The Nuclear Collective WSp(6,R) Model

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ABSTRACT

The sp(6,R) algebra of the nuclear symplectic model is extended to wsp(6,R) = w(3) $\oplus$ sp(6,R) by including the electric dipole operator among the generators. Labels characterizing the wsp(6,R) irreducible representations (irreps) are defined, and the branching rule for wsp(6,R) $\supset$ sp(6,R) is determined. Two bases of a wsp(6,R) irrep carrier space are constructed and used to calculate matrix representations of the wsp(6,R) generators. Finally the choice of a physically relevant wsp(6,R) irrep is reviewed.

1. INTRODUCTION

Over the past few years, the symplectic model, based upon a non-compact Sp(6,R) group, has proved an appropriate framework for a many-body theory of nuclear collective motion, and has found various applications in light and heavy rotational nuclei [1]. Since the monopole and quadrupole mass tensors belong to the sp(6,R) algebra, the model can correctly predict the position of the giant isoscalar monopole and quadrupole resonances, as well as the corresponding transition strengths. It can also reproduce the low-energy collective states and the transition probabilities between them without effective charges.

Unfortunately, the giant isovector dipole resonance, which is the best known and most thoroughly investigated giant resonance, does not fall so easily within the scope of the model because the latter does not contain the electric dipole operator among the generators of its dynamical group. To eliminate this undesirable feature, an extension of the Sp(6,R) model to a WSp(6,R) one was suggested some years ago by Rowe and Iachello [2]. However, Rowe and Iachello's idea was pursued no further, until quite recently the same extension of Sp(6,R) was independently considered again [3,4]. The purpose of the present talk is to review the main results of Refs. 3 and 4.

2. THE wsp(6,R) ALGEBRA

To define the algebra of the Sp(6,R) model, one first replaces the A nucleon coordinates and momenta x'_{is}, p'_{is}, $i = 1,2,3$, $s = 1,2,\ldots,A$, by Jacobi coordinates and momenta x_{is}, p_{is}, $i = 1,2,3$, $s = 1,2,\ldots,A$, then one eliminates the centre-of-mass motion by restricting the range of index s to $s = 1,2,\ldots,n$, where $n = A-1$, and finally one introduces boson creation and annihilation operators η_{is}, ξ_{is}, associated with the latter:

$$\eta_{is} = (x_{is} - i\,p_{is})/\sqrt{2}\ ,\quad \xi_{is} = (x_{is} + i\,p_{is})/\sqrt{2}\ ,\quad i=1,2,3,\ s=1,2,\ldots,n. \tag{1}$$

In terms of η_{is} and ξ_{is}, the sp(6,R) generators are given by

$$D^\dagger_{ij} = \sum_{s=1}^{n} \eta_{is}\,\eta_{js}\ ,\quad D_{ij} = \sum_{s=1}^{n} \xi_{is}\,\xi_{js}\ ,$$

$$E_{ij} = \sum_{s=1}^{n} \eta_{is}\,\xi_{js} + (n/2)\,\delta_{ij}\ ,\quad i,j = 1,2,3, \tag{2}$$

where E_{ij}, $i,j = 1,2,3$, span the u(3) subalgebra.

The electric dipole operator and its corresponding momentum operator are given by

$$\mathcal{D}_i = \sum_s{}' \,(x'_{is} - X_i)\ ,\qquad \mathcal{P}_i = \sum_s{}' \,(p'_{is} - P_i), \tag{3}$$

where the primed summations run only over the Z proton coordinates and momenta, and X_i, P_i are the components of the centre-of-mass coordinate and momentum respectively. They can be rewritten as

$$\mathcal{X}_i = -\gamma\,(B_i^\dagger + B_i)/\sqrt{2}\ , \qquad \mathcal{P}_i = -i\gamma\,(B_i^\dagger - B_i)/\sqrt{2}, \tag{4}$$

in terms of some new operators

$$B_i^\dagger = \gamma^{-1} \sum_{s=1}^{n} t_{3s}\, \eta_{is}\ , \qquad B_i = \gamma^{-1} \sum_{s=1}^{n} t_{3s}\, \xi_{is}\ . \tag{5}$$

Here $\gamma = [Z(A-Z)]^{1/2}$, and the operators t_{3s}, $s = 1,2,\ldots,n$, are obtained from the standard isospin operators t'_{3s}, $s = 1,2,\ldots,A$ (with eigenvalues $+1/2$ and $-1/2$ for neutrons and protons respectively) by the Jacobi transformation.

The operators $B_i^\dagger$, and B_i satisfy boson commutation relations, and therefore, together with I, span a w(3) Heisenberg-Weyl algebra. Moreover they are vector operators with respect to sp(6,R). Hence, by adding the operators $B_i^\dagger$, B_i, and I to the sp(6,R) generators, we obtain the semidirect sum algebra w(3) $\oplus$ sp(6,R), that we denote by wsp(6,R).

3. IRREDUCIBLE REPRESENTATIONS OF wsp(6,R)

In the realization (2), (5), all the wsp(6,R) irreps have a lowest weight state (LWS) $|\mathbf{\Omega}\rangle$, defined by

$$E_{ii}\,|\mathbf{\Omega}\rangle = \Omega_{4-i}\,|\mathbf{\Omega}\rangle\ ,\ \ E_{ij}\,|\mathbf{\Omega}\rangle = 0\ \ (i>j)\ ,\ \ D_{ij}\,|\mathbf{\Omega}\rangle = 0\ , \tag{6a}$$

$$B_i\,|\mathbf{\Omega}\rangle = 0\ , \tag{6b}$$

where $\mathbf{\Omega}$ is a shorthand notation for $\Omega_1\Omega_2\Omega_3$, and $[\Omega_1-n/2, \Omega_2-n/2, \Omega_3-n/2]$ is some partition into non-negative integers. From (6), it follows that the state $|\mathbf{\Omega}\rangle$ is the LWS of an sp(6,R) irrep $\langle\mathbf{\Omega}\rangle$, satisfying the additional condition (6b). The same labels $\mathbf{\Omega}$ may therefore characterize the wsp(6,R) irrep built on it. We shall denote this wsp(6,R) irrep by $\langle\langle\mathbf{\Omega}\rangle\rangle$ to distinguish it from the sp(6,R) irrep $\langle\mathbf{\Omega}\rangle$ with the same LWS. The irrep $\langle\langle\mathbf{\Omega}\rangle\rangle$ can be alternatively characterized by the eigenvalues of three Casimir operators $\hat{G}_3$, $\hat{G}_6$, and $\hat{G}_9$, of degree 3, 6, and 9 in the wsp(6,R) generators respectively.

The reduction of a wsp(6,R) irrep into a direct sum of sp(6,R) irreps has been studied by the raising operator technique. The raising operators

R_i, i = 1,2,3, for wsp(6,R) ⊃ sp(6,R) are defined by

$$R_i \, |\boldsymbol{\Omega}\ \boldsymbol{\omega}\rangle = N_i(\boldsymbol{\omega},\boldsymbol{\omega}') \, |\boldsymbol{\Omega}\ \boldsymbol{\omega}'\rangle \ , \tag{7}$$

where $|\boldsymbol{\Omega}\ \boldsymbol{\omega}\rangle$ denotes a state in the carrier space of $\langle\langle\boldsymbol{\Omega}\rangle\rangle$, which is the LWS of some sp(6,R) irrep characterized by $\langle\boldsymbol{\omega}\rangle$, $N_i(\boldsymbol{\omega},\boldsymbol{\omega}')$ is some normalization coefficient, and $\omega'_j = \omega_j + \delta_{ji}$. These raising operators have been explicitly constructed and their normalization coefficient calculated. For such purposes, two different methods have been used: the first one is the method developed by Bincer to determine shift operators for the unitary, orthogonal, and symplectic algebras [5]; the second one is a new technique based upon the converse procedures of shift operator contraction and expansion [6]. From such work, the following branching rule for wsp(6,R) ⊃ sp(6,R) has been obtained:

$$\langle\langle\boldsymbol{\Omega}\rangle\rangle \downarrow \sum_{\omega_1=\Omega_1}^{\infty} \ \sum_{\omega_2=\Omega_2}^{\Omega_1} \ \sum_{\omega_3=\Omega_3}^{\Omega_2} \oplus \langle\boldsymbol{\omega}\rangle \ . \tag{8}$$

Note that in (8), there are no multiplicities.

4. BASES OF wsp(6,R) AND MATRIX REPRESENTATIONS OF THE GENERATORS

Two bases have been constructed in the $\langle\langle\boldsymbol{\Omega}\rangle\rangle$ carrier space. The first one is the monomial basis, whose states are defined by

$$|\boldsymbol{\Omega}\ (\Omega)\ \mathbf{k}\ \boldsymbol{\ell}\rangle = \Big\{ \prod_{i\leq j} (k_{ij}!)^{-1/2} \left[(1+\delta_{ij})^{-1/2} D^\dagger_{ij}\right]^{k_{ij}} \Big\}$$

$$\times \ \Big\{ \prod_{i} (\ell_i!)^{-1/2} (B^\dagger_i)^{\ell_i} \Big\} \ |\boldsymbol{\Omega}\ (\Omega)\rangle \ , \tag{9}$$

where k_{ij} ($i\leq j$) and ℓ_i run over all non-negative integers and (Ω) over all Gel'fand patterns of the u(3) irrep $[\boldsymbol{\Omega}]$. The states (9) do not belong to definite u(3) nor sp(6,R) irreps. The second basis is the sp(6,R) ⊃ u(3) one, whose states are defined by

$$|\boldsymbol{\Omega}\ \boldsymbol{\omega}\ \boldsymbol{\nu}\ \rho\ \mathbf{h}\ \zeta\rangle = \alpha_{\boldsymbol{\nu}} \left[P_{\boldsymbol{\nu}}(\mathbf{D}^\dagger) \times |\boldsymbol{\Omega}\ \boldsymbol{\omega}\rangle \right]^{\rho \mathbf{h}}_{\zeta} \ , \tag{10}$$

where $P_{\boldsymbol{\nu}\zeta'}(\mathbf{D}^\dagger)$ is a polynomial in $D^\dagger_{ij}$ characterized by a given u(3) irrep

$[\boldsymbol{\nu}] = [\nu_1\nu_2\nu_3]$ and a given row ζ', the square bracket denotes a coupling of the u(3) irreps $[\boldsymbol{\nu}]$ and $[\boldsymbol{\omega}]$ to an irrep $[\mathbf{h}] = [h_1h_2h_3]$ and row ζ, ρ distinguishes between repeated irreps $[\mathbf{h}]$, and $\alpha_{\boldsymbol{\nu}}$ is some coefficient. For ζ, we may choose either a Gel'fand pattern (h) or the angular momentum L, its component M_L, and some multiplicity label κ.

Since both bases (9) and (10) are not orthonormal, the corresponding matrix representations of the wsp(6,R) generators have been determined in two steps. First the recursion relations satisfied by the overlap matrices of both bases have been established. Second, the matrix elements of the wsp(6,R) generators between a basis state and a dual basis one have been calculated. For such purpose, it is convenient to use a boson realization of wsp(6,R), generalizing a well known boson realization of sp(6,R) [7]. The former is defined in terms of two independent sets of boson creation and annihilation operators $a^\dagger_{ij} = a^\dagger_{ji}$, $a_{ij} = a_{ji}$, and $b^\dagger_i$, b_i (i,j = 1,2,3), and of some intrinsic u(3) generators E^0_{ij} (i,j = 1,2,3), commuting with the boson operators, and only acting in the carrier space of the u(3) irrep $[\boldsymbol{\Omega}]$. In matrix notation, it is given by

$$D^\dagger_D = \mathbf{a}^\dagger \ , \ B^\dagger_D = \mathbf{b}^\dagger \ , \ E_D = \mathbf{a}^\dagger\mathbf{a} + \mathbf{b}^\dagger\mathbf{b} + \mathbf{E}^0 \ ,$$

$$D_D = \mathbf{a}(\mathbf{a}^\dagger\mathbf{a} - 5\mathbf{I}) + \mathbf{a}(\mathbf{b}^\dagger\mathbf{b} + \mathbf{E}^0) + (\tilde{\mathbf{b}}\tilde{\mathbf{b}}^\dagger + \tilde{\mathbf{E}}^0)\mathbf{a} + \tilde{\mathbf{b}}\mathbf{b} \ , \tag{11}$$

$$B_D = \tilde{\mathbf{b}}^\dagger\mathbf{a} + \mathbf{b} \ .$$

This is a Dyson boson realization since the Hermiticity properties of the generators are not preserved.

5. APPLICATIONS TO NUCLEI

In applications to nuclei, not all the mathematically realized wsp(6,R) irreps are physically relevant. Pauli principle indeed imposes some additional conditions. In the lowest approximation, we may restrict ourselves to a single wsp(6,R) irrep, namely that containing the sp(6,R) irrep corresponding to the lowest approximation of the symplectic model. For closed shell nuclei, for instance, this wsp(6,R) irrep has the closed shell state as LWS; it is characterized by $\Omega_1 = \Omega_2 = \Omega_3 = \Omega$ or, in Elliott's notations, by $N_{00} = 3\Omega$ and $(\lambda_{00}\mu_{00}) = (00)$. Here N_{00}, λ_{00}, and μ_{00} are respectively defined by $N_{00} = \Sigma_i\ \Omega_i$, $\lambda_{00} = \Omega_1 - \Omega_2$, $\mu_{00} = \Omega_2 - \Omega_3$. The lowest two sp(6,R) irreps belonging to ⟨⟨N_{00} (00)⟩⟩ are characterized by ⟨N_{00} (00)⟩ and ⟨N_{00}+1 (10)⟩, and are of opposite parity. In Ref. 3, the reduced matrix elements of the dipole operator between basis states of these two irreps have been determined in the case of ^{16}O for which N_{00} = 69/2.

In conclusion, we have shown that the Sp(6,R) model can be extended to deal with giant dipole transitions. The techniques needed to perform dynamical calculations in the extended model are now available, and it is hoped that such calculations will be performed in a near future.

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