STRINGS, CONDUCTIVE AND OTHERWISE

Jorge Willemsen

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

1. The Physical Nature of the String

In his lectures, Dr. Dragon has reviewed for us the theory of the relativistic string, discussing the formal problems associated with incorporation of the constraints of the theory, and the unphysical features encountered in attempting a canonical quantization.

In the course of this seminar I want to describe a model worked out in collaboration with C. Carlson, L. N. Chang, and F. Mansouri, which was motivated by an attempt to circumvent some of these difficulties in quantization by identifying canonical variables different from the coordinates of the string, already at the classical level.

Before I get to our model, however, I would like to take time to remind you of some of the physical pictures which have been proposed as candidates for the underlying structure of the string. The "string model" itself is, after all, an attempt to understand what physical structures can give rise to Veneziano-type scattering amplitudes, and it is natural to proceed a step further and ask, "What makes up the string, and how does it hold together?" Hopefully, this discussion will provide physical motivation for our "conductive string" model.

The first conceptual problem one is faced with is in visualizing how a hadron could look like a one-dimensionally extended object at all. It is reasonable to expect that what something looks like depends upon how we look at it, and I want to remind you that field-theoretic models have already been around for a long time, in which the hadrons do appear as one-dimensionally extended matter distributions. The hadrons are supposed to "look" this way to the extent that in a high energy, low momentum transfer collision, the inclusive particle distribution in the central region provides us with a "snapshot" of the hadrons' constituent matter distribution. A convenient way to display this information is by drawing a $p_{\perp}$ - $p_{\parallel}$ phase space plot

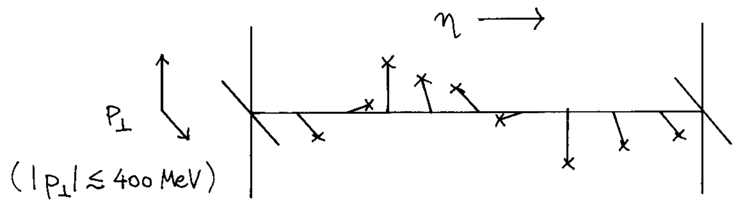

Figure 1

If, as the number of partons increases, we can smoothly interpolate between the points on the plot with a single curve, we will see the hadron as a string of partons.

Does this parton string have anything to do with the dualists' string? If we transform $p_\perp$ to $x_\perp$, but keep the "length" axis as the longitudinal momentum fraction, we do indeed get just the picture that emerges from the Goddard, Goldstone, Rebbi, and Thorn (GGRT) parametrization of the Nambu string action. The original papers on the subject should be consulted to see that this is so (here and elsewhere), because I will not have time to show you the details. But this very nice mesh between the intuitive parton picture and the actual mathematical formalism of the string model should be kept in mind when thinking about the physics of the string.

This simple physical picture is not without its faults, of course. As indicated in the drawing, we cannot really make contact with what is happening to the "leading particles" in the hadrons. Phenomenologically, current induced reactions suggest that partons with the quantum numbers of the quarks carry the most significant fractions of the hadron's momentum. Although Harari-Rosner diagrams suggest a picture where the ends of the string are like quarks and antiquarks, with an essentially "neutral" middle, the mathematical string formalism does not genuinely reflect this fact in any way.

In addition, this picture reflects only one component of the transverse momentum distribution, the exponentially bounded part dying off at $\simeq$ 400 MeV. We now know that there are other parts of the hadronic wavefunction, extending out much further than this with a power-law behavior. The string picture is no more able to account for this region than the simple "wee", "soft" parton picture that motivates the Feynman-Wilson phase space plot.

I want to milk this soft parton picture for two more features.

The first is that, as the energy increases, the number of produced particles increases logarithmically. On a rapidity plot, this means, of course, that the central region is filled uniformly. We expect, in the snapshot, that there will be a small likelihood for multiple occupancy at a given rapidity value. This is necessary for our hadron to extend out in one dimension, rather than filling up like a three-dimensional jelly. However, this still leaves a question about whether the hadron is one big string, or a series of short strings, i.e., whether there is any correlation in $x_\perp$ space between neighboring points in longitudinal momentum. I will return to this question momentarily.

My second point is not really derived from the parton picture, but rather an input to it originating from the notion of duality. Duality tells us there is an intimate relationship between the behavior in the Regge region, which the soft parton concepts describe, and the resonance region. That is, there should be a connection between the extended, "wee sea" component of the hadronic wave-function, and the spectrum of hadrons that is observed. It is natural to anticipate that, if there is such a relationship, it arises from the collective behavior of the wee partons in the sea. The collective excitations of the sea give rise to the observed spectrum of hadrons. To do this, of course, one cannot adhere too closely to the strictly "free" notion of partons. Allowance must be made for parton-parton interactions, which can give rise to sequences of excitations, but do not destroy the basic linear extension of the hadrons.

At this point, it is natural to wonder if there is any model, no matter how simple, that can actually give rise to behaviors of the type we have been discussing. Many of the concepts of the parton model itself are realized in the field theoretic multiperipheral model, and this provides a natural point of departure for a first attempt to describe strings. To go from an initial Regge picture to a dual picture, one should generalize the class of graphs that are considered:

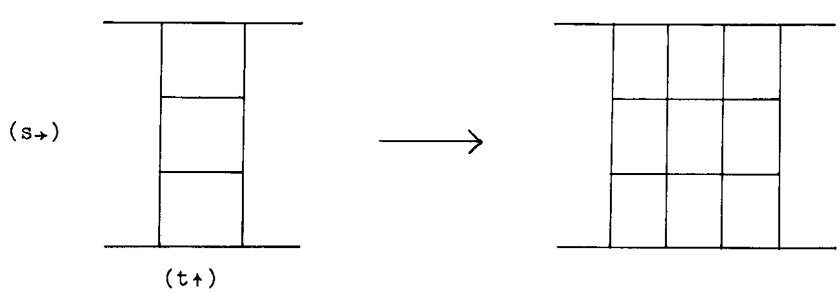

Figure 2

Kraemmer, Nielsen, and Susskind, and separately Gervais, Sakita, and Virasoro studied the properties of such "fishnet" diagrams, and elaborated the approximations under which such diagrams give rise to Veneziano-type scattering amplitudes. Not unexpectedly, this kind of behavior is obtained under the assumption that momentum flows smoothly and uniformly throughout the graph, so that propagators can be sensibly approximated by Gaussians. By so doing, one loses information about the short-distance structure of the constituent interactions, and this is already a strong hint as to the inherent limitations of the string picture.

The physical assumptions involved in jumping from a quantum field theory to a string picture are even more vividly portrayed in an old model due to Bjorken (Tel-Aviv lecture). This model is also discussed in detail by Kogut and Susskind in their Physics Report about partons. The basic idea is that one can examine the hadronic wavefunction in terms of the constituents, at a given time, in ϕ^3 theory,

$$\psi_{(n)}(t) \sim \langle \eta_1, \underset{\sim}{K}_{\perp,1}; \ldots \eta_n, \underset{\sim}{K}_{\perp,n} | u(t,-\infty) \quad \psi_0 | 0 \rangle \tag{1a}$$

$$\sim \prod_{j=1}^{n-1} \left[\frac{M^2}{2} - \sum_{i=1}^{j} \frac{K_i^2 + m^2}{2\eta_i} + \frac{L_j^2 + m^2}{2\beta_j} \right]^{-1} \left[\frac{\beta_{n-1}}{\beta_j} \right] \quad . \tag{1b}$$

Here η_j is the longitudinal momentum fraction of the jth parton, $(p_\parallel)_j/\Sigma_j(p_\parallel)_j$; and $(K_{\perp,j})$ is the transverse momentum of the jth parton. The quantities β_i are the sequential longitudinal momentum transfers down the chain, and $(L_{\perp,i})$ are the transverse momentum transfers. Equation (1b) is obtained using the rules of old-fashioned perturbation theory, for a single time ordering in which the cascade occurs as one long sequence. (Further, the calculation is performed in the infinite momentum frame. Alternately, one may work directly using "light-cone quantization". I will use the languages of these treatments interchangeably where there is no real ambiguity.)

Now, if we further <u>order</u> the momentum flow as in the multiperipheral model, $\beta_j \ll \eta_j$, and Fourier transform to transverse configuration space, we obtain

$$\psi_{(n)}(t) \to \left[\prod_{j=1}^{n-1} K_0(m|\underset{\sim}{x}_{\perp,j} - \underset{\sim}{x}_{\perp,j+1}|) \right] F(n) \tag{2}$$

Here K_0 is a Bessel function, which dies exponentially for transverse separations on the order of the length associated with a parton mass.

Let us examine the important qualitative features of this result:

1) Near neighbors in η are nearby in $\perp$ configuration space. The second parton "orbits" about the first, the third "orbits" about the second, etc., with the net result that the whole configuration random walks out in transverse configuration space.

This simple calculation provides, therefore, a justification, albeit loose, for the notion that the hadron is a single long string, since indeed the transverse coordinates of neighboring elements on the string are tightly correlated.

2) The transverse and longitudinal momentum dependence of the wavefunction factorize. This is a dynamical result, strongly dependent on the trivial nature of the ϕ^3 coupling. It is nonetheless a useful notion to hang onto, since the longitudinal momentum can then serve purely as a label for the points along the string, and one can as well write the transverse coordinates $x_{\perp,j}$ as $x_\perp(\eta_j)$.

3) As already noted, these results are obtained from looking at a special graph under a special approximation. To "derive" stringlike behavior from any field theory, one must of course make some special approximations. The value of having a simple model is, equally obviously, that one can see clearly just what the nature of the special approximations is. In addition to the approximations above, one needs still further assumptions plus a "leap of faith" to finally arrive at the string. However, these further steps suggest themselves naturally from the qualitative picture we've been pursuing. They are that:

a) Since the near-neighbors in rapidity are close together in configuration space, the residual soft interactions needed for the system to produce a spectrum are short range, and in fact between nearest neighbors;

b) Since the graph that motivates the thing neglects virtual pair formation in any link along the chain (these processes are actually down for the usual reasons in the $p_z \to \infty$ frame), and since the characteristic property of the partons that appears is the transverse coordinate, one can think of the $x_\perp(\eta_j)$ as the relevant dynamical variables in terms of which the residual dynamics can be described. We will work in first-quantization.

c) In the Regge region, it should be really the infinite "wee sea" that mediates the dynamics. We shall neglect, therefore, any

$[\Sigma_n P(n)]$, i.e., assume the parton configuration with an infinite number of partons is in some sense "dominant", so that a continuum approximation is possible. This is really a terrible assumption on two counts.

First of all, we seem to close the door on being able to go back and account sensibly for the "valence" configurations, where an infinite sea plays no role. Field theoretically, the wavefunction must be a sum over resolutions into configurations with all possible numbers of partons compatible with the overall quantum numbers of the hadron. Actually, the "conductive string" model suggests a way to deal with this situation, and I will discuss it further in Section III.

The second problem does not relate to whether we are doing sensible physics vis-a-vis field theory, but is a problem of internal consistency. It is that if we random-walk out in $x_\perp$-space with an infinite number of steps, we fill out all of the space, i.e., the hadron is infinitely big. Scaling the momentum from 0 to π, $[\theta/\pi = \int_0^\theta d\theta' \, P_{\parallel}(\theta')/ \int_0^\pi d\theta' \, P_{\parallel}(\theta')]$, one finds $< (x_\perp(\pi) - x_\perp(0))^2 >$ diverges logarithmically. Going backwards now, one naturally says any physical string is made out of constituents, and is not a true continuum. The paradox is that if we ask how fine-grained we should make our string so that the size of a "hadron" turns out to be the size of a hadron, we come up with a spatial cut-off so small as to be physically meaningless. That's why I say this is a problem of consistency.

Nevertheless, this problem is not troublesome for hadron-hadron-scattering. As I will remind you again later, scattering is involved with the overlap of the $P_{\parallel}$ ends of separate strings, and it is only when we really try to look inside the string that we find the inside is infinitely big. To discuss interactions with currents, for example, one has to subtract off $[<x_\perp^2(\pi)> + <x_\perp^2(0)>]$, each of which are also logarithmically divergent, to obtain finite results. Once this is done, the effective size of the hadron reduces to something like $[\ln 2]$. This is just an aside, but again points to the limitations we must remain aware of.

d) The final thing we must do is make the dynamics stringlike. Using the null-plane Hamiltonian to describe the residual interactions, we have

$$H_{eff} \sim \sum_i \frac{p_{\perp,i}^2 + m^2}{2\eta_i} + (x_{\perp,i} - x_{\perp,i+1})^2$$

$$\to \int_0^\pi d\theta \left[\left(\frac{\partial x_\perp}{\partial \theta}\right)^2 + \left(\frac{\partial x_\perp}{\partial \tau}\right)^2\right] + (\text{const.}) \qquad (3)$$

provided that the density $d\eta/d\theta$ = const. Only then do the (η_i) come out in the proper fashion to give a uniform string Hamiltonian in the continuum limit. It is gratifying that this requirement is also a mathematical property of the GGRT treatment of the string.

I stress once again that it is because of the rather precise manner in which the GGRT results fit the physical picture suggested by the parton model that I have dwelled on this model for so much time. Progress in attempting to understand hadrons as one dimensionally extended objects has not ended here, of course, and a lot of effort is currently going into incorporating more physics (such as hard, short range forces) into the structure of the theory from the very beginning. I prepared a set of (hopefully) pedagogic notes on the string model for the SLAC Summer Institute, entitled "The Beginner's String", in which references to the stimulating works of numerous authors may be found. I will not be able to go into details of these works here.

II. Lorentz Invariance

The parton model discussions may be helpful in providing some basis for insight into how a hadron can be a string, but in its mathematical formulation it hardly looks like it could be a Lorentz covariant theory. One logical possibility, the one initially explored by Nambu, is to complete the process of abstraction by postulating an action principle for the string dynamics that incorporates simultaneously the 1d extension and the Lorentz invariance of the system. The string is, after all, imbedded in the four-dimensional Minkowski space. Dr. Dragon has been lecturing on the consequences of this elegant postulate.

With the benefit of hindsight, however, we are now in a position to ask whether a set of ten Poincare generators for the dynamical string system could have been guessed if one had been very clever. The motivation for attempting to invent the generators rather than derive them is what is lost in elegance may be made up for in flexibility.

I will now discuss one route to guessing the desired generators, because this is the way we constructed them in the conductive string model.

Actually, the method is not really too much guesswork, since the approach was discussed in detail by Bacry and Chang, and by Bardakci and Halpern in their works on light-cone quantization. More recently, other relevant articles have appeared in Phys. Rev. by Biedenharn and van Dam, and by Staunton. Rather than trudging through the formal arguments, however, I would like to give you a simple mnemonic device which conveys the idea.

Recall one nice thing about the l.c. quantization is that the dynamics has a non-relativistic appearance to it. The Hamiltonian is $(p_\perp^2 + m^2/2\eta)$, two of the boosts are transverse Galilei boosts, a longitudinal boost is a scaling operation, etc. But simply using the metric $A_\mu B^\mu = A_+B_- + A_-B_+ - A_\perp B_\perp$ does <u>not</u> give this simple structure to the Dirac system. The non-relativistic structure only emerges if we first decompose the Dirac field as $\psi = \psi_+ + \psi_-$, using projectors $(\gamma^\pm\gamma^\mp)$, and then observe that the Dirac equation for ψ_- involves only a "spatial" derivative. The components ψ_- are not canonical dynamical variables, but can be eliminated in favor of the true independent degrees of freedom ψ_+.

If, further, we prudently choose the Bjorken, Kogut, Soper representation for the γ^μ, we obtain their expression for ψ_+ (free),

$$\psi_+(x) \sim \sum_{\lambda=\pm 1/2} \int d^2p_\perp \int \frac{d\eta}{\sqrt{\eta}}$$

$$\left[b(p_\perp,\eta;\lambda)\, e^{-ipx}\, \omega(\lambda) + d^\dagger(p_\perp,\eta;\lambda)\, e^{ipx}\, \omega(-\lambda)\right] , \qquad (4)$$

where $\omega(1/2) = \binom{1}{0}$, $\omega(-1/2) = \binom{0}{1}$. We can use two component spinors with no loss of generality.

I've gone into these elementary results to remind you that once ψ_- is eliminated, and Eq. (4) used for ψ_+, the ten Poincare generators of the free Dirac theory may be written as follows:

$$G \sim \int dx\, \psi^\dagger(x)\, g\psi(x) \qquad , \qquad (5)$$

where the first-quantized forms for the generators, g, are:

$$p_\perp = -i\partial_\perp, \qquad P^+ \equiv \eta, \qquad P^- \equiv H = \frac{p_\perp^2 + m^2}{2\eta} \; ; \tag{6a}$$

$$K_3 = \frac{i}{2}\{\eta, \partial\eta\}; \tag{6b}$$

$$J_3 = \varepsilon_{ab}\, x_a\, p_b + \frac{\sigma_3}{2} \; ; \tag{6c}$$

$$B_\perp = \eta x_\perp \; ; \tag{6d}$$

$$S_k = \frac{1}{2}\{\, x_k, H\,\} - \frac{1}{2}\{\,\frac{1}{\eta}, K_3\,\}\, P_k + \frac{\varepsilon_{k\ell}}{2\eta}\left[\frac{\sigma_3}{2} P_\ell - m\frac{\sigma_\ell}{2}\right] \tag{6e}$$

These generators obey the Poincare algebra under the first-quantization canonical commutation rules $[x_a, P_b] = i\delta_{ab}$.

For N free particles, we have $g(N) = \sum_i^N g_i$. It is convenient to use CM and relative coordinates, e.g.,

$$H_{1+2} = \frac{p_\perp^2}{2M} + \frac{\underline{\pi}^2 + m^2}{2\mu} .$$

If, to the two free particle terms, we add an interaction term between them, it is convenient to introduce a (mass)^2 operator in which the interaction is buried. What follows is simply a definition:

$$H = H_{1+2} + V_{12} \equiv \frac{p_\perp^2 + \mathcal{M}^2}{2M} , \tag{7}$$

with

$$\mathcal{M}^2 = 2M\, V_{12} + \frac{M}{\mu}(\underline{\pi}^2 + m^2)$$

Now, the mnemonic is quite simple, and consists of making the following replacements in the generators Eq. (6):

$$m^2 \text{ (parameter)} \rightarrow \mathcal{M}^2 \text{ (operator)} \tag{8a}$$

$$\frac{\sigma_i}{2} \text{ (Pauli)} \quad \rightarrow \quad j_i \text{ (operators)} \tag{8b}$$

Also $\eta \to M$; and $x_\perp$, $p_\perp$ are C.M. operators which commute with $\mathcal{M}^2$ and j_i. All the algebraic properties of Eq. (6) are to be preserved. Thus the j_i satisfy the spin algebra, and $\mathcal{M}^2$ must be a rotational scalar. The idea is that the mass, which is a parameter in an elementary particle theory, becomes an operator in a composite particle theory. Similarly the spin is not an intrinsic property, but arises from the dynamical configuration of the system. All of the information of the state of internal excitation of the particle is carried by $\mathcal{M}^2$ and the j_i, which are to be expressed in terms of some set of appropriate internal degrees of freedom of the system. In the string, these degrees of freedom are $x_\perp(\theta,\tau)$ and $p_\perp(\theta,\tau)$, or equivalently, their Fourier coefficients, the boson operators a_n and $a_n^\dagger$:

$$x_\perp(\theta,\tau) = x_\perp^{(0)} + p_\perp^{(0)}\tau + \sum_n \sqrt{\frac{2}{n}}\,[\cos n\theta\,(a_{n\perp}(\tau) + a_{n\perp}^\dagger(\tau))]\ ;$$

$$p_\perp(\theta,\tau) = \partial x_\perp/\partial\tau; \qquad (9)$$

$$[a_{n_i}, a_{m_j}^\dagger] = \delta_{nm}\,\delta_{ij}\ .$$

As it turns out, this is not quite right for the string model. Following Gursey and Orfanidis, and Ramond, introduce operators which <u>transform</u> as $m\,\sigma^i$

$$T^i \sim \sqrt{\mathcal{M}^2}\,\frac{\sigma^i}{2}\ ,\ \text{i.e.,} \qquad (10a)$$

$$[T^i, T^j] = i\,\varepsilon^{ij}\,\mathcal{M}^2\,J^3\ ; \qquad (10b)$$

$$[J^3, T^i] = i\,\varepsilon^{ij}\,T^j\ . \qquad (10c)$$

The combination $(m\sigma^i)$ appears in the generators S_k, Eq. (6e), and from GGRT we learn that the structures that emerge in the string model in those generators has the algebra of the T^i. Dr. Dragon has discussed for us the difficulties with 26 dimensions and tachyons that arise from the structure of those generators.

III. The Conductive String

I have gone to some length to provide you with a non-formal background on strings, most of which is well known to specialists, because the conductive string model does not really follow from any pretty formalism, but rather arose as a tentative step away from the rather close-knit formal structure of string theory.

One specific mathematical motivation for this particular way to move away from the string model arises from asking why the canonical quantization procedure fails for this theory. Why is it that we run into these troubles with dimension and with tachyons? Perhaps one has not chosen the dynamical variables properly, for which quantization rules are to be prescribed. At the classical level, another choice for the variables suggests itself quite naturally. As Dr. Dragon has noted, the equations of motion become the string equations, $\ddot{x} = x''$, if the coordinate conditions

$$\left(\frac{\partial x^{\mu}}{\partial u^{\mp}}\right)^{2} = 0, \tag{11}$$

with $u^{\pm} = \tau \pm \theta$, are imposed. That is, $(\partial x^{\mu}/\partial u^{+})$ and $(\partial x^{\mu}/\partial u^{-})$ are null vectors.

Now, any such null vector may be represented as

$$\partial x^{\mu}/\partial u^{\pm} = \xi_{\pm}^{\dagger}\, \sigma^{\mu}\, \xi_{\pm} \tag{12}$$

already at the classical level. We shall try, therefore, to take the pair of two component spinors $\xi_{\pm}$ as our basic dynamical variables.

The string equation of motion will then be satisfied if

$$\partial_{u_{\pm}} \xi_{\mp} = i\, B_{\mp}\, \xi_{\mp} \tag{13}$$

where $B_{\mp}$ are arbitrary Hermitian functions. However, by Eq. (12) the $\xi_{\pm}$ enjoyed a phase invariance under $\xi \to e^{i\lambda(\theta,\tau)}\xi$. This invariance can be preserved in Eq. (13) if the $B_{\mp}$ simultaneously transform as gauge fields,

$$\begin{aligned} \xi &\to e^{i\lambda}\, \xi \\ B &\to B - \partial\lambda \end{aligned} \tag{14}$$

We now depart from our strict adherence to the string model by treating $B_{\pm}$ as gauge fields. Specifically, this means that we will derive Eq. (13) from a new <u>effective</u> Lagrangian, and include kinetic energy terms for $B_{\pm}$ as well.

Before displaying this effective Lagrangian and plunging into the details of the spectrum, etc., I want to jump the gun a little and confirm your suspicion that we will be doing two-dimensional electrodynamics.

The point I want to make right now is that there are physical motivations for doing this. The argument regarding the choice of proper classical variables could not guide one into making $B_{\pm}$ gauge fields, but it is reasonable to try this nonetheless, for different reasons.

These physical motivations stem in part from work done by Nielsen and Olesen, who observed that the electrodynamics of scalar fields, cum Higgs mechanism, could give rise to filamentary solutions at the classical level. These filaments are analogous to trapped magnetic flux lines in a type II superconductor. Nambu has argued that if these flux lines terminate on (abelian) magnetic poles, the static, classical expression for the energy contains two pieces,

$$E \sim a\,L + b\ (\text{Yukawa}). \tag{15}$$

The first piece is proportional to the length of filament between the poles, and should represent something like the ground state energy of of the unexcited string. The second piece contributes for short wavelengths, and is desirable for producing power-law fall-offs in form factors.

In addition, we have heard Professor Susskind's lecture on the hadron-wurst picture he has been working on with J. Kogut. Following K. Wilson, one examines the current-current correlation function

$$\langle J_\mu(x)\ J_\nu(o)\rangle \sim \int DA\ D\psi\ D\psi^+\ J_\mu(x)\ J_\nu(o)\ \exp i \int d^4x \mathcal{L}(x;A,\psi,\psi^+).$$

The factor $[\exp i \int dx_\mu\ A^\mu]$ leads one to believe that if the q and $\bar{q}$ in a loop exchange photons in a fairly uniform manner,

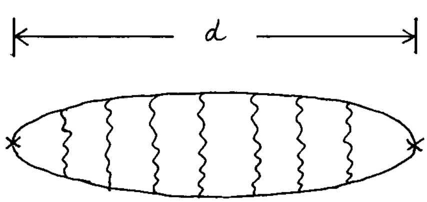

Figure 3

the contribution to the action will go as $[2d\langle A\rangle]$, where (2d) is the perimeter of the loop.

But if for some reason the exchanges do not work out this way, but rather conspire to make the effective action proportional to the area, something interesting occurs. Let me try to give a crude argument for how this works.

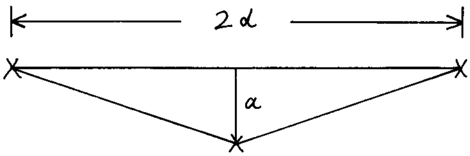

Figure 4

First, if the action $I \sim d + \sqrt{d^2 + a^2}$, clearly $dI/da \sim a/\sqrt{d^2 + a^2} \sim a/d$. It does not cost much to separate the quarks further and further apart ($d \to \infty$). On the other hand, if $I \sim da$, $dI/da \sim d$, and we lose a great deal. This was why Wilson wanted to get the action to go like the area.

In addition, however, in a static situation $L = -H$, and so $I \propto E$, whence $E \propto ad$. Here "a" is, in some frame, the spatial separation between the members of a pair created at the origin and moving toward the point x. So again we have a situation where the potential energy grows linearly with the separation of the pair.

There are, to summarize, various ways in which conventional field theories may support solutions in which the effective interaction is just like the interaction in two-dimensional electrodynamics, or perhaps like its non-Abelian brother. One may view the conductive string either as an abstraction from such models, on the same footing as deciding on harmonic forces between partons in Bjorken's illustrative model; or as an approximation to the full field theory which may be appropriate for studying a special class of properties of the hadron. Let me now continue with the main line of development of the model.

We have

$$\mathcal{L} = i\left[\bar{\psi}\Gamma^a\,(\partial_a - ig\,B_a)\psi\right] - \frac{1}{4}\,F_{ab}\,F^{ab}, \qquad (16)$$

with "a" and "b" running over 0,1. The spinor ψ has four components, consisting of the two components each of ξ_+ and ξ_-. The 4x4 matrices Γ^a satisfy the algebra $\{\Gamma^a,\Gamma^b\} = 2\eta^{ab}$, with $\eta^{oo} = -\eta^{11} = 1, \eta^{o1} = 0$. In terms of the usual Dirac matrices, we may write $\Gamma^0 = i\gamma^o\gamma^5, \Gamma^1 = i\gamma^5$. As usual, $F_{ab} = \partial_b B_a - \partial_a B_b$, where the two components B_a are linear combinations of B_+ and B_-. It is easily seen that with these choices, variation of $\mathcal{L}$ gives the Eqs. (13), plus the Maxwell equations for B.

Working in analogy with the string model, we confine our spatial domain to $[0,\pi]$, and impose boundary conditions on ψ such that $(\partial x^\mu/\partial\theta)$ vanishes at the boundaries, using Eq. (12). These boundary conditions also lead to the conditions $j'(0) = j'(\pi) = (\partial j^o/\partial\theta)|_o = (\partial j^o/\partial\theta)|_\pi = 0$, where the currents

$$j^a \equiv \bar{\psi}\Gamma^a\psi. \qquad (17)$$

For our purposes we are not interested in the Green's functions of the theory, but rather in the physical spectrum of excitations supported by the system. As is well known, TDQED has no genuine radiation field, and in the gauge $B_1 = 0$, the timelike field can be solved for in terms of the charge density,

$$B_0 = -\frac{g}{2}\int_0^\pi d\theta' |\theta-\theta'|\; j^0(\theta',\tau). \tag{18}$$

Forming the Hamiltonian, then, we have

$$H = -i\int d\theta \bar{\psi}\Gamma^1\partial_1\psi - \frac{g^2}{4}\iint d\theta\, d\theta'\; j^0(\theta,\tau)|\theta-\theta'|\; j^0(\theta',\tau). \tag{19}$$

The idea is to diagonalize this Hamiltonian, and display the energy eigenstates. Our task is somewhat simplified by the consistency condition on TDQED first discussed by Zumino, which says that all the physical states of the system must be neutral.

To perform the diagonalization, it is useful to introduce a set of coupled fermion operators which satisfy Bose commutation relations. These "plasmons" are the Fourier components of the vector current,

$$\rho(p) = \frac{1}{\sqrt{2p}}\int_0^\pi d\theta\colon \left[j_0 \cos p\theta + ij_1 \sin p\theta\right]\colon \tag{20}$$

It is easily seen that the δ' Schwinger term in the equal time j_0, j_1 commutator provides

$$\left[\rho(p),\, \rho^+(q)\right] = \delta_{pq}. \tag{21}$$

Also, $[\rho(p),Q] = 0$, so acting with plasmons does not destroy the neutrality of a state. Inverting Eq. (20) for j_0 and inserting into (19), one obtains

$$H = H_0 + \frac{\mu^2}{4}\sum_{n=1}^{\infty}\frac{1}{n}\left[2\rho_n^{\;+}\rho_n + \rho_n\rho_n + \rho_n^{\;+}\rho_n^{\;+}\right], \tag{22}$$

where $\mu^2 \equiv 2g^2/\pi$. In this form, it is straightforward to diagonalize H by means of a Bogoliubov transformation. The details are presented in SLAC-PUB-1418, to be published in Phys. Rev., and I will give only the results:

1) The ground state is

$$|\Omega\rangle = \exp\left[-\sum_{n=1}^{\infty}\left(\frac{1}{2}\tanh^{-1}\frac{\mu^2}{\mu^2+2k^2}\right)(\rho_k^{\;+}\rho_k^{\;+} - \rho_k\rho_k)\right]|0\rangle \tag{23}$$

$$\equiv e^{-is}|0\rangle.$$

This state clearly contains indefinite numbers of quarks and anti-quarks, and it is because of this that it acts like a "conductive" medium, any excess test charge being screened in the interior and only reappearing on the boundary.

Notice the $q\bar{q}$'s do not pair off to make bosons localized in space. Rather, there are correlated pairs of fixed total momentum. The ground state $|\Omega\rangle$ has a finite negative definite energy with respect to the Fock vacuum $|0\rangle$,

$$\varepsilon_0 = \frac{1}{2} \sum_n [\varepsilon_n - n - \frac{\mu^2}{2n}], \tag{24}$$

with

$$\varepsilon_n = \sqrt{\mu^2 + n^2}. \tag{25}$$

The fact this energy is negative reflects, of course, that the correlated state is favored over the no-particle state.

We can recognize in this result a possible answer to the problem posed earlier as to whether a string is really only the $N = \infty$ parton configuration in the wave function. In this type of model, where correlations are extremely important, the stationary states will project onto all possible (neutral) bare parton states.

2) The presence of doubled spinors allows for the presence of neutral "filled Fermi sea" states in the spectrum. These are of the form

$$|F\rangle = \prod_{n=1}^{F} b_i^{+}(n)\, c_j^{+}(n)|0\rangle, \quad (i \neq j), \tag{26}$$

where b^+ and c^+ are particle-antiparticle creation operators. The essential property of these states is that

$$\rho(n) \quad |F\rangle = 0, \text{ (all n)}.$$

Then

$$H(e^{-is}|F\rangle) = \varepsilon_F(e^{-is}|F\rangle),$$

where

$$\varepsilon_F = 2 \sum_{n=1}^{F} (n - \frac{1}{2}) = F^2$$

comes entirely from the non-interacting part of the Hamiltonian. (We are now shifting the energy by ε_0 so $|\Omega\rangle$ has zero energy.)

3) Plasmons may be added onto the filled sea states,

$$|N_p,P;F\rangle = \prod_{m=1}^{P} \frac{[\rho^+(m)]^{N_m}}{\sqrt{N_m!}} |F\rangle. \tag{27}$$

These states are also energy eigenstates, the plasmons contributing as massive bosons, by Eq. (25),

$$H[e^{-is}|N_p,P;F\rangle] = (\varepsilon_F + \varepsilon_p)[e^{-is}|N_p,P;F\rangle],$$

$$\varepsilon_p = \sum_{m=1}^{p} N_m\sqrt{m^2 + \mu^2}.$$

In this model, then, we see clearly that the states of excitation consist of collective excitations of the constituent fermions.

To complete the story, we have to construct the Poincare generators, and discuss how physical states transform in the full four-dimensional Minkowski space. We can do this with the machinery erected in Section II, in two steps.

First, we can identify the dynamical (mass)^2 operator to be some function of the Hamiltonian of our theory above. It turns out by a simple counting argument that the Hagedorn degeneracy is reproduced if we choose

$$\mathcal{M}^2 = H_{TDQED} \tag{28}$$

We will need center of mass four-momenta for our particles, subject to the mass-shell conditions

$$P_\mu P^\mu = \mathcal{M}^2. \tag{29}$$

This of course relates the "true" Hamiltonian P^- to the spectrum of internal excitations, as in Eq. (7).

The state of a single free particle in motion may then be labelled

$$|k^+, \underset{\sim}{k}^\perp, \mathcal{M}^2, J, \lambda\rangle = e^{ikx}|N_p,P;F\rangle. \tag{30}$$

The second stage in our construction is to interpret the spin and helicity labels J and λ by producing operators j_i. This can be done very naturally in our model, because the TDQED with four component spinors enjoys an extra SU(2) symmetry generated by

$$j^k = \frac{1}{2} \int d\theta \; : \psi^+ \begin{pmatrix} \sigma^k & 0 \\ 0 & \sigma^k \end{pmatrix} \psi :.$$

One then finds that the plasmons are Lorentz scalar excitations,

$$[j^i, \rho(m)] = 0, \qquad (31a)$$

while

$$j^3 | F> = (\pm)F \; | F >.$$

Only the "filled sea" states carry spin, and by means of the ladder operators $(j^1 \pm ij^2)$ one can complete the multiplets of spin F. Said differently, one easily checks that

$$W_\mu W^\mu |F\rangle = m^2 \; F(F+1)|F\rangle \quad ,$$

where W_μ is the Pauli-Lubanski vector.

The net result of all this is that the model describes a system of parabolic trajectories, $J = \sqrt{m^2}$, with sea states providing the leading trajectory. The plasmons then shift this trajectory to the right to form an infinite family of particles. Unfortunately, this spectrum does not appear to be particularly realistic, although we are always in four dimensions and we have no tachyon and no ghosts. It has to be admitted that our Lorentz generators are constructed in an ad hoc fashion, and other ways of proceeding may exist that would lead to a different trajectory structure. For the present, however, I have no light to shed on this question.

IV. Speculations

The great virtue of the string model, as compared with other theories of composite hadrons, is that scattering amplitudes already exist, and a great deal is known about the structure of these amplitudes. Especially in the last year, a lot of progress has been made in formulating the theory of interacting strings, both as a particle theory in the sense of Feynman path integrals, and as an interacting multilocal field theory.

While I think that the conductive string model illustrates many good features that a more realistic model of this genre should possess, it remains to be seen whether enough can be learned from the string model to be able to discuss interactions of conductive strings. These closing remarks are speculations on approaches to this problem.

To describe string-string interactions as a second quantized field theory, Kaku and Kikkawa have introduced master fields describing entire strings, which are functionals of the first quantized coordinates $x_\perp(\theta,\tau)$,

$$\psi = \psi[x_\perp(\theta,\tau)] \ .$$

Now, in addition to the original string model, there are other string models in which the constituents are endowed with intrinsic spins, the Neveu-Schwarz and Ramond models. The master field would then have to be a functional of <u>two</u> sets of "fields" on the two-dimensional submanifold,

$$\psi = \psi[x_\perp(\theta,\tau),\ \phi(\theta,\tau)]$$

where $\phi(\theta,\tau)$ is a spinor field describing the spin excitations.

Structures like this are reminiscent of the supergauge fields that Professor Wess told us about, with $x_\perp$ and ϕ playing the role of the gauge parameters. Here these fields themselves obey free equations of motion, Klein-Gordon and Dirac respectively. What we would want for our conductive string model is to extend this even further and allow the functional arguments of the master field to be <u>interacting</u> quantities. The speculation consists of the conjecture that if we hold on to the guiding principle of gauge invariance, the consistent formulation of such a theory will already contain the allowed forms of the master field interactions. This hope is bolstered by the observation that there are, in fact, residual vortex-vortex interactions in type II superconductors, whose form is determined from the original Landau-Ginzburg equations.

In any case, the study of multilocal field systems is just beginning, and it may be valuable to investigate master fields with quite general field arguments, $\psi = \psi(x,\phi_j(x))$, where "j" is any space-time and/or internal symmetry index, and x are the Minkowski space coordinates, in addition to the forms suggested by the string models. Such master fields would represent "particles" whose constituents' dynamics is itself field-theoretic in nature. One intuitively expects the particles' dynamics to follow from the form of the constituents' dynamics, and an interesting problem is how much of this can be deduced from general considerations such as gauge invariance.