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THE MASSES OF THE NEUTRINOS IN THE UNIFICATION GAUGE THEORIES AND THE NEUTRINO-ANTINEUTRINO OSCILLATIONS*

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p. 15, second sentence from top: "The SU(2) instanton may not change SU(2) doublet to singlet, so it cannot split the double line by a very small space.[21]" should read "The SU(2) instanton may not split the double line by a very small space.[21]"

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** On leave from the Institute of High Energy Physics, P. O. Box 918, Beijing, China.

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ABSTRACT

We discuss the two types of neutrino oscillation: (I) the $\nu_e \leftrightarrow \nu_\mu$ like oscillations and (II) the $\nu \leftrightarrow \nu^c$ like neutrino-antineutrino oscillations. To connect the oscillation phenomenology with mass differences of the neutrinos, we discuss possible neutrino mass patterns in a general form including the Dirac mass and the Majorana mass, when there are both left- and right-handed neutrinos. Then we extract four extreme and interesting cases of the mass patterns. We connect these mass patterns with the W-S model, the grand unification models such as SU(5), SO(10) and E_6, and some constituent models. At the end of this paper we deduce that one more neutral intermediate boson with the mass at the same order of the mass of the W boson may exist from one of the interesting neutrino mass patterns.

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I. INTRODUCTION

Three species of neutrino have been found experimentally. All three are consistent with being left-handed neutrinos. We do not know whether these neutrinos are massless neutrinos. We do not know whether they have right-handed partners. Whether they have family number unconserved and/or lepton number unconserved interactions, we also do not know. We can imagine how difficult it is to answer these questions, if we remember that the neutrino was invented by Pauli[1] in the beginning of the 1930's, some 35 years later than the discovery of the β type radioactivity (1896), and its existence was "verified" by Cowan and Reines _et al_.[2] some 20 years after its invention. Of course, the point is that the interactions of the neutrinos are so weak, especially for right-handed neutrinos if ever they exist. Also, the masses of the left-handed neutrinos are extremely small.[3] In this situation the neutrino oscillation experiments[4,5] may play a big role. A naive estimate says that neutrino oscillation experiments may be sensitive to the mass differences Δm^2 in the following region

$$10^{-12}(\mathrm{eV})^2 < \Delta m^2 < 10^2(\mathrm{eV})^2 \quad . \qquad (1)$$

A given experiment may cover a part of this region[6] depending upon the character of the neutrino source and the distance between the source and the detector. But most of this region cannot be reached by spectrometer experiments.[3]

The neutrino oscillation experiments may also help to answer whether there are right-handed neutrinos if the relative mass difference falls in the Δm^2 region in Eq. (1). Actually we may have two types of neutrino oscillation: The first type of oscillation is among the left-handed neutrinos[4] (or among their antineutrinos)

$$\text{type I} \qquad \begin{array}{ccc} \nu_{eL} & \leftrightarrow & \nu_{\mu L} \\ & \nwarrow\!\!\searrow \quad \swarrow\!\!\nearrow & \\ & \nu_{\tau L} & \end{array} \quad . \tag{2}$$

These oscillations conserve the lepton number but break the family numbers, whereas the second type of oscillation[7] is between the neutrino and the antineutrino with the same helicity

$$\text{type II} \qquad \nu \leftrightarrow \nu^c \quad . \tag{3}$$

These oscillations change the lepton number by $\Delta L = \pm 2$. We shall refer to the type II as the "neutrino-antineutrino oscillations" and the type I "neutrino-neutrino oscillations."

II. NEUTRINO OSCILLATIONS IN GENERAL[4,8]

The left-handed neutrinos have the $SU(2) \times U(1)$ weak gauge interactions[9] which can be shown by putting the neutrinos in doublets:

$$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L \quad . \tag{4}$$

Here we have three families (or generations) of leptons. Because the gauge bosons meet only the particles in the same representation of the gauge group in an interaction vertex, the family quantum number, which differentiates different representations of the same dimension, is crucial for specifying the weak gauge interactions of the neutrinos. Here the family quantum number is e, μ, or τ. Every neutrino in the table of Eq. (4) has fixed quantum numbers T, T_3 and family number. Thus we call these neutrinos the eigenstates of interactions. Because the $SU(2) \times U(1)$ gauge interactions are the dominant interaction of neutrinos, in a reaction a neutrino in a pure eigenstate of interactions is produced or annihilated. The right-handed neutrinos (if they exist) are in SU(2) singlet with the hypercharge of U(1) being zero. They belong to the other kind of eigenstates of interactions in the sense that they have fixed quantum numbers of $SU(2) \times U(1)$ gauge group, though they do not have any gauge interactions of the $SU(2) \times U(1)$ type.

However, besides the main interaction there may be some other weaker interactions which break the $SU(2) \times U(1)$ quantum numbers, family numbers, even lepton number. For instance, the Yukawa interaction between Fermions and Higgs (we will call them "superweak" here; the typical strength of the effective four Fermion superweak interactions are $(m_F^2/m_H^2)G_F$) violates family numbers and some grand unification interactions mediated by extremely heavy boson (we will call them "grand weak" here) transfer leptons to antileptons. These interactions will produce nondiagonal masses among different neutrino eigenstates of interactions. The contribution of the superweak and grand weak interactions to the mass matrix is much bigger than the reactions which violate the conservation laws of the main $SU(2) \times U(1)$ gauge interactions because, roughly speaking, the former is an S-matrix but the latter is the absolute value square of the S-matrix In this way, the eigenstates of interactions become different states from the eigenstates of masses.

Let ν_α be the eigenstates of interactions and ν_i be the eigenstates of masses

$$|\nu_\alpha\rangle = \sum_i V_{\alpha i}|\nu_i\rangle \tag{5}$$

where $V_{\alpha i}$ are the elements of the unitary matrix V, $VV^{\dagger} = 1$. V has also been called the mixing matrix with the mixing angles and phases. By a physical reaction we produce an eigenstate of interaction $|\nu_{\alpha}>$ which is a special combination of $|\nu_i>$, as shown in Eq. (5). $|\nu_i>$ with different i's are different eigenstates of masses, i.e., different eigenstates of propagation. So there will be an evolution of the neutrino state $|\nu_{\alpha}>$ in its propagation process. Let $|\nu_{\alpha}(x,t)>$ be the evolution state, $|\nu_{\alpha}(o,o)> = |\nu_{\alpha}>$, then

$$|\nu_{\alpha}(x,t)> = \sum_i V_{\alpha i}|\nu_i> e^{i(p_i x - E_i t)}$$

$$= \sum_{i,\beta} V_{\alpha i} V^*_{i\beta}|\nu_{\beta}> e^{i[E_i(x-t) - (m_i^2/2E_i)x]} \qquad (6)$$

where we use $m_i << E_i$ and $p_i = E_i - m_i^2/2E_i$. To measure the neutrino flux is to pick up an eigenstate of interaction if the corresponding spectrometer cannot differentiate the different neutrino masses.[3] From the number of the events

$$\nu + A \rightarrow \ell_{\beta}^- + \cdots$$

we know the flux of ν_{β} in the neutrino beam

$$N_{\alpha\rightarrow\beta}(x,E) = \int_t^{t+\Delta t} |<\nu_{\beta}|\nu_{\alpha}(t')>|^2 dt'$$

$$= \sum_{i,j} V_{\alpha i} V^*_{i\beta} V_{j\beta} V^*_{\alpha j} \cos \frac{\Delta m^2_{ij}}{2E} x \qquad (7)$$

where Δt is the time resolution of the detector, and we have assumed that Δt is large enough so that E_i and E_j have to be almost the same, $E_i = E_j = E$.

$$\Delta m^2_{ij} = m_i^2 - m_j^2 \qquad (8)$$

$N_{\alpha\to\beta}(x,E)$ means the transition probability of a neutrino ν_α with energy E at x = 0 to be a neutrino ν_β at x. We notice that

$$\sum_\beta N_{\alpha\to\beta}(x,E) = 1 \tag{9}$$

This is the expression of the conservation of the flux of probability.

In an ordinary case, only left-handed neutrinos are produced and only left-handed neutrinos can be detected. Suppose ν_α is a left-handed neutrino and in Eq. (9) we sum over only left-handed neutrinos, then we get the number of the left-handed neutrinos in total

$$N_\alpha^{NC}(x,E) = \sum_{\beta(L)} N_{\alpha\to\beta}(x,E) \tag{10}$$

Of course we have

$$N_\alpha^{NC}(x,E) \leq 1 \tag{11}$$

The equality happens only when there are no right-handed neutrinos in the set labeled by β or mixings between left-handed neutrinos and right-handed antineutrinos are zero. Equation (10) is also for the number of neutral current events because the neutral current interactions are diagonal and proportional to the flux of left-handed neutrinos in total.

A typical function N(x,E) normed by the spectral function f(E) is shown in Fig. 1 when only one mass difference is concerned. The curve is a triangle function of x/2E. This curve will be measurable if

(1)
$$\Delta\left(\frac{x}{2E}\right) << \frac{1}{\Delta m^2} \tag{12}$$

where Δ(x/2E) is the uncertainty of the measurement of x/2E, which is caused mainly by the size of the source and the uncertainty of the energy measurement. If this condition is not satisfied, i.e., the Δm^2 is too big for given x and E, the oscillation part will be wiped out and the flux becomes a constant relative to the mixing angles as follows

$$N_{\alpha\to\beta}(x,E) = \sum_i |V_{i\alpha}|^2 |V_{i\beta}|^2 = \text{const.} \tag{13}$$

and the diagonal transition probability has a low bound

$$N_{\alpha\to\alpha}(x,E) = \text{const.} \geq \frac{1}{N} \tag{14}$$

where N is the number of the species of neutrinos. For instance, if there are three species of left-handed neutrinos, then $N = 3$; if for each left-handed neutrino we have also a right-handed neutrino, then $N = 6$. If the oscillation part of the number of the neutral current events in Eq. (10) is wiped out, then the flux of the neutral current becomes a constant and has a low bound

$$N_\alpha^{NC}(x,E) = \text{const.} \geq \frac{1}{2} \tag{15}$$

Of course, if the oscillation part of the flux is wiped out, we will not be able to get any information about the mass of neutrinos from the flux measurement, except a low bound of mass if we know the intensity and the size of the source beforehand.

(2) The relative mixing is not too small as the oscillation terms escape from the sensitivity of the detector. By increasing the statistics of the experiment, we can measure smaller and smaller mixing angles.

(3)

$$\frac{x}{2E} \sim \frac{1}{\Delta m^2} \tag{16}$$

If x/2E is too small, then the oscillation cannot be seen.

If all ν_α in Eq. (5) are left-handed neutrinos, then we get oscillations of type I only. The unitary transformation between set ν_α and set ν_i is similar to the Kobayashi-Maskawa matrix[10] between (d',s',b') and (d,s,b). We cannot see oscillations but mixing angles (or the K-M matrix[10]) in the quark case because the mass differences are too big here and the condition of Eq. (12) is not satisfied.

In the case of only one left-handed and one right-handed neutrino, we have pure type II oscillation. This oscillation is quite similar to the well known K^0-$\overline{K}^0$ oscillation[11] and the recently discussed neutron-antineutron oscillation.[12] If there is more than one family, we get generally mixed oscillations of types I and II.

III. POSSIBLE MASS PATTERNS OF NEUTRINOS[7,13,14]

The situation of oscillations and its measurability depend on the mass pattern of the neutrinos. Let us discuss the case when there is only one left-handed neutrino and one right-handed neutrino. The case of more than one family of neutrinos is an extension of this discussion and the case without right-handed neutrinos is a special example.

How can ν-ν^c system have two masses? Suppose ν and ν^c are Dirac neutrinos

$$\nu^c = \mathbb{C}\ \nu\ \mathbb{C}^{-1} \tag{17}$$

and

$$\mathbb{P} \begin{pmatrix} \nu \\ \nu^c \end{pmatrix} \mathbb{P}^{-1} = \begin{pmatrix} \nu \\ -\nu^c \end{pmatrix} \tag{18}$$

where $\mathbb{C}$ and $\mathbb{P}$ are charge conjugation and space inverse operator, respectively. We take normal definition of these operators as follows

$$\mathbb{C}\ \psi\ \mathbb{C}^{-1} = C\ \overline{\psi}^T \ , \ C = i\gamma^2\ \gamma^0 \tag{19}$$

and

$$\mathbb{P}\ \psi(\vec{x},t)\mathbb{P}^{-1} = \gamma_0\ \psi(-\vec{x},t) \tag{20}$$

also the definition of chirality states as follows

$$\nu_{\substack{L\\R}} = \frac{1}{2}(1 \mp \gamma_5)\nu$$

$$\bar{\nu}_{\substack{L\\R}} = \frac{1}{2}\,\nu^{\dagger}(1 \mp \gamma_5)\gamma_0 \tag{21}$$

We notice that at the limit $m \to 0$, a particle with left-handed chirality ν_L has left-handed helicity whereas an antiparticle with left-handed chirality $(\nu_L)^c$ has right-handed helicity.

$$(\nu_L)^c = (\nu^c)_R \quad , \quad (\nu_R)^c = (\nu^c)_L \quad .$$

We have the most general mass terms of one-family neutrinos in the Lagrangian as follows

$$a\,\bar{\nu}_R\,\nu_L + b(\overline{\nu^c})_R\,\nu_L + c\,\bar{\nu}_R(\nu^c)_L + \text{h.c.} \tag{22}$$

Here as a convention we call the term a the Dirac mass and the terms b and c the left-handed and the right-handed Majorana masses, respectively. The Majorana mass terms violate the lepton number conservation with $\Delta L = \pm 2$. Defining

$$\psi_L = \begin{pmatrix} \nu_L \\ (\nu^c)_L \end{pmatrix} \quad , \quad \psi_R = \begin{pmatrix} \nu_R \\ (\nu^c)_R \end{pmatrix} \tag{23}$$

as the eigenstates of interactions, we can rewrite Eq. (23) as

$$\bar{\psi}_R\, M\, \psi_L + \text{h.c.} \tag{24}$$

where

$$M = \begin{pmatrix} a & c \\ b & a \end{pmatrix} \tag{25}$$

is the mass matrix. The diagonal elements are the same because of the CPT theorem. In order to make M diagonalized, let us define two unitary matrices U_R and U_L (m_- and m_+ are positive numbers) such that

$$U_R \, M \, U_L^{\dagger} = \begin{pmatrix} m_- & 0 \\ 0 & m_+ \end{pmatrix} e^{i\phi} \tag{26}$$

we also define eigenstates of masses

$$\chi_L = \begin{pmatrix} \chi_- \\ \chi_+ \end{pmatrix}_L = U_L \, \psi_L \tag{27}$$

$$\chi_R = \begin{pmatrix} \chi_- \\ \chi_+ \end{pmatrix}_R = U_R \, \psi_R \tag{28}$$

If $\phi = 0$, the mass term Eq. (22) becomes

$$m_- \bar{\chi}_- \chi_- + m_+ \bar{\chi}_+ \chi_+ \tag{29}$$

where

$$\chi_{\mp} = \chi_{\mp L} + \chi_{\mp R} \tag{30}$$

If there are three families of neutrinos, our mass matrix will be a 6×6 matrix

$$M = \begin{pmatrix} A & C \\ B & A \end{pmatrix} \tag{31}$$

where A is the Dirac mass matrix and B and C are the Majorana mass matrices. When diagonalizing Eq. (31) to $\begin{pmatrix} m_- & 0 \\ 0 & m_+ \end{pmatrix}$ with m_- and m_+ 3×3 diagonal matrices, the general solutions are very complicated, but there are four very interesting extreme cases. The discussion of these four mass patterns will give us a good insight into the physics.

<u>Pattern a)</u> A = C = 0, if there are no right-handed neutrinos. Then χ_- is a linear combination of the left-handed neutrino and its antineutrino

$$\chi_- = \nu_L + (\nu_L)^c \tag{32}$$

and χ_- is a Majorana neutrino $\chi_-^c = \chi_-$. The mass spectrum of neutrinos has only three lines (Fig. 2a). Of course, in this case only oscillations of type I are possible.

Pattern b) $B = C = 0$, if there are no lepton number violated interactions. The mass spectrum of neutrinos is shown in Fig. 2b, which is similar to case a), but every line is doubly degenerate.

Pattern c) $B = 0$, $C >> A$. Here we compare two matrices by comparing all nonzero elements. In this case we have

$$m_- \simeq (A^d)^2/C^d \quad , \quad \chi_- \simeq \nu_L + (\nu_L)^c \tag{33}$$

$$m_+ \simeq C^d \quad , \quad \chi_+ \simeq \nu_R + (\nu_R)^c \tag{34}$$

The mass spectrum is shown in Fig. 2c.

Pattern d) all A, B and C are small as to match the present bounds[3] on neutrino mass. The shape of the spectrum is ugly. There is a much more interesting special case of this pattern when each doubly degenerate lines in Fig. 2b splits into two lines with very small space (Fig. 2d) as the fine structures in optics.

Only in pattern d) may there exist observable oscillations of type II.

IV. MORE PHENOMENOLOGY OF THE OSCILLATIONS[4,7,8]

Let us return to our discussion with one family, see Eq. (22). To simplify the discussion, we concentrate on the case when $b = c << a$. From Eq. (30), we get

$$\chi_{\mp} = \frac{1}{\sqrt{2}} (\nu \pm \nu^c)$$

$$|m_- - m_+| = 2|b| \equiv \Delta m \tag{35}$$

$$\mathbb{C}\ \chi_{\mp}\ \mathbb{C}^{-1} = \mp\ \chi_{\mp}$$

$$\mathbb{P}\ \chi_{-}\ \mathbb{P}^{-1} = \chi_{+} \tag{36}$$

According to Eq. (7), we have

$$N_L(x,E) = \frac{1}{2}\left(1 + \cos\frac{\Delta m^2}{2E}\,x\right) \tag{37}$$

Because the right-handed neutrino escapes from our detection, we do not write it here.

Going back to the three family case, choosing spectrum Fig. 2d and the simplifying form Eq. (35) as an example, we get

$$N^{*}_{\alpha\to\beta} = \frac{1}{2}\sum_{i=1}^{3}|V_{i\alpha}|^2|V_{i\beta}|^2\left(1 + \cos\frac{\Delta m_i^2}{2E}\,x\right)$$

$$+\ 2\sum_{i<j} V_{\alpha i}\ V^{*}_{i\beta}\ V_{j\beta}\ V^{*}_{\alpha j}\ \cos\frac{\Delta m_{ij}^2}{2E}\,x \tag{38}$$

where α and β are left-handed neutrinos. $V_{i\alpha}$ are transformation matrices between the eigenstates of interaction and the eigenstates of masses when omitting the fine structures. $\Delta m_i^2 = (m_i^+)^2 - (m_i^-)^2$, $\Delta m_{ij}^2 = m_i^2 - m_j^2$, $m_i = \frac{1}{2}(m_i^+ + m_i^-)$. $\Delta m_k^2 << \Delta m_{ij}^2$ for any $i \neq j$ and any k is the characteristic of the spectrum in Fig. 2d. According to Eq. (10), the number of the neutral current events is

$$N_\alpha^{NC} = 1 - \frac{1}{2}\sum_{i,\beta}|V_{i\alpha}|^2|V_{i\beta}|^2\left(1 - \cos\frac{\Delta m_i^2}{2E}\,x\right) \tag{39}$$

It is distance dependent because there are also type II oscillations. However, if $\Delta m_i^2 = 0$ (i.e., there are no type II oscillations), Eq. (39) becomes identical to Eq. (9), the measurable flux conservation. The dependence of the number of the neutral current events on the distance is the characteristic of the neutrino-antineutrino oscillations. This characteristic is preserved in general, as was shown in Eq. (10).

The observation of neutrino oscillations of both types I and II will be simplified if the neutrino mass pattern is that shown in Fig. 2d. In this case we can use the advantage $\Delta m_k^2 \ll \Delta m_{ij}^2$ for any $i \neq j$ and any k. For instance, first we use small source (like reactor or accelerator) and short distance to measure Δm_{ij}^2 and find (compare the condition of Eq. (16))

$$N_{\alpha\to\beta} = \delta_{\alpha\beta} - 2 \sum_{i<j} V_{\alpha i} V_{i\beta}^* V_{j\beta} V_{\alpha j}^* \left(1 - \cos \frac{\Delta m_{ij}^2}{2E} x \right)$$

and

$$N_\alpha^{NC} = 1$$

Then we use big source (like the sun) and large distance to measure Δm_i^2 and find (compare the condition of Eq. (12)) in the simplest case Eqs. (35) and (36)

$$N_{\alpha\to\beta} = \frac{1}{2} \sum_{i=1}^{3} |V_{i\alpha}|^2 |V_{i\beta}|^2 \left(1 + \cos \frac{\Delta m_i^2}{2E} x \right) \tag{40}$$

and

$$N_\alpha^{NC} = \text{Eq. (39)}$$

Will nature repeat the same trick that happened in the spectrum of an atom, then in the spectrum of the nuclei? We do not know.

V. MODELS WHICH GIVE DIFFERENT NEUTRINO MASS PATTERNS

Now we are going to give a brief discussion about the neutrino mass pattern in some grand unification models of weak, electromagnetic and strong interactions. The constituent models of leptons and quarks are proliferating recently.[15-18] We will discuss the neutrino mass pattern in some of these models also.

In the original SU(5) model of Georgi-Glashow,[19] where the Fermions are in 5 and 10^* representations, only left-handed neutrinos, which are in 5-plets, and Higgs 24 and 5-plets are involved. Incidentally, there is a global B-L conservation[20] (B is the baryon number, L is the lepton number) which makes neutrino massless. However, if we have Higgs 10 or 15-plet, B-L will be broken[20] and a Majorana neutrino mass for the left-handed neutrinos will exist. In Fig. 3 we show a two-loop diagram which involves the Higgs 10-plet and contributes a Majorana mass to the neutrino

$$m_\nu \sim g \frac{m_d^2}{m_L} \lambda \left(\frac{\alpha}{\pi}\right)^2 \sim 10^{-7} \frac{m_d^2}{m_L} \tag{41}$$

where g is the gauge coupling constant, $\alpha = g^2/4\pi$, m_d and m_L are masses of the down quarks (d,s,b) and the W bosons, respectively. λ is the coupling constant between Higgs 10 and 5-plets. The Yukawa coupling constant is nearly $g \frac{m_d}{m_L}$. Because of the chirality change of the Fermion line, the S-matrix is proportional to m_d anyway. Thus the SU(5) model is the model that suits mass pattern a). Especially for the mass of the τ neutrino, Eq. (41) gives a mass ~60 eV. We notice here that a Majorana mass term comes from the violation of B-L conservation.

The Weinberg-Salam model may suit mass pattern b) with doubly degenerate lines if there are Yukawa couplings between the right-handed neutrinos and the left-handed doublets. A lot of authors drop these terms by assuming no right-handed neutrinos. The SU(2) instanton may not change SU(2) doublet to singlet, so it cannot split the double line by a very small space.[21]

The SO(10) model,[22] where all Fermions are in 16 spinoral representations can, in principle, suit any mass pattern one wants, especially mass patterns c) and d). Most authors doing SO(10) model favor pattern c). Giving one neutral color singlet component of Higgs 126, which transforms as $(1,3,10^*)$ under the subgroup $SU(2)_L \times SU(2)_R \times SU(4)$ and is responsible for the Majorana mass of the right-handed neutrino, a huge vacuum expectation value 10^{14} GeV or so, whereas the other neutral color singlet component of 126, which transforms as (3,1,10) under the subgroup $SU(2)_L \times SU(2)_R \times SU(4)$ and is responsible for the Majorana mass of the left-handed neutrino, a zero vacuum expectation value, we break the SO(10) down to $SU(3) \times SU(2) \times U(1)$. Then we use 10-plet Higgs to break $SU(2) \times U(1)$ down to U(1) at 300 GeV and give the Fermions Dirac masses. This is the simplest way to make mass pattern c) in the SO(10) model. We notice that the components which may get VEV in $(1,3,10^*)$ and (3,1,10) of 126 have B-L = ±2 respectively. Thus giving any one of them VEV means to break B-L gauge symmetry.

However, we may have two methods to arrange mass pattern d) in the SO(10) model.[14]

Method A. We make the model by following three steps: 1) give the neutral color singlet components of Higgs 45, which transform as (1,1,15) and (1,3,1) under the subgroup different VEV's to break SO(10) down to $SU(3) \times SU(2)_L \times U(1)_R \times U(1)$; 2) give the neutral components of Higgs 10 and 126, which transform as (2,2,1) and (2,2,15), respectively, under the subgroup $SU(2)_L \times SU(2)_R \times SU(4)$ VEV's close to 300 GeV and make the Dirac mass of neutrino zero at tree level. Here we take the extreme smallness of the neutrino mass as an ansatz for the assignment of the VEV's and the Yukawa couplings. Because there are 4 VEV's (two of them are relevant to neutrino masses) and 2 Yukawa coupling constants, we can definitely make such arrangement; 3) give the neutral components in $(2,2,10)$ and $(2,2,10^*)$ of Higgs 210, which have non-zero B-L, VEV's close to 300 GeV.

In such a model, the Dirac mass of the neutrino is given at one loop level in Fig. 4a and is nearly equal

$$m_\nu \sim \frac{m_L}{m_R}\left(\frac{\alpha}{\pi}\right) m_\ell \lesssim 10^{-5}\, m_\ell \tag{42}$$

where m_R is the mass of the right-handed w-boson. The Majorana mass of the neutrino is given at two loop level in Fig. 4b, which is extremely small

$$\Delta m_\nu \sim g\,\frac{m_u^2}{m_L}\,\frac{V^2}{M^2}\,\lambda_1\,\lambda_2\left(\frac{\alpha}{\pi}\right)^2 \sim (10^{-20}\text{-}10^{-34})\,\frac{m_u^2}{m_L} \tag{43}$$

where M is the mass scale of grand unification or the mass of the relative Higgs. $V \sim 300$ GeV. For the τ neutrino, $\Delta m_\nu \sim 10^{-9}\text{-}10^{-23}$ eV.

<u>Method B</u>. The first two steps are the same as those in Method A, but the third becomes: 3) give the two neutral components of Higgs 16, which have non-zero B-L also, VEV's close to 300 GeV. The Dirac mass of the neutrino is the same as Eq. (42). The Majorana mass is given in the two loop diagram[23] shown in Fig. 5, and we have

$$\Delta m_\nu \sim g\,\frac{m_u^2}{m_L}\,\frac{V^2}{M^2}\,\lambda\left(\frac{\alpha}{\pi}\right)^2 \sim (10^{-18}\text{-}10^{-32})\,\frac{m_u^2}{m_L} \tag{44}$$

The neutrino mass pattern in the E_6 model[24] is quite similar to that in the SO(10) model. Especially, we can also get mass pattern d) in the simplest E_6 model. In the simplest E_6 model, all fermions are in 27 and Higgs in $27 \times 27 = (27^* + 351)_S + 351_A$ only. To make mass pattern d), the principles of the model building are: 1) The neutrino mass at the tree level equals zero, whether it is the Dirac mass or the Majorana mass. 2) All the neutral components which transform under the subgroup $SU(2)_L \times SU(2)_R \times SU(3)_C$ as the representation (2,2,1), (1,2,1) or (2,1,1) get the vacuum expectation values near 300 GeV. The neutrino will get a Dirac mass at the one loop level similar to Eq. (42) and a Majorana mass at the two loop level similar to Eq. (44).

We notice that in both the SO(10) model and the E_6 model, a Majorana mass of the neutrino (whether left- or right-handed) also comes from the violation of B-L gauge symmetry.

Mass and mixing patterns among neutrinos in different families could be discussed using the horizontal symmetry (discrete[25] or continuous[26] groups) or huge grand unification models.[27] Some results about neutrino masses and mixings have been found in this area.[28] We are not going to details here. What we want to say is that in any huge grand unification models, the interaction among different families is gauged and unified with the interactions within a family. At the same time, the original family structure, which is our first insight to the nature, is destroyed and reorganized according to the way it fills in the representation of the huge groups.

What do neutrino masses look like in the constituent models? This may be another point the constituent models[15-18] can touch on at their primary stage of development. Let us take the Tarazawa model[16] and the Harari model[17] as two examples.

In the Tarazawa model,[16] the constituents are three kinds of Fermions, all with spin $\frac{1}{2}$. They are the carriers of weak SU(2) quantum number W_i (i=1,2), the carriers of color SU(4) quantum number C_α ($\alpha = 0,1,2,3$) and the carriers of generation numbers h_g (g=1,2,3,...). Leptons and quarks are made up by picking up one from each kind and combining them together like

$$(W_i \; C_\alpha \; h_g) \tag{45}$$

In the Harari model,[17] there are only two spinor constituents, T and V, with the electric charges $-\frac{1}{3}$ and 0, respectively. The construction of the leptons and quarks is given in Eq. (46)

$$\begin{array}{lll} e^+(T\,T\,T) & & \\ \nu(V\,V\,V) & & \\ u_1(T\,T\,V) \;, & u_2(T\,V\,T) \;, & u_3(V\,T\,T) \\ \bar{d}_1(T\,V\,V) \;, & d_2(V\,T\,V) \;, & d_3(V\,V\,T) \end{array} \tag{46}$$

The next generations are some exitation of the first generation. Both of these models give very interesting results in a very simple way. For instance, in the Harari model the proton decay is just a rearrangement of the constituents

$$p\{(T\,T\,V)(T\,V\,T)(\overline{V}\,\overline{V}\,\overline{T})\} \rightarrow \underset{e^+}{(T\,T\,T)}\underset{\pi^o}{\{(\overline{V}\,\overline{V}\,\overline{T})(V\,V\,T)\}} \tag{47}$$

Although these models suffer from some difficulties (e.g., the Harari model has a statistical problem), the constituent models are attractive and interesting both from the viewpoint of physics and philosophy. If we take them seriously, and go ahead as far as possible, we would say that all these models[16-18] give neutrinos mass pattern d) in Section III, probably like what is shown in Fig. 2d with fine structures.

The reasons are: First, they have right-handed neutrinos in their models because the neutral constituents have to have two chiralities to get correct quantum numbers for the quarks; second, if we assume that the main contribution to the mass of the composite particle is from its inner interactions, then the Dirac mass of a neutrino must be much bigger than the Majorana mass of the neutrino.

Some constituent models[17-18] see particles grouped in families as a basic fact and see the interaction among families as leaking interactions. So in the zeroth order of the masses of the fermions (leptons and quarks), they have at least SO(10) symmetry.[18]

VI. THE IMPLICATIONS OF NEUTRINO OSCILLATIONS

A complete neutrino oscillation experiment may give us the information about:

1) The mass pattern of the neutrinos,

2) The mixing matrix among different families,

3) The mixing matrix between the left-handed neutrinos and the right-handed antineutrinos if the right-handed neutrinos exist.

However, if we find only neutrino-neutrino oscillations (type I), we can say nothing definite about the right-handed neutrinos because there are two possibilities: 1) There are no right-handed neutrinos, or 2) there are right-handed neutrinos, but the $\Delta L = 2$ interactions are too weak as to give almost mass pattern b) or the right-handed neutrinos are too heavy to give mass pattern c). If we also find neutrino-antineutrino oscillations (both type I and type II), then we can say definitely that there are right-handed neutrinos and there are $\Delta L = 2$ interactions.

To have an insight on the mass pattern of neutrinos and the interactions among neutrinos, we would like to cite t'Hooft's words[29] here:

"At any energy scale μ, a physical parameter or set of physical parameters $\alpha_i(\mu)$ is allowed to be very small only if the replacement $\alpha_i(\mu) = 0$ would increase the symmetry of the system."

We notice that if the Majorana mass of the neutrino equals zero, we will have B-L conservation and may be a massless B-L gauge boson. If all the masses of the neutrinos equal zero, or the mixing matrix is trivial, then we will have the family lepton number conservation to a very high extent, for instance, the decay rate of $\mu \to e\gamma$ is (see Fig. 6)

$$\Gamma(\mu \to e\gamma) \propto \sin^2\theta_c \frac{m_\mu^5}{M^4} \left(\frac{\alpha}{\pi}\right)^6 \frac{m_F^4}{M^4} \tag{48}$$

where θ_c is the Cabibbo like angle in quark-lepton interactions,[30] m_F is the typical mass of fermions and M is the mass scale of grand unification. The extra $(m_F/M)^4$ factor comes from the GIM mechanism.[31] Experimentally, the masses of the left-handed neutrinos are very small, so we need not acquire the mixing matrix of left-handed neutrinos too much to meet the demands of a very good family lepton number conservation law.

However, if we want to have an extremely good B-L conservation (much better than B and L separate conservations at any energy scales), we should give neutrinos (left- or right-handed) very small Majorana masses, as we did in the SU(5), SO(10) and E_6 models (see Figs. 4 and 5). From the viewpoint of cosmology, to get net survived baryon number in the universe, a B-αL, where α is a constant, conservation law is wanted at the temperature 10^8-10^9 GeV.[32] The models giving mass pattern in Fig. 2d with fine structures satisfy this acquirement automatically. The exciting low energy physics of these models, which give neutrino mass pattern fine structures, is that we would expect two neutral gauge bosons[33] with masses at nearly the same energy level as that of w boson.[9] In the near future, we will be able to answer whether or not this is true.

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   $m_\nu < 60$ eV in β decay of nucleus

   $m_\nu < 0.5$ MeV in μ decay or π decay

   $m_\nu < 250$ MeV in τ decay

This means the masses of the neutrinos are so small as to escape from detection by spectrometers. However, a β spectrometer using H^3 decay as the source got a positive result

$$14 \text{ eV} < m_{\nu_e} < 46 \text{ eV} \quad .$$

See: V. A. Lyubimov, E. G. Novikov, V. Z. Nozik, E. F. Tretyakov and V. S. Kosik, ITEP-62, Moscow. Of course, this is a big progress. But as electron neutrino may not be a mass eigenstate, the result suffers the ambiguity and the value may be some weighted average value of a few masses. See the text.

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FIGURE CAPTIONS

1. A typical function N(x,E), see Eqs. (7) and (10).
2. The mass spectrum of the neutrinos: Pattern a) Three lines, if there are only left-handed neutrinos. Pattern b) Three double degenerated lines in the W-S model. Pattern c) Neutrino masses in the SO(10) model with terrible heavy right-handed leptons. Pattern d) An extreme case with double line fine structures.
3. A two loop diagram which contributes a Majorana mass to the neutrino in the SU(5) model with 10-plet Higgs. See caption of Fig. 4b.
4. The neutrino mass in the SO(10) model, Method A: a) The one loop diagram that gives neutrino a Dirac mass through the mixing between right- and left-handed gauge bosons. b) Two loop diagram that gives neutrino a Majorana mass. The waved lines are gauge bosons. The dotted lines are Higgs with their dimensions of SO(10) representations along the lines. The notation "x" means VEV. The numbers in parentheses show the transformation properties of the components under the subgroup $SU(2)_L \times SU(2)_R \times SO(6)$ which develop VEV. Φ is the Higgs linear combination of 10 and 126 which gives neutrino zero mass at tree level.
5. A two loop diagram which gives the neutrino a Majorana mass in the SO(10) model, Method B. See the caption of Fig. 4b.
6. $\mu \to e\gamma$ process when masses of neutrinos are zero.

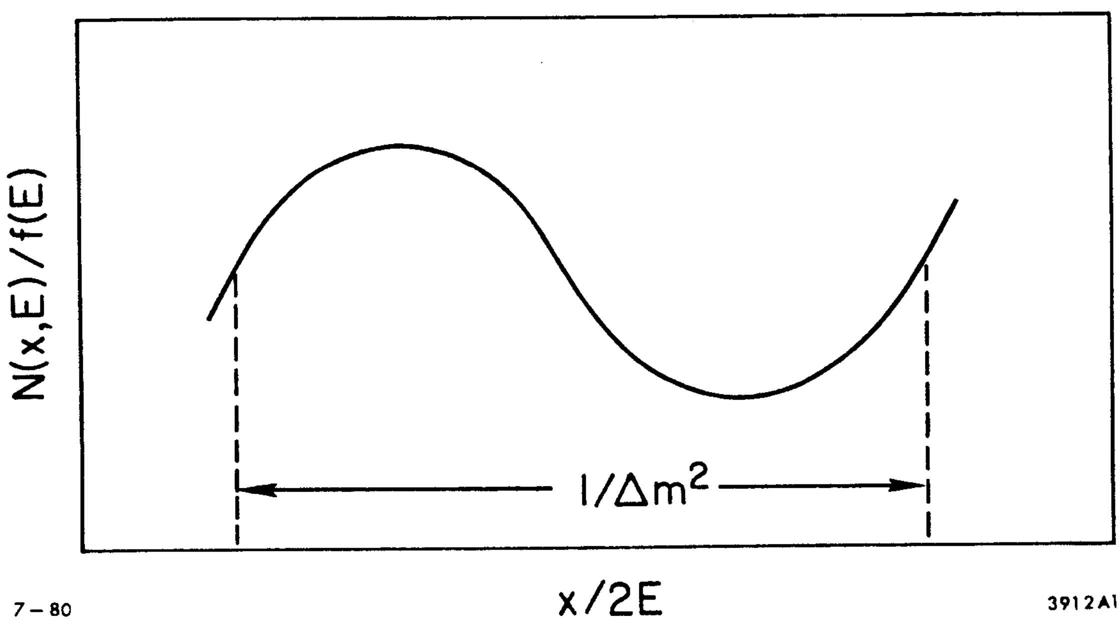


Fig. 1

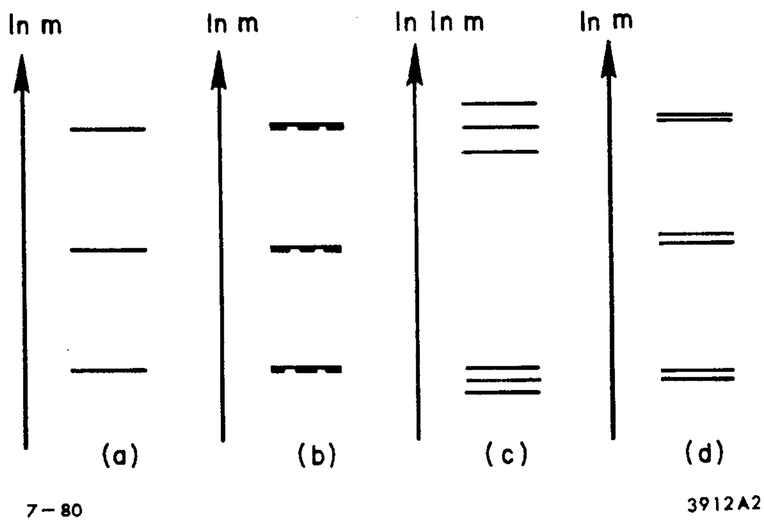


Fig. 2

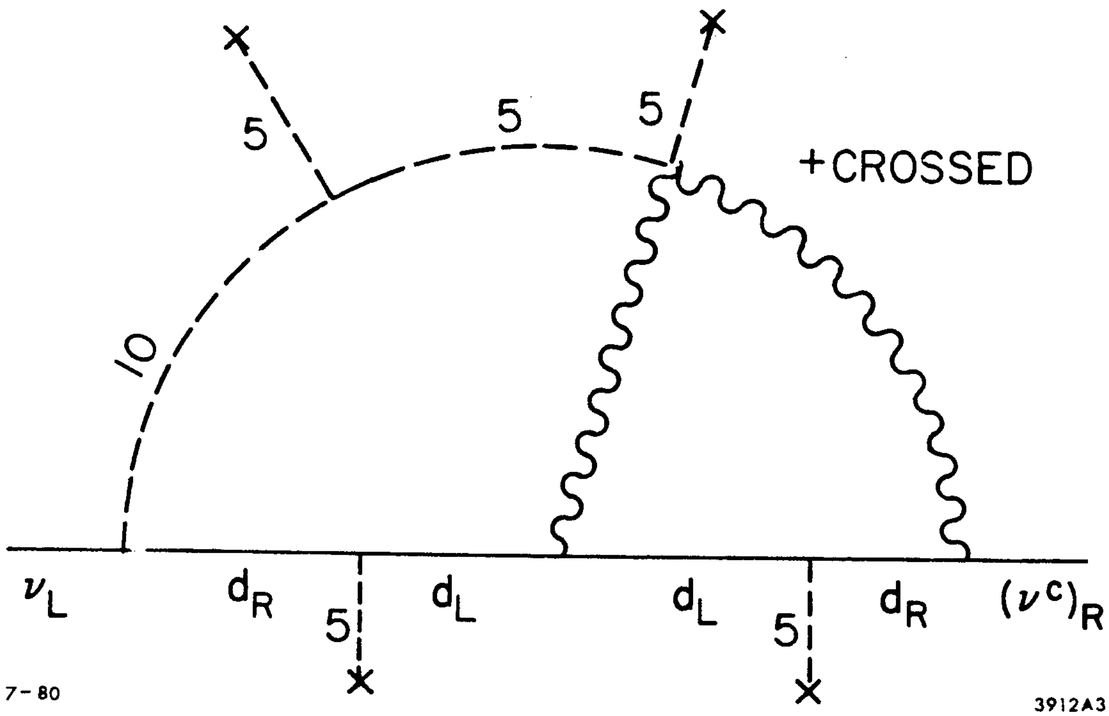
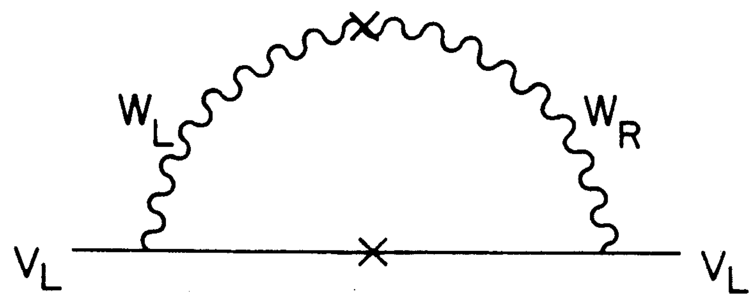
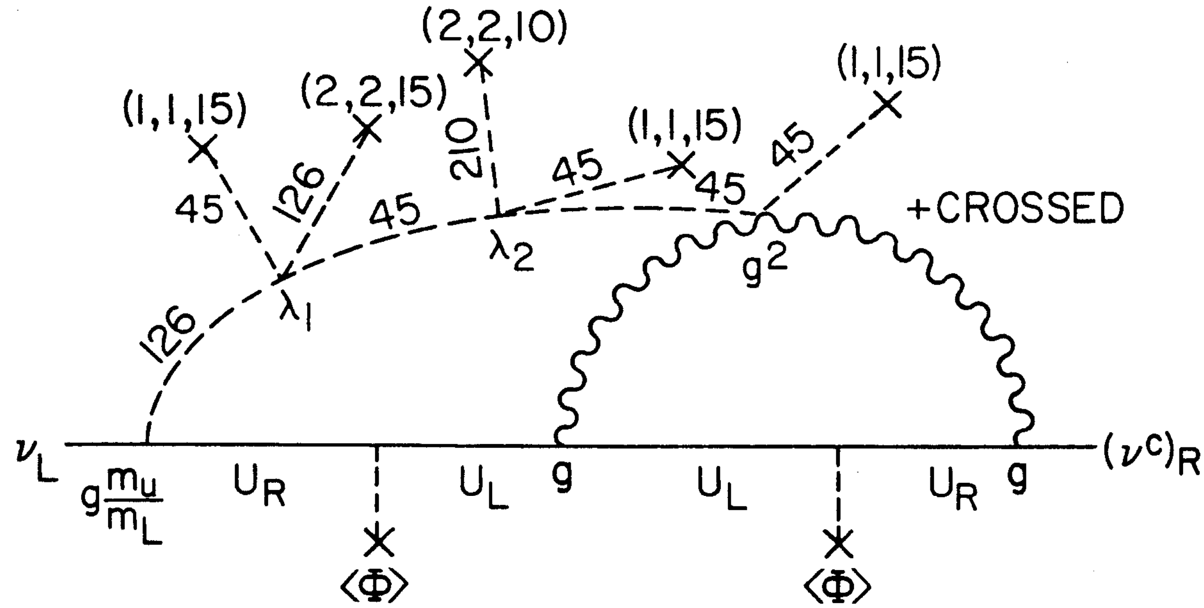


Fig. 3



(a)



7-80

(b)

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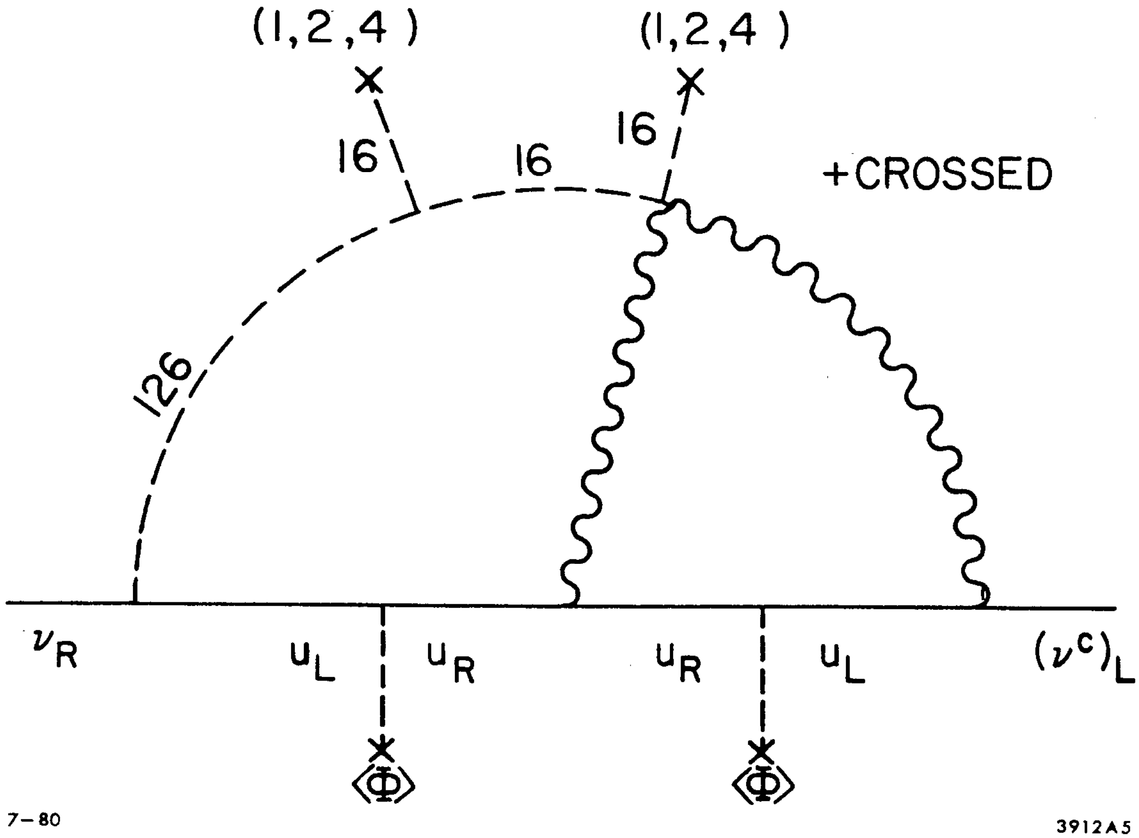
Fig. 4



(1,2,4)
(1,2,4)
16
16
16
+CROSSED
126
ν_R
u_L
u_R
u_R
u_L
$(\nu^c)_L$
$\langle\Phi\rangle$
$\langle\Phi\rangle$
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3912A5

Fig. 5