{2}$, Q being the (electric) charge of a fermion, being the 3rd component of weak isospin and Y being the fermion hypercharge with respect to $U(1)_Y$, is fixed by the model and can be chosen arbitrarily. Thus, the model restricts only the difference between charges of the pair of fermions in a weak $SU(2)_L$ doublet :

$$Qu - Qd = 1, Q_\nu - Q_e = 1, \tag{1.5}$$

The absolute values of quark charges remain arbitrary. In other words, the SM does not explain the quantisation of electric charge.

Higgs Scalar

In the SM, the Higgs scalar is a complex doublet ϕ under $SU(2)_L$ having $Y(\phi) = 1$. This scalar is needed to obtain massive gauge bosons through spontaneous symmetry breaking (SSB) of $SU(2)_L \times U(1)_Y \rightarrow U(1)_{e.m}$, and Higgs mechanism.

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} (2,\frac{1}{2},1) \tag{1.6}$$

The real part of ϕ^0 acquires vacuum expectation value (VEV) and becomes massive physical Higgs scalar. The VEV is expressed as

$$\langle\phi\rangle_0 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix} = \phi \tag{1.7}$$

In the SM, there are no charged scalar fields. The charged components of ϕ are absorbed by the massless $W^{\pm}$ gauge bosons as their longitudinal modes whereas the imaginary part by the neutral Z^0 boson. The electroweak part of the Lagrangian is written as

$$L = L_{gauge} + L_{lepton} + L_{quarks} + L_{scalar} + L_{yukawa} \tag{1.8}$$

where

$$L_{gauge} = -\frac{1}{4}Tr(F_{\mu\nu}F^{\mu\nu}) - \frac{1}{4}f_{\mu\nu}f^{\mu\nu} \tag{1.9}$$

$$f^i_{\mu\nu} = \partial_\nu b^i_\mu - \partial_{\mu^i} b^i_\nu + g_2\varepsilon^{ijk}b_{\mu j}b_{\nu k} \tag{1.10}$$

where $b^i_\mu (i = 1,2,3)$ = gauge bosons for $SU(2)_L$ and

$$f_{\mu\nu} = \partial_\nu b_\mu - \partial_\mu b_\nu$$

where B_μ is the gauge boson for $U(1)_Y$

$$L_{lepton} = \overline{R}i\gamma^\mu(\partial_\mu + i\frac{g_1}{2}B_\mu Y)R + \overline{L}i\gamma^\mu(\partial_\mu + i\frac{g_1}{2}B_\mu Y + i\frac{g_2}{2}\overline{\tau}\overline{b}_\mu)L \tag{1.11}$$

where g_2 and g_1 are the gauge couplings of $SU(2)_L$ and $U(1)_Y$ respectively

$$L_{scalar} = (D^{\mu}\phi)^{\dagger}(D_{\mu}\phi) - V(\phi^{\dagger}\phi), \quad (\phi^{\dagger}\phi) = \mu^2(\phi^{\dagger}\phi) + \lambda(\phi^{\dagger}\phi)^2 \tag{1.12}$$

$$L_{Yukawa} = -h_f[\overline{R}(\phi^{\dagger}L) + (\overline{L}\phi)R] \tag{1.13}$$

For 1st-family, L_{Yukawa} term becomes

$$L_{Yukawa} = -[h_u\overline{\Psi}_{qL}\widetilde{\phi}u_R + h_d\overline{\Psi}_{qL}\phi d_R + h_e\overline{\Psi}_{eL}\phi e_R] + h.c \tag{1.13a}$$

where

$$\Psi_{qL} = \begin{pmatrix} u \\ d \end{pmatrix}_L, \Psi_{eL} = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L), \phi = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}, \widetilde{\phi} = i\tau_2\phi^* = \begin{pmatrix} \frac{v}{\sqrt{2}} \\ 0 \end{pmatrix}, \phi^* = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$$

$$L_{Yukawa} = -[(h_u\frac{v}{\sqrt{2}})\overline{u}_L u_R + (h_d\frac{v}{\sqrt{2}})\overline{d}_L d_R + (h_e\frac{v}{\sqrt{2}})e_L^{+} e_R] + h.c \tag{1.13b}$$

where the masses of the fermions are identified as

$$m_u = h_u\frac{v}{\sqrt{2}}, \quad m_d = h_d\frac{v}{\sqrt{2}}, \quad m_e = h_e\frac{v}{\sqrt{2}}, \tag{1.13c}$$

From L_{scalar} part , one can identify the masses of gauge bosons

$$M_W^{\pm} = \frac{1}{2}vg_2, \quad M_Z = \frac{1}{2}vg_2\sec\theta_W, \quad M_{A\mu} = 0$$

Mixing angle

The electroweak mixing between W_{μ}^3 and B_{μ} gauge bosons at the Z^0-mass scale is defined as

$$A_\mu = \cos\theta_W B_\mu + \sin\theta_W W^3_\mu$$

$$Z_\mu = -\sin\theta_W B_\mu + \cos\theta_W W^3_\mu$$

where the mixing angle (Weinberg angle θ_w) is given by

$$\sin^2\theta_W(M_Z) = \frac{e^2(M_Z)}{g_2^2(M_Z)} = \frac{g_1^2}{g_1^2 + g_2^2} \equiv \frac{\alpha(M_Z)}{\alpha_{2L}(M_Z)}$$

$$e(M_Z) = \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}}, M_W = M_Z \cos\theta_W \tag{1.14}$$

$$\alpha_i(M_Z) = \frac{g_i^2(M_Z)}{4\pi}, \quad i = 1,2,3 \tag{1.15}$$

The (relative) strength of neutral currents is determined by the angle θ_w. However, in the framework of SM it has to be extracted from experiment. The reason is that, $SU(2)_L \times U(1)_Y$ is not simple but it is the product of two simple groups, therefore there are two independent coupling constants g_1 and g_2 in the theory. However, if one embeds $SU(2)_L \times U(1)_Y$ into a bigger simple group (say, into SU(5)) with a single gauge coupling g_{GU}, then both coupling constants g_1 and g_2 will have to be proportional to g_{GU}, so that $\sin^2\theta_w$ may allow to predict the value of θ_w

The electric charge is related to $SU(2)_L$ and $U(1)_Y$ generators in the standard model as

$$Q = T_{3L} + Y/2 \tag{1.16}$$

here Q is the electric charge and T_{3L} is the third generator of $SU(2)_L$. The $U(1)_Y$ generators of SM when embeded in a GUT is

$$I_Y = (\frac{3}{5})^{\frac{1}{2}}(\frac{Y}{2}) \tag{1.17}$$

The neutral interaction through Z^0 exchange is a unique prediction of standard electroweak theory. The ratio of neutral to charged current interaction strength is denoted by ρ-parameter,

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2\theta_W} \equiv 1 \tag{1.18}$$

Physics beyound standard model is indicated by a significant deviation of ρ from unity. By comparing the charge current interaction of the electroweak Lagrangian with the conventional weak interaction for muon decay, one can obtain the relation;

$$\frac{g_2^2}{8M_W^2} = \frac{G_F}{\sqrt{2}} \tag{1.19}$$

Using eq (1.14), the mass of $W^{\pm}$ boson is predicted as

$$M_W^2 = \frac{\pi\alpha_e}{\sqrt{2}G_F \sin^2\theta_W} \tag{1.20}$$

Putting the values $\alpha_e^{-1} = 127.9, G_F = 1.166 \times 10^{-5}/m_p^2, M_W = \frac{37.3}{\sin\theta_w} GeV$,

$$M_Z = \frac{M_W}{\cos\theta_W} = \frac{74.6}{\sin 2\theta_W} GeV \tag{1.21}$$

From the relations $M_W = \frac{1}{2}vg_2$ and $\frac{G_F}{\sqrt{2}} = g_2^2/8M_W^2$

the VEV can also be estimated as

$$v = (\sqrt{2}G_F)^{-1/2} = [\frac{10^5}{1.166 \times \sqrt{2}}]^{1/2} = 246 GeV$$

$$\langle\phi\rangle_0 = \frac{v}{\sqrt{2}} = 174 GeV \tag{1.22}$$

The electromagnetic fine structure constant at Z^0 mass scale can be estimated using the evolution

$$\frac{\partial e^2(\mu)}{\partial(ln\mu)} = \frac{1}{6\pi^2}\sum_f Q_f^2 e^4 \tag{1.23}$$

$$\frac{1}{\alpha(M_Z)} \cong \frac{1}{\alpha(0)} - \frac{2}{3\pi}\sum_f Q_f^2 ln\frac{M_Z}{m_f}, \frac{1}{\alpha(0)} \cong 137 \tag{1.24}$$

The experimental value of α^{-1} at M_Z is given by [9]

$$\alpha^{-1}(M_Z) = 127.9 \pm 0.1 \tag{1.25}$$

If the hypothesis of grand unified (GUT) is true, we have a matching relation,

$$\frac{1}{\alpha_i(M_Z)} = \frac{5}{3}\frac{1}{\alpha_1(M_Z)} + \frac{1}{\alpha_2(M_Z)} \tag{1.26}$$

The values of $\alpha_{1Y}(M_z)$ and $\alpha_{2L}(M_z)$ can be estemated by using eqs (1.14) and (1.27) as

$$\frac{1}{\alpha_i(M_Z)} = \frac{3}{5}\frac{1-S^2(M_Z)}{\alpha(M_Z)}; \frac{S^2(M_Z)}{\alpha(M_Z)}; \frac{1}{\alpha_3(M_Z)} for \quad i = 1,2,3 \tag{1.27}$$

where $S^2(M_Z) = \sin^2\theta_w(M_Z)$

In eq.(1.13) the fermion masses in SM arise due to Yukawa interaction. A single Yukawa coupling h_f cannot account for the quarks and leptons masses of all three generations. Using eq.(1.13c), the Yukawa coupling h_f for every fermion is determined by its known mass

$$h_f(M_Z) = m_f(M_Z)/(\frac{v}{\sqrt{2}}) \tag{1.28}$$

This is not a prediction, but an input parameter to 'Standard Model' through known fermion mass. A fundamental theory has to predict the low energy values of these Yukawa couplings having diverse numeric values. This is one of the intriguing problems of modern particle physics. The quarks and charged leptons receive Dirac masses from the Yukawa term whereas the neutrinos of the three families remain massless(1.13c) in SM. The masslessness of neutrino in the SM is due to the absence of right handed neutrinos. If neutrinos are to acquire a Dirac mass by tree level interaction, the presence of right handed neutrino for each generation is necessary. There are several experimental indications leading to the fact that neutrinos could have a mass much smaller than the quarks or charged leptons.

According to Fermi theory neutrino is assumed to be a Dirac particle without mass and charge, the V-A theory of weak interaction demands the existence of chiral fermions. It is neccessary to follow that a chiral fermion has to remain massless. The masslessness of neutrino predicts that the Fermi-Kurie plot should be a straight line in the process, but for a massive neutrino,the tail expected to be curved. The Standard Model, $SU(3)_C \times SU(2)_L \times U(1)_Y$, which provides an adequate understanding of low energy electorweak and strong interaction, has been tested with numerous experimental observations including the deep inelasitic lepton-nucleon scattering [10] neutral currents [11], detection of $W^{\pm}$ and Z bosons [12] etc. Despite the success of the SM, it is not able to answer some important questions which arise naturally. Some of these limitations are :

(1) The model only partially unifies three of the basic forces and gravitation is left out.

(2) SM could not explain the strong hierarchy between the masses of the fermions.

(3) It does not explain why there is parity violation in the electroweak sector.

(4) The parameterisation of CP violation in the weak interaction involves unknown parameters whose origins are never explained by the model

(5)SM does not explain the generation of small neutrino masses.

(6)It does not explain dark matter and dark energy.

(7) The model has too many unknown parameters.

Although superstring theories are conjectured to be ultimate basic theories of nature, some or most of the problems are solved through gauge theories of higher rank and finally to unified theories with or without supersymmetry.

1.2.1 Phenomenological status of Grand Unification

(A)**Pati-Salam Model**

Pati and Salam[13], first attempted to unify all the three coupling constant g_1, g_2, and g_3 of the standard model $SU(3)_C \times SU(2)_L \times U(1)_Y$. They unified quarks and leptons under the group $SU(2)_L \times SU(2)_R \times SU(4)_C$ ($\equiv G_{224p}, p = parity$), extending the colour gauge group to include leptons and imposing left- right discrete symmetry. Pati Salam group $SU(2)_L \times SU(2)_R \times SU(4)_C$ is subgroup of $SO(10)$, so it automatically embeds the minimal supersymmetric Left-Right models [14]. Partial unification of electroweak and strong interaction $G_{224p}(g_{2L} = g_{2R})$ gauge symmetry were achieved by Pati and Salam which has two gauge couplings $g_{2L}(\mu) = g_{2R}\mu$ and $g_{4C}(\mu)$. The grand unification of electroweak and strong forces have been proposed in Pati-Salam model through SU(4) and other gauge groups involving single coupling constant above the GUT scale.Subsequently, the standard model of electroweak and strong interaction based on $SU(2)_L \times SU(R)_L \times SU(4)_C$ ($\equiv G_{224p}, p = parity$) was extended in the gauge, Higgs and fermion sectors to left right symmetry [15] $SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C$ ($\equiv G_{2213p}, p = parity$) . In such left-right symmetric model, the fermion sector for every generation contains a right-handed neutrino in addition to 15 fermions of the standard model. This makes non-zero Dirac neutrino masses very naturally. In addition, if the neutrinos are Majorana particles, Majorana mass term for the left and right handed neutrinos are also allowed quite naturally. A very attractive feature of left-right symmetric model is the generation of small Majorana masses for neutrinos by see-saw mechanism, in the presence of large right-handed Majorana neutrino masses[16,17], where the left-handed triplets of Higgs are used to break the left-right symmetry along with $SU(2)_R$ [15,18].

(B) **SU(5) GUT and SUSY SU(5)GUT**

Georgi and Glashow[19] first proposed minimal $SU(5)$ GUT as an extension of standard model with dimension $N = 5$ and rank $(N-1) = 4$. As a consequence of gauge theory, there are 24 vector bosons associated with 24 generators, out of which 12 are for standard model and other 12 new X,Y bosons are vector bosons which mediate GUT interactions and proton decay. Neutrino cannot develop a Dirac mass term through Higgs mechanism, because right handed neutrino is not included in the theory,

The fundamental representation of $SU(5)$ is a quintet which has the following decomposition in terms of $SU(3)_C$ and $SU(3)_L$:

$$5 = (3,1) + (1,2) \tag{1.29}$$

The product of two such fundamental representations is given by

$$5 \times 5 = 15(symm) + 10(antisymm) \tag{1.30}$$

With the following decomposition in terms of $SU(3)_C$ and $SU(2)_L$

$$\begin{aligned} 15 &= (6,1) + (1,3) + (3,2) \\ 10 &= (\overline{3},1) + (1,1) + (3,2) \end{aligned} \tag{1.31}$$

15 matter fields in eq.(1.33) can be accommodated in the representations $\overline{5}$ (Symmetric) and 10 (antisymmetric):

$$\overline{5} = (\overline{3},1) + (1,2) \equiv (\overline{d}_\alpha)_L + (\nu_e, e^-)_L \tag{1.32}$$

$$10 = (\overline{3},1) + (1,1) + (3,2) \equiv (\overline{u}_\alpha)_L + e_L^+ + (u,d)_{\alpha L} \tag{1.33}$$

Where α is colour quantum number, and the left and right handed fields are connected by charge conjugation

$$(\Psi_R^C)^a = C^{-T}\psi_a$$

The Weinberg parameter $\sin^2\theta_w$ in $SU(5)$ model at or above unification mass scale M_X is obtained as

$$\sin^2\theta_w = \frac{TrT_{3L}^2}{TrQ^2} \equiv \frac{3}{8} \tag{1.34}$$

The value of the Weinberg angle is fixed in this exact SU(5) symmetry, and is different from the one obtained in GWS theory because in latter case spontaneous symmetry breaking has been introduced where isospin and hypercharge operators are mixed and some of the bosons $(W^\pm, Z^0)$ acquire masses, and the Lagrangian is used to determine its value. The value $\sin^2\theta_w(\mu)$ at an arbitrary scale can be computed using renormalization group equation (RGE) [7] and this shall be compared with experimental values.

Normalization of Coupling Constants

The embedding of a gauge group G_i into a large group G imposes a relation among their coupling constants [7] . The coupling constants of SU(5), $SU(3)_C, SU(2)_L$ and $U(1)$ are denoted by $g_5, g_3, g_2,$ and g_1 respectively. The invariance of SU(5) above the unification mass scale M_X^2 implies $g_5 = g_3 = g_2 = g_1$. However, GWS $-SU(2)_L \times U(1)_Y$ model is considered separately and the coupling constants for $SU(2)_L$ and $U(1)_Y$ are denoted by the usual notation g and $g\prime$ respectively. The embedding of $SU(2)_L \times U(1)_Y$ into SU(5) results in the normalization of coupling constants. This is expressed by relation,

$$g = g_2, g' = \sqrt{\frac{3}{5}} g_1 \quad or \quad g_1 = \sqrt{\frac{5}{3}} g' \tag{1.35}$$

where $\sqrt{\frac{5}{3}}$ is the normalization factor involved between the charge generator Q and $SU(2)_L \times U(1)_Y$ generators T and T_0 as

$$Q = T^3 - \sqrt{\frac{5}{3}} T_0$$

Symmetry Breaking Pattern

Symmetry breaking occurs in two stages

$$SU(5) \longrightarrow SU(3)_C \times SU(2)_L \times U(1)_Y \longrightarrow SU(3)_L \times U(1)_{Y\,em} \tag{1.36}$$

In the first stage of symmetry breaking, 24.-plet of scalar ϕ of the adjoint representation with the vacuum expectation value of the form

$$\langle\phi\rangle = v_{diag}(1,1,1,\frac{-3}{2},\frac{-3}{2}) \tag{1.37}$$

is used to give masses to lepto-quarks X_i and Y_i,

$$M_X^2 = M_Y^2 = \frac{25}{8} g^2 v^2 = M_{ij}^2 \tag{1.38}$$

$$\langle\phi\rangle \cong M_X \sim 10^{14} GeV \tag{1.39}$$

and all other gauge bosons remain massless.

The second stage of symmetry breaking is provided by the scalar field transforming as (1,2) with respect to $SU(3)_C \times SU(2)_L$. The minimal dimension representation with such properties is $\overline{5}$, with the VEV of the form,

$$\langle H \rangle = (0,0,0,0,\frac{v}{\sqrt{2}}) \tag{1.40}$$

which gives masses to $W^{\pm}, Z^0$

$$\langle H \rangle \sim M_w \sim 10^2 GeV \tag{1.41}$$

The SU(5) model with the scalar sector consisting of only the representations $\underline{5}$ and $\underline{24}$ is called for simplicity of the minimal SU(5). We see that

$$\langle H \rangle / \langle \phi \rangle \sim 10^{-12} \tag{1.42}$$

which is a very small number. The origin of this great difference in VEVs is usually referred to as a “hierarchy problem".

SUSY SU(5) GUT

Dimopoulos, Georgi [20] and Sakai[21] independently developed the supersymmetric version of Georgi- Glashow minimal SU(5) GUT, where all particles of the simple SU(5)GUT have partners known as ‘super partners’ having the same SU(5) quantum numbers but spins differ by $\frac{1}{2}$ unit. This will resolve the question of divergences by cancelling corresponding fermion and boson loops. These superpartners also carry handedness . Only supermultipletes of same handedness can have Yukawa couplings to one another. The model includes chiral scalar $(J = 0)$ superfields of each quark and lepton of the family $\overline{5}$ and 10-plets, having the same SU(5) quantum numbers. In the Higgs sector, the known Higgs fields of ordinary SU(5) model, transforming as 24 and 5 representations, are replaced by a complex $\underline{24}(\sum_{Y}^{*}), a\overline{5}(H_x)'$ and a $5(H^*)$ in the SUSY case. There are various arguments that there must be at least 2 Higgs doublets in SUSY. Presence of 4 Higgs doublets and enlargement of Higgs content i.e. $(10_H + \overline{10}_{H'})$ in the theory, are also discussed in the literature in order to give compatible values of $\sin^2\theta_w, \frac{m_t}{m_b}$ and τ_p of this model. The most uncertain and important part of the particle contents of SUSY is the Higgs contents and this can lead to different SUSY SU(5) models. For completeness, the gauge fermions, superpartners of gauge bosons and new fermions in the Higgs supermultiplets are also included

It is generally accepted that in the pattern of SUSY breaking, the SUSY SU(5) model breaks down to SUSY $SU(3)_C \times SU(2)_L \times U(1)_Y$ (known as minimal supersymmetric standard Model, MSSM [22] at unification mass scale M_X preserving supersymmetry. The mechanism of breaking at this stage is same as that of ordinary SU(5)GUT . The physics behind this stage is that the unbroken SUSY keeps non-Goldstone massless bosons exactly massless, the matter fields are also massless because they are protected by an unbroken chiral gauge symmetry of the full theory. The gauge symmetry also protects the gauge fields from acquiring masses . Only the colour triplet "chiral superfields" develop masses of the order of M_X

In the second stage of SUSY breaking, both SUSY and electroweak group break up from $SU(2)_L \times U(1)_Y$ at the mass scale $\sim M_w$ via Higgs doublets. In fact, these doublets originate from those non-Goldston bosons ϕ occurred in the first stage, where they are protected from acquiring heavy mass M_X due to radiative corrections by the existence of the non-normalization theorems. Further, Yukawa couplings of matter – supermultiplets and Higgs-multiplets give rise to quark – and lepton-masses. Low energy SUSY breaking can be achieved in different approaches leading to different SUSY SU(5) models [23–26]

One of the successes of Standard Model (SM) namely the automatic conservation of baryon number (B) and lepton number (L) by the renormalizable interactions is not stated by the minimal supersymmetric standard model (MSSM). In SM, the conservation of B and L follows from the particle content and the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge invariance. In MSSM, baryon and lepton number violation can occur at tree level with catastrophic consequence unless the corresponding couplings are very small. The most common way to eliminate these tree level B and L violating terms is to impose a discrete Z_2 symmetry [27,28] known as a matter parity for superfields $(= (-1)^{3(B-L)})$ or equivalently R- parity on component fields $(R_P = (-1)^{3(B-L)+2s})$, (s being the spin of the particle), where all the standard Model particles having $R_P = +1$, while all superpartners have $R_P = -1$. However, the assumption of R-parity conservation appears to be adhoc, since it is not required for the internal consistency of the minimal supersymmetric standard model, and the problem becomes acute for low energy supersymmetric models [29]. The subject is of current interest.

Phenomenological Status of SU(5)GUT

There are at least three distinct conceptual predictions which can be tested experimentally. They are

1) The value of weak angle at the Z-pole, $\sin^2\theta_W(M_Z)$

2) Meeting of the three gauge couplings α_1, α_2, and α_3 at a point when extrapolated to high energy,

3) The scale of meeting point of three couplings is high enough to prevent a very fast proton decay rate via vector-boson exchange.

4) The prediction of lepto-quark unification for understanding of the origin of fermion masses

To satisfy the above criteria, there should be only one underlying gauge coupling in grand unified theories, so the properly normalized running couplings

$$\alpha_1 \equiv \frac{5g'^2}{34\pi}, \alpha_2 \equiv \frac{g^2}{4\pi}, \alpha_3 \equiv \frac{{g_s}^2}{4\pi}$$

are expected to meet at unification scale M_X. One can predict M_X and $\sin^2\theta_w \cong g'^2/(g^2+g'^2)$ from the observed ratio of $\frac{\alpha}{\alpha_s}$ and the renormalization group equations (RGE) provided.

(a) There are only two mass scales (M_W and M_X), with a grand desert in between, so that the three couplings all meet at one point
(b) The quantum numbers of a full G-multiplet are known(so that the normalization of the α_i can be computed)
(c) The light particle quantum numbers are known (so that the RGE coefficients can be computed)
(d) The above observations are true for a whole class of GUTs- SU(5), SO(10) and E_6 which have the same relative renormalization of the Gi generators

In particular, the minimal grand unification model based on the SU(5) has made very precise predictions for the proton decay lifetime of τ_p within 1.6×10^{30} years to 2.5×10^{28} years against the present experimental bound $\tau_p \geq 10^{33}$ years . Failure to observe proton decay at this level seems to rule out the simple minimal SU(5) models. The three couplings α_1, α_2 and α_3 do not exactly meet at a single point consistent with the low energy input values, the grand unification scale is predicted at $M_X \cong 4.6 \times 10^{14}$ GeV which is responsible for very fast proton decay rate (i.e. rate of proton decay is inversely proportional to the 4th power of the grand unification scale) .

A revival of interest in the grand unified occurred after the idea of supersymmetry became a part of phenomenology of particle physics in the early 80's. Two points were realized that to this first is that a theoretical understanding of the large hierarrchy between the weak scale and the GUT scale possible only within the framework of supersymmetry as discussed earlier. Secondly, on a more phenomenological level, measured values of $\sin^2\theta_W$ from the accelerators coupled with the observed values for α_{strong} and $\alpha_{e.m}$, that could be reconciled with the unification of gauge couplings and the supersymmetry breaking scale was assumed to be near the weak scale, which was indecently motivated anyway [30]. This suggests SUSY SU(5) model as a serious candidate for unified theory for further investigations in some other aspects e.g. Yukawa coupling unification.

It should be however made clear that supersymmetry is not the only well motivated beyond standard model physics which leads to coupling constant unification consistent with the measured value of $\sin^2\theta_W$. If the neutrinos have masses in the micro-milli-eV range, then the seesaw mechanism is given by the formula

$$m_{\nu_1} \cong \frac{m_{u_i}^2}{M_{B-L}} \tag{1.43}$$

where the M_{B-L} scale is around 10^{11} GeV or so . It was shown in the early 80's that coupling constant unification can take place without any need for supersymmetry if it is assumed that above the M_{B-L} gauge symmetry becomes $SU(2)_L \times SU(2)_R \times SU(1)_{B-L} \times SU(3)_C$ or $SU(2)_L \times SU(2)_R \times SU(4)_C$ which may emerge from higher rank groups such as SO(10),E6 etc.

C. SO(10)GUT

There are many other interesting unification models motivated for different reasons. They are based on higher rank groups SO(10), E_6, $SU(5) \times U(1)$ and $SU(5) \times SU(5)$. We discuss here very briefly the simplest one i.e.SO(10), unification group and its supersymmetric version

SO(10) model of grand unification [31] is based on the orthogonal group of rank 5 and order 45. It has a number of additional desirable features over SU(5) model. For instance, all the matter fermions fit into one spinor representation of SO(10). Secondly, the SO(10) spinor being 16-dimensional, it contains the right handed neutrino leading to non-zero neutrino masses. The gauge group of SO(10) is left-right symmetric which has the consequences that it can solve SUSY CP problem and R-parity problem etc. Since SU(5) is a sub-group of SO(10), the 16-component representation can be reduced under SU(5) as

$$16 = 10 + \overline{5} + 1 \tag{1.44}$$

Where $\overline{5}$ and 10-plets cancel the corresponding anomalies. The fermions are Yukawa interacting with the following scalar multiplets

$$16 \times 16 = 10 + 120 + 126 \tag{1.45}$$

All the scalar multiplets contain neutral color singlet and so they may give masses to fermions . Under SU(5) we have the following decompositions

$$120 = 5 + \overline{5} + 10 + \overline{10} + 45 + \overline{45}$$

$$126 = 1 + \overline{5} + 10 + \overline{15} + 45 + \overline{50} \tag{1.46}$$

Further lepton-quark boson X_s does not contribute to nucleon decay in tree approximation. The Y' generator coincides with (B-L), and (B-L) is now thus a gauge generator and therefore it has to be violated, otherwise there would be massless photon coupled to it . The presence of ν_R allows for a Dirac mass (ν_R, ν_L) and the violation of (B-L) allows the Majorana term $(\overline{\nu}_R, \nu_L^C)$

If SO(10) is broken down to SU(5) as

$$SO(10) \longrightarrow SU(5) \longrightarrow SU(3)_C \times SU(2)_L \times U(1)_Y \longrightarrow SU(3)_C \times U(1)_{em} \qquad (1.47)$$

then one will have a SU(5) model and theoretical predictions with additional scalar multiplets. In another class of symmetry with minimal set of Higgs multiplets, SO(10) breaks to the standard model via only one intermediate stage, that consisists of the left-right symmetric gauge group with or without the parity symmetry [32],depending on the Higgs multiplet chosen to break SO(10). The discrete Z_2 symmetry is denoted by the symbol P. This lead to the following four possibilities for the intermediate gauge symmetry

I. $G_{224p} = SU(2)_L \times SU(2)_R \times SU(4)_C \times P$

II .$G_{224} = SU(2)_L \times SU(2)_R \times SU(4)_C$

III. $G_{2213p} = SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C \times P$

IV. $G_{2213} = SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times SU(3)_C$

Case-I arises if the Higgs multiplet used to break is a single 54- dimensional one [33],

$$54 = (1,1,1) + (3,3,1) + (1,1,20) + (2,2,6)$$

Case-II and III arise if a single 210-Higgs multiplet is used. Depending upon the range of the parameters in the Higgs potential,either case –II or III arises as the intermediate symmetry [34] : In case II decomposition we have

$$210 = (1,1,15) + (1,1,1) + (2,2,10) + (2,2,\overline{10}) + (1,3,15) + (3,1,15) + (2,2,6)$$

Case –iV arises when one uses a combination of 45- and 54- dimensional Higgs multiplet [35] .The rest of the symmetry breaking is implemented to a single 126- dimensional representation,

$$126 = (3,1,10) + (1,3,\overline{10}) + (2,2,15) + (1,2,6)$$

To break $SU(2)_R \times U(1)_{B-L}$ as well as to understand neutrino masses and a single10 to break the electroweak $SU(2)_R \times U(1)_{B-L}$ down to $U(1)_{em}$. These cases therefore represent the four simplest and completely realistic minimal SO(10) models. The prediction for the proton lifetime in these four SO(10) models (I-IV) have not yet been ruled out by the present experimental bound [36]

For cases I-IV respectively, the uncertainties include the threshold uncertainties in both the intermediate at the unification scales. Case I is very important as it can be tested by the next generation experiments.

The Yukawa couplings in SO(10) are of the form $H(\underline{16},\underline{16})$, where H denotes the irreducible representation according to which the Higgs field transforms [37]. For example in the model G_{224P} , the masses of the third generation came from a single coupling of the form stated above, where H is a complex $\underline{10}$ of the Higgs field which forms a coupling $h_3\underline{10}(\underline{16},\underline{16})$. The up quark couples to a $SU(2)L$ doublet (φ_u) inside the $\underline{10}$ and the down quarks and leptons to the second $SU(2)_L$ doublet(φ_d) These are two cases that have to be treated rather differently: the one-Higgs doublet case and the two-Higgs-doublet case. In the two –Higgs doublet case both (φ_u) and (φ_d) remain light and the fermion masses arise from Yukawa terms in the low-energy theory of the form

$$h_t(\bar{t}t)\varphi_u + [(h_b(\bar{b}b) + h_\tau(\overline{\tau}\tau)]\varphi_d$$

In the one-Higgs- doublet case φ_u and φ_u can be written as

$$\varphi_u = \varphi \cos\theta - \varphi_H \sin\theta, \quad \varphi_d = \varphi \sin\theta + \varphi_H \cos\theta$$

where φ is the light Higgs doublet with no vacuum expectation value (VEV). The relevant Yukawa terms of the low-energy are then just

$$[h_t(\bar{t}t) + (h_b(\overline{b}b) + h_\tau(\overline{\tau}\tau)]\varphi$$

where $\langle\varphi\rangle = v = 2^{-3/4}G^{-1/2} = 174$ GeV for the one-Higgs doublet. In case of two-Higgs doublet, we do not know the values $\langle\varphi\rangle = v_1$ and $\langle\varphi\rangle = v_2$, but we know only the $\sqrt{v_1^2 + v_2^2} = 2^{-3/4}G_F^{-1/2} = v = 174$ GeV with the ratio $v_1/v_2 = \tan\beta$,

, The unification of third generation Yukawa coupling (h_t, h_b, h_τ) at the intermediate scale in both cases is a subject of current interest and it shed light on the origin of the fermion masses [37]. In the conventional SUSY SO(10) employing the Higgs Supermultiplets 54, $16_H \oplus \overline{16}_H$ and 10 in the usual fashion, it is impossible to achieve an intermediate scale of the symmetry breaking substantially lower than unification scale M_U [38]. When $126_H \oplus \overline{126}_H$ are used instead of $16_H \oplus \overline{16}_H$, no intermediate gauge group contain $SU(4)_C$ has been found to be possible in Ref [39]. But the possibilities of other intermediate gauge symmetries in string inspired SUSY SO(10) including G_{2213} have been demonstrated [39,40] by using extra light G_{2213} submultiplets not needed for spontaneous Symmetry breaking but predicted to be existing in the spectrum [41]. Further extensive discussion on SUSY SO(10) is given in Ref.[42].

1.3 Scope of the present thesis

The discovery of neutrino masses and mixing in neutrino oscillation experiments in 1998 has greatly increased the interest in a mechanism of baryogenesis through leptogenesis , a model of baryogenesis which is a cosmological consequence. The most popular way to explain why neutrinos are massive but at the same time much lighter than all other fermions, is the the see-saw mechanism.

Thus, leptogenesis realises a highly non-trivial link between two completely independent experimental observations: the absence of antimatter in the observable universe and the observation of neutrino mixings and masses. Therefore, leptogenesis has a built-in double sided nature. It describes a very early stage in the history of the universe characterised by temperature $T_{lep} > 100$ GeV, much higher than those probed by the Big Bang nucleosynthesis $(T_{BBN} \sim 1MeV)$ on one side, and on another side it complements low energy neutrino experiments, providing a completely independent phenomenological tool to test models of new physics, embedding the see-saw mechanism. The discovery of Higgs boson of mass 125GeV having properties consistent with the SM, further supports for leptogenesis mechanism.

In the introductory Chapter, we present a brief sketch on the phenomenological status of Standard Model (SM) and its extension to GUT with or without SUSY. In Chapter 2 we review on neutrino oscillation and its implications with latest experiments. In Chapter 3, we discuss the outline of the possible explanation of the generation of observed baryon asymmetry of the Universe. We discuss baryogenesis via leptogenesis through the decay of heavy Majorana neutrinos. We also discuss formulation of thermal leptogenesis. A brief discussion on the relation between Inflaton mass and non- thermal leptogenesis is presented in the last section

. In Chapter 4 we try to explore the possibilities for the discrimination of the six kinds of Quasi-Degenerate Neutrino(QDN)mass models in the light of baryogenesis via leptogenesis.

To predict baryon asymmetry of the Universe (BAU) for six kinds of quasi-degenerate neutrino (QDN) mass models, both in normal hierarchy (NH) and inverted hierarchy (IH), we need light left-handed Majorana neutrino mass matrix m_{LL}. These textures of m_{LL} are taken from the ref [152].

In Chapter 5, we present a summary and conclusion of the thesis. At the end of this chapter we outline a scope for possible extension of the present work.

2

A brief review on Neutrino Oscillation

2.1 Introduction

In this chapter we review on neutrino oscillation and its implication with latest experiments. Neutrino oscillation is a peculiar quantum mechanical effect.

2.2 Discovery of neutrino oscillation

According to SM neutrinos are massless. All other fermions like charged leptons and quarks have mass, so masslessness of neutrinos gives them a different status. Various charged and neutral current process involving neutrinos are successfully explained in SM. Therefore it is believed for a long time that the neutrinos are indeed massless. During the last several years, compelling evidence, for existence, of non-zero masses have been obtained, in the experiments studying oscillations of solar,atmospheric, reactor and accelerator neutrinos, which changed the scenario completely. Let us consider that neutrinos are massive. A neutrino can have either a specific flavour or specific mass but not both at the same time. So the flavour eigenstates of the neutrinos are different from the mass eigenstates, and we can treat a flavour eigenstate as to be a linear combination of the mass eigenstates[43- 47].The neutrinos are created by charged current weak interaction which is denoted as

$$\mathscr{L}_{cc} = \frac{g}{\sqrt{2}}\overline{l'}_{\alpha L}\gamma^{\mu}\nu'_{\alpha L}W^{-}_{\mu} + h.c \tag{2.1}$$

where $l'_{\alpha L}$ denotes left handed charged lepton field and $\nu'_{\alpha L}$ corresponds to left handed neutrino field. Prime signifies that they are in the flavour eigenbasis and α is the flavour index ($\alpha = e, \mu, \tau$). The relationship between the flavour eigenstates and mass eigenstates can be written as,

$$\overline{l'}_{\alpha L} = (V_L)_{\alpha j} l_{jL}$$

$$\nu'_{\alpha L} = (U_L)_{\alpha j}\nu_{jL} \tag{2.2}$$

where sum over repeated index is understood. In general V_L, U_L are 3×3 matrices in the above relation (2.2) and fields in the R.H.S. denotes the corresponding fields in the mass basis. Using these transformation relations, the charged current interaction can be rewritten as,

$$-\mathscr{L}_{cc} = \frac{g}{\sqrt{2}}\overline{l'}_{iL}\gamma^{\mu}(V_L^{\dagger}U_L)_{ij}\nu_{jL}W_{\mu}^{-} + h.c \tag{2.3}$$

The matrix $U = (V_L^{\dagger}U_L)$ is called the leptonic mixing matrix or PMNS matrix named after Pontecorvo[48], Maki, Nakagawa and Sakata[49] who have introduced this matrix (it is analogous to the well known quark mixing matrix or CKM matrix). The flavour eigenstates are related to the mass eigen states through U matrix as

$$|\nu'_{\alpha}\rangle = U_{\alpha i}^{*}|\nu_i\rangle \tag{2.4}$$

Now, we exclude primes and use greek indices for flavour and latin for mass eigenstates $(\alpha = e, \mu, \tau; i = 1, 2, 3)$. Now, we are going to study the time evolution of different flavours. Let us consider initially (t = 0) we have a definite flavour eigenstates i.e $|\nu(t=0)\rangle = |\nu_{\alpha}\rangle$. So after a time t this state will evolve to a state

$$|\nu(t)\rangle = U_{\alpha j}^{*}e^{-iE_jt}|\nu_j\rangle \tag{2.5}$$

The amplitude of that neutrino in a flavour state ν_{β} after a time t is given by

$$A(\nu_{\alpha} \to \nu_{\beta}; t) = \langle\nu_{\beta}|\nu_{\alpha}\rangle = U_{\beta j}e^{-iE_jt}U_{\alpha j}^{*} \tag{2.6}$$

The corresponding probability is obtained as,

$$P(\nu_{\alpha} \to \nu_{\beta}; t) = |A(\nu_{\alpha} \to \nu_{\beta}; t)|^2 = |U_{\beta j}e^{-iE_jt}U_{\alpha j}^{*}|^2 \tag{2.7}$$

Two flavour oscillation

Let us consider two flavour scenario $(\nu_e and \nu_\mu)$. The neutrino mixing U can be written as,

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \tag{2.8}$$

The probability of finding a muon neutrino (ν_μ) from a (ν_e), evolves in time t is obtained by using the expression (2.7) as follows,

$$P(\nu_e \to \nu_\mu; t) = P(\nu_\mu \to \nu_e) = \sin^2 2\theta \sin^2(\frac{t\Delta m^2}{4E}) \tag{2.9}$$

where $\Delta m^2 = m_2^2 - m_1^2$.The following assumptions are used in deriving (2.9).The energy eigenvalue

$$E_i = \sqrt{p^2 + m_i^2} \cong p + \frac{m_i^2}{2E} \tag{2.10}$$

Here m_i is the mass eigenvalue of neutrinos, taking $p \simeq E$ with $p^2 \gg m^2$, since mass of the neutrino is negligible compared to its momentum. For relativistic neutrinos $L \simeq t$ the transition probability can be written as,

$$P(\nu_e \to \nu_\mu; L) = \sin^2 2\theta \sin^2(\pi \frac{L}{l_{osc}}) \tag{2.11}$$

where the new parameter $L_{osi}(= \frac{4\pi E}{\Delta m^2})$ is called oscillation length which is equal to the distance between any two closest minima or maxima of the transition probability. The probability of conversion from ν_e to ν_μ flavour or vice versa can be obtained from the expression (2.9,2.11). Similarly, the probability for flavour non changing interaction after a time interval is given by

$$P(\nu_\alpha \to \nu_\alpha; t) = 1 - P(\nu_\alpha \to \nu_\beta; t) \tag{2.12}$$

here α and β denotes e or μ

Three flavour oscillation

There are three neutrino flavours, the flavour and mass eigenstates are related by a 3×3 mixing matrix,

$$\begin{pmatrix} \nu_{eL} \\ \nu_{\mu L} \\ \nu_{\tau L} \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_{1L} \\ \nu_{2L} \\ \nu_{3L} \end{pmatrix} \tag{2.13}$$

The parametrization of the U matrix will be different for Dirac and Majorana type. We need three mixing angle $(\theta_{12}, \theta_{13}, \theta_{23},)$ and one CP violating Dirac phase δ for Dirac neutrino to parametrize the mixing matrix. The standard parameterization of PMNS matrix as recommended by PDG [50]

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \tag{2.14}$$

where $s_{ij} = \sin\theta_{ij}$ and c_{ij}. The transition probability between different flavours in three neutrino case is more complicated than the two flavour case. For practical concern , we can get simple expression in several limiting case. Solar and atmospheric neutrino oscillation data indicates $\Delta m^2_{21} \sim 10^{-5}$ and $\Delta m^2_{31} \sim 10^{-3}$ respectively i.e

$$|\Delta m^2_{21}| \ll |\Delta m^2_{31}| \simeq |\Delta m^2_{32}| \tag{2.15}$$

It can be concluded from the above pattern of mass squared differences that there exist a hierarchy of neutrino masses either $m_1 \leq m_2 \ll m_3$ (normal hierarchy) or $m_3 \ll m_1 \leq m_2$ (inverted hierarchy)

Consider a baseline length L for which

$$\frac{\Delta m_{21}^2}{2E} L \ll 1 \tag{2.16}$$

Above condition is applicable to atmospheric, reactor and accelerator neutrino experiments. If the solar mass squared difference is taken to be vanishing, then oscillation due to small mass squared difference (Δm_{21}^2) actually does not take place. Then, one can obtained the simplified form of oscillation probability as

$$P(\nu_\alpha \to \nu_\beta; L) = 4|U_{\alpha 3}|^2 |U_{\beta 3}|^2 \sin^2(\frac{\Delta m_{31}^2}{4E} L) \tag{2.17}$$

Using suitable values of the flavour indices α and β, the probability of oscillation between e, μ and τ flavours can be written accordingly.

For long baseline reactor neutrino experiments, we have to use the condition

$$\frac{\Delta m_{31}^2}{2E} L \simeq \frac{\Delta m_{32}^2}{2E} L \gg 1 \tag{2.18}$$

Applying the above condition along with normal mass hierarchy, the ν_e survival probability is given by

$$P(\nu_e \to \nu_e; L) \simeq c_{13}^4 P + s_{13}^4 \tag{2.19}$$

Since the oscillations due to the mass difference Δm_{31}^2 and Δm_{32}^2 are fast, so they give an averaged effect . P in the R.H.S. of eq.(2.19) represent the two flavour ν_e survival probability in this case, which is given by

$$P = 1 - \sin^2 2\theta_{12} \sin^2(\frac{\Delta m_{21}^2}{4E} L) \tag{2.20}$$

Lastly, when we consider the limit $U_{e3} \to 0$, the expressions of transition probability between different flavours is obtained as

$$P(\nu_e \to \nu_\mu; L) = c_{23}^2 \sin^2 2\theta_{12} \sin^2 \Delta_{21} \tag{2.21}$$

$$P(\nu_e \to \nu_\tau; L) = s_{23}^2 \sin^2 2\theta_{12} \sin^2 \Delta_{21} \tag{2.22}$$

$$P(\nu_\mu \to \nu_\tau; L) = \sin^2 2\theta_{23}(-s_{12}^2 c_{12}^2 \sin^2 \Delta_{21} + s_{12}^2 \sin^2 \Delta_{31} + c_{12}^2 \sin^2 \Delta_{32}) \tag{2.23}$$

where $\Delta m_{ij} \equiv (\frac{\Delta m_{12}^2}{4E})L$, and we have made no assumption about the mass hierarchy in deriving these expressions.

2.2.1 Neutrino Oscillation in matter

When neutrinos propagate through vacuum, then the expressions of oscillation probability shown in the previous section are valid. There is substantial change in the oscillation probabilities when neutrinos propagate through matter. Matter effect can greatly enhance neutrino mixing, resulting in a large oscillation probability even for small mixing angle in vacuum. This resonance enhancement of the mixing in the presence of matter with varying density is termed as [51,52] "Mikheyev-Smirnov-Wolfenstein (MSW)" effect.

Among three charged leptons (e, μ, τ), normal matter contains only electron(e). While all flavour neutrinos are allowed to pass through matter, they interact with electrons, protons and neutron through neutral current process mediated by Z^0 bosons. Only electron neutrinos can interact via $W\pm$ mediated charged current process with electrons. The CC contribution to effective Hamiltonian is given by

$$(V_e)_{CC} \equiv V_{CC} = \sqrt{2}G_F N_e \tag{2.24}$$

Here N_e is the electron number density. For all three flavours V_{NC} are same which implies NC contribution are flavour independent. In an electrically neutral medium, the number density of protons and electrons are equal. So their contribution towards V_{NC} cancels each other. The only neutrino neutrino scattering contribution is given by

$$(V_e)_{NC} = (V_\mu)_{NC} = (V_\tau)_{NC}) = -\frac{G_F N_n}{\sqrt{2}} \tag{2.25}$$

where N_n is the number density of neutrinos. The matter induced potential are given by

$$\begin{aligned} V_e &= \sqrt{2}G_F(N_e - \frac{N_n}{2}) \\ V_\mu = V_\tau &= \sqrt{2}G_F(-\frac{N_n}{2}) \end{aligned} \tag{2.26}$$

where G_F is the Fermi constant

Taking two flavour system, we are able to find the time evolution of the neutrinos in matter. The time evolution equation of two mass eigenstates (corresponding to the flavour eigenstates ν_e and ν_μ) is given by

$$i\frac{\partial}{\partial t}\begin{pmatrix} \nu_1(t) \\ \nu_2(t) \end{pmatrix} = H \begin{pmatrix} \nu_1(t) \\ \nu_2(t) \end{pmatrix} \tag{2.27}$$

with diagonal basis of H as

$$H = \begin{pmatrix} E_1 & 0 \\ 0 & E_2 \end{pmatrix} = E + \begin{pmatrix} \frac{m_1^2}{2E} & 0 \\ 0 & \frac{m_1^2}{2E} \end{pmatrix} \tag{2.28}$$

The mixing matrix U connect the flavour and mass eigenstates, the evolution of the neutrinos in flavour basis is given by

$$i\frac{\partial}{\partial t}\begin{pmatrix} \nu_e(t) \\ \nu_\mu(t) \end{pmatrix} = H' \begin{pmatrix} \nu_e(t) \\ \nu_\mu(t) \end{pmatrix} \tag{2.29}$$

where

$$\begin{pmatrix} (E+\frac{m_1^2+m_2^2}{4E}) - \frac{\Delta m^2}{4E}\cos 2\theta & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & (E+\frac{m_1^2+m_2^2}{4E}) - \frac{\Delta m^2}{4E}\cos 2\theta \end{pmatrix} \tag{2.30}$$

We have to include V_e and V_μ in the diagonal elements of H' to incorporate the matter effect and modified H is as follows

$$\begin{pmatrix} (E+\frac{m_1^2+m_2^2}{4E}) - \frac{\Delta m^2}{4E}\cos 2\theta + & \\ \sqrt{2}G_F(N_e - N_{\frac{n}{2}}) & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & (E+\frac{m_1^2+m_2^2}{4E}) - \frac{\Delta m^2}{4E}\cos 2\theta - \sqrt{2}G_F(\frac{n}{2}) \end{pmatrix} \tag{2.31}$$

After omitting the common terms in the diagonal elements of the Hamiltonian, the final evolution equation of the flavour states in presence of matter is given by

$$i\frac{\partial}{\partial t}\begin{pmatrix} \nu_e(t) \\ \nu_\mu(t) \end{pmatrix} = \begin{pmatrix} -\frac{\Delta m^2}{4E}\cos 2\theta + \sqrt{2}G_F N_e & \frac{\Delta m^2}{4E}\sin 2\theta \\ \frac{\Delta m^2}{4E}\sin 2\theta & \frac{\Delta m^2}{4E}\cos 2\theta \end{pmatrix} \begin{pmatrix} \nu_e(t) \\ \nu_\mu(t) \end{pmatrix} \tag{2.32}$$

The above equation describes $\nu_e \leftrightarrow \nu_\mu$ in matter and it is easy to realize that evolution equation for $\nu_e \leftrightarrow \nu_\tau$ oscillation has exactly the same form. Since $V_\mu = V_\tau$, hence two flavour oscillation $\mu_e \leftrightarrow \nu_\tau$ is not modified in presence of matter.

After diagonalization of effective Hamiltonian of eq.(2.32), we obtain modified mass eigenstates are given by

$$\nu_A = \nu_e \cos\theta_m + \nu_\mu \sin\theta_m$$

$$\nu_B = -\nu_e \sin\theta_m + \nu_\mu \cos\theta_m \tag{2.33}$$

where mixing angle θ_m is given by

$$\tan 2\theta_m = \frac{\frac{\Delta m^2}{2E}\sin 2\theta}{\frac{\Delta m^2}{2E}\cos 2\theta - \sqrt{2}G_F N_e} \tag{2.34}$$

and difference in energy eigenvalues is

$$E_A - E_B = \sqrt{(\frac{\Delta m^2}{2E}\cos 2\theta - \sqrt{2}G_F N_e)^2 + (\frac{\Delta m^2}{2E})^2 \sin^2 2\theta} \tag{2.35}$$

Thus the $\nu_e \leftrightarrow \nu_\mu$ oscillation probability in matter is written as

$$P(\nu_e \to \nu_\mu) = \sin^2 2\theta_m \sin^2(\pi \frac{L}{l_m}) \tag{2.36}$$

Now we examine oscillation amplitude

$$\sin^2 2\theta_m = \frac{(\frac{\Delta m^2}{2E})^2 \sin^2 2\theta}{(\frac{\Delta m^2}{2E}\cos 2\theta - \sqrt{2}G_F N_e)^2 + (\frac{\Delta m^2}{2E})^2 \sin^2 2\theta} \tag{2.37}$$

which attain maximum value of unity when the resonance condition

$$\sqrt{2}G_F N_e = \frac{\Delta m^2}{2E}\cos 2\theta \tag{2.38}$$

is satisfied. When resonance condition is fulfilled, the condition demand that the R.H.S of eq.(2.38) must be positive.

$$\Delta m^2 \cos 2\theta = (m_2^2 - m_1^2)(\cos^2\theta - \sin^2\theta) > 0 \tag{2.39}$$

C, CP and CPT Symmetries in the context of Neutrino Oscillation)

When a charge conjugation operator act on a left handed neutrino field, it will transform into a left handed antineutrino, which even does not exist. But the combined operation of charge conjugation(C) and parity, (i.e CP operator) transforms a left handed neutrino ν_L to right handed anti-neutrino (which is the actual antiparticle of ν_L). Thus CP acts as a particle-antiparticle conjugate operator. If CP is conserved, then we have

$$P(\nu_\alpha \to \nu_\beta; t) = P(\overline{\nu}_\alpha \to \overline{\nu}_\beta; t) \tag{2.40}$$

When act on lepton mixing matrix U it becomes U^*. If the U matrix is real, CP is conserved in the leptonic sector. There are $\frac{n(n-1)}{2}$ angle parameters and $\frac{n(n+1)}{2}$ phase parameters in a $n \times n$ unitary matrix. If the neutrinos are assumed to be Dirac particle of which $(2n-1)$ phase can be taken out by redefining phases of the left handed fields,then in Dirac case, U matrix consists of $\frac{n(n+1)}{2}$ and $\frac{n(n-1)(n-2)}{2}$ physical phases which suggests that CP violation is not possible unless $n \geq 3$. On the other hand, if neutrinos are Majorana particles, only n phases can be removed from the mixing matrix and we are left with $\frac{n(n-1)}{2}$ physical phases, of which $\frac{n(n-1)(n-2)}{2}$ are usual Dirac phases and the remaining $(n-1)$ are Majorana type phases. Neutrino flavour oscillation[53-56] is not affected by Majorana phases. The phase redefinition freedom is less in Majorana case, since the mass term is of the type $\overline{\nu}_L(\nu_L)^c$ rather than $\overline{\nu}_L\nu_R$. The Majorana phases will appear in the neutrino-antineutrino oscillation phenomena[57] which is at present too far to realize through experiment.

The CPT transformation is realized as: transformation $(particle \Longleftrightarrow antiparticle) + Timereversal(initialstate \Longleftrightarrow finalstate)$. Therefore under CPT: $P(\nu_\alpha \to \nu_\beta;t)$ goes into $P(\overline{\nu}_\beta \to \overline{\nu}_\alpha;t)$.Under CPT: $U \to U^*$ and $t \to -t$ and thus from eq.(2.6) it can be seen easily that the amplitude of oscillation becomes complex conjugate when acted upon by CPT. Therefore the oscillation probability being the modulus squared of amplitude remains invariant under CPT.

$$CPT : P(\nu_\alpha \to \nu_\beta;t) = P(\overline{\nu}_\beta \to \overline{\nu}_\alpha;t) \tag{2.41}$$

Now we study the effect of CP violation in $\nu(\overline{\nu} \to \nu(\overline{\nu}$ oscillation. If CP is not conserved then we have $P(\nu_\alpha \to \nu_\beta;t) \neq P(\overline{\nu}_\alpha \to \overline{\nu}_\beta;t)$ and this demands presence of atleast one removable phase in the mixing matrix (U). In standard PMNS parametrization the three generation lepton mixing matrix U contains one unremovable phase δ which is responsible for CP violation. The CP asymmetry is defined through the expression as

$$\Delta P_{\alpha\beta} \equiv P(\nu_\alpha \to \nu_\beta;t) - P(\overline{\nu}_\alpha \to \overline{\nu}_\beta;t) \tag{2.42}$$

The CPT invarience gives $\Delta P_{\alpha\beta} = -\Delta P_{\alpha\beta}$. Following eq.(2.14) it can be shown that

$$\begin{aligned}\Delta P_{e\mu} = \Delta P_{\mu\tau} = \Delta P_{\tau e} = 4 s_{12} c_{12} s_{13} c_{13}^2 s_{23} c_{23} \sin\delta \\ \times [\sin(\frac{\Delta m_{12}^2}{2E} t) + \sin(\frac{\Delta m_{23}^2}{2E} t) + (\frac{\Delta m_{31}^2}{2E} t)]\end{aligned} \tag{1.43}$$

In the context of CP violation, it is worthwhile to mention few words about rephasing invariants [58-62]. The symmetric neutrino mass matrix is diagonalized by a unitary matrix as

$$U^T_{i\alpha} (M_\nu)_{\alpha\beta} U_{\beta j} = K_i^2 (M_\nu)_i \delta_{ij} \tag{2.44}$$

Here $(M_\nu)_i$ is the real diagonal matrix and K is the diagonal phase matrix and the unitary matrix U relates the flavour and mass eigenstate of neutrinos. Any phase redefinition of the physical neutrino field ($\nu_i \to e^{-i\theta_i}\nu_i$) should not have any observable effect in charged current interaction and mass matrix which in turn demands that

$$\{U_{\alpha i}, K_i\} \to e^{-i\theta_i}\{U_{\alpha i}, K_i\} \tag{2.45}$$

Since physical processes are independent of phase redefinition of the neutrino fields, U and K must enter in any physical observable in a rephasing invariant combination like

$$s_{\alpha ij} = Im\{U_{\alpha i}U^*_{\alpha j}K^*_i K_j\}$$

$$t_{\alpha i\beta j} = Im\{U_{\alpha i}U_{\beta j}U^*_{\alpha j}U^*_{\beta i}\} \tag{2.46}$$

$s_{\alpha ij}$ and $t_{\alpha i\beta j}$ are correlated through

$$t_{\alpha i\beta j} = s_{\alpha ij}s_{\beta ji} \tag{2.47}$$

A minimal set of rephasing invariant is

$$J_{CP} = \{t_{\alpha i13}; J_1 = s_{113}; J_2 = s_{123}\} \tag{2.48}$$

This J_{CP} is equivalent to the Jarlskog invariant which is defined in the case of quark mixing. The other two CP violating measures do not appear in lepton number conserving processes. The quantities $\Delta P_{\alpha\beta}$ are free of J_1 and J_2 and proportional to J_{CP} only, because there is no question of lepton number violation in the neutrino flavour oscillations. In the standard CKM parametrization (eq.(2.14))

$$J_{CP} = -s_{12}c_{12}s_{13}c_{13}^2 s_{23}c_{23}\sin\delta \tag{2.49}$$

Hence, eq(1.66) reduces to

$$\Delta P_{e\mu} = \Delta P_{\mu\tau} = \Delta P_{\tau e} = -4J_{CP}[\sin(\frac{\Delta m_{12}^2}{2E}t) + \sin(\frac{\Delta m_{232}^2}{2E}t) + \sin(\frac{\Delta m_{31}^2}{2E}t)] \tag{2.50}$$

which indicates that, for vanishing J_{CP} there will be no CP violation in neutrino oscillation.

2.2.2 Neutrino Experiment

Solar neutrino experiments

The Homestake experiment [63] of Raymond Davis and John N. Bachcall was the first pioneering experiment to detect solar neutrinos. Its purpose was to collect and count neutrinos emitted by the sun. The experiment which started in 1965, after running for few years, produced a result of average capture rate of solar neutrinos as 2.56 ± 0.25 Solar Neutrino Unit (SNU) ($1SNU = 10^{-36}$ capture per target atom per second). But the capture rate predicted by Standard Solar Model (SSM)[64-67] is 8.1 ± 1.25 SNU. Hence there is a deficit in solar neutrinos measured by the experiment, which is about three times less as predicted by SSM. This discrepancy is known as the Solar Neutrino Problem. To find a solution of the above problem, several experiment have been done with solar neutrinos.

Super Kamiokande : The Kamiokande experiment is the second solar experiment started in 1987. It confirmed the long standing solar neutrino problem. In Super Kamiokande [68], detector was a large water Cherenkov detector. The mode of solar neutrino detection in Super Kamiokande is the elastic scattering channel $\nu_e + e^- \rightarrow \nu_e + e^-$ which has threshold energies of 5 Mev. Kamiokande is very unique among the other solar neutrino experiments as it is able to measure the direction of incident neutrinos and can provide recoil energy information. The capture rate obtained from Super Kamiokande was about 0.45 ± 0.02 SNU, whereas the SSM prediction was 1.0 ± 0.2 SNU which shows that the experientially measured rate is about half of the value as predicted by the model.

SAGE and GALLEX : Soviet American Gallium Experiment (SAGE) [69] is a collaborative experiment. In the experiment 50-57 tonnes of liquid Gallium is kept deep underground at the Bakson Neutrino Observatory in Russia. SAGE observed a capture rate of 70.9 ± 5.0 SNU compared to 129 ± 9 SNU predicted by different SSM. Gallium Experiment or GALLEX [70] is also a solar neutrino experiment based on radio chemical method and the detector is liquid Gallium. GALLEX observed a rate of 77.5 ± 8 SNU, which were lower than the prediction and this time by about 40%. It shows that deficit is energy dependent.

Solar neutrinos were detected through charged current interactions $(\nu_e + X \rightarrow e^- + Y)$ in the detectors of all the solar neutrino experiments .In radiochemical experiments to generate unstable ion, charged current interaction were used; and in water Cherenkov experiment the final state electron is needed as a tag that ν_e had interacted in the detector. The charged muon mass is 105 MeV, whereas the solar neutrino energies are less than about 30 MeV. So this is a drawback in the method of detection of solar neutrinos. To create the charged leptons, there must be sufficient energy to interact via charged current. It can be concluded from all the solar neutrino experiments that electron neutrinos might be changing to muon or tau neutrinos. Due to lack of sensitivity to detect muon and tau neutrinos, all the experiments would not be able to see the ν_μ or ν_τ part of the flux. So the detectors should be sensitive to all the three flavours of neutrinos and this was done by the SNO detector.

SNO : Sudbury Neutrino Observatory (SNO)[71] was designed to detect solar neutrinos through their interactions with heavy water (D_2O). The detector tank was a acrylic vessel which contained 1000 tonnes of heavy water. The tank was connected with approximately

9,600 photomultiplier tubes. The deuteron is a fragile nucleus and it takes only 2MeV to break it apart into a proton and a neutron. Solar neutrinos have energies up to 30 MeV and so any of the neutrino ν_e, ν_μ, ν_τ can break apart a deuteron in a neutral current interaction. All of the solar neutrino detectors prior to SNO are primarily sensitive to electron neutrinos, while SNO was sensitive to all the three flavours of neutrinos. The detecting technique was based on three different interactions

Charged Current (CC) :

$$\nu_e + d \to p + p + e^- \tag{2.51}$$

In this interaction a neutrino converts the neutron of a deuteron into a proton and an electron which is detectable. The emitted electron carries off most of the neutrinos energy. The interaction is sensitive to electron neutrinos.

Neutral current(NC):

$$\nu + d \to n + p + \nu \tag{2.52}$$

This interaction is sensitive to all the three neutrino flavours.

Elastic scattering : A neutrino collides with an atomic electron and scattered off elastically

$$\nu + e^- \to \nu + e^- \tag{2.53}$$

This is predominantly sensitive to electron neutrinos, but has also sensitivity to other two flavours of neutrinos.

The neutrino fluxes measured in units of $10^{-8} cm^2 s^{-1}$ in three different kind of interactions are given by

$$\phi_{CC} = \phi(\nu_e) = 1.76 \pm 0.01$$

$$\phi_{ES} = \phi(\nu_e) + 0.15(\phi(\nu) + \phi(\nu_\tau)) = 2.39 \pm 0.26$$

$$\phi_{NC} = \phi(\nu_e) + \phi(\nu_\mu) + \phi(\nu_\tau) = 5.09 \pm 0.63 \tag{2.54}$$

The total flux of muon and tau neutrinos from the sun is two times larger than flux of ν_e . Solar neutrinos are created as electron neutrinos,therefore it can be concluded that those ν_e must have oscillated to other flavours. Further the SSM predict a total flux of neutrinos with energies greater than 2 MeV of

$$\phi_{SSM} = (5.05 + 0.01) \times 10^{-8} cm^{-2} s^{-1} \tag{2.55}$$

Which is a very good agreement with the NC flux measured by SNO. thus the solar neutrino problen is solved satisfactorily by the SNO experiment.

Atmospheric neutrino experiment

Atmospheric neutrinos are produced in the collision of primary cosmic rays (typically protons) with different types of air nuclei in the upper atmosphere which creates a shower of hadrons, mainly pions. In the decay process these hadrons creates neutrinos. The decay chain is

$$\pi^+ \to \mu^+ + \nu_\mu, \quad \mu^+ \to e^+ + \nu_e + \overline{\nu_\mu}$$

$$\pi^- \to \mu^- + \overline{\nu_\mu}, \quad \mu^- \to e^- + \overline{\nu_e} + \nu_\mu \tag{2.56}$$

Neutrinos can be obtained from kaon at high energies. The spectrum of these neutrinos peaks at 1 Gev and extends up to 100 GeV. At moderate energies the ratio $R = \frac{(\nu_\mu + \overline{\nu_\mu}}{(\nu_e + \overline{\nu}_e)}$ should be ~ 2. The disappearance of μ neutrinos was indicated from Kamiokande and Super -Kamiokande water Cherenkov detectors. This was first seen in the ratio of the $\frac{\nu_\mu}{\nu_e}$ fluxes, and later confirmed by the zenith angle distribution of the ν_μ events. This result was later confirmed by other experiments like MACRO [72] and Soudan [73] and the long base line KEK experiment. These experiments have established the fact that ν_μ indeed oscillate into ν_τ with nearly maximal mixing and the difference between the squared mass eigenvalues $|\Delta m^2_{atm}| \sim 2 \times 10^{-3}$eV.

Reactor neutrino experiments

The major source of human generated neutrinos are nuclear reactors. Anti-neutrinos are produced in the beta decay of neutron rich daughter fragments in the fission process. The emitted neutrinos have energy of the order of few MeV.

Reactor neutrino experiments are

(a) CHOOZ Experiment in France (1990)

(b) Palo-Verde which supports CHOOZ results

(d) KARMEN Experiment which contradicts LSND results

(e) MiniBooN experiment, which tries to resolve conflict between LSND and KARMEN results

CHOOZ Experiment : In the long baseline reactor experiments, CHOOZ and Palo verde, the $\overline{\nu}_e$ disappearance due to neutrino oscillations in the atmospheric range of Δm^2 was searched. In these experiments electrons antineutrinos were detected via the observation the process.

$$p + \overline{\nu}_e \longrightarrow e^+ + n$$

No indication of the disappearance of reactor $\overline{\nu}_e$ was found . The ratio R of the measured and expected numbers of $\overline{\nu}_e$ events in the CHOOZ and the Palo Verde experiments are respectively-

$$R = 1.01 \pm \frac{2.8}{100} \pm \frac{2.7}{100} R = 1.01 \pm \frac{2.8}{100} \pm \frac{2.7}{100}$$

and

$$\Delta m^2_{CHOOZ} = 2.5 \times 10^{-3} eV^2$$

LSND Experiments : The Liquid Scintillator Neutrino Detector gives the evidence for neutrino oscillation of $\overline{\nu}_e$ appearance in a $\overline{\nu}_\mu$ beam. In LSND experiment at Los Alamos, the neutrino source was the beam of an intense 800 MeV proton beam where a large number of charged pions were created an stopped. Since π^- capture on nuclei with very high probability, essentially the π^+ decay, producing ν_μ and then $\overline{\nu}_\mu$ and $\nu_e(\mu^+)$. These neutrinos have very well defined energy spectra (from decay of particle at rest) and there are no $\overline{\nu}_e$ produced in this process. The 160 ton detector is then used to search for $\overline{\nu}_e$ events vis inverse beta decay on protons at a distance of 30 m from the neutrino source.. The experiment gives the probability of $\overline{\nu}_\mu$ oscillation $\overline{\nu}_e$ as

$$P_{\overline{\nu}_\mu \to \overline{\nu}_e} = (0.34 \pm 0.020 \pm 0.07) \times 10^{-2} \tag{2.57}$$

The dominant process in this experiments are

$$\pi^- \longrightarrow \mu^+ \nu_\mu ; \mu^+ \longrightarrow e^+ + \overline{\nu}_\mu + \nu_e$$

with e^+ energy between 36 and 60 Mev, followed by a γ-ray from $np \rightarrow d\gamma$(2.2 Mev). LSND ovserved $9e^+$ events with energy within $36 \leq E_\nu \leq 60 MeV$. The back-ground events is estimated to be $2.1 + 0.03$ events. Thus, there is an excess of the number of observed events. If the excess obtained is due to $\overline{\nu}_\mu \rightarrow \overline{\nu}_e$, then it corresponds to an oscillatiom probability given by eq.(2.57). The 90 percent confidence limit obtained from the analysis of data are

$$\Delta m^2 \leq 0.07 eV^2 for \sin^2 2\theta \cong 1$$

$$\Delta m^2 \geq 20 eV^2 for \sin^2 2\theta \leq 6 \times 10^{-3}$$

$$\Delta m^2 \cong (0.2 - 10) eV^2$$

KARMEN Experiments : In the KARMEN experiments, ν_e are detected by the exclusive charged current reaction.

$$\nu_\mu \longrightarrow \nu_e + C^{12} \longrightarrow N^{12} + e^-$$

,

$$N^{12} \longrightarrow C^{12} + e^+ \nu_e$$

This experiments provides a flux independent measurement of the oscillation probability $P(\nu_\mu \rightarrow \nu_e)$ for oscillation of monoenergetic ν_μ to ν_e of the same energy $(E_\nu = 29.8 MeV)$

The KARMAN group also, like the LSND group,have performed two kinds of appearance experiments $\nu_\mu \rightarrow \nu_e$ and $\overline{\nu}_\mu \rightarrow \overline{\nu}_e$. The second one is more sensitive, since the source has no $\overline{\nu}_e$. They do not find any indication of oscillation in either experiments. They find only upper limits of conversion probabilities.

Besides, the above cited experiments, recently, θ_{13} has been obtained by reactor experiments Daya Bay in China, RENO in South Korea and Double Chooz in France.

Accelerator neutrino experiments

Some particle accelerator have been used to make neutrino beams. The technique is is to smash proton into fixed target, producing charged pions or kaons.

There are a number of experiments like K2K, MINOS, T2K, NoνA, OPERA and ICARUS. We discuss few of them

K2K : K2K means KEK to Kamioka. K2K is first long baseline accelerator neutrino experiment where the ν_μ beam is produced from a proton synchrotron located at KEK and ν_μ s are detected at Kamioka observatory. Baseline L = 250 Km and average neutrino energy E = 1.3GeV. It seek for $\nu_\mu \longrightarrow \nu_\tau$ oscillation.

MINOS : MINOS (or Main injector Neutrino oscillator Search). It is a long base line experiments, to study the phenomenon of neutrino oscillation. MINOS experiment sheds light on the mystery of muon neutrino disappearance.

T2K : T2K means Tokai to kamika. It is a "Next generation" long baseline experiment where the ν_μ beam is produced at Japan Proton Accelerator Research Complex (J-PARC in Tokai and ν_μs are detected by the existing Superkamiokande (SK) water Cerenkov detector at a distance L = 295km. The main goal of T2K is to discover the transformation of muon neutrinos into electron neutrinos.

OPERA and ICARUS : These are long baseline accelerator neutrino experiments designed to study $\nu_\mu \rightarrow \nu_\tau$ oscillations.

2.2.3 Present status of oscillation experiment

Many neutrino oscillation experiments are conducted across the globe.

We focus on few experiment

Deep Under-ground Neutrino Experiment (DUNE) [74]: DUNE is a long base line neutrino experiment. DUNE is expected to be capable of measuring δ up to a precision of 10^0 to 20^0 and observing CP violation at 3σ for 67% of δ. India is a member of the DUNE collaboration. T2K in Japan [75] and Fermilab NOνA [76] are two important ongoing neutrino oscillation experiments of this kind.

In 2012, Daya Bay [77] a multinational China based short-baseline experiment, and the RENO[78](Reactor Experiment for Neutrino oscillations),located in South Korea, collaborations reported one of the most significant discoveries of contemporary particle physics experiments,namely the non-zero θ_{13} upto 5.2 and 4.9 standard deviation respectively.

3

Baryogenesis via leptogenesis

3.1 INTRODUCTION

In this section we present a brief review on baryogenesis through leptogenesis. Leptogenesis is a scenario where a Baryon Asymmetry of the Universe (BAU) is generated from decay of heavy majorana neutrinos. Leptogenesis is appealing because of its simplicity and connection to neutrino physics. The formalism of thermal leptogenesis will be discussed in this chapter. Some of the recent developements on flavoured leptogenesis are also discussed in the following sections. Finally in the last section we briefly mention inflation and non thermal leptogenesis.

3.2 Matter-antimater asymmetry

The puzzle of the pre-dominance of matter over antimatter in the universe and all the solutions provided till date, are rather speculative. The Universe should contain the same amount of baryons and antibaryons according to the prediction of standard cosmological theory. So assuming that both evolved in an identical way (as the hot Big Bang theory suggests), there would be no apparent reason for baryons to exist in such a large amount whereas antibaryons are so rare in the universe today A plausible explanation to this asymmetry within the framework of the standard cosmological model is known as baryogenesis.

3.2.1 Matter antimatter asymmetry through observational evidences

The studies of anti-proton flux in cosmic ray experiments conducted on earth is one the foremost observation of the matter dominance within our own galaxy the Milky way. It has been observed that the ratio of antiprotons to protons is about $\overline{p}/p \sim 10^{-4}$ [79,80,81]. From these results, it can be inferred that these antiprotons are only byproducts produced in cosmic ray collisions with the inter-stellar media, and cannot be originated from a non-negligible antiproton density in the galaxy.

The presence of intra-cluster hydrogen gas clouds(as indicated by their x-ray emissions) within the galaxy clusters implies that the patches containing large amount of antibaryons cannot exist there, as the existence of such patches would give rise to strong γ-ray emission from baryon-antibaryon annihilations near the interfaces with the gas clouds. But no such γ ray flux has been seen. The possibility that our universe contains the same amount of matter and antimatter today is excluded, because there may exist large patches of space with antimatter as indicated in literature,which is beyond the observation [82].

Recent measurements of the temperature anisotropy of the Cosmic Microwave Background (CMB) radiation by the WMAP probe [83] together with studies of large scale structure [84], have given us a reliable estimate of the baryon-to-photon ratio at the current epoch of

$$\eta_B^{CMB} \equiv \frac{\eta_B}{\eta_\gamma} = (6.1 \pm 0.2) \times 10^{-10} \tag{3.1}$$

We see that the above Eqn (3.1) is very much consistent with the standard Big Band Nucleosynthesis (BBN) analysis of primordial abundances of He^3, He^4, Deuterium, Li^6 and Li^7 which depends crucially on the value of the baryon-to-photon ratio. In fact, astrophysical observations have inferred that [85,86]

$$\eta_B^{BBN} \approx (4.7 - 6.5) \times 10^{-10} \tag{3.2}$$

Thus, inspite of their different origin both the standard cosmology, Eqn.(3.1) and BBN analysis analysis Eqn(3.2) are in good agreement with one another and hence their validity is proved. From observation, the ratio of η_B is very significant; in the sense that at the Big Bang epoch (radiation dominated) if the universe was baryon-antibaryon symmetric at $T \sim 100MeV$, the annihilation process $B + \overline{B} \to 2\gamma$ would significantly reduce both the value of $\frac{\eta_B}{\eta_\gamma}$ and $\frac{\eta_{\overline{B}}}{\eta_\gamma}$ before they subsequently froze out at $T \sim 22MeV$ when the annihilations became ineffective. By studying the Boltzmann evolution of the number density of the (anti) baryons in this scenario, one can estimate the expected baryon-to-photon ratio for today to be [87]

$$\frac{\eta_B}{\eta_\gamma} = \frac{\eta_{\overline{B}}}{\eta_\gamma} \simeq 10^{-18} \tag{3.3}$$

but the observed ratio η_B is of the order 10^{-10} in eq.(3.1). This discrepancy suggests that there must have been a primordial baryon asymmetry in the early universe and the present day dominance of matter over antimatter is just a manifestation of this fact. As $\eta_B \gg \eta_{\overline{B}}$, so the observed value of the baryon-to-photon ratio also characterizes the amount of asymmetry between matter and antimatter in the Universe.

$$\eta_B \simeq \frac{\eta_B - \eta_{\overline{B}}}{\eta_\gamma} \tag{3.4}$$

In understanding the evolution of the universe, the prime question that needs to be answered is the physical origin of primordial baryon asymmetry. At the Big Bang if there existed any asymmetry, then these pre-existing asymmetry would be washed out during reheating. So, excluding this possibility, it is expected that this excess must have been dynamically generated in the course of evolution of the universe . The study of generating this baryon-to-photon ratio from an initial asymmetry is called baryogenesis and there are different scenarios that can lead to this asymmetry and leptogenesis is one such mechanism.

3.3 Basic ingredients for baryon asymmetry generation

The three ingredients required to dynamically generate a baryon asymmetry were given by Sakharov [88]. They are

(i). Baryon number violation

(ii). C and CP violation and

(iii). Out of thermal equilibrium.

The first condition requires that the interactions must be B number violating which will either increase or decrease the number of baryons. Since for every baryon number violating process $X \to qq$, there would be a mirror process for the corresponding anti-baryon $\overline{X} \to \overline{qq}$, and no net baryon asymmetry may result if both the processes are equally likely. Hence, the second condition also becomes necessary so that $\Gamma(X \to qq) \neq \Gamma(\overline{X} \to \overline{qq})$. The third condition requires these processes to be out-of-thermal equilibrium. This is obvious from the fact that as baryon number is CPT-odd (i.e. $\Theta^{-1}B\Theta = -B$), it can be easily demonstrated that no baryon asymmetry can be generated in thermal equilibrium. The Hamiltonian of the system under consideration is usually assumed to be CPT invariant (i.e. $\Theta^{-1}\mathscr{H}\Theta = \mathscr{H}$), where Θ is the CPT operator and $\mathscr{H}$ denotes Hamiltonian). For a system in thermal equilibrium, we can define the thermal average B(t) at temperature T by

$$\langle B(t) \rangle_T = Tr[B(t)e^{-\mathscr{H}/T}] \tag{3.5}$$

where B(t) is the baryon number at time t and $e^{-\mathscr{H}/T}$ is the density operator. Given that the time evolution of baryon number is $B(t) = e^{-i\mathscr{H}t}B(0)e^{i\mathscr{H}t}$, then it can be shown that a system in thermal equilibrium must be stationary i.e $\langle B(t) \rangle_T = 0$

So, to take place successful baryogenisis, there must be departure from thermal equilibrium.

In the Standard Model (SM), all the three ingredients of Sakharov's conditions are present but no SM mechanism generating enough baryon has been found. Baryon number is violated in the Standard Model, and the resulting baryon number violating process are fast in the early universe [89]. The violation is due to the triangle anomaly, and leads to process that involve nine left-handed quarks (three from each generation). At zero temperature, the amplitude of the baryon number violating process is proportional to $e^{-8\pi^2/g^2}$ [90], which is too small to have any observable effect. At high temperatures, however, these transitions become unsuppressed [88]. Again the electroweak interactions in the SM also violate C and CP but is of the order of 10^{-20}; which is very small to generate the required baryon asymmetry. In SM, interactions at the electroweak phase transition go out of thermal equilibrium. The experimental lower bound on Higgs Mass implies that this transition is not strongly first order, as required for successful baryogenesis. This shows that baryogenesis requires new physics that extends beyond the SM. Some of the baryogenesis scenarios [91] are the following

(a) GUT baryogenesis:For the development of realistic model of baryogenesis[92], Grand Universe Theories(GUT) are of particular importance. These theories provide natural heavy particle candidate whose decays can be source of baryon asymmetry. The B- number violation is an unavoidable consequence in grand unified models as quarks and leptons are unified in the same representation of a single group. Furthermore, CP violation can be incorporated naturally in GUT models, as there exist many possible complex phases , in addition to those that are present in the SM. The relevant time scales of the decays of heavy gauge bosons or scalars are slow , compared to the expansion rate of the universe at early epoch of the cosmic evolution. The decays of these heavy particles thus inherently out-of-equilibrium . But the GUT baryogenesis scenario has difficulties with the non-observation of proton decay, which puts a lower bound on the mass of the decaying boson, and therefore on the reheat temperature after inflation. Simple inflation models do not give such a high reheat temperature, which in addition might regenerate unwanted relics.

(b) Electroweak Baryogenesis: The strong first order phase transition provides the departure from thermal equilibrium in electroweak baryogenesis [93,94,95]. The nice feature of this mechanism is that it can be probed in collider experiments . On the other hand, the allowed parameter space is very small. It requires more CP violation than what is provided in the SM. Even though there are additional source of CP violation than what is provided in SM. Even though there additional sources of CP violation in MSSM, the requirement of strong first order phase transition translates into a stringent bound on the Higgs mass, $m_H \leq 120$ GeV. To obtain Higss mass of this order , the stop mass needs to be smaller than, or of the order of, the top quark mass which implies fine tuning in the model parameter.

(c) Affleck-Dine baryogenesis [96,97] implemented in SUSY theories. This mechanism faces same challenges as in GUT baryogenesis and in EW baryogenesis.

(d) Leptogenesis: Fukugita and Yanagita [98] put forwarded this mechanism, where decay of the lightest heavy Majorana neutrino produces a CP violating out-of-decay. We focus on the motivated realisation of leptogenesis: both thermal leptogenesis with and without flavour as well as " non-standard scenerios" like non-thermal leptogenesis.

3.3.1 Sphaleron effect

: A brief overview of the non-perturbative baryon number violation interactions known as Sphaleron processes is presented in this section. The Baryon number and lepton number are accidental symmetries in the SM. Thus, it is not possible to violate symmetries at the tree level. But baryon number violation occurs naturally in Grand Unified Theories (GUT), because quark and leptons are unified in the same irreducible represenions. It is thus possible to have gauge bosons and scalars mediating interactions among fermions having different baryon numbers, t'Hooft realized that [99] the non-perturbative instanton effects may give rise to processes that violet $(B+L)$, but conserve $(B-L)$. Due to the chiral anomaly, there are non-perturbative gauge field configurations [90,99,100] which can act as sources for $B+L_e+L_\mu+L_\tau(B+L_e+L_\mu+L_\tau)$ is conserved). In the early universe, at temperatures above the electroweak phase transition (EWPT), such configurations occur frequently [89,101,101], and lead to rapid B+ L violation. These configurations are commonly referred to as “sphalerons” [103,104,105]. A study of the electroweak theory at the classical and quantum level shows how B and L violation occurs at the quantum level but still reconciling their conservation at low energies. Since in SM we have global $U(1)_B$ and $U(1)_L$ symmetries, Noether's theorem implies that classical J^B_μ and J^L_μ currents are conserved.

$$J^B_\mu = \frac{1}{3}\sum_i(\overline{q}_{L_i}\gamma_\mu q_{L_i} - \overline{u}^c_{L_i}\gamma_\mu u^c_{L_i} - \overline{d}^c_{L_i}\gamma_\mu d^c_{L_i}) \tag{3.6}$$

$$J_\mu^L = \sum_i (\bar{l}_{L_i}\gamma_\mu l_{L_i} - \bar{e}^c_{L_i}\gamma_\mu e^c_{L_i}) \tag{3.7}$$

where we have conveniently defined the baryon and lepton numbers for quarks and leptons as :

$B_{quark} = 1/3, B_{lepton} = 0, L_{quark} = 0, L_{lepton} = 1$ Here q_L refers to the $SU(2)_L$ doublet quarks, while u_L and d_L refers to the $SU(2))_L$ singlet quarks. Similarly, l_L refers to the $SU(2)$ charged lepton singlets. The subscript i is the generation index. Even though B and L are individually conserved at the tree level, the Adler-Bell-Jackiw (ABJ) triangular anomalies [106] nevertheless do not vanish and thus B and L are anomalies[102] at the quantum level through the interactions with the electroweak gauge fields in the triangle diagrams[107]. In other words, the divergence of the currents associated with B and L do not vanish at the quantum level and they are given by

$$\partial_\mu J_B^\mu = \partial_\mu J_L^\mu = \frac{N_f}{32\pi^2}(g^2 W^p_{\mu\nu}\widetilde{W}^{p\mu\nu} - g'^2 B_{\mu\nu}\widetilde{B}^{\mu\nu}) \tag{3.8}$$

where $W_{\mu\nu}$ and $B_{\mu\nu}$ are the $SU(2)_L$ $U(1)_Y$ field strengths ,

$$W^p_{\mu\nu} = \partial_\mu W^p_\nu - \partial_\nu W^p_\mu \tag{3.9}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \tag{3.10}$$

respectively, with corresponding gauge coupling constants being g and g' and N_f is the number of fermion generations. As $\partial^\mu(J^B_\mu - J^L_\mu) = 0, (B-L)$ is conserved. However$(B+L)$ is violated with the divergence of the current given by

$$\partial^\mu(J^B_\mu + J^L_\mu) = 2N_F\partial_\mu K^\mu \tag{3.11}$$

where

$$K^{\mu} = -\frac{g^2}{32\pi^2} 2\varepsilon^{\mu\nu\rho\sigma} W_{\nu}^{\rho} (\partial_p W_{\sigma}^{p} + \frac{g}{3}\varepsilon^{pqr} W_p^q W_{\sigma}^r) + \frac{g'^2}{32\pi^2}\varepsilon^{\mu\nu\rho\sigma} B_{\nu} B_{\rho\sigma} \tag{3.12}$$

The change in baryon number over some duration t is given in terms of the Chern-Simons number/winding number

$$N_{CS}(t) = \frac{g^3}{96\pi^2}\int d^3x \varepsilon^{ijk}\varepsilon_{abc} W_i^a W_j^b W_k^c(t) \tag{3.13}$$

$$\begin{aligned} B(t) - B(0) &= \int_0^t \int d^3x \partial^{\mu} J_{\mu}^B \\ &= N_f[N_{CS}(t) - N_{CS}(0)] \end{aligned} \tag{3.14}$$

An analogous equation exist for lepton number

This theory has an infinite number of quasi-degerate vacua, in each of which the W_i^a are pure gauge and consequently N_{CS} becomes an integer. Eqn.(3.14) tells us that vacuum-to-vacuum transitions involve $\Delta B = \Delta L = N_f \Delta N_{CS}$, so the minimum change for such a transition in the SM, where $N_f = 3$, is $\Delta B = \Delta L = 3$,

$$O_{B+L} = \prod_i (q_{L_i} q_{L_i} q_{L_i} l_i), \tag{3.15}$$

where we have included the $(B+L)$ subscript to draw attention to another important consequence of Eqn. (3.14) : since the same equation describes the evolution of both baryon and lepton number, the combination $(B-L)$ is conserved in any process of the form Eqn.(3.14) while $(B+L)$ is violated by at least 6 units. These effective interactions serve as the primary means of conversion between B and L in leptogenesis. At zero temperature, gauge field configurations that give non-zero $\int d^4x\widetilde{W}_{\mu\nu}W^{\mu\nu}$ correspond to tunnelling configurations, and are called instantons [108]. They change fermion number by an integer N , so the istantons [70]. They change fermions number by an integer N, so the instanton action is large

$$\left|\frac{1}{4g^2}\int d^4xW^{\mathscr{A}}_{\mu\nu}W^{\mu\nu\mathscr{A}}\right| \geq \left|\frac{1}{4g^2}\int d^4xW^{\mathscr{A}}_{\mu\nu}\widetilde{W}^{\mathscr{A}}\right| \geq \frac{64\pi^2N}{4g^2}$$

The first inequality follows from the Schwartz inequality [109]. Consequently, the associated rate is highly suppressed,

$$\Gamma\infty e^{-(instantonAction)} \sim e^{-4\pi}/\alpha_w$$

and the mediated $B+L$ violation is unobservably small. Moreover, the instantons do not threaten the stability of the proton [90], because an instanton acts as a source for three leptons (one from each generation), and nine quarks (all colours and generations). So it induces $\Delta B = \Delta L = 3$ processes. Since the three quantum numbers $B/3 - L_\alpha$ are not anomalous, so they are conserved in the SM.

For finite temperatures, transition between gauge vacua happens at a much greater rate because of (non-perturbative) thermal fluctuations over the barrier [89,103]. Depending on whether the temperature T is above or below the critical temperature, for electroweak symmetry restoration, the transitions will proceed at significantly different rates. It has been shown that for $T < T_{ew} \simeq 100GeV$, the rate is Boltzmann suppressed by e^{-Esph}/T [66], whereas for $T > T_{ew}$ the rate is proportional to T^4 (at leading order) [105]. Therefore, in the early universe where $T \gg T_{ew}$ these sphaleron process are potent, while at low temperatures such as those accessible in conventional experiments, baryon and lepton violations due to quantum corrections are physically irrelevant B and L can be regarded as conserved quantities to good approximation.

By comparing the sphaleron rate $\Gamma^T_{sph} > T_{ew}$ with the Hubble expansion rate at T

$$H \simeq 1.66\sqrt{g_s^*}\frac{T^2}{M_{pl}} \tag{3.16}$$

: where g_s^* is the number of relativistic degrees of freedom and $M_{pl} \approx 1.22 \times 10^{19}$GeV is the Planck mass, one can check that for T in the range:

$$T_{ew} \simeq 100GeV < T \ll 10^{12}GeV \tag{3.17}$$

$B+L$ violating sphaleron interactions are in thermal equilibrium. This observation is important because Sakharov's 3rd condition then implies that any baryogenesis mechanism which operates above T_{ew} can't generate an excess of B and L unless they also violate $B-L$. Since $B-L$ is conserved in the sphaleron process, therefore, any asymmetry in $B-L$ generated from interactions in the model will not be erased.

3.3.2 C and CP violation in heavy particle decays

C and CP violation is another essential ingredient for generation of baryon asymmetry. During the epoch of interest B+L violating processes are in thermal equilibrium, and hence would not generate any baryon B-asymmetry. To generate a net CP asymmetry we must invoke physics beyond the SM.

One such is to expand the particle content with exotic heavy particles and include new (B violating) interaction terms that couple them to other constituents of the model. Such heavy particles could be by product of the enlargement in the models symmetry as typical in grand unification theories (GUT's) and supersymmetry (SUSY) models . CP violation can arise in the decay of such heavy particles. Consider a toy model with a set of exotic particles X_k which can interact with other fermions q_j and scalars ξ through the Yukawa terms:

$$\mathscr{L}_{int} = h_{jk}\overline{q}_j\xi X_k + h.c \tag{3.18}$$

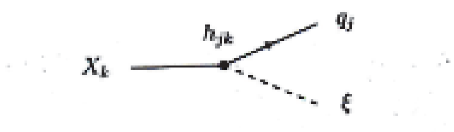

Fig. 3.1 Tree-level Feynman diagram for the heavy particle decay of $X_k \to q_j\xi$ where the arrow denotes the flow of baryon number

here indices $j,k = 1,2,......$ are labels for the different particles within a set and h_{jk} denotes the coupling which is a complex quantity in general. The tree-level Feynman diagram for the decay of X_k induced by this term is shown in fig.3.1 Suppose there are other interactions besides eqn.(3.18) and that they link X_k to a final state with a different baryon number to the state $q_j\xi$, then the decay of X_k must violate B. Let us assume that $X_k \to q_j\xi$ gives a change of $\Delta B_X = +1$, while the antiparticle decay ; $\overline{X}_k \to \overline{q}_j\overline{\xi}$ has $\Delta B_{\overline{X}} = -1$, then the CP asymmetry in baryon number produced by these decays can be quantified by

$$\varepsilon_{CP} = \frac{\Delta B_x \Gamma(X_k \to q_j\xi)}{\Gamma_{tot}} + \frac{\Delta B_{\overline{x}} \Gamma(\overline{X}_k \to \overline{q}_j\overline{\xi})}{\Gamma_{tot}}$$

$$= \frac{(+1)\Gamma(X_k \to q_j\xi) + (-1)\Gamma(\overline{X}_k \to \overline{q}_j\overline{\xi})}{\Gamma_{tot}}$$

$$= \frac{\Gamma - \overline{\Gamma}}{\Gamma_{tot}} \tag{3.19}$$

where $\Gamma_{tot} = \Gamma + \overline{\Gamma}$ is the total decay rate with $\Gamma \equiv \Gamma(X_k \to q_j\xi)$ and $\overline{\Gamma} \equiv \Gamma(\overline{X}_k \to \overline{q}_j\overline{\xi})$. As expected, Eqn.(3.19) confirms the requirements of unequal rates for particle and antiparticle decays in order to produce an asymmetry in baryon number. Therefore, we seek the general condition s under which Γ and $\overline{\Gamma}$ can be different.

Because of CPT invariance, there can never be a difference between Γ and $\overline{\Gamma}$ if one only considers the tree-level process depicted in Fig 3.1 as $\Gamma = |h_{jk}|^2 I_{tree} = |h_{jk}|^2 \overline{I}_{tree} = \overline{\Gamma}$, where the kinematics factors I_{tree} and $\overline{I}_{tree}$, which come from integrating over phase space are

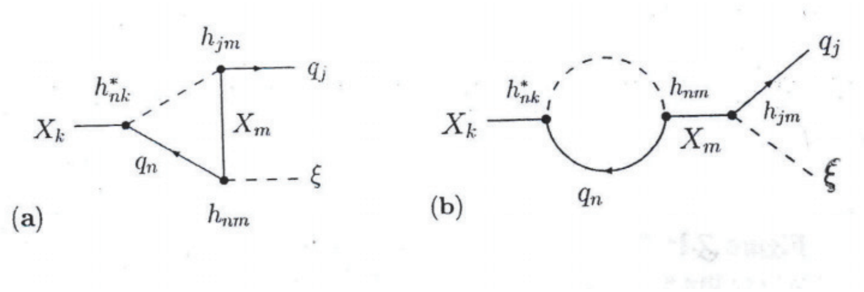

Fig. 3.2 Feynman diagram of the (a) one loop vertex correction for $X_k \to q_j\xi$ and (b) the corresponding one- loop self-energy correction.for the heavy particle decay of $X_k \to q_j\xi$ where the arrow denotes the flow of baryon number
.

necessarily equal. As a result, one must go beyond the lowest order i.e. beyond tree level. The first nonzero contribution to ε_{CP} comes from the interference between the tree-level graph and the one-corrections shown in Fig.3.2. Writing out the terms up to order of h^4 in the couplings, we have:

$$\Gamma = |h_{jk}|^2 I_{tree} + h^*_{jk} h_{jm} h_{nm} h^*_{nk} I_{loop} + h_{jk} h^*_{jm} h^*_{nm} h_{nk} I^*_{loop} + O(h^6) \tag{3.20}$$

where I_{loop} denotes the kinematic factor associated with the one-loop diagrams in Fig.3.2 that accounts for integration over the phase space of final states, as well as any internal loop momenta. Repeating this for the antiparticle decay , one obtains

$$\overline{\Gamma} = |h_{jk}|^2 I_{tree} + h_{jk} h^*_{jm} h^*_{nm} h_{nk} I_{loop} + h^*_{jk} h_{jm} h_{nm} h^*_{nk} I^*_{loop} + O(h^6) \tag{3.21}$$

where we have used the fact that $\overline{I}_{loop} \equiv I_{loop}$. So, putting Eqn.(3.20) and Eqn.(3.21) into the definition for ε_{CP} in (3.19) and ignoring the higher order terms, we have

$$\varepsilon_{CP} = \frac{1}{\Gamma_{tot}} (\mathscr{A}_h I_{loop} + \mathscr{A}^*_h I^*_{loop} - \mathscr{A}^*_h I_{loop} - \mathscr{A}_h I^*_{loop}) \tag{3.22}$$

where $\mathscr{A} \equiv h^*_{jk} h_{jm} h_{nm} h^*_{nk}$ and $\Gamma_{tot} \simeq 2|h_{jk}|^2 I_{tree}$ to the lowest order. Thus,

$$\varepsilon_{CP} = \frac{1}{\Gamma_{tot}}(\mathscr{A}_h - \mathscr{A}_h^*)(I_{loop} - I_{loop}^*$$

$$= \frac{1}{\Gamma_{tot}} 2iIm(\mathscr{A}_h) 2iIm(I_{loop})$$

$$= -\frac{1}{\Gamma_{tot}} Im(h_{jk}^* h_{jm} h_{nm} h_{nk}*) Im(I_{loop}) \tag{3.23}$$

Eqn.(3.23) highlights the three ingredients required for any model to have nonzero CP asymmetry. • Firstly, at least two heavy particles X_k must be present in the model because if $k = m$ in Eqn.(3.23) then $Im(h_{jk}^* h_{jm} h_{nm} h_{nk}^*) = Im(|h_{jk}|^2 |h_{nk}|^2) = 0$ and no asymmetry can be generated;

• Secondly, the couplings h must be complex so that the imaginary part of their products are in general non-vanishing;

• Thirdly, the $Im(I_{loop})$ term demands that the mass of X_k must be greater than the combined mass of the two intermediate state particles: q_n and ξ. This is because the imaginary part of the internal loop function corresponds to the on shell contribution of the diagram, and hence kinematical restriction implies that X_k must be heavy enough in order to give $\varepsilon_{CP} \neq 0$

Also, the dependence on $Im(I_{loop})$ indicates why self-energy loop graphs of Fig.3.2b are actually relevant and can contribute to the overall asymmetry [110]

3.4 Thermal leptogenesis: Relating baryon and lepton asymmetries

The SM sphaleron strictly conserve the $B - L$ quantum number and thus the minimal SM fails to dynamically generate the correct amount of baryon asymmetry. So one possible way to solve the baryon asymmetry problem is to look for new physics that can violate lepton number L. In the seesaw model, we see that if neutrinos are Majorana, then it will violate

L by two units. Therefore, it is natural to ask whether such lepton violating interactions can actually lead to the observed baryon asymmetry. In order to relate these baryon and lepton number asymmetries to each other in a quantitative way, we can take advantage of the set of conditions among the chemical potentials μ_i implied be equilibrium conditions among the species i present in the thermal bath. If chemical equilibrium is established between particle species $a_1, a_2, ...a_m, b_1, b_1...b_n$ by sufficiently rapid interactions of the form $a_1......a_1 \leftrightarrow a_1 a_1.....b_f$, their chemical potentials obey the relation

$$\sum_{i=1}^{m} \mu_{a_i} = \sum_{i=1}^{n} \mu_{b_i} \tag{3.24}$$

The sphaleron induced 12 fermion interactions are in equilibrium for $10^2 \leq T \leq 10^{12}$ GeV, then Eqn.(3.15) implies that

$$\sum_{i=1} (3\mu_{q_i} + \mu_{l_i}) = 0 \tag{3.25}$$

Moreover, since electroweak symmetry is restored in the epoch of the early universe, the sum of hypercharge of all particle species must vanish, and thus:

$$\sum_{i=1} (2\mu_{q_i} - \mu_{u_i} - \mu_{d_i}) = 0 \tag{3.26}$$

$$\sum_{i=1} (\mu_{q_i} + 2\mu_{u_i} - \mu_{d_i} - \mu_{d_i} - \mu_{d_i} + \frac{2}{N_f}\mu_H) = 0 \tag{3.27}$$

and from the fermions Yukawa interactions

$$\sum_{i=1} (2\mu_{q_i} - \mu_H - \mu_{d_i}) = 0 \tag{3.28}$$

$$\sum_{i=1} (2\mu_{q_i} - \mu_H - \mu_{u_i}) = 0 \tag{3.29}$$

$$\sum_{i=1}(2\mu_{L_i} - \mu_H - \mu_{e_i}) = 0 \tag{3.30}$$

As particles of different generations will also be in equilibrium with one another at high temperatures, so we can take $\mu_{q_i} = \mu_{q_L}, \mu_{u_i} = \mu_u, \mu_{d_i} = \mu_d, \mu_{l_i} = \mu_l, \mu_{e_i} = \mu_e$ and solve this system of equations for one of the μ_i (we choose μ_l). The resulting chemical potential are

$$\mu_e = \frac{2N_f+3}{6N_f+3}\mu_l, \quad \mu_d = \frac{6N_f+1}{6N_f+3}\mu_l, \mu_u = \frac{2N_f-1}{6N_f+3}\mu_l$$

$$\mu_e = \frac{1}{3}\mu_l, \quad \mu_H = \frac{4N_f}{6N_f+3}\mu_l$$

.

Since the values of B and L are related to these chemical potentials by

$$B = \frac{1}{g_{*s}T}\sum_{i=baryon} g_i\mu_i = \frac{N_f}{g_{*s}T}(2\mu_q + \mu_u + \mu_d) \tag{3.31}$$

$$L = \frac{1}{g_{*s}T}\sum_{i=baryon} g_i\mu_i = \frac{N_f}{g_{*s}T}(2\mu_l + \mu_e) \tag{3.32}$$

one finds that

$$B = \frac{8N_f+4}{22N_f+13}(B-L) \qquad L = -\frac{14N_f+9}{22N_f+13}(B-L)$$

in the Standard Model. From this, we can state the relationship between baryon and number when sphaleron interactions are in equilibrium :

$$B = \frac{28}{51}L \tag{3.33}$$

Since $B-L$ is conserved by sphaleron and violated only by the lepton-number-producing decays of N_R in standard leptogenesis, the initial value $(B-L)_{int} = -L_{int}$ generated during the leptogenesis epoch can be partially converted into a B asymmetry by sphalerons and other SM processes and the universe receives a nonzero baryon number. Thus it is fruitful to study Massive Majorana neutrino mass models which violate the lepton number violation and may simultaneously explain the baryon asymmetry problem.

3.4.1 Thermal leptogenesis

One of the main reasons to study physics beyond the Standard Model is the experimental evidence for neutrino masses. In the theoretical context, the seesaw mechanism becomes very important as it can generate small neutrino mass and can explain the baryon asymmetry of the Universe(BAU).
Fukugita and Yanagida [98] , first put forward the "standard" thermal leptogenesis scenario in the year 1986 which involves taking the type I seesaw Lagrangian Eqn.(2.37) with three heavy RH Majorana neutrinos

$$-\mathscr{L}_{type-I} = Y_\nu \bar{l}_L \phi \nu_R + \frac{M_R}{2}\overline{(\nu_R)^c}\nu_R + h.c \tag{3.34}$$

Here, during the primordial time, the Yukawa interactions between the RH neutrinos and the ordinary LH leptons violate L that can generate $(B-L)$ asymmetry. Furthermore, in thermal leptogenesis scenario the spectrum of the RH neutrino masses is assumed to be hierarchical in general, and therefore the asymmetry created will be dominated by the decays of the lightest RH neutrinos (denoted N_i) due to the efficient washout of any $N_{2,3}$ generated asymmetries by N_i mediated $\Delta L \neq 0$ scattering process in equilibrium. Rewriting the seesaw Lagrangian Eqn.(3.34) in the mass eigenbasis of the heavy RH neutrinos:

$$-\mathscr{L}_{int} = -Y_{\alpha\beta}\bar{l}_\alpha \widetilde{\phi} e_\beta - h_{jk}\bar{l}_j \phi N_k - \frac{1}{2}\overline{N}_k M_k N_k + h.c \tag{3.35}$$

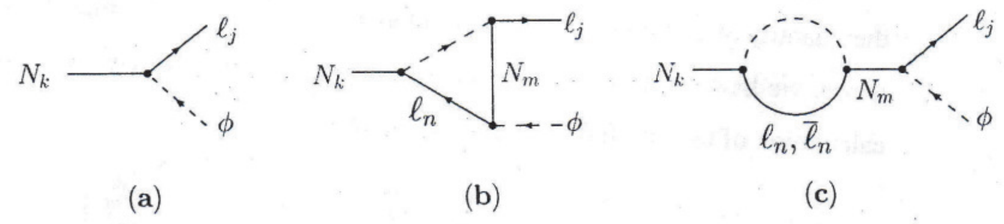

Fig. 3.3 The (a) tree-level, (b) one-loop vertex correction and (c) one-loop self energy correction graph for the decay of $N_k \rightarrow l_j\phi$

where flavour indices α, β, j can be one of e, μ, or τ and $k = 1,2,3$ are labels for the lightest to heaviest RH neutrinos (with mass M_R). The $SU(2)_L$ doublets: $l_\alpha = (\nu_L, e_L)^T_\alpha$ and $\phi = (\phi^0, \phi^-)^T$ have their usual meanings, with $\widetilde{\phi} = i\sigma_2\phi^*$ being the charge conjugate Higgs. The Yukawa couplings $h_{jk}\bar{l}_j\phi N_k$in Eqn.(3.36) can then induce heavy RH neutrino decays via two channels:

$$N_k \rightarrow l_j + \overline{\phi} \quad N_k \rightarrow \bar{l} + \phi \tag{3.36}$$

which violate lepton number by one unit. If these decays also violate CP and go out of equilbrium at some stage during the evolution of the early universe then all Sakharov's conditions for leptogenesis will be satisfied. As we have already shown in section 3.2.2 Eqn.(3.23) the requirement for CP violation means that coupling matrix h must be complex and the mass of N_k must be greater than the combined mass of l_j and ϕ, so that interferences between the tree-level process (Fig.3.3a) and the one-loop corrections (Fig3.3b,c) with on shell intermediate states will be non-zero. Both of these are possible in type I seesaw mechanism since it implies a very large M_k in order to induce small LH neutrino masses, while it does not forbid the presence of CP violating phase in the RH neutrino sector. The condition of thermal equilibrium is achieved when the expansion rate of the universe exceeds the decay rate of N_k . Therefore, as we see form section (3.2.2) Eqn.(3.20) for L, the CP asymmetry in the lepton number production due to N_k decays can be re written as :

$$\varepsilon_{kj} = \frac{\Gamma(N_k \to l_j\overline{\phi}) - \Gamma(N_k \to \overline{l_j}\phi)}{\Gamma(N_k \to l_j\overline{\phi}) + \Gamma(N_k \to \overline{l_j}\phi)} \tag{3.37}$$

In this section, we shall consider the evolution of the L asymmetry to be "flavour blinds" (flavour case we discuss in the next subsection), and we are only interested in the quantity after summing over lepton flavour j . In addition , for hierarchical RH neutrinos, we have the N_1-dominated scenario, and therefore, we can set $k = 1$. Explicit calculation of the interference terms will then result in [111]:

$$\varepsilon_i = \frac{1}{8\pi(h^\dagger h)_{11}} \sum_{m\neq 1} I_m[(h^\dagger h)_{1m})]^2[f_V \frac{M_m^2}{M_1^2} + f_S \frac{M_m^2}{M_1^2}] \tag{3.38}$$

where f_V and f_S corresponds to the vertex and self energy correction.

$$f_V(x) \equiv \sqrt{x}[-1 + (x+1)ln(1+\frac{1}{x})], \quad f_S(x) \equiv \frac{\sqrt{x}}{x-1} \tag{3.39}$$

which denote the vertex and self-energy contributions respectively. The tree-level N_1 decay rate (at T= 0) used to calculate the denominator of Eqn.(3.37) with j summed is given by:

$$\Gamma(N_1 \to l\overline{\phi}) \equiv \Gamma(N_1 \to \bar{l}\phi) = \frac{(h^\dagger h)_{11}}{16\pi} M_1 \tag{3.40}$$

For the temperature of the Universe less than the mass of the decaying lightest of the heavy RH neutrino, the out-of-equilibrium condition is reached as the inverse decay is blocked. The CP asymmetry which is caused by the interference of tree level with one-loop corrections for the decays of lightest of heavy right-handed Majorana neutrino N_1, for standard model (SM) case, is defind by[98,112,113]

$$\varepsilon_i = -\frac{3}{16\pi}[\frac{I_m[(h^\dagger h)_{12}^2]}{(h^\dagger h)_{11}} \frac{M_1}{M_2} + \frac{I_m[(h^\dagger h)_{13}^2]}{(h^\dagger h)_{11}} \frac{M_1}{M_3}] \tag{3.41}$$

where $h = \frac{m_{LR}}{v}$ is the Yukawa coupling of the Dirac neutrino mass matrix defined in the basis where the right –handed neutrino mass matrix is diagonal. Here, M_1, M_2, M_3 are the physical right-handed Majorana masses taken in hierarchical order $(M_1 < M_2 < M_3)$. However, for quasi-degenerate spectrum i.e., for $M_1 \simeq M_2 < M_3$ the asymmetry is largely enhanced by a resonance factor [112, 113, 114, 115]. In such situation, the lepton asymmetry is modified [112,113,114,115] to

$$\varepsilon_i = \frac{1}{8\pi}\frac{I_m[(h^\dagger h)^2_{12}]}{(h^\dagger h)_{11}}R \tag{3.42}$$

$$R = \frac{M_2^2(M_2^2 - M_1^2)}{M_1^2 - M_2^2)^2 + \Gamma_2^2 M_1^2} \quad \Gamma_2 = \frac{(h^\dagger h)_{22} M_2}{8\pi}$$

Again, lepton asymmetry is further converted into baryon asymmetry via a non-perturbative sphaleron process[89]. The ratio of baryon asymmetry to entropy Y_B is related to the lepton asymmetry through the relation, $Y_B = wY_{B-L} = \frac{w}{w-1}Y_L$ where $w = \frac{8N_f + 4N_H}{22N_f + 13N_H}$. The baryon asymmetry of the Universe Y_B is defined as the ratio of the baryon number density η_γ to the photon density where $s = 7.04\eta_\gamma$ This can be compared with the observational data $Y_B = (6.21 \pm 0.160) \times 10^{-10}$. In SM it can be expressed as

$$Y_B^{SM} \simeq dk_1\varepsilon_1 \tag{3.43}$$

where $d = 7.04 \times \frac{w}{g_i}{}^*(w-1)$ For SM with $N_F = 3, N_H = 1, g_i^* = 106.75$,we have $d \simeq 1.62 \times 10^{-2}$. This value [78] can be compared with lower value $d = 0.98 \times 10^{-2}$ used by other authors [117,118] In the expression for baryon-to-photon ratio, the efficiency factor (also known as dilution factor) k_i describes the washout factor of the lepton asymmetry due to various lepton number violating processes. This factor mainly depends on the effective neutrino mass $\widetilde{m}_1$defined by

$$\widetilde{m}_1 = \frac{(h^\dagger h)_{11} v^2}{M_1}$$

where v is the vev of the standard model Higgs field $v = 174GeV$.

. For $10^{-2}eV < \widetilde{m}_1 < 10^3 eV$, the washout factor k_i can be well approximated by[50,78]

$$k_1(\widetilde{m_1}) = 0.3[\frac{10^{-3}}{\widetilde{m_1}}][log\frac{\widetilde{m_1}}{10^{-3}}]^{-0.6} \tag{3.44}$$

It appears that leptogenesis should be able to produce the correct baryon-to-photon ratio of $\eta_B \simeq 10^{-10}$ quite easily. While the freedom to adjust ε_1 (partly) ensures the eventual success of baryogenesis via leptogenesis quantitatively, the solution to the problem is actually quite subtle and involves careful tracking the evolution of the N_1's abundance in the thermal plasma, as well as the $B-L$ asymmetry they generate.

In this section we have pointed some of the essential features in quantitative understanding of the classic leptogenesis scenario of [98] which has the type I seesaw setup as its backbone. Specifically, we have discussed the 'standard" thermal leptogenesis scenario where the heavy RH Majorana neutrinos are strongly hierarchical. As a result, only the lightest of the three RH Majorana neutrinos, N_i ,is expected to contribute significantly to the final asymmetry. This is because the $B-L$ violating interactions mediated by N_1 would still be in thermal equilibrium when $N_{2,3}$ decayed away, and therefore any excess $B-L$ produced by $N_{2,3}$ would be erased. When the N_i eventually decay out-of-equilibrium, an excess of $B-L$ is created through CP violating loop effects. Subsequently, this excess is converted into a B asymmetry by sphaleron.

The exact amount of B generated in this way depends crucially on the interplay between the decay and washout processes, as well as the CP asymmetry in the neutrino model under consideration. By studying the Boltzmann evolution of the particle species and then explicitly calculating the loop diagrams, both of these crucial ingredients may be conveniently encapsulated into the efficiency factor k_1 and CP asymmetry ε_1 respectively. Consequently, variations to the standard scenario can be quantified by changes in these values. As the main objective of this thesis is using leptogenesis as an essential tool for discriminating neutrino mass models we shall not go into the details of Boltzmann transport equation.

3.4.2 Flavoured leptogenesis

In this section, we review the flavour effects in leptogenesis [119]. Earlier leptogenesis calculations were done by studying Boltzmann Equations (BE) for the $B-L$ asymmetry. But later [120] studied flavour $B-L$ asymmetries where the results were significantly different from the "single flavour approximation". Subsequently, many authors [121,122,123] have included flavour effects to enhance the baryon asymmetry in particular model. In thermal leptogenesis the importance of flavour effects comes from the wash-out effects , where scattering produces N_1 population of neutrinos at temperature $T \simeq M_1$. When T drops below M_1, then N_1 population decays to leptons and if these decays are CP violating, it can produce asymmetry in all the lepton favours. If the interactions are "out-of-equilibrium", then the above asymmetries would survive.

In thermal leptogesis the Yukawa coupling constant related to the production of N_i also controls the decay of N_1. Initially it seems that both the CP asymmetry will be washed out leaving no lepton asymmetry . However, a net asymmetry survives after the potential cancellation of CP asymmetry between processes with N_1 and l_α in the final state such as $X \to N l_\alpha$ scattering and N in the initial state and l_α in the final state, such as $N \to \phi l_\alpha$. Only processes with l_α in the final state can produce the asymmetry. There is no cancellation between asymmetries produced in the decays and inverse decays . Any initial asymmetry produced with the N population is depleted by scattering, decays and inverse decays. This depletion is called washout. The initial state of washout contains a lepton, so it is important to know which leptons are distinguishable. It is always assumed that interactions whose timescale is very different from the leptogenesis scale are dropped out from the Boltzmann Equations(BE).

In the interaction Lagrangian the different flavours are distinguished by their Yukawa couplings h_α. Thus if the h_α mediated interactions are fast compared to the leptogenesis scale and the universe expansion rate, then there distinguishable h_α will have induced differently in the thermal masses of different leptons as each of $h_{e,\mu,\tau}$ has different strengths. Thus, when the charge leptons Yukawa interactions are fast then flavour basis is correct basis for the BE, otherwise leptogenesis has no knowledge of the lepton flavour for 'slow interactions'.

3.5 Non-thermal leptogenesis

In this sections, we discuss leptogenesis scenarios via inflaton decay. Here the heavy Majorana neutrinos responsible for lepton asymmetry generation, are produced by the non-thermal decays of the 'inflaton' –the scalar field of the vacuum energy that dominated during the inflationary epoch of timescale $\sim 10^{-36}$ sec after the Big Bang $\sim 10^{-33}$ sec . In the next subsection, we briefly present an overview of inflaton and inflationary epoch of the early universe. Secondly, we present leptogenesis via non-thermal decay of inflaton to right-handed neutrinos

3.5.1 Inflationary epoch and the inflaton

It is strongly supported that the early universe underwent an epoch of accelerated expansion called inflation. Inflation [124,125,126] was introduced to solve certain problems of standard model of cosmology. Some of these problems are the horizon problem, flatness problem and monopole problem. These refer to the fact that why the universe is homogeneneous and isotropic on large scales, and why the present value of Ω is very close to unity. Inflation solves these problems as exponential expansion leaves the homogeneous and isotopic at large scales with exponentially small curvature. Inflation not only solves the problems of the standard model of cosmology but also provides seeds for the observed CMB anisotropy in the univese [127,128,129,130,131]. Inflation predicts super horizon correlations of temperature anisotropy in CMB which were first confirmed by Cosmic Background Explorer (COBE)[132]. The precise measurements of CMB anisotropy, are being done by Wilkinson Microwave Anisotropy Probe (WMAP)[83] and other ground based, balloon based and satellite based experiments, are also consistent with the early period of inflation.

Exponential expansion during inflation is achieved by scalar field called inflaton. During this period the potential energy of inflaton dominates the energy density of the universe . For inflation to occur kinetic energy of the inflaton should be smaller than the potential energy of the inflation field and the curvature of the inflaton potential should be smaller than the Hubble constant. These conditions are called as slow-roll conditions. For this inflaton potential should be very flat. During inflation, the inflaton rolls slowly through its potential and it ends when the slow-roll conditions break down. The coupling of inflaton to other particles is extremely weak during inflation. Due to exponential expansion the temperature of the universe decreases and it becomes effectively zero at the end of inflation. For a successful model of inflation the universe should again reheat to the temperature required for hot big-Bang picture i.e. it should be higher than the BBN.

After inflation, inflaton oscillates near the minima of its potential and it produces standard model particles . These particles interact with each other and they come to a thermal equilibrium at temperature T. This is called as reheating and the temperature T is called as reheating temperature.

3.5.2 Non-thermal leptogenesis: Leptogenesis in inflaton decay

In this section, we discuss the leptogenesis scenario where the right–handed neutrino are produced non-thermally in the inflaton decays. The basic difference between this scenario and the case of the thermally produced right-handed neutrino discussed in the earlier sections is that, the heavy right-handed neutrino N can be produced with relatively low reheating temperature T_R. We find that the required baryon asymmetry can be obtained even for $T_R < 10^6$GeV. in some of the inflation models, and hence there is no cosmological gravitino problem in the interesting wide region of the gravitino mass $m_{3/2} \simeq 10MeV - 10TeV$. on the other hand, the amount of the produced lepton asymmetry (and hence baryon asymmetry) in the present scenario crucially depends on the physics of the inflation, such as the mass of the inflaton and the reheating temperature T_R. At the end of inflationary epoch, when the Hubble parameter H of the universe becomes comparable to the decay width of the inflaton then vacuum energy of the inflaton ϕ field is completely released into decay products, and the universe is reheated through thermal scattering. The temperature at this time, the reheating temperature T_R usingΓ_ϕ is given by

$$T_R = 0.55\sqrt{M_*\Gamma_\phi} \tag{3.45}$$

where $M_* = 2.4 \times 10^{18}$GeV is the reduced Planck mass. If N_i are produced 'non-thermally' the ratio between the number density of produced N_i and the entropy density s is estimated to be

$$\frac{nN_i}{s} = Br_i\frac{3T_R}{2m_\phi} \tag{3.46}$$

where Br_i denotes the branching ratio of the decay channel $\phi \to N_iN_i$. When the decay rate of N_i is much larger than Γ_ϕ the N_i decays immediately after being produced by the inflaton decays. As discussed in the earlier sections the asymmetric decay of N_1 into leptons and anti-leptons are CP violating and can produces a lepton asymmetry. There decay channels are;

$$N_i \longrightarrow \phi + l, \quad N_i \longrightarrow \phi^\dagger + \bar{l} \tag{3.47}$$

where ϕ and l denote the Weinberg-Salam Higgs and lepton doublets in the Standard Model, respectively. The lepton asymmetry generated by N_i decays can be expressed by

$$\varepsilon_{kj} = \frac{\Gamma(N_i \to \phi + l) - \Gamma(N_i \to \phi^\dagger + \bar{l})}{\Gamma(N_i \to \phi + l) + \Gamma(N_i \to \phi^\dagger + \bar{l})} \tag{3.48}$$

After the completion of the reheating, the lepton asymmetry induced by N_i decay is given by

$$\frac{n_L}{s} = \sum_{i=1}^{3} \varepsilon_i Br_i \frac{3T_R}{2m_\phi} \tag{3.49}$$

If there is a $B-L$ conservation and if it is considered that the lepton asymmetry in Eqn.(3.49) is produced well before the electroweak phase transition of the thermal history,i.e., $T_R \gg 100$ GeV then the B and L conversion by the sphaleron process is active, and it brings a part of this lepton asymmetry into the baryon asymmetry as [95]

$$\frac{n_B}{s} = -\frac{28}{79}\frac{n_L}{s} \tag{3.50}$$

If we consider that there is only one weak Higgs doublet the final expression for the produced baryon asymmetry [134]

$$\frac{n_B}{s} = -\frac{42}{79}\sum_{i=1}^{3} \varepsilon_i Br_i \frac{T_R}{m_\phi} \tag{3.51}$$

Here, it can be shown that the production of heavy neutrino N_i in the inflaton decays is possible only when

$$m_\phi > 2M_i \tag{3.52}$$

Second, the estimation of the lepton asymmetry in (3.51) is obtained under the requirement that the N_i are produced non-thermally by the inflaton decay, which leads to the following condition on the reheating temperature

$$T_R < M_i \tag{3.53}$$

where $i = 1, 2, 3$, Therefore, considering the inflation model satisfying the conditions (3.52) and (3.53), the decays of the Majorana neutrinos N_i generate the lepton asymmetry which is given in Eqn.(3.51) just after the reheating. Later in Chapter 4, these equations (3.52) and (3.53) are used to check the consistency of the neutrino mass models. However, the lepton asymmetry might be washed out, after it is produced, by the lepton-number violating process. The most dangerous ones are the processes mediated by N_1, since N_1 is the lightest Majorana neutrino, so that it survives and still can be produced in the thermal bath after N_2 and N_3 have disappeared. If those processes are well in thermal equilibrium, the produced lepton asymmetry is washed-out strongly [135] . To avoid these wash-out process, we have to invoke that the production of the Majorana neutrinos takes place at a sufficiently late time so that the wash-out processes have already decoupled and been ineffective. Thus, we have to consider sufficiently low reheating temperatures,

$$T_R < M_1 \tag{3.54}$$

With such low reheating temperatures, the lighter Majorana neutrino(s) are completely decoupled from the thermal bath of the Universe, and the $B-L$ becomes a good 'accidental symmetry' for $T < T_R$. The conditions in Eqn.(3.52) and (3.54) justify our estimation of the baryon asymmetry in Eqn.(3.51), by explaining why there is no wash-out effect. It should be noted that Eqn.(3.53) ensures the non-thermal production of the heavier Majorana neutrinos N_2 and N_3 if $m_\phi > 2M_3$. This means that the decays of N_2 and N_3 can be dominant sources of the present baryon asymmetry $(if \varepsilon_2 Br_2, \varepsilon_3 Br_3 \gg \varepsilon_1 Br_1)$. Further, this mechanism works even if there is no N_1 i.e., if $Br_1 = 0$. This feature is completely different from the conventional thermal leptogenesis [136] where the lepton asymmetry is generated by the decays of $N_2(N_3)$ at the temperature of $T \sim M_2(M_3) \gg M_1$ and hence the produced lepton asymmetry may be easily washed out and the resultant baryon asymmetry comes from the decays of the lightest Majorana neutrinos N_1.

4

Leptogenesis in Quasi-degenerate neutrino Mass Models with non-zero θ_{13}

4.1 INTRODUCTION

In this chapter we try to explore the possibilities for the discrimination of the six QDN mass models in the light of Baryogenesis via leptogenesis.

The discovery of flavour conversion of solar, atmospheric, reactor and accelerator neutrinos have provided evidences for the existance of neutrino oscillations. This implies that neutrinos have non-zero masses and they mix among themselves much like quarks. According to Standard Model (SM), neutrino is massless. So one has to go beyond the SM to measure neutrino mass and mixing. The see-saw mechanism is characterized by the existence of right-handed neutrino and can explain the smallness of neutrino masses. When one invokes the see-saw mechanism to explain smallness of neutrino masses, then leptogenesis[137] is almost unavoidable scenario.The presence of heavy right-handed neutrinos with complex Yukawa couplings in see-saw mechanism generates the lepton asymmetry through out-of-equilibrium decays. This lepton asymmetry is then partly transformed into baryon asymmetry through non-perturbative sphaleron processes [138].

The three conditions required to dynamically generate baryon asymmetry are given by Sakharov [139]: (i) the existence of baryon number violating interaction (ii) C and CP violations and (iii) the deviation from thermal equilibrium. The first condition is satisfied by the Majorana nature of heavy neutrinos and sphaleron effect in the SM at high temperature[138]. CP violating decay of heavy right-handed Majorana neutrino provided second condition . The expansion of the Universe provides deviation from thermal equilibrium.

Leptogenesis is the most attractive mechanism of baryogenesis because of its simplicity and connection to neutrino physics. In recent years [140,141]connection between the low energy neutrino mixing parameters and high energy leptogenesis parameters have received much attention.The discovery of Higgs boson of mass 125 GeV [142] having properties consistent with the SM further support for leptogenesis mechanism.

Leptogenesis is of two type, thermal with and without flavour effect and non-thermal. Very high reheating temperature after inflation [143,144] is required in thermal leptogenesis, which is problematic because of the gravitino constraint.The range of gravitino mass in a gravity mediated SUSY breaking is $m_{\frac{3}{2}} = 100GeV - 1TeV$ and gravitino is unstable with life time larger than Nucleosynthesis time $t_N \sim 1$ sec which is dangerous for cosmology.If the reheating temperature after inflation is bounded from above is $T_R \leq (10^6 - 10^7)GeV$ [145], then gravitino problem [146] can be avoided.

To enhance the baryon asymmetry over the single flavour approximation [147-148], one cannot avoid the flavour effect in thermal leptogenesis.

Right-handed neutrinos are produced in non-thermal leptogenesis by the direct non-thermal decay of the inflation ϕ . Using standard procedure[151] and bounds from below and from above on inflation mass (M_ϕ) and reheating temperature (T_R) after inflation, models were excluded using bounds. The expression $T_R = (2Y_B/3C\varepsilon_1)M_\phi$ connect T_R and M_ϕ.Two more boundary conditions[151] which supplement this expression are (i) lower bound on inflation mass, $M_\phi > 2M_1$ coming from allowed kinematics of inflation decay to two right-handed Majorana neutrinos N_1, and (ii) an upper bound for the reheating temperature, $T_R \leq 0.01M_1$ coming from out-of-equilibrium decay of N_1 . One can establish the relation between T_R and M_ϕ for each neutrino mass model by using the observed central value of the baryon asymmetry Y_B and theoretical prediction of CP asymmetry ε_1. In the calculation, the lightest right-handed neutrino mass M_1 , the CP assymetry ε_1 and theoretical bounds in ref.[151]: are $T_R^{min} < T_R \leq T_R^{max}$ and $M_\phi^{min} < M_\phi \leq M_\phi^{max}$ and along with other two boundary conditions mentioned above. In the non-thermal leptogenesis only those models can survive which satisfy the constraints $T_R^{max} > T_R^{min}$ and $M_\phi^{min} < M_\phi^{max}$.

To predict baryon asymmetry of the Universe (BAU) for six quasi-degenerate neutrino (QDN) mass models both in NH and IH, we need light left-handed Majorana neutrino mass matrix m_{LL}. This m_{LL} is considered from our earlier work [152].So for details see the ref.[152]

The chapter is organized as follows : In section 4.2 we discuss the Methodology and classification of neutrino mass models for non-zero θ_{13}. Section 4.3 gives an outline of leptogenesis. The numerical analysis and result for neutrino mass models m_{LL} with non-zero θ_{13} are given in section 4.3. Finally in section 4.5 we summarize the results and discussion. Important expression related to m_{LL} are illustrated to Appendix A.

...

4.2 METHODOLOGY AND CLASSIFICATION OF NEUTRINO MASS MODELS

... To estimate the baryon asymmetry $\eta_B = 6.5 \times 10^{-10}$ [153] from a given mass model, we start with the new light left-handed Majorana neutrino mass matrix (m_{LL}) [152] and then relates it with heavy right-handed Majorana neutrinos (M_{RR}) and Dirac neutrino mass matrix (m_{LR}) through the inversion of Type-I see-saw mechanism in an elegant way. Here we apply new mass matrices (m_{LL}), and hence results of baryon asymmetry are new. We also briefly discuss the construction of (m_{LL})

As we do not know the structure of Yukawa matrix for Dirac neutrino, so we consider the texture of Dirac neutrino mass matrix (m_{LR}) as either the down-quark mass matrix,the charged lepton mass matrix or up-quark mass matrix as allowed by SO(10) GUT models, for phenomenological analysis.The texture of Charged lepton, down-quark,and up-quark is given below.

$$\text{Down-quark type } m_{LR} = \begin{pmatrix} \lambda^4 & 0 & 0 \\ 0 & \lambda^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\text{Up-quark type } m_{LR} = \begin{pmatrix} \lambda^8 & 0 & 0 \\ 0 & \lambda^4 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\text{Charged lepton type } m_{LR} = \begin{pmatrix} \lambda^6 & 0 & 0 \\ 0 & \lambda^2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

The origin of complex CP violating phases are derived from the right-handed Majorana neu-

trino mass matrix (M_{RR}) in the estimation of baryon asymmetry of the Universe. Although it is a theoretical possibility, but it is the main agenda in our present investigations.
Two of our authors derive certain fruitful texture in quasi-degenerate neutrino models based on $\mu-\tau$ symmetry and charged lepton correction where the free parameter (α,η) and standard Wolfenstein parameter λ is used. At first, m_{LL}^{ν} is assumed to follow $\mu-\tau$ symmetry see ref[152], then expected deviation at, $\theta_{13}=0$ and $\theta_{23}=\frac{\pi}{4}$ are obtained from charged lepton sector. When charged lepton correction is considered, the $\mu-\tau$ symmetry is perturbed. The texture of broken $\mu-\tau$ symmetry is described below.

For QD-NH-IA $m_{LL}=(I_0+\Delta I_0^{\lambda})-(2\eta-\frac{\alpha}{2})(I_1-\Delta I_0^{\lambda})+2\alpha(\eta^2-\frac{1}{4})(I_2+\Delta I_2^{\lambda})+\alpha\eta(1-2\eta^2)^{\frac{1}{2}}(I_3+\Delta I_3^{\lambda})$

where the texture of the invariant building blocks $I_{1=0,1,2,3}$ and texture of perturbation to the respective building block matrices, I_is are estimated in terms of ΔI_is are given below

$$I_0=\frac{1}{2}\begin{pmatrix}0&0&0\\0&1&1\\0&1&1\end{pmatrix}\quad\text{and}\quad \Delta I_0^{\lambda}=\frac{1}{2}\lambda\begin{pmatrix}\lambda&1-\frac{1}{2}\lambda&1+\frac{1}{2}\lambda\\1-\frac{1}{2}\lambda&-1-\frac{1}{4}\lambda&-\lambda\\1+\frac{1}{2}\lambda&-\lambda&1-\frac{3}{4}\lambda\end{pmatrix}$$

$$I_1=\frac{1}{2}\begin{pmatrix}2&0&0\\0&1&-1\\0&-1&1\end{pmatrix}\quad\text{and}\quad \Delta I_1^{\lambda}=-\Delta I_0^{\lambda}$$

$$I_2=\frac{1}{2}\begin{pmatrix}-2&0&0\\0&1&-1\\0&-1&1\end{pmatrix}\quad\text{and}\quad \Delta I_2^{\lambda}=\frac{1}{2}\lambda\begin{pmatrix}\lambda&-\frac{1}{2}\lambda&1+\frac{1}{2}\lambda\\-\frac{1}{2}\lambda&1-\frac{3}{4}\lambda&0\\1+\frac{1}{2}\lambda&0&-1-\frac{1}{4}\lambda\end{pmatrix}$$

$$I_3 = \frac{1}{2}\begin{pmatrix} 0 & -1 & 1 \\ -1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \quad \text{and} \quad \Delta I_3^{\lambda} = \frac{1}{2}\lambda\begin{pmatrix} 0 & -1+\lambda & -1-\frac{1}{2}\lambda \\ -1+\lambda & 2 & 2\lambda \\ -1-\frac{1}{2}\lambda & 2\lambda & -2 \end{pmatrix}$$

Similarly the texture for other broken $\mu-\tau$ of QDN models are described below

QD-NH-IB $m_{LL} = (I_0+\Delta I_0^{\lambda}) + (2\eta - \frac{\alpha}{2})(I_1 - \Delta I_0^{\lambda}) - 2\alpha(\eta^2 - \frac{1}{4})(I_2+\Delta I_2^{\lambda}) - \alpha\eta(1-2\eta^2)^{\frac{1}{2}}(I_3+\Delta I_3^{\lambda})$

QD-NH-IC $m_{LL} = -(I_0+\Delta I_0^{\lambda}) + (2\eta - \frac{\alpha}{2})(I_1 - \Delta I_0^{\lambda}) - 2\alpha(\eta^2 - \frac{1}{4})(I_2+\Delta I_2^{\lambda}) - \alpha\eta(1-2\eta^2)^{\frac{1}{2}}(I_3+\Delta I_3^{\lambda})$

QD-IH-IA $m_{LL} = 2\eta(I_0+\Delta I_0^{\lambda}) - (1 - \frac{\alpha}{2})(I_1 - \Delta I_0^{\lambda}) + 2\alpha(\eta^2 - \frac{1}{4})(I_2+\Delta I_2^{\lambda}) + \alpha\eta(1-2\eta^2)^{\frac{1}{2}}(I_3+\Delta I_3^{\lambda})$

QD-IH-IB $m_{LL} = 2\eta(I_0+\Delta I_0^{\lambda}) + (1 - \frac{\alpha}{2})(I_1 - \Delta I_0^{\lambda}) - 2\alpha(\eta^2 - \frac{1}{4})(I_2+\Delta I_2^{\lambda}) - \alpha\eta(1-2\eta^2)^{\frac{1}{2}}(I_3+\Delta I_3^{\lambda})$

QD-IH-IC $m_{LL} = -2\eta(I_0+\Delta I_0^{\lambda}) + (1 - \frac{\alpha}{2})(I_1 - \Delta I_0^{\lambda}) - 2\alpha(\eta^2 - \frac{1}{4})(I_2+\Delta I_2^{\lambda}) - \alpha\eta(1-2\eta^2)^{\frac{1}{2}}(I_3+\Delta I_3^{\lambda})$

Here QD means quasi-degenerate where all the three absolute masses are of similar order,$m_1 \sim m_2 \sim m_3$, NH and IH refers Normal Hierarchy and Inverted hierarchy respectively. Again, each model has three subclasses. Based on CP parity patterns in the three absolute neutrino masses, the subclasses are: Type IA: $m_{LL}^d = diag(+m_1, -m_2, +m_3)$, Type IB: $m_{LL}^d = diag(+m_1, +m_2, +m_3)$, and Type IC: $m_{LL}^d = diag(+m_1, +m_2, -m_3)$, .

The texture of six QDN $\mu-\tau$ symmetry mass matrix and corrected mass matrix are numerically shown in Appendix A. This estimation of baryon asymmetry of the Universe on the basis of thermal and non-thermal leptogenesis may serve as an additional criterion to discriminate the correct patterns of neutrino mass models and also shed light on the structure of Dirac neutrino mass matrix.

4.3 OUTLINE OF LEPTOGENESIS

...In order to estimate lepton asymmetry, we consider the conventional leptogenesis model[154], Type-I see-saw mechanism [155,156] which relates the left-handed Majorana mass matrix m_{LL} and heavy right handed Majorana mass matrix (M_{RR}) in a simple way is :

$$m_{LL} = -m_{LR} M_{RR}^{-1} m_{LR}^{T} \tag{4.1}$$

where (m_{LR}) is the Dirac neutrino mass matrix

4.3.1 Unflavoured Thermal leptogenesis

...In thermal leptogenesis right-handed neutrinos are produced through scattering(mainly inverse decays) in the thermal plasma. CP-violating out-of-equilibrium decay of the lightest of the heavy right-handed Majorana neutrinos can contribute to lepton asymmetry,like $N_R \longrightarrow l_L + \phi, N_R \longrightarrow \bar{l}_L + \phi^{\dagger}$ where l_L is the lepton, $\bar{l}_L$ is the antilepton,and ϕ is the Higgs doublets chiral supermultiplets. The branching ratio is likely to be different for these two decay modes. The interference between the tree level and one-loop level decay amplitude give us the leading contribution and the CP-violating parameter is found to be [157].

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$$\varepsilon_i = \frac{1}{8\pi(h^\dagger h)_{ii}} \sum_{j=2,3} I_m[(h^\dagger h)_{ij})]^2[f\frac{M_j^2}{M_i^2} + g\frac{M_j^2}{M_i^2}] \tag{4.2}$$

where f(x) and g(x) corresponds to the vertex and self energy correction.

$$f(x) \equiv \sqrt{x}[-1+(x+1)ln(1+\frac{1}{x})], \quad g(x) \equiv \frac{\sqrt{x}}{x-1} \tag{4.3}$$

respectively, and both are reduced to $\sim -\frac{1}{2\sqrt{(x)}}$ for $x >> 1$. So in this approximation, ε_i becomes

$$\varepsilon_i = -\frac{3}{16\pi}[\frac{I_m[(h^\dagger h)_{12}^2]}{(h^\dagger h)_{11}}\frac{M_1}{M_2} + \frac{I_m[(h^\dagger h)_{13}^2]}{(h^\dagger h)_{11}}\frac{M_1}{M_3}] \tag{4.4}$$

where m_{LR}/v is the Yukawa coupling of the Dirac neutrino mass matrix in the diagonal basis of M_{RR} and v= 174 GeV is the vev of the standard model. In terms of light Majorana neutrino mass matrix m_{LL} the above expression can be simplified to

$$\varepsilon_i = -\frac{3}{16\pi}\frac{M_1}{(h^\dagger h)_{ii}} I_m(h^\dagger m_{LL} h^\star)_{11} \tag{4.5}$$

The CP-asymmetry parameter ε_i is related to leptonic asymmetric parameter through Y_L as

$$Y_L \equiv \frac{\eta_L - \widetilde{\eta}_L}{s} = \sum_i^3 \frac{\varepsilon_i k_i}{g_{*i}} \tag{4.6}$$

Where η_L is the lepton number density, $\overline{\eta_L}$ is the anti-lepton number density, s is the entropy density,k_i is the dilution factor for the CP asymmetry ε_i and g_{*i} is the effective number of degrees of freedom at temperature $T = M_i$. The baryon asymmetry Y_B is produced through the sphaleron transition of Y_L, while the quantum number B-L remains conserved, and is given by[154]

$$Y_B = \frac{\eta_B}{s} = CY_{B-L} = CY_L \tag{4.7}$$

where $C = \frac{8N_F+4N_H}{22N_F+13N_H}$, N_F is the number of fermion families and N_H is the number of Higgs doublets. Since $s = 7.04\eta_\gamma$ is the baryon number density over photon number density η_γ corresponds to the observed baryon asymmetry of the Universe[158,159].

$$\eta_B^{SM} = (\frac{\eta_B}{\eta_\gamma})^{SM} \approx dk_1\varepsilon_1 \tag{4.8}$$

Where $d \approx 0.98 \times 10^{-2}$ is used in the present calculation . In the expression for baryon to- photon ratio in equation(4.8), k_1 describes the washout of the lepton asymmetry due to various lepton number violating processes. This efficiency factor(also known as dilution factor) mainly depends on the effective neutrino mass

$$\widetilde{m_i} = \frac{(h^\dagger h)_{11} v^2}{M_1} \tag{4.9}$$

Where v is the electroweak vev, v = 174 GeV. For $10^{-2}eV < \widetilde{m_1} < 10^3 eV$, the washout factor k_1 can be well approximated by[160]

$$k_1(\widetilde{m_1}) = 0.3[\frac{10^{-3}}{\widetilde{m_1}}][log\frac{\widetilde{m_1}}{10^{-3}}]^{-0.6} \tag{4.10}$$

we adopt a single expression for k_1 valid only for given range of $\widetilde{m_1}$ [161-163]. . . .

4.3.2 Flavoured thermal leptogenis

. . . When flavour effect is included in thermal leptogenesis[147-150], there is an enhancement in baryon asymmetry over the single flavour approximation is observed. In the flavour basis the equation for lepton asymmetry in $N_1 \longrightarrow l_\alpha\phi$ decay where $\alpha = (e, \mu, \tau)$ becomes

$$\varepsilon_{\alpha\alpha} = \frac{1}{8\pi}\frac{1}{(h^\dagger h)_{11}}[\sum_{j=2,3} Im[h^\star_{\alpha 1}(h^\dagger h)_{1j}h_{\alpha j}]g(x_j) + \sum_j Im[h^\star_{\alpha 1}(h^\dagger h)_{j1}h_{\alpha j}]\frac{1}{(1-x_j)}] \tag{4.11}$$

Here we have $x_j = \frac{M_j^2}{M_i^2}$ and $g(x_j) \sim \frac{3}{2}\frac{1}{\sqrt{x_j}}$ The efficiency factor for the out-of-equilibrium situation is given by $k_\alpha = \frac{m_\star}{\widetilde{m}_{\alpha\alpha}}$

Here $m_\star = \frac{8\pi H v^2}{M_1^2} \sim 1.1\times 10^{-3} eV$ and $\widetilde{m}_{\alpha\alpha} = \frac{h^\dagger_{\alpha 1} h_{\alpha 1}}{M_1} v^2$

This leads to the baryon asymmetry of the Universe,

$$\eta_{3B} = \frac{\eta_B}{\eta_\gamma} \sim 10^{-2}\sum_\alpha \varepsilon_{\alpha\alpha} k_\alpha \sim 10^{-2} m_\star \sum_\alpha \frac{\varepsilon_{\alpha\alpha}}{\widetilde{m}_{\alpha\alpha}} \tag{4.12}$$

For single flavour case the 2nd term in $\varepsilon_{\alpha\alpha}$ vanishes when summed over all flavours. Thus

$$\varepsilon_1 \equiv \sum_\alpha \varepsilon_{\alpha\alpha} = \frac{1}{8\pi}\frac{1}{(h^\dagger h)_{11}}[\sum_j Im[(h^\dagger h)^2_{1j}]g(x_j) \tag{4.13}$$

this leads to baryon asymmetry

$$\eta_{1B} \approx 10^{-2} m_\star \frac{\varepsilon_1}{\widetilde{m}} = 10^{-2} k_1 \varepsilon_1 \tag{4.14}$$

where $\varepsilon_1 = \sum_\alpha \varepsilon_{\alpha\alpha}$ and $\widetilde{m} = \sum_\alpha \widetilde{m}_{\alpha\alpha}$

...

Non-thermal leptogenesis

...In non-thermal leptogenesis [164-167] where the right handed neutrinos are produced through direct non-thermal decay of the inflaton ϕ, interact only with the leptons and Higgs through Yukawa couplings. In supersymmetric models the superpotentials which describes their interactions with leptons and Higgs is[168]

$$W_1 = Y_{ia} N_i L_\alpha H_u \tag{4.15}$$

where Y_{ia} is the matrix for the Yukawa couplings,H_u is the superfield of the Higgs doublet that couples up-type quarks and $L_\alpha(a = e, \mu, \tau)$ is the superfield of the lepton doublets. For supersymmetric models, the interaction between inflaton and right-handed neutrinos is described by superpotential[167] as :

$$W_2 = \sum_{i=1}^{3} \lambda_i S N_i^c N_i^c \tag{4.16}$$

where λ_i are the Yukawa couplings for this type of interaction and S is a gauge singlet chiral superfield for the inflation. With such a superpotential, the inflaton decay rate Γ_ϕ is given by[167]

$$\Gamma_\phi = \Gamma(\phi \to N_i N_i) \approx \frac{|\lambda_i|^2}{4\pi} M_\phi \tag{4.17}$$

where M_ϕ is the mass of inflation ϕ.The reheating temperature T_R after inflation is given by[169]

$$T_R = (\frac{45}{2\pi^2 g_\star})^{\frac{1}{4}} (\Gamma_\phi M_P)^{\frac{1}{2}} \tag{4.18}$$

where $M_P \approx 2.4 \times 10^{18} GeV$ is the reduced Planck mass [170] and $g_\star$ is the effective number of relativistic degree of freedom at reheating temperature. For SM, we have $g_\star = 106.75$ and for MSSM, $g_\star = 228.75.$, The branching ratio of this decay process is taken as $BR \sim 1$, when the inflation dominantly couples to N_i, and the produced baryon asymmetry of the Universe can be calculated by the following relation [164-167],

$$Y_B = \frac{\eta_B}{s} = C Y_L = C \frac{3}{2} \frac{T_R}{M_\phi} \varepsilon \tag{4.19}$$

where Y_L is the lepton asymmetry generated by CP-violating out-of-equilibrium decays of heavy neutrino N_1 and T_R is the reheating temperature. The fraction C has the value $C = -\frac{28}{79}$ for SM and $C = -\frac{8}{15}$ in the MSSM. The observed baryon asymmetry measured in WMAP data, $\eta_B = \frac{\eta_B}{\eta_\gamma} = 6.5 \times 10^{-10}$ [153], where $s = -7.0 \eta_\gamma$, is related to $Y_B = \frac{\eta_B}{s} = 8.7 \times 10^{-11}$

in eq.(4.17). From eq.(4.17) the connection between T_R and M_ϕ is expressed as,

$$T_R = (\frac{2Y_B}{3C\varepsilon})M_\phi \tag{4.20}$$

Two more boundary conditions is supplemented in the above expression : (i) lower bound on inflaton mass, $M_\phi > 2M_1$ coming from allowed kinematics of inflaton decay to two right-handed Majorana neutrinos N_1, and (ii) an upper bound for the reheating temperature, $T_R \leq 0.01M_1$ coming from out-of-thermal equilibrium decay of N_1. The relation between T_R and M_ϕ for each neutrino mass model can be established using the observed central value of the baryon asymmetry Y_B and theoretical prediction of CP asymmetry ε, in eq.(4.18). The lightest right-handed neutrino mass M_1 and ε, are used in the calculation of theoretical bounds: $T_R^{min} < T_R \leq T_R^{max}$ and $M_R^{min} < M_R \leq M_R^{max}$ following eq(4.7), along with other two boundary conditions cited above. Those models which satisfy the constraints $T_R^{max} > T_R^{min}$ and $M_\phi^{min} < M_\phi^{max}$ simultaneously, can only survive in the non-thermal leptogenesis. ...

...

4.4 Numerical Analysis

For the numerical calculations, different neutrino mass matrices with non-zero θ_{13} employed are given in Appendix A. The predictions of mass-squared differences and mixing angles are given in Table-4.1. These values are consistent with the observed neutrino oscillation data at global best fit value at 1σ level. The complex CP-violating phases necessary for leptogenesis are usually derived from Majorana phases appearing in PMNS lepton mixing matrix U_{PMNS}. But in our work, we derived the complex CP-violating Majorana phases from M_{RR} and are used in the estimation of baryon asymmetry of the universe. The approach which we used is also used by ref.[171]. M_{RR} is generated from m_{LL} and m_{LR} through inversion of type-1 see-saw formula,$M_{RR} = -m_{LR}^T m_{LL}^{-1} m_{LR}$. We choose a basis U_R where

Table 4.1 Predicted values of the solar and atmospheric neutrino mass-squared differences for $\sin^2\theta_{12} = 0.32$ calculated from m_{LL} with non-zero θ_{13} in Appendix A

Type	Δm^2_{21}	Δm^2_{31}	$\sin^2\theta_{12}$	$\sin^2\theta_{23}$	$\sin^2\theta_{13}$
QD-NH-IA	7.605	2.497	0.319	0.3943	0.0252
QD-NH-IB	7.605	2.497	0.319	0.3943	0.0252
QD-NH-IC	7.605	2.497	0.319	0.3943	0.0252
QD-IH-IA	7.60	-2.464	0.3195	0.3943	0.0252
QD-IH-IB	7.60	-2.464	0.3195	0.3943	0.0252
QD-IH-IC	7.60	-2.464	0.3195	0.3943	0.0252

$M_{RR}^{diag} = U_R^T M_{RR} U_R = diag(M_1, M_2, M_3)$ with real and positive eigenvalues [160,161]. We then convert diagonal form of Dirac mass matrix, $m_{LR} = diag(\lambda^m, \lambda^n, 1)v$ to the basis $m_{LR} \to m'_{LR} = m_{LR} U_R Q$. where $Q = diag(1, e^{i\alpha}, e^{i\beta})$ is the complex matrix containing CP-violating Majorana phases α and β derived from M_{RR}. We choose some arbitrary values of α and β other than $\frac{\pi}{2}$, π, and 0. Here λ is the Wolfenstein parameter and the choice (m,n) in m_{LR} gives the type of Dirac neutrino mass matrix.The value of the vacuum expectation values(vev) is taken as v= 174 GeV.

In our case, phenomenologically three possible forms of Dirac neutrino mass matrices are considered, such as (i) (m,n)=(4,2) for the down quark mass matrix,(ii) (6,2) for the charged lepton type mass matrix and (iii) (8,4) for up-quark type mass matrix. Dirac neutrino Yukawa coupling becomes $h = \frac{m'_{LR}}{v}$ in the prime basis which is entered in the expression of CP-asymmetry ε in eq.(4.4). The term $Im(h^\dagger h)_{1f}$ appearing in lepton asymmetry ε_1 gives a non-zero contribution, because the new Yukawa coupling matrix h also becomes complex. We fix the Majorana phase $\alpha = \frac{\pi}{4}, \beta = \frac{\pi}{4}$ which leads highest baryon asymmetry.

In Table 4.2, we give numerical predictions on three right-handed Majorana neutrino masses from these neutrino mass models under consideration for the case of $\sin^2\theta_{12} = 0.32$. The three right handed Majorana mass matrices which are constructed through the inversion of Type-I see-saw mechanism , for three choices of diagonal Dirac neutrino mass

Table 4.2 Heavy right-handed neutrino masses M_J for quasidegenerate models with normal and inverted ordering mode for $sin^2\theta_{12} = 0.32$ using neutrino mass matrices given in Appendix A . The entry (m,n) indicates the type of Dirac neutrino mass matrix, as explained in the text.

Type	(m,n)	$M_1(GeV)$	$M_2(GeV)$	$M_3(GeV)$
QD-NH-IA	(4,2)	8.0597×10^9	-1.8622×10^{12}	9.043×10^{13}
	(6,2)	2.0912×10^7	-1.8493×10^{12}	9.0425×10^{13}
	(8,4)	5.388×10^4	-4.8799×10^9	8.8299×10^{13}
QD-NH-IB	(4,2)	3.0626×10^9	1.0880×10^{12}	4.0730×10^{14}
	(6,2)	7.8912×10^6	1.0880×10^{12}	4.0730×10^{14}
	(8,4)	2.0332×10^4	2.8035×10^9	4.0730×10^{14}
QD-NH-IC	(4,2)	3.2682×10^9	1.0659×10^{13}	-3.8961×10^{13}
	(6,2)	8.4220×10^6	1.0658×10^{13}	-3.8960×10^{13}
	(8,4)	2.1700×10^4	3.7270×10^{10}	-2.8707×10^{13}
QD-IH-IA	(4,2)	6.4231×10^9	-9.4457×10^{11}	1.6604×10^{14}
	(6,2)	1.6689×10^7	-9.3667×10^{11}	1.6603×10^{14}
	(8,4)	4.300×10^4	-2.4327×10^9	1.6472×10^{14}
QD-IH-IB	(4,2)	2.4136×10^9	1.0081×10^{12}	4.1397×10^{14}
	(6,2)	6.2190×10^6	1.0081×10^{12}	4.1397×10^{14}
	(8,4)	1.6023×10^4	2.5977×10^9	4.1396×10^{14}
QD-IH-IC	(4,2)	2.5157×10^9	3.2928×10^{12}	-1.2160×10^{14}
	(6,2)	6.4827×10^6	3.2925×10^{12}	-1.2160×10^{14}
	(8,4)	1.6703×10^4	8.7098×10^9	-1.1844×10^{14}

matrix discussed before. The corresponding baryon asymmetry η_f are estimated for both unflavoured η_{1B} and flavoured η_{3B} leptogenesis respectively in Table 4.3. As expected, there is enhancement in baryon asymmetry in case of flavoured leptogenesis η_{3B} as shown in Table 4.3 . We also observe the sensitivity of baryon asymmetry predictions on the choice of Dirac neutrino mass matrix (m,n). Out of the eighteen possibilities of six QDN mass models, six possibilities , one in each of models are consistent with observational data.
A comparative study of efficiency factor of unflavoured ε_{uf} and flavoured ε_{fl} is presented in Table 4.4. Besides this we also present a comparative study of baryon asymmetry in unflavoured η_{uf} , single flavoured approximation η_{1B} and full flavoured consideration η_{3B} in Table 4.5.
Our search on BAU will be completed after calculation on non-thermal leptogenesis. Such studies will also differentiate or impose an additional constraint on the models of neutrino mass matrices.Formulae given in section III will be used for the calculation. Taking the lightest right-handed Majorana neutrino mass M_1 from Table 4.2 and the CP asymmetry $\varepsilon_1$from Table 4.3, for all the mass models in the calculation of theoretical bounds $T_R^{min} < T_R \leq T_R^{max}$ and $M_\phi^{min} < M_\phi \leq M_\phi^{max}$ following eqns.(4.17) and (4.18) along with other two boundary conditions $M_\phi > 2M_1$ (ii) $T_R \leq 0.01M_1$.Only those models which fulfil the constraints $T_R^{max} > T_R^{min}$ and $M_\phi^{min} < M_\phi^{max}$ simultaneously, can survive in the non-thermal leptogenesis. Results are given in Table 4.6.

From Table 6, the neutrino mass models with (m,n) which are compatible with $M_\phi \sim (10^{10} - 10^{13})GeV$ and $T_R \approx (10^4 - 10^8)GeV$. are listed as QD(NH-IA,IB,IC and IH-IA,IH-IB,IH-IC)with down-quark(4,2)type. Again in order to avoid gravitino problem[146] in supersymmetric models, one has the bound on reheating temperature, $T_R \approx (10^6 - 10^7)GeV.$, and this bound is also satisfied by all six QDN mass model with down-quark type. These findings agree with six QDN mass models with down-quark(4,2)type in the context of 3

Table 4.3 Values of CP asymmetry ε and baryon asymmetry(η_{1B}, η_{3B}) for all quasi-degenerate models, with $sin^2\theta_{12} = 0.32$ using light neutrino mass matrices given in Appendix A. The entry (m,n) indicates the type of Dirac mass matrix as explained in the text.

Type	(m,n)	$(h^{\dagger}h)_{11}$	ε_1	η_{1B}	η_{3B}
QD-NH-IA	(4,2)	4.57×10^{-5}	7.07×10^{-6}	4.53×10^{-10}	1.003×10^{-9}
	(6,2)	1.20×10^{-7}	1.85×10^{-8}	1.17×10^{-12}	2.58×10^{-12}
	(8,4)	3.10×10^{-10}	4.99×10^{-11}	3.15×10^{-15}	6.93×10^{-15}
QD-NH-IB	(4,2)	6.64×10^{-6}	7.97×10^{-10}	1.33×10^{-13}	1.64×10^{-10}
	(6,2)	1.71×10^{-8}	2.05×10^{-12}	3.43×10^{-16}	4.23×10^{-13}
	(8,4)	4.41×10^{-11}	5.29×10^{-15}	8.85×10^{-19}	1.09×10^{-15}
QD-NH-IC	(4,2)	7.57×10^{-6}	5.58×10^{-7}	8.75×10^{-11}	9.83×10^{-10}
	(6,2)	1.95×10^{-8}	1.44×10^{-9}	2.26×10^{-13}	2.92×10^{-12}
	(8,4)	5.02×10^{-11}	6.40×10^{-12}	1.00×10^{-15}	1.29×10^{-14}
QD-IH-IA	(4,2)	4.66×10^{-5}	4.21×10^{-6}	2.10×10^{-10}	5.05×10^{-10}
	(6,2)	1.23×10^{-7}	1.11×10^{-8}	5.46×10^{-13}	1.30×10^{-12}
	(8,4)	3.17×10^{-10}	2.89×10^{-11}	1.42×10^{-15}	1.32×10^{-15}
QD-IH-IB	(4,2)	6.64×10^{-6}	3.13×10^{-10}	4.13×10^{-14}	8.41×10^{-11}
	(6,2)	1.71×10^{-8}	8.08×10^{-13}	1.06×10^{-16}	2.17×10^{-13}
	(8,4)	4.41×10^{-11}	2.08×10^{-15}	2.74×10^{-19}	5.59×10^{-16}
QD-IH-IC	(4,2)	7.21×10^{-6}	9.22×10^{-8}	1.16×10^{-11}	2.21×10^{-10}
	(6,2)	1.86×10^{-8}	2.38×10^{-10}	3.02×10^{-14}	5.69×10^{-13}
	(8,4)	4.79×10^{-11}	6.46×10^{-13}	8.18×10^{-17}	1.54×10^{-15}

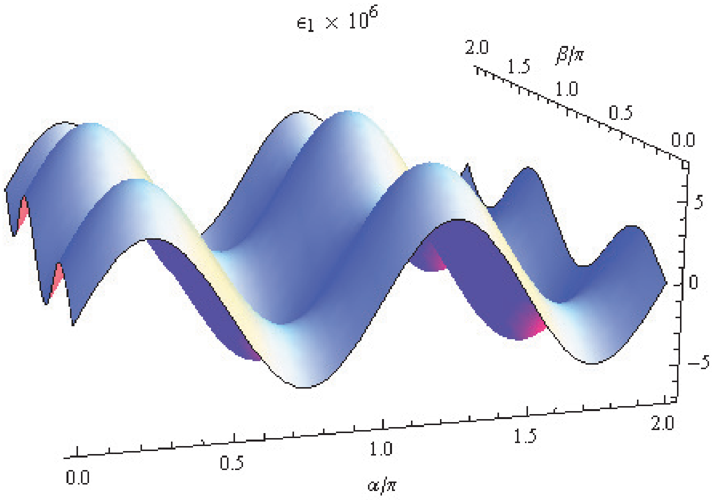

Fig. 4.1 Variation of ε_1 with α and β for QD-NH-IA(4,2) model

Table 4.4 A comparative study of efficiency factors of unflavoured ε_{uf} and flavoured ε_{fl} leptogenesis is shown. ε_{fl} corresponds to both single approximation and three flavoured regimes. Among them, only the values of QD(NH-IA(4,2),NH-IC(4,2),IH-IA(4,2),IH-IC(4,2) are nearly consistent with Davidson-Ibarra upper bound on the lightest RH neutrino CP asymmetry $|\varepsilon_1| \leq 3.4 \times 10^{-7}$[172]

Type	(m,n)	ε_{uf}	ε_{fl}	k_{uf}	$k_{1f} = \frac{m_\star}{m_f}$
QD-NH-IA	(4,2)	2.32×10^{-6}	7.07×10^{-6}	6.53×10^{-4}	6.40×10^{-3}
	(6,2)	5.99×10^{-9}	1.85×10^{-8}	6.43×10^{-4}	6.31×10^{-3}
	(8,4)	1.58×10^{-11}	4.99×10^{-11}	6.43×10^{-4}	6.31×10^{-3}
QD-NH-IB	(4,2)	7.97×10^{-10}	7.97×10^{-10}	1.93×10^{-3}	1.67×10^{-2}
	(6,2)	2.05×10^{-12}	2.05×10^{-12}	1.93×10^{-3}	1.67×10^{-2}
	(8,4)	5.29×10^{-15}	5.29×10^{-15}	1.93×10^{-3}	1.67×10^{-2}
QD-NH-IC	(4,2)	6.88×10^{-8}	5.58×10^{-7}	1.79×10^{-3}	1.56×10^{-2}
	(6,2)	6.33×10^{-10}	1.44×10^{-9}	1.79×10^{-3}	1.56×10^{-2}
	(8,4)	4.59×10^{-13}	6.40×10^{-12}	1.79×10^{-3}	1.56×10^{-2}
QD-IH-IA	(4,2)	2.39×10^{-6}	4.21×10^{-6}	4.95×10^{-4}	4.99×10^{-3}
	(6,2)	6.36×10^{-9}	1.11×10^{-8}	4.87×10^{-4}	4.91×10^{-3}
	(8,4)	1.64×10^{-11}	2.89×10^{-11}	4.87×10^{-4}	4.91×10^{-3}
QD-IH-IB	(4,2)	3.13×10^{-10}	3.13×10^{-10}	1.47×10^{-3}	1.31×10^{-2}
	(6,2)	8.08×10^{-13}	8.08×10^{-13}	1.47×10^{-3}	1.31×10^{-2}
	(8,4)	2.08×10^{-15}	2.08×10^{-15}	1.47×10^{-3}	1.31×10^{-2}
QD-IH-IC	(4,2)	2.80×10^{-8}	9.22×10^{-8}	1.40×10^{-3}	1.26×10^{-2}
	(6,2)	7.24×10^{-11}	2.38×10^{-10}	1.40×10^{-3}	1.26×10^{-2}
	(8,4)	1.82×10^{-13}	6.46×10^{-13}	1.40×10^{-3}	1.26×10^{-2}

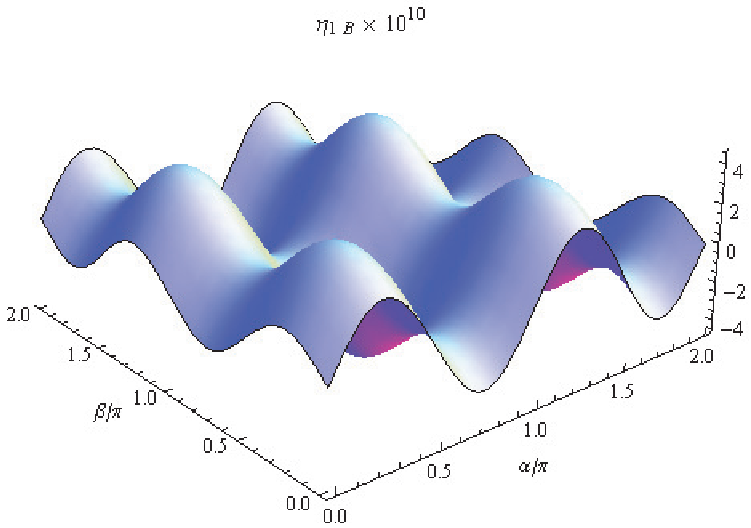

Fig. 4.2 Variation of η_{1B} with α and β for QD-NH-IA(4,2) model

Table 4.5 A comparative study of baryon asymmetry in unflavoured(η_{uf}), single flavoured approximation (η_{1B}) and full flavoured consideration (η_{3B}). Among them, the prediction of QD(NH-IA(4,2),NH-IC(4,2),IH-IA(4,2),IH-IC(4,2)) shows enhancement by one order in magnitude and consistent with observational data. Other models too indicate enhancement by order of one in magnitude but they are too small to be consistent with experimental data.

Type	(m,n)	$(h^{\dagger}h)_{11}$	η_{uf}	η_{1B}	η_{3B}
QD-NH-IA	(4,2)	4.57×10^{-5}	1.51×10^{-11}	4.53×10^{-10}	1.003×10^{-9}
	(6,2)	1.20×10^{-7}	3.85×10^{-14}	1.17×10^{-12}	2.58×10^{-12}
	(8,4)	3.10×10^{-10}	1.02×10^{-16}	3.15×10^{-15}	6.93×10^{-15}
QD-NH-IB	(4,2)	6.64×10^{-6}	1.54×10^{-14}	1.33×10^{-13}	1.64×10^{-10}
	(6,2)	1.71×10^{-8}	3.97×10^{-17}	3.43×10^{-16}	4.23×10^{-13}
	(8,4)	4.41×10^{-11}	1.02×10^{-19}	8.85×10^{-19}	1.09×10^{-15}
QD-NH-IC	(4,2)	7.57×10^{-6}	1.23×10^{-12}	8.75×10^{-11}	1.13×10^{-9}
	(6,2)	1.95×10^{-8}	1.13×10^{-14}	2.26×10^{-13}	2.92×10^{-12}
	(8,4)	5.02×10^{-11}	8.24×10^{-18}	1.00×10^{-15}	1.29×10^{-14}
QD-IH-IA	(4,2)	4.66×10^{-5}	1.18×10^{-11}	2.10×10^{-10}	5.05×10^{-10}
	(6,2)	1.23×10^{-7}	3.09×10^{-14}	5.46×10^{-13}	1.30×10^{-12}
	(8,4)	3.17×10^{-10}	7.98×10^{-17}	1.42×10^{-15}	3.40×10^{-15}
QD-IH-IB	(4,2)	6.64×10^{-6}	4.62×10^{-15}	4.13×10^{-14}	8.41×10^{-11}
	(6,2)	1.71×10^{-8}	1.19×10^{-17}	1.06×10^{-16}	2.17×10^{-13}
	(8,4)	4.41×10^{-11}	3.07×10^{-20}	2.74×10^{-19}	5.45×10^{-16}
QD-IH-IC	(4,2)	7.21×10^{-6}	3.94×10^{-13}	1.16×10^{-11}	2.21×10^{-10}
	(6,2)	1.86×10^{-8}	1.02×10^{-15}	3.02×10^{-14}	5.69×10^{-13}
	(8,4)	4.79×10^{-11}	2.56×10^{-18}	8.18×10^{-17}	1.54×10^{-15}

Table 4.6 Theoretical bound on reheating temperature T_R and inflaton masses M_ϕ in non-thermal leptogenesis are calculated using M_1, ε_f and $\eta_{uf}(orY_B)$ from Table 4.2, 4.4, 4.5 for all neutrino mass models with $\sin^2\theta_{12} = 0.32$. The two boundary conditions $M_\phi > 2M_1$ and $T_R \leq 0.01M_1$ and the other two constraints $T_R^{max} > T_R^{min}$ and $M_\phi^{min} < M_\phi^{max}$ are fulfilled simultaniously. Acceptable results are identified as QD(NH-IA(4,2),NH-IB(4,2), NH-IC(4,2), IH-IA(4,2), IH-IB(4,2), IH-IC(4,2)

Type	(m,n)	$T_R^{min} < T_R \leq T_R^{max}(GeV)$	$M_\phi^{min} < M_\phi \leq M_\phi^{max}(GeV)$
QD-NH-IA	(4,2)	$6.55\times10^4 < T_R \leq 8.06\times10^7$	$1.61\times10^{10} < M_\phi \leq 1.98\times10^{13}$
	(6,2)	$1.65\times10^2 < T_R \leq 2.09\times10^5$	$4.18\times10^7 < M_\phi \leq 5.28\times10^{10}$
	(8,4)	$0.4195 < T_R \leq 5.38\times10^2$	$1.07\times10^5 < M_\phi \leq 1.38\times10^8$
QD-NH-IB	(4,2)	$2.25\times10^5 < T_R \leq 3.06\times10^7$	$6.13\times10^9 < M_\phi \leq 8.32\times10^{11}$
	(6,2)	$5.82\times10^2 < T_R \leq 7.89\times10^4$	$1.57\times10^7 < M_\phi \leq 2.13\times10^9$
	(8,4)	$1.49 < T_R \leq 2.03\times10^2$	$4.07\times10^4 < M_\phi \leq 5.54\times10^6$
QD-NH-IC	(4,2)	$2.74\times10^4 < T_R \leq 3.26\times10^7$	$6.53\times10^9 < M_\phi \leq 7.78\times10^{12}$
	(6,2)	$2.51\times10^2 < T_R \leq 8.42\times10^4$	$1.68\times10^7 < M_\phi \leq 5.63\times10^9$
	(8,4)	$0.1064 < T_R \leq 2.17\times10^2$	$4.34\times10^4 < M_\phi \leq 8.84\times10^7$
QD-IH-IA	(4,2)	$6.85\times10^4 < T_R \leq 6.42\times10^7$	$1.28\times10^{10} < M_\phi \leq 1.20\times10^{13}$
	(6,2)	$1.76\times10^2 < T_R \leq 1.67\times10^5$	$3.34\times10^7 < M_\phi \leq 3.14\times10^{10}$
	(8,4)	$0.4523 < T_R \leq 4.30\times10^2$	$8.6\times10^4 < M_\phi \leq 8.17\times10^7$
QD-IH-IB	(4,2)	$1.35\times10^5 < T_R \leq 2.41\times10^7$	$4.82\times10^9 < M_\phi \leq 8.58\times10^{11}$
	(6,2)	$3.48\times10^2 < T_R \leq 6.22\times10^4$	$1.24\times10^7 < M_\phi \leq 2.22\times10^9$
	(8,4)	$0.9009 < T_R \leq 1.60\times10^2$	$3.20\times10^4 < M_\phi \leq 5.69\times10^6$
QD-IH-IC	(4,2)	$4.09\times10^4 < T_R \leq 2.52\times10^7$	$5.03\times10^9 < M_\phi \leq 3.09\times10^{12}$
	(6,2)	$1.05\times10^2 T_R \leq 6.48\times10^4$	$1.29\times10^7 < M_\phi \leq 7.94\times10^9$
	(8,4)	$0.2521 < T_R \leq 1.67\times10^2$	$3.34\times10^4 < M_\phi \leq 2.21\times10^7$

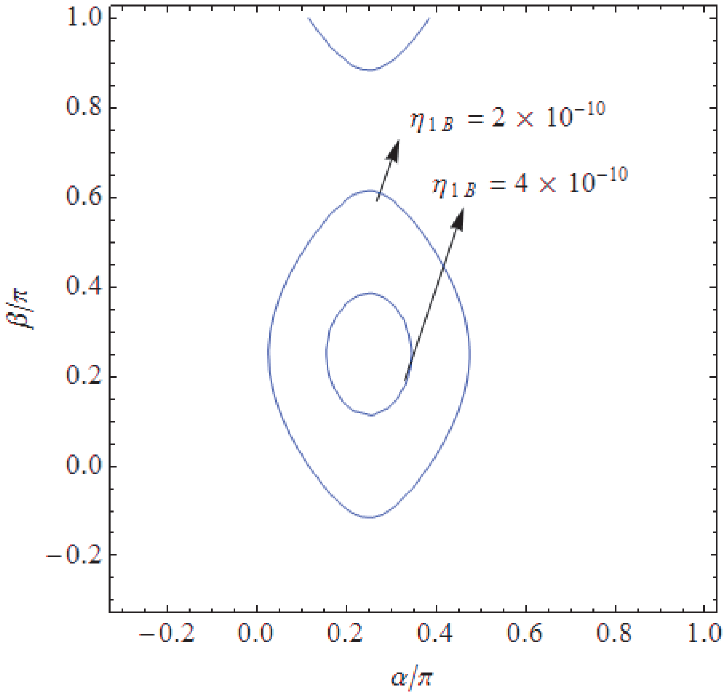

Fig. 4.3 Variation of η_{1B} with α and β for QD-NH-IA(4,2) model

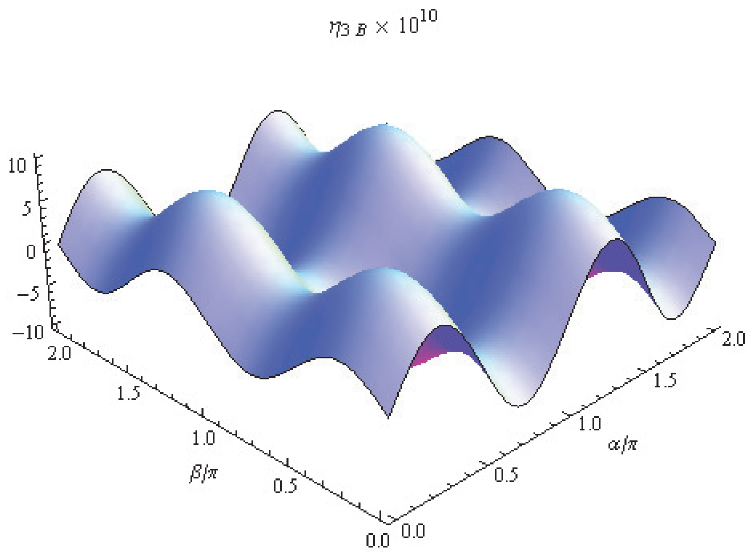

Fig. 4.4 Variation of η_{3B} with α and β for QD-NH-IA(4,2) model

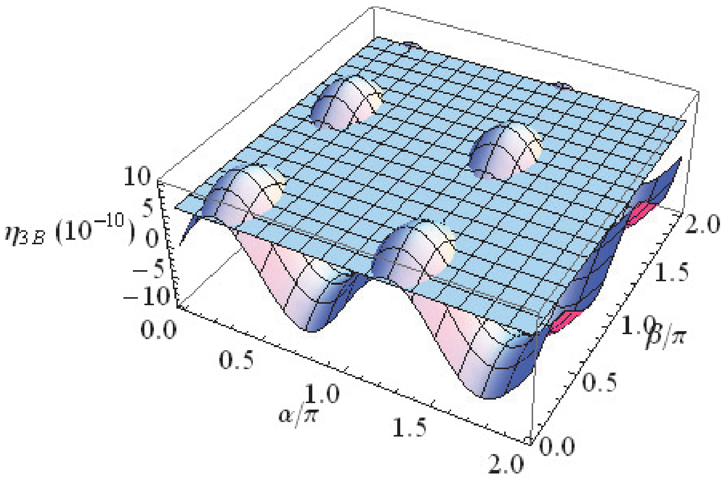

Fig. 4.5 Variation of η_{3B} with α and β for QD-NH-IA(4,2) model

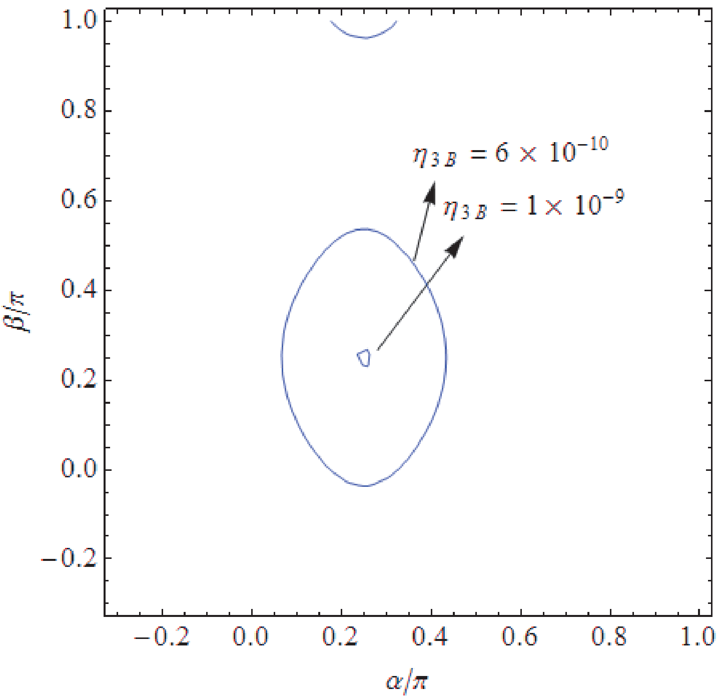

Fig. 4.6 Variation of η_{3B} with α and β for QD-NH-IA(4,2) model

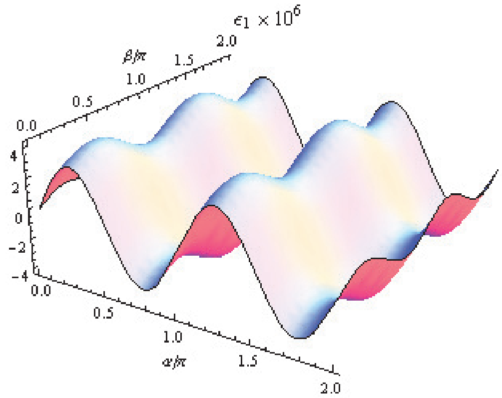

Fig. 4.7 Variation of ε_1 with α and β for QD-IH-IA(4,2) model

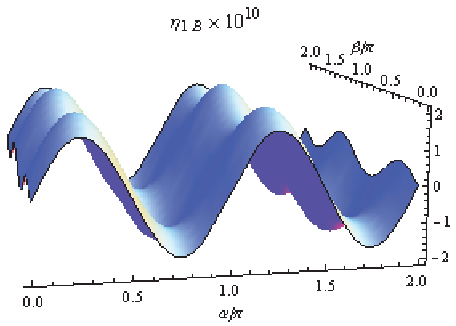

Fig. 4.8 Variation of η_{1B} with α and β for QD-IH-IA(4,2) model

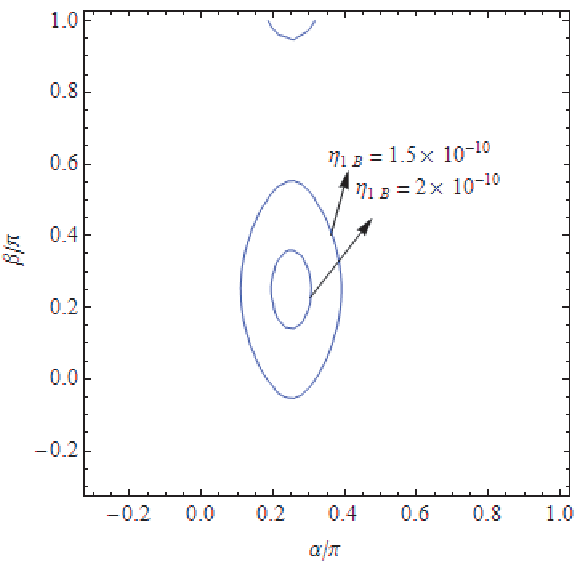

Fig. 4.9 Variation of η_{1B} with α and β for QD-IH-IA(4,2) model

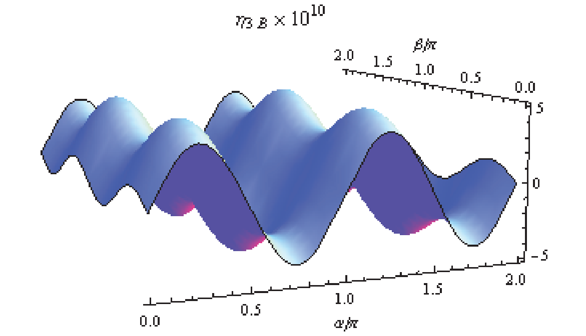

Fig. 4.10 Variation of η_{3B} with α and β for QD-IH-IA(4,2) model

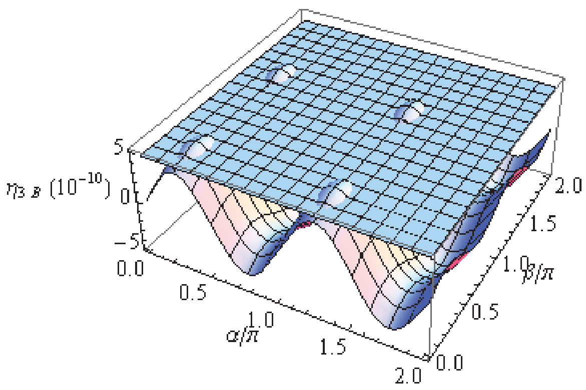

Fig. 4.11 Variation of η_{3B} with α and β for QD-IH-IA(4,2) model

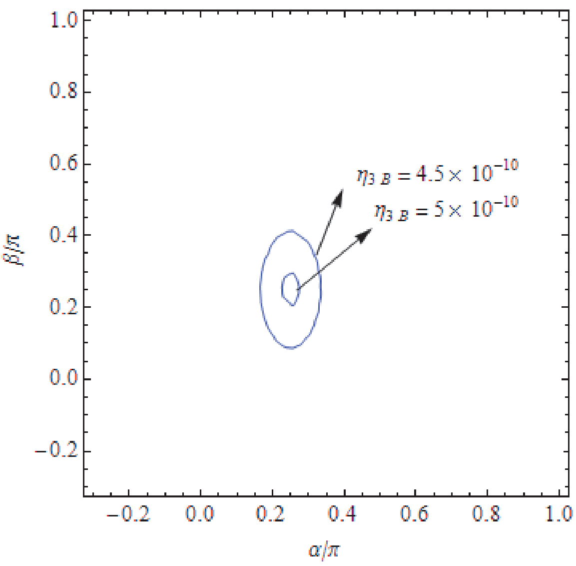

Fig. 4.12 Variation of η_{3B} with α and β for QD-IH-IA(4,2) model

flavoured thermal leptogenesis in Table 3.

Although the Majorana phases are fixed in our analysis, we can show the variation of ε_1, η_{1B} and η_{3B} by keeping the phases arbitrary. To exemplify, we consider two models, QD-NH-IA and QD-IH-IA (with Dirac neutrino mass matrix as down type quark(4,2)). The variation of the observable quantities in the context of the above mentioned models are shown in figures (4.1)-(4.12).

4.5 Summary and Discussions

After overall analysis of the six quasi-degenerate neutrino(QDN) mass models in normal hierarchical and inverted hierarchical patterns with non-zero θ_{13} for $sin^2\theta_{12} = 0.32$ by considering three forms of Dirac mass matrices: down-quark(4,2),charged lepton(6,2) and up-quark (8,4)type , we have seen that all the six QDN mass models with down quark type Dirac matrix are relevant in the context of flavoured leptogenesis. But if it is unflavoured or single flavoured, scenario is little different,where we see that only QD-NH-IA and QD-IH-IA are fairly consistent. While the rest of the models with charged lepton(6,2) and up-quark(8,4) type Dirac matrices lead very small baryon asymmetry, but we can't discard other models' which are subjected to sensitivities of future experiment.In our work QD-NH-IA and QD-IH-IA with down-quark(4,2) type are identified as favourable models, since these two models consistently agree with observational data in all three stages of leptogenesis. The predicted inflaton mass needed to produce baryon asymmetry of the Universe is found to be $M_\phi \sim (10^{11} - 10^{13})GeV$ corresponding to the reheating temperature $T_R \approx (10^6 - 10^7)GeV$.. In ref.[173] non-zero θ_{13} is generated by perturbing the $\mu - \tau$ symmetric matrix using type II see-saw.But we perturbed $\mu - \tau$ symmetric mass matrix by taking charged lepton contribution.

We might ask whether QDN model are valid or not in the context of oscillations experiments, cosmological observation and baryon asymmetry. It was seen in our earlier work in ref.[152], that QDN mass model are self sufficient to comply with the observation obtained from oscillation experiment. In the present literature we have tried to discriminate those models further and we have found that QDN mass models parameterized by us shown better results than those ref.[173],where QDN mass models are found less relevant in the context of baryogenesis. In comparision to the ref.[173], present work sees better visualization in 3 flavoured context. Our present analysis also strengthen those models which were almost rule out in the work of ref.[174]. The results presented in this article have important implications to discriminate the correct neutrino mass models and also shed light on the structure of the Dirac mass matrix as well as quasi-degenerate pattern of neutrinos could be natural neutrinos in the neutrino oscillation experiments.

5

Summary and Outlook

One of the primary questions in particle physics is the determination of individual neutrino masses. The origin of matter is another important puzzle of modern cosmology.This puzzle of cosmology may be related to the existence of tiny but non-zero neutrino masses, which are now established. As a matter of fact, a simple extension of Standard Model(SM) naturally leads to small neutrino masses via the see-saw mechanism. Its cosmological consequence is the leptogenesis which elegantly produces the required baryon asymmetry of the universe. Therefore, the see-saw mechanism and leptogenesis have very attractive features, relating low energy of neutrino physics to high energy like cosmology.

In the introductory Chapter, we present a brief sketch on the phenomenological status of Standard Model (SM) and its extension to grand unified theories (GUT) with or without SUSY. There are several gauge models based on GUT symmetries such as $SU(5), SO(10), E_6, SO(18)$ with or without supersymmetry. The models are briefly discussed on how different predictions arises from different symmetry breaking patterns. Model based on SO(10) combined with a continuous or discrete, flavour symmetry group, have been constructed to understand flavour problem, especially the small neutrino masses and large leptonic mixing angles. Supersymmetry helps us to understand the hierarchy problem - why the scale of EW symmetry breaking is so much smaller than the scales of grand unification. SU(5) is the simplest GUT group. The basic matter representations of $SU(5), \overline{5} \oplus 10$, do not contain the right-handed neutrino, and to describe neutrino masses, one must extend the model by adding three right-handed neutrinos, one per generation. Then the Majorana mass of the gauge singlet right-handed neutrino is unconstrained and can be the same as the Planck mass, which causes problem, as it makes it difficult to accommodate the neutrino data. If one considers the SO(10) group, then its basic representation automatically contains the right-handed neutrino along with the other 15 fermions of the SM (for each family). Therefore one must break SO(10) symmetry [or more precisely the B-L subgroup of SO(10)], to give mass to the right-handed neutrino, which naturally solves the right-handed neutrino mass fine-tuning problem. Thus, to proceed beyond the SM, SO(10)GUT is the most natural way.

In Chapter 2, we review on neutrino oscillation and its implication with latest experiments. In section 2.2, a brief review on the discovery of neutrino oscillation is presented. Then a short account of neutrino oscillation in vacuum as well as in medium, followed by a review on solar, atmospheric, accelerator and reactor type neutrino experiments, is presented in section 2.2.1 and 2.2.2. At the end, in section 2.2.3 we present implications with latest experiments.

In Chapter 3, we discuss the outline of the possible explanation of the generation of observed baryon asymmetry of the universe. We discuss baryogenesis via leptogenesis through the decay of heavy Majorana neutrinos. We also discuss formulation of thermal leptogenesis. A brief discussion on the relation between Inflaton mass and non- thermal leptogenesis is presented in the last section.

In Chapter 4, we try to explore the possibilities for the discrimination of six kinds of Quasi-degenerate (QDN) mass models in the light of baryogenesis via leptogenesis. Leptogenesis realises a highly non-trivial link between two completely independent experimental observations: the absence of antimatter in the observable universe and the observation of neutrino mixings and masses. Therefore, leptogenesis has a built-in double sided nature. The discovery of Higgs boson of mass 125GeV having properties consistent with the SM, further supports for leptogenesis mechanism. In this chapter, we discuss leptogenesis from decay of right-handed neutrino to lepton and Higgs via Yukawa couplings. The CP asymmetry arises due to the interference of the loop diagrams and tree-level diagrams, which leads to a small difference of leptons and anti-leptons in the final state per RH neutrino decay.

Assuming strong hierarchical RH neutrino masses, $M_1 \ll M_2 \ll M_3$, the CP asymmetry is a function of the lightest RH neutrino mass and Dirac Yukawa couplings. In the see-saw model, the light neutrino masses can be expressed in terms of masses of RH neutrinos and Yukawa couplings, so the CP asymmetry of RH neutrino decay is constrained by the masses of light neutrino. The so-called Davidson-Ibarra bound on CP asymmetry requires the mass of the lightest RH neutrino $M_1 \geq 10^{8-9}$GeV to generate the correct baryon-entropy ratio. In the super-gravity theories, the temperature of leptogenesis $T \sim M_1 \geq 10^{8-9}$ GeV results in a catastrophe, since the decay of gravitino can dilute the light elements from BBN. We find that enough lepton/baryon asymmetry can be generated in the hot plasma in the universe, when the exotic Yukawa couplings are relatively large and the CP asymmetry of RH neutrino decay can be $\sim 10^{-6}$. Thus, we can avoid the problem of gravitino-over-production. We have considered this only for leptogenesis in N_1 dominated scenario.

QD-NH (IA,IB,IC) and QD-IH (IA,IB,IC) with down quark (4,2)type, do overcome the gravitino problem in supersymmetric models. The bound on the reheating temperature $T_R \approx (10^6 - 10^7)$ is fulfilled. While the rest of the models with charged leptons (6,2) and up-quark(8,4) type Dirac matrices have lower inflaton masses. The predicted inflaton mass needed to produce the observed BAU is found to be $M_\phi \sim (10^{11} - 10^{13}) GeV$ corresponding to the reheating temperature $T_R \approx (10^6 - 10^7) GeV$..

We have seen that all the six QDN mass models are relevant in the context of flavoured leptogenesis .But if it is unflvoured or single flavoured, scenario is little different,where we see that only QD-NH-IA and QD-IH-IA are dominant. In order to get specific results, the choice of Dirac neutrino mass matrix as down-quark type, is found to be most favourable.

The present work could be extended to normal hierarchical (NH) and inverted hierarchical (IH) mass models.

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A

Classification of Neutrino Mass Models with non-zero θ_{13}

1. A list of $\mu - \tau$ symmetric mass matrix $M_{\mu\tau}$ and corrected neutrino mass matrix after taking charged lepton contribution are shown for six quasi-degenerated neutrino models viz.,QD(NH-IA.NH-IB, NH-IC, IH-IA, IH-IB, IH-IC) in Table A1 and Table A3 respectively. All the neutrino mass matrices given below predict $\sin^2\theta_{12} = 0.32$. The values of three input parameters used in $\mu - \tau$ symmetric matrix are given in Table A2.

In our case, charged lepton mixing matrix $\widetilde{U}_{eL}$ is : ref.[152].

$$\widetilde{U}_{eL} = \widetilde{R}_{12}^{-1}(\frac{\lambda}{2})\widetilde{R}_{31}^{-1}(\frac{\lambda}{2})\widetilde{R}_{23}(\frac{\lambda}{2})$$

$$\widetilde{U}_{eL}^{\dagger} = \begin{pmatrix} 1-\frac{\lambda^2}{4} & \frac{\lambda}{2} & \frac{\lambda}{2} \\ -\frac{\lambda}{2}+\frac{\lambda^2}{4} & 1-\frac{\lambda^2}{4} & -\frac{\lambda}{2} \\ -\frac{\lambda}{2}-\frac{\lambda^2}{4} & \frac{\lambda}{2}-\frac{\lambda^2}{4} & 1-\frac{\lambda^2}{4} \end{pmatrix}$$

where λ, is the Wolfenstein parameter, $\lambda = 0.2253$

$$\widetilde{U}_{eL} = \begin{pmatrix} 0.98731 & -0.0993273 & -0.12391 \\ 0.111933 & 0.98874 & 0.0993237 \\ 0.11265 & -0.111933 & 0.98731 \end{pmatrix}$$

Type	$M_{\mu\tau}(\alpha,\eta)/m_0$	$\frac{m_i}{m_0}$
QD-NH-IA	$\begin{pmatrix} \alpha-2\eta-2\alpha\eta^2 & -\alpha\eta(1-2\eta^2)^{\frac{1}{2}} & \alpha\eta(1-2\eta^2)^{\frac{1}{2}} \\ -\alpha\eta(1-2\eta^2)^{\frac{1}{2}} & \frac{1}{2}-\eta+\alpha\eta^2 & \frac{1}{2}+\eta-\alpha\eta^2 \\ \alpha\eta(1-2\eta^2)^{\frac{1}{2}} & \frac{1}{2}+\eta-\alpha\eta^2 & \frac{1}{2}-\eta+\alpha\eta^2 \end{pmatrix}$ $= I_0-(2\eta-\frac{\alpha}{2})I_1+2\alpha(\eta^2-\frac{1}{4})I_2+\alpha\eta(1-2\eta^2)^{\frac{1}{2}}I_3$	$\alpha-2\eta,-2\eta,1$
QD-NH-IB	$\begin{pmatrix} 2\eta+2\alpha\eta^2-\alpha & \alpha\eta(1-2\eta^2)^{\frac{1}{2}} & -\alpha\eta(1-2\eta^2)^{\frac{1}{2}} \\ \alpha\eta(1-2\eta^2)^{\frac{1}{2}} & \frac{1}{2}+\eta-\alpha\eta^2 & \frac{1}{2}-\eta+\alpha\eta^2 \\ -\alpha\eta(1-2\eta^2)^{\frac{1}{2}} & \frac{1}{2}-\eta+\alpha\eta^2 & \frac{1}{2}+\eta-\alpha\eta^2 \end{pmatrix}$ $= I_0+(2\eta-\frac{\alpha}{2})I_1-2\alpha(\eta^2-\frac{1}{4})I_2-\alpha\eta(1-2\eta^2)^{\frac{1}{2}}I_3$	$2\eta-\alpha,2\eta,1$
QD-NH-IC	$\begin{pmatrix} 2\eta+2\alpha\eta^2-\alpha & \alpha\eta(1-2\eta^2)^{\frac{1}{2}} & -\alpha\eta(1-2\eta^2)^{\frac{1}{2}} \\ \alpha\eta(1-2\eta^2)^{\frac{1}{2}} & \eta-\alpha\eta^2-\frac{1}{2} & \alpha\eta^2-\frac{1}{2}-\eta \\ -\alpha\eta(1-2\eta^2)^{\frac{1}{2}} & \alpha\eta^2-\frac{1}{2}-\eta & \eta-\alpha\eta^2-\frac{1}{2} \end{pmatrix}$ $= -I_0+(2\eta-\frac{\alpha}{2})I_1-2\alpha(\eta^2-\frac{1}{4})I_2-\alpha\eta(1-2\eta^2)^{\frac{1}{2}}I_3$	$2\eta-\alpha,2\eta,-1$
QD-IH-IA	$\begin{pmatrix} \alpha-2\alpha\eta^2-1 & -\alpha\eta(1-2\eta^2)^{\frac{1}{2}} & \alpha\eta(1-2\eta^2)^{\frac{1}{2}} \\ -\alpha\eta(1-2\eta^2)^{\frac{1}{2}} & \eta+\alpha\eta^2-\frac{1}{2} & \frac{1}{2}+\eta-\alpha\eta^2 \\ \alpha\eta(1-2\eta^2)^{\frac{1}{2}} & \frac{1}{2}+\eta-\alpha\eta^2 & \eta+\alpha\eta^2-\frac{1}{2} \end{pmatrix}$ $= 2\eta I_0-(1-\frac{\alpha}{2})I_1-2\alpha(\eta^2-\frac{1}{4})I_2+\alpha\eta(1-2\eta^2)^{\frac{1}{2}}I_3$	$\alpha-1,-1,2\eta$
QD-IH-IB	$\begin{pmatrix} 1-\alpha-2\alpha\eta^2 & \alpha\eta(1-2\eta^2)^{\frac{1}{2}} & -\alpha\eta(1-2\eta^2)^{\frac{1}{2}} \\ \alpha\eta(1-2\eta^2)^{\frac{1}{2}} & \frac{1}{2}+\eta-\alpha\eta^2 & \eta+\alpha\eta^2-\frac{1}{2} \\ -\alpha\eta(1-2\eta^2)^{\frac{1}{2}} & \eta+\alpha\eta^2-\frac{1}{2} & \frac{1}{2}+\eta-\alpha\eta^2 \end{pmatrix}$ $= 2\eta I_0+(1-\frac{\alpha}{2})I_1-2\alpha(\eta^2-\frac{1}{4})I_2-\alpha\eta(1-2\eta^2)^{\frac{1}{2}}I_3$	$1-\alpha,1,2\eta$
QD-IH-IC	$\begin{pmatrix} 1-\alpha+2\alpha\eta^2 & \alpha\eta(1-2\eta^2)^{\frac{1}{2}} & -\alpha\eta(1-2\eta^2)^{\frac{1}{2}} \\ \alpha\eta(1-2\eta^2)^{\frac{1}{2}} & \frac{1}{2}-\eta-\alpha\eta^2 & \alpha\eta^2-\eta-\frac{1}{2} \\ -\alpha\eta(1-2\eta^2)^{\frac{1}{2}} & \alpha\eta^2-\eta-\frac{1}{2} & \frac{1}{2}-\eta-\alpha\eta^2 \end{pmatrix}$ $= -2\eta I_0+(1-\frac{\alpha}{2})I_1-2\alpha(\eta^2-\frac{1}{4})I_2-\alpha\eta(1-2\eta^2)^{\frac{1}{2}}I_3$	$1-\alpha,1,-2\eta$

Table A.1 $\mu-\tau$ symmetric matrix (using the building blocks)

.

Type	α	η	m_0
QD-NH-IA	1.5929	0.40	0.082
QD-NH-IB	0.0071	0.40	0.082
QD-NH-IC	0.0071	0.40	0.082
QD-IH-IA	1.9946	0.3987	0.084
QD-IH-IB	0.0054	-0.3987	0.084
QD-IH-IC	0.0054	-0.399687	0.084

Table A.2 Three input parameters α,η,m_0 as used in $\mu-\tau$ symmetric matrix

.

Type	$m_{LL}^{\nu} = \widetilde{U}_{eL}^{\dagger} M_{\mu\tau} \widetilde{U}_{eL}$
QD-NH-IA	$\begin{pmatrix} 0.0247635 & -0.0410168 & 0.0449261 \\ -0.0410168 & 0.0267516 & 0.0530573 \\ 0.0449261 & 0.0530573 & 0.0299028 \end{pmatrix}$
QD-NH-IB	$\begin{pmatrix} 0.0656274 & 0.00186231 & 0.00188115 \\ 0.00186231 & 0.0717453 & 0.00788961 \\ 0.00188115 & 0.00788961 & 0.0752451 \end{pmatrix}$
QD-NH-IC	$\begin{pmatrix} 0.0614915 & -0.0142848 & -0.0181301 \\ -0.0142848 & 0.00870457 & -0.0702373 \\ -0.0181301 & -0.0702373 & -0.215783 \end{pmatrix}$
QD-IH-IA	$\begin{pmatrix} 0.0310301 & -0.0563559 & 0.0529383 \\ -0.0563559 & 0.0197294 & 0.0499092 \\ 0.0529383 & 0.0499092 & 0.0159344 \end{pmatrix}$
QD-IH-IB	$\begin{pmatrix} 0.0832739 & -0.00146641 & -0.00214974 \\ -0.00146641 & 0.0773984 & -0.00796848 \\ -0.00214974 & -0.00796848 & 0.0740215 \end{pmatrix}$
QD-IH-IC	$\begin{pmatrix} 0.0798871 & -0.0146888 & -0.0185364 \\ -0.0146888 & 0.0257762 & -0.0719444 \\ -0.0185364 & -0.0719444 & -0.00526434 \end{pmatrix}$

Table A.3 Numerically corrected left-handed Majorana neutrino mass matrices for six QDN mass models

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