

Locally Rotationally Symmetric Bianchi Type-I Cosmological Model in $f(R, T)$ Gravity

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Abstract. In this paper, we have investigated a spatially homogeneous locally rotationally symmetric Bianchi type-I space-time with cosmological term Λ in presence of perfect fluid distribution in $f(R, T)$ gravity theory. We have derived explicitly the field equations of the theory and obtained the exact solution of field equations by employing a periodic varying deceleration parameter, which is a unique feature of the model. We have also performed the analysis of the model such as the equation of state parameter, pressure, energy density, density parameter and jerk parameter which are significant in the discussion of cosmology. Some physical and geometrical properties of the model have also been discussed along with the graphical representation of various parameters. We obtained the presence of quintessence and phantom regions based on chosen parameters. It is observed that the deceleration parameter exhibits a smooth transition from early deceleration to late time acceleration of the universe and oscillate based on chosen parameters. We have observed that the presented model is compatible with the recent cosmological observations.

1. Introduction

In cosmology, the late-time accelerated expansion of the universe has been a major subject of investigation. Modified gravity approach is one of the best ways to explain the cosmic acceleration and ultimate fate of universe. It seems attractive to explain the phenomena of dark energy and late-time acceleration. Hence, the modified theories of gravity is attracting currently several researchers to investigate dark energy (DE) models. Among these geometrically modified theories, $f(R, T)$ theory has attracted a lot of attention of many cosmologists and astrophysicists in recent times because of its ability to explain several issues in cosmology and astrophysics [1, 2]. The evolution of the universe from early deceleration to late time acceleration is effectively described by $f(R, T)$ theory of gravity. The $f(R, T)$ modified theory of gravity developed [3], where the gravitational Lagrangian is given by an arbitrary function of the Ricci scalar R and the trace T of the energy-momentum tensor. It is to be noted that the dependence of T may be induced by exotic imperfect fluid or quantum effects. They have obtained the gravitational field equations in the metric formalism, as well as, the equations of motion of test particles, which follow from the covariant divergence of the stress-energy tensor. They have derived some particular models corresponding to specific choices of the function $f(R, T)$.

In this theory, the interactions of matter with space-time curvature become a well motivation to consider cosmological consequence with different matter components [4]. Some cosmological models in $f(R, T)$ gravity were reconstructed [5] where it was proved that the dust fluid reproduced Λ CDM. Friedmann-Lemaitre-Robertson-Walker (FLRW) space-time cosmological models have explored [6, 7]. FRW cosmological model in $f(R, T)$ gravity have been investigated along with perfect fluid matter and



linearly varying deceleration parameter with magnetized strange quark matter and Λ [8, 9]. Moreover, a periodic time varying deceleration parameter (PVDP) has been introduced [10] in order to account for an oscillating cosmological models with quintom matter. Since these models are very natural to resolve the coincidence problem due to periods of acceleration [11]. Some Bianchi space-time have studied [12, 13]. Cosmological models and solar system consequences of the theory have reconstructed [14]. The Bianchi type- I cosmological model have studied in [15, 16]. Dark energy Bianchi type- I cosmological models have been explored [17]. Aspects of anisotropic cosmological models were studied [18]. Recently, the investigation from the transition of deceleration to acceleration [19].

Various researchers have studied locally rotational symmetric (LRS) Bianchi-type models. An inhomogeneous LRS model investigated by [20, 21], which was later continued [22–28]. In this study, we have explored the LRS Bianchi type- I space-time in $f(R, T)$ theory of gravity. On the other hand, the cosmological term Λ has an important role in the study of the accelerating universe, and which is also a candidate for dark energy. The cosmological constant in the gravitational Lagrangian is a function of the trace of the stress-energy tensor, and consequently the model was denoted $\Lambda(T)$ gravity. It was argued that recent cosmological data favor a variable cosmological constant, which are consistent with $\Lambda(T)$ gravity, without the need to specify an exact form of the function $\Lambda(T)$ [29, 30].

The investigation of Bianchi-type models in modified or alternative theories of gravity is another interesting topic of discussion. Perfect fluid solutions using a Bianchi type- I space-time in scalar tensor theory have been explored [31]. With the above motivation, we have investigated a class of LRS Bianchi type- I model with variable Λ term within the framework of $f(R, T)$ gravity theory by choosing $f(R, T) = R + 2f(T)$, where $f(T) = \lambda T$, and λ is an arbitrary constant. The paper is organized as follows. The field equations in $f(R, T)$ gravity are derived in section 2. In section 3, we present the metric and field equations. The solution of the field equations has been explored in section 4. In section 5, some physical and geometrical properties of the model are also investigated. Finally, conclusions are given in section 6.

2. Field equations in $f(R, T)$ theory of gravity

The $f(R, T)$ theory of gravity is one of the important modifications of general theory of gravity proposed [3]. Here in this theory, the gravitational Lagrangian is described by an arbitrary function of the Ricci scalar R and the trace T of the energy-momentum tensor T_{ij} . Following [3], let us consider the action of the form in the units $8\pi G = 1 = c$

$$S = \frac{1}{2} \int f(R, T) \sqrt{-g} d^4x + \int L_m \sqrt{-g} d^4x, \quad (1)$$

where g is the determinant of the metric tensor g_{ij} , $f(R, T)$ is the function of Ricci scalar, R and trace of energy-momentum tensor, T and L_m represents the matter Lagrangian density. The energy-momentum tensor of the matter is defined as

$$T_{ij} = -\frac{2}{\sqrt{-g}} \frac{\delta(\sqrt{-g} L_m)}{\delta g^{ij}}, \quad (2)$$

so that trace $T = g^{ij} T_{ij}$.

Considering Lagrangian density L_m of matter depends only on the metric tensor components g_{ij} , equation (2) becomes

$$T_{ij} = g_{ij} L_m - 2 \frac{\partial L_m}{\partial g^{ij}}. \quad (3)$$

Varying the action S mention in equation (1) with respect to the metric tensor components g^{ij} , the field equations of $f(R, T)$ gravity can be written by [3] as

$$f_R(R, T) R_{ij} - \frac{1}{2} f(R, T) g_{ij} + (g_{ij} \square - \nabla_i \nabla_j) f_R(R, T) = T_{ij} - f_T(R, T) T_{ij} - f_T(R, T) \Theta_{ij}, \quad (4)$$

where $\square \equiv g^{ij}\nabla_j\nabla_i \equiv \nabla^i\nabla_i$ is d'Alembert operator, $f_R(R, T) = \frac{\partial f(R, T)}{\partial R}$, $f_T(R, T) = \frac{\partial f(R, T)}{\partial T}$ and ∇_i denotes the covariant derivative.

The expansion tensor Θ_{ij} is given by

$$\Theta_{ij} = -2T_{ij} + g_{ij}L_m - 2g^{\alpha\beta} \frac{\partial^2 L_m}{\partial g^{ij} \partial g^{\alpha\beta}}. \quad (5)$$

For the perfect fluid, the energy-momentum tensor T_{ij} is given by

$$T_{ij} = (\rho + p)u_i u_j - pg_{ij}, \quad (6)$$

where $u^i = (0, 0, 0, 1)$ is the four velocity in co-moving coordinates which satisfies the conditions $u^i u_i = 1$ and $u^i \nabla_j u_i = 0$. Here ρ is the energy density and p the pressure of the fluid. Moreover, the matter Lagrangian is not uniquely specified. So, the source term is described as a function of the Lagrangian matter through different choices of it. Here we choose the matter Lagrangian as $L_m = -p$, so that equation (5) becomes

$$\Theta_{ij} = -2T_{ij} - pg_{ij}. \quad (7)$$

Since the field equations in $f(R, T)$ gravity also depend on the physical nature of the matter field (through the tensor Θ_{ij}), for each choice of f , we obtain several theoretical models. Among these, we assume $f(R, T)$ gravity as suggested by [3]

$$f(R, T) = R + 2f(T), \quad (8)$$

where $f(T)$ is an arbitrary function of trace T . Using equations (7) and (8) into (4), we have obtained

$$R_{ij} - \frac{1}{2}Rg_{ij} = T_{ij} + 2f'(T)T_{ij} + [2f'(T)p + f(T)]g_{ij}, \quad (9)$$

where a prime denotes derivative with respect to the argument.

We also wish to consider the following choice of $f(T)$

$$f(T) = \lambda T, \quad (10)$$

where λ is an arbitrary constant.

Using equation (10) in (9) and re-arranging, the field equations become

$$R_{ij} - \frac{1}{2}Rg_{ij} = (1 + 2\lambda)T_{ij} + (2p + T)\lambda g_{ij}. \quad (11)$$

Let us recall Einstein's equations with cosmological constant on the right side,

$$R_{ij} - \frac{1}{2}Rg_{ij} = T_{ij} - \Lambda g_{ij}. \quad (12)$$

By comparing equations (11) and (12), and taking the coupling parameter λ to be small, we see that an effective cosmological parameter as a function of T may be defined in $f(R, T)$ as

$$\Lambda = \Lambda(T) = -(2p + T)\lambda = (p - \rho)\lambda. \quad (13)$$

For this correspondence further details of it given [29]. Thus, we can also regard this form of $f(R, T)$ theory for the case $f(R, T) = R + 2\lambda T$ for a perfect fluid as equivalent to general relativity with an effective cosmological parameter.

3. Metric and field equations

We consider the spatially homogeneous and anisotropic LRS Bianchi type-*I* spacetime as

$$ds^2 = dt^2 - A^2 dx^2 - B^2 (dy^2 + dz^2), \quad (14)$$

where A and B are functions of cosmic time t only.

The energy-momentum tensor for a perfect fluid is taken as:

$$T_{ij} = (\rho + p)u_i u_j - p g_{ij}. \quad (15)$$

Now assuming the co-moving coordinate system, the field equations (11) for the metric (14) with the help of (15) can be written as

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} = \rho\lambda - (1 + 3\lambda)p, \quad (16)$$

$$2\frac{\ddot{B}}{B} + \frac{\dot{B}^2}{B^2} = \rho\lambda - (1 + 3\lambda)p, \quad (17)$$

$$\frac{\dot{B}^2}{B^2} + 2\frac{\dot{A}\dot{B}}{AB} = (1 + 3\lambda)\rho - \lambda p, \quad (18)$$

where dot denotes ordinary differentiation with respect to cosmic time t .

From equation (16) and (17) we get

$$\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{B}^2}{B^2} = 0. \quad (19)$$

Integrating equation (19), we obtain

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = \frac{c_1}{AB^2}, \quad (20)$$

where c_1 is a constant of integration.

We define the following physical parameters for the LRS Bianchi type-*I* model: The average scale factor a and the volume scale factor V are defined as

$$a = \sqrt[3]{AB^2}, \quad V = a^3 = AB^2. \quad (21)$$

The average Hubble parameter H is given in the form

$$H = \frac{\dot{a}}{a} = \frac{1}{3}(H_1 + H_2 + H_3) = \frac{1}{3}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right), \quad (22)$$

where $H_1 = \frac{\dot{A}}{A}$, $H_2 = H_3 = \frac{\dot{B}}{B}$ are the directional Hubble parameters along the respective axes. The physical quantities of observational interest in cosmology, which are the expansion scalar θ , the shear scalar σ^2 and the average anisotropy parameter A_m , are defined as

$$\theta = u^i_{;i} = 3H = \frac{\dot{A}}{A} + \frac{2\dot{B}}{B}, \quad (23)$$

$$\sigma^2 = \frac{1}{2}\left(\sum_{i=1}^3 H_i^2 - 3H^2\right) = \frac{1}{3}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right)^2, \quad (24)$$

$$A_m = \frac{1}{3}\sum_{i=1}^3 \left(\frac{\Delta H_i}{H}\right)^2 = \frac{2}{3}\frac{\sigma^2}{H^2}, \quad (25)$$

where $\Delta H_i = H_i - H$ and H_i for $i = 1, 2, 3$ are directional Hubble's parameters in the directions of x , y and z respectively.

4. Solutions of the field equations

We can observe that the field equations (16)–(18) are a system of three independent differential equations with four unknowns, namely A , B , p and ρ . Hence, in order to solve this inconsistent system we require some additional conditions to get viable cosmological model with determinate solutions.

Periodically varying deceleration parameter

The deceleration parameter (q) in cosmology is a measure of the cosmic acceleration of the universe's expansion and is defined as

$$q = -1 - \frac{\dot{H}}{H^2}, \quad (26)$$

where the overhead dots denote derivatives with respect to cosmic time. It is the geometrical parameters through which the dynamics of the universe can be quantified. Many researchers have used a constant deceleration parameter to obtain the solutions of the model which gives a power law for the metric potentials [32–35].

Based on the late time cosmic speed up phenomena with a cosmic transit from a phase of deceleration to acceleration at some redshift (z). It can be a speculate signature flipping of the deceleration parameter. Geometrical parameter such as jerk parameter is usually extracted from observation of high z supernova. However, the exact time dependence of these parameter is not known to a satisfactory extent. In the absence of any explicit form of parameters, many authors have used parametrized forms especially that of the deceleration parameter to address different cosmological issues. Many parametrized forms of deceleration parameters such as linearly and quadratic varying deceleration parameter are studied by [36]. A special law of variation of Hubble parameter in FLRW-spacetime, which yields a constant form of deceleration parameter [37–39]. This law of variation for Hubble's parameter is valid for slowly varying deceleration parameter models [32, 40].

Linear parametrization of the deceleration parameter shows quite natural phenomena toward the future evolution of the universe whether it expands forever or ends up with Big rip in finite future. Such a parametrization has been used frequently by [41, 42]. It is to mention here that the general dynamical behavior can be assessed through the values of the deceleration parameter in the negative domain. While de-Sitter expansion occurs for $q = -1$, accelerating power-law expansion can be achieved for $-1 < q < 0$. A super-exponential expansion of the universe occurs for $q < -1$. There is an apprehension in determining the deceleration parameter but from observational data most of the studies in recent times constrain this parameter in the range $-1 \leq q < 0$ [43–45].

Considering in view the signature flipping nature of q , we assume a periodic time varying deceleration parameter [10].

$$q = h_1 \cos k_1 t - 1, \quad (27)$$

where h_1 and k_1 are positive constants. Here k_1 decides the periodicity of the periodic varying deceleration parameter and can be considered as a cosmic frequency parameter. h_1 is an enhancement factor that enhances the peak of the periodic varying deceleration parameter. This model simulates a positive deceleration parameter $q = h_1 - 1$ (for $h_1 > 1$) at an initial epoch and evolves into a negative peak of $q = -h_1 - 1$. After the negative peak, it again increases and comes back to the initial states. The evolutionary behavior of q is periodically repeated. In other words, the universe in the model starts with a decelerating phase and evolves into a phase of super-exponential expansion in a cyclic history.

Integration of equation (27) and assuming constant of integration equal to zero yields the Hubble function becomes

$$H = \frac{k_1}{h_1 \sin k_1 t}. \quad (28)$$

Using equation (28) and $\dot{a} = aH$, we get $\dot{H} = -h_1 H^2 \cos k_1 t$. The scale factor a is obtained by integrating

the Hubble function in equation (28) as

$$a = a_0 \left[\tan \left(\frac{1}{2} k_1 t \right) \right]^{\frac{1}{h_1}}. \quad (29)$$

where a_0 is the scale factor at the present epoch and taking $a_0 = 1$.

From the above equations (28) and (29), we obtain volume of the scale factor V and expansion scalar θ as

$$V = \left[\tan \left(\frac{1}{2} k_1 t \right) \right]^{\frac{3}{h_1}}, \quad (30)$$

$$\theta = \frac{3k_1}{h_1 \sin k_1 t}. \quad (31)$$

The redshift z is given by

$$z = \frac{1}{a} - 1 = \left[\tan \left(\frac{1}{2} k_1 t \right) \right]^{-\frac{1}{h_1}} - 1. \quad (32)$$

From equation (32), we obtain

$$t = \frac{2 \tan^{-1} [(z+1)^{-h_1}]}{k_1}. \quad (33)$$

Integrating equation (20)

$$\frac{A}{B} = c_2 \exp \left[c_1 \int \frac{dt}{AB^2} \right] = c_2 \exp \left[c_1 \int \left[\tan \left(\frac{1}{2} k_1 t \right) \right]^{-\frac{3}{h_1}} dt \right], \quad (34)$$

where c_2 is integration constant.

From equations (20), (21) and (34), the values of metric potentials are

$$A = c_2^{\frac{2}{3}} \left[\tan \left(\frac{1}{2} k_1 t \right) \right]^{\frac{1}{h_1}} \exp \left[\frac{2c_1}{3} \int \left[\tan \left(\frac{1}{2} k_1 t \right) \right]^{-\frac{3}{h_1}} dt \right], \quad (35)$$

and

$$B = c_2^{-\frac{1}{3}} \left[\tan \left(\frac{1}{2} k_1 t \right) \right]^{\frac{1}{h_1}} \exp \left[\frac{-c_1}{3} \int \left[\tan \left(\frac{1}{2} k_1 t \right) \right]^{-\frac{3}{h_1}} dt \right]. \quad (36)$$

where c_1 and c_2 are integrating constants.

The metric (14) can now be written as

$$ds^2 = dt^2 - c_2^{\frac{4}{3}} a^2 \exp \left[\frac{4c_1}{3} \int a^{-3} dt \right] dx^2 - c_2^{-\frac{2}{3}} a^2 \exp \left[\frac{-2c_1}{3} \int a^{-3} dt \right] (dy^2 + dz^2), \quad (37)$$

where $a = \left[\tan \left(\frac{1}{2} k_1 t \right) \right]^{\frac{1}{h_1}}$.

By using equations (28), (35) and (36), we obtain the value of directional Hubble parameters for our model as

$$H_1 = \frac{k_1}{h_1 \sin k_1 t} + \frac{2c_1}{3} \left[\tan \left(\frac{1}{2} k_1 t \right) \right]^{-\frac{3}{h_1}}, \quad (38)$$

$$H_2 = \frac{k_1}{h_1 \sin k_1 t} - \frac{c_1}{3} \left[\tan \left(\frac{1}{2} k_1 t \right) \right]^{-\frac{3}{h_1}}. \quad (39)$$

The shear scalar (σ^2) and anisotropy parameter (A_m) become

$$\sigma^2 = \frac{c_1}{3} \left[\tan \left(\frac{1}{2} k_1 t \right) \right]^{\frac{-6}{h_1}}, \quad (40)$$

$$A_m = \frac{2c_1}{9k_1^2} [h_1 \sin(k_1 t)]^2 \left[\tan \left(\frac{1}{2} k_1 t \right) \right]^{\frac{-6}{h_1}}. \quad (41)$$

5. The physical and geometrical properties of the model

In this section, we will discuss the physical and geometrical properties of the model which are important for descriptions of cosmology. From the above equations (28), (31), (38) and (39), it can be noticed that Hubble parameter, scalar expansion and directional Hubble parameters diverge at $t = \frac{n\pi}{k_1}$, where n is a positive integer including zero and they all tend to constants as $t \rightarrow \infty$. The directional Hubble parameters differ from H by certain dynamical parameters. It can be mentioned here that the anisotropy condition, i.e., $\frac{\sigma^2}{\theta^2} \neq 0$ as $t \rightarrow \infty$, when $c_1 \neq 0$. If $c_1 = 0$, our model becomes isotropic and shear scalar vanishes.

It can be observed from equation (30) that the spatial volume is zero at $t = \frac{2n\pi}{k_1}$ for n is a positive integer including zero. It suggests that the universe starts evolving with zero volume at $t = \frac{2n\pi}{k_1}$, i.e. it has the big bang scenario. It can be observed that the average scale factor is zero at the epoch $t = \frac{2n\pi}{k_1}$. Within the time frame, the scale factor increases with cosmic time whereas the Hubble parameter decreases with cosmic time. However, the evolutionary behavior of the scale factor is governed by a tangent function and that of the Hubble parameter is governed by a sine function. Hence the model has a point type singularity [46]. As $t \rightarrow \infty$, both the metric potentials A and B tend to infinity. It shows that the universe expands constantly at later times.

An equivalent present epoch can be derived from redshift relation given in equation (32) as $t = \left(\frac{8n+1}{k_1} \right) \frac{\pi}{2}$, where n is a positive integer including zero. Therefore, it is possible to express the deceleration parameter of equation (27) in terms of redshift. In figure 1, we have shown the evolutionary aspect of the

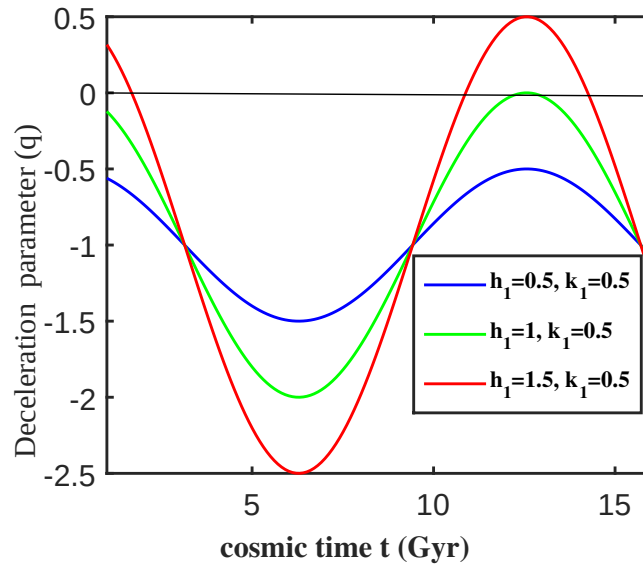


Figure 1: Plot of deceleration parameter q versus cosmic time t for $h_1 = 0.5$, $h_1 = 1$, $h_1 = 1.5$ and $k_1 = 0.5$.

deceleration parameter as a function of cosmic time for three different domain of the parameter h_1 namely

$h_1 = 0.5$, $h_1 = 1$ and $h_1 = 1.5$. The periodic nature of the periodically varying deceleration parameter is clearly depicted in this figure. Figure 2 shows the plot of deceleration parameter q versus redshift z . The evolutionary behavior of the periodically varying deceleration parameter is affected by the choice of the parameter h_1 . Hence, the deceleration parameter oscillates in between $h_1 - 1$ and $-h_1 - 1$. For $h_1 = 0$, deceleration parameter becomes a constant quantity with a value of -1 and can lead to a de-Sitter kind of expansion. For $0 < h_1 \leq 1$, it varies periodically in the negative domain and provides accelerated models. However, for $h_1 > 1$, q evolves from a positive region to a negative region showing a signature flipping at some redshift z . It is interesting to mention here that, the transition redshift depends on the choice of the parameter h_1 . This can be constrained from the cosmic transit behavior and transit redshift z . Figure 2 shows the behavior of deceleration parameter versus redshift z for different values of h_1 . It

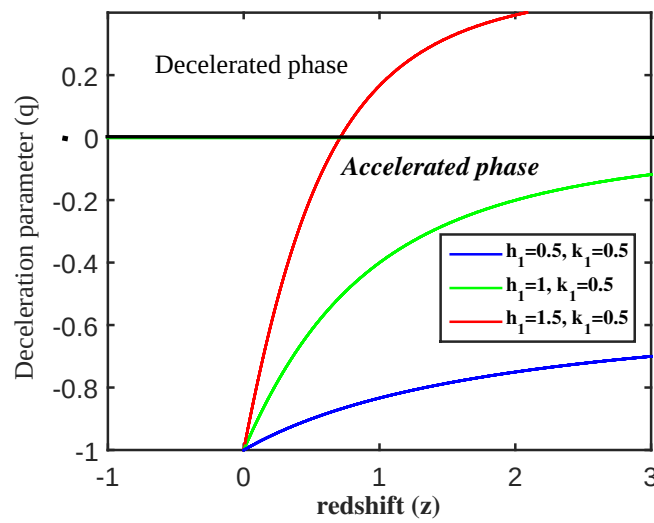


Figure 2: Plot of deceleration parameter q versus redshift z for $h_1 = 0.5$, $h_1 = 1$, $h_1 = 1.5$ and $k_1 = 0.5$.

may be noted that for $h_1 = 0.5$ and 1 the model exhibits completely accelerating universe while $h_1 = 1.5$ exhibits a smooth transition from decelerated phase to the accelerated phase of the universe. It may be seen that the model enters the accelerated phase for $h_1 = 1.5$ at $z \approx 0.71$. This is quite in accordance with recent cosmological observations [19, 47–49]. In the event of non-availability of any observational data regarding cosmic oscillation and corresponding frequency, we consider k_1 as a free parameter. Here in this work, we are interested for a time varying deceleration parameter that oscillates in between the decelerating and accelerating phase to simulate the cosmic transit phenomenon. In order to assess the dynamical features of the model through numerical plots, we assume a small value for k_1 , say $k_1 = 0.5$.

The physical properties of the model from the assumed dynamics of the universe with a periodic varying deceleration parameter helps us to study the energy density and pressure of the universe. From equations (16), (17) and (18), we can get the energy density ρ and pressure p of the fluid as

$$\rho = \frac{(5\lambda + 2) \frac{\dot{B}^2}{B^2} + (4 + 11\lambda) \frac{\dot{A}}{A} \frac{\dot{B}}{B} - \lambda \left(\frac{\ddot{A}}{A} + 3 \frac{\ddot{B}}{B} \right)}{2 \left((1 + 3\lambda)^2 - \lambda^2 \right)}, \quad (42)$$

$$p = \frac{(\lambda + 1) \frac{\dot{B}^2}{B^2} + (1 - \lambda) \frac{\dot{A}}{A} \frac{\dot{B}}{B} + (1 + 3\lambda) \left(\frac{\ddot{A}}{A} + 3 \frac{\ddot{B}}{B} \right)}{2 \left(\lambda^2 - (1 + 3\lambda)^2 \right)}. \quad (43)$$

Applying the corresponding metric potentials and their derivatives, for a periodic varying deceleration

parameter as defined in equation (27) we get the density ρ and pressure p of fluids as

$$\rho = \frac{k_1^2 (3 + 2\lambda (3 + h_1)) (\tan(\frac{1}{2}k_1 t))^{-2} (\sec(\frac{1}{2}k_1 t))^4}{4h_1^2 [(1 + 3\lambda)^2 - \lambda^2]} - \frac{\lambda k_1^2 (\sec(\frac{1}{2}k_1 t))^2}{2h_1 [(1 + 3\lambda)^2 - \lambda^2]} - \frac{(1 + 4\lambda) c_1^2 (\tan(\frac{1}{2}k_1 t))^{-\frac{6}{h_1}}}{3 [(1 + 3\lambda)^2 - \lambda^2]}, \quad (44)$$

$$p = -\frac{k_1^2 (1 + 2(1 + 3\lambda)(1 - h_1)) (\tan(\frac{1}{2}k_1 t))^{-2} (\sec(\frac{1}{2}k_1 t))^4}{4h_1^2 ((1 + 3\lambda)^2 - \lambda^2)} - \frac{(1 + 3\lambda) k_1^2 (\sec(\frac{1}{2}k_1 t))^2}{2h_1 ((1 + 3\lambda)^2 - \lambda^2)} - \frac{(1 + 4\lambda) c_1^2 (\tan(\frac{1}{2}k_1 t))^{-\frac{6}{h_1}}}{3 ((1 + 3\lambda)^2 - \lambda^2)}. \quad (45)$$

The cosmological parameter Λ obtained from equation (13) as

$$\Lambda = \frac{[3\frac{\ddot{B}^2}{B^2} + 5\frac{\dot{A}\dot{B}}{AB} + \frac{\ddot{A}}{A} + 3\frac{\ddot{B}}{B}](1 + 2\lambda)\lambda}{2(\lambda^2 - (1 + 3\lambda)^2)}. \quad (46)$$

Substituting corresponding metric potentials A and B with their respect derivatives, we get

$$\Lambda = \frac{[k_1^2 (3 - h_1) (\tan(\frac{1}{2}k_1 t))^{-2} \sec^4(\frac{1}{2}k_1 t) + k_1^2 h_1 \sec^2(\frac{1}{2}k_1 t)](1 + \lambda)\lambda}{2h_1^2 (\lambda^2 - (1 + 3\lambda)^2)}. \quad (47)$$

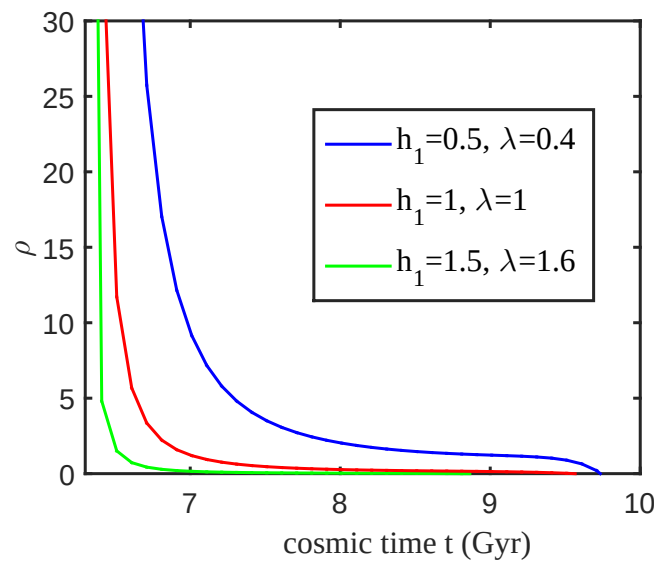


Figure 3: Plot of energy density ρ versus cosmic time t for $k_1 = 0.5$ and $c_1 = 1$.

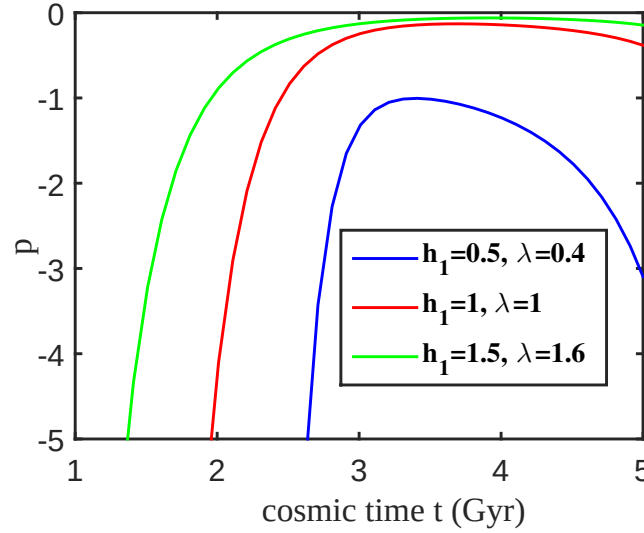


Figure 4: Plot of pressure p versus cosmic time t for $k_1 = 0.5$ and $c_1 = 1$.

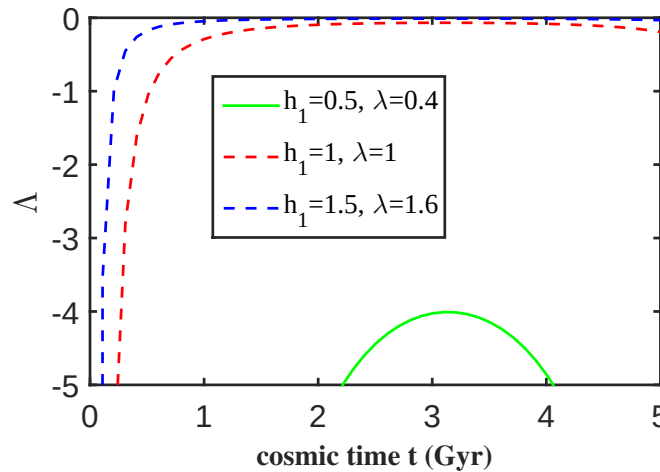


Figure 5: Plot of cosmological constant (Λ) versus cosmic time t for $k_1 = 0.5$ and $c_1 = 1$.

A signature flipping behavior of the deceleration parameter fixes h_1 to be greater than 1 (see figure 2). In view of this, one may take λ as a free parameter with positive values only. Here, we have considered three moderate values, $\lambda = 0.4, 1$ and 1.6 for numerical calculations of the dynamical parameters.

Figure 3 represents the behavior of energy density ρ versus cosmic time t . It can be seen from the graph that it decreases as the cosmic time increases. In this case we take the values of the parameters $h_1 = 0.5, 1, 1.5$, $\lambda = 0.4, 1, 1.6$, $k_1 = 0.5$ and $c_1 = 1$. For the above choice of parameters the energy densities are positive throughout the evolution of the model. It is observed that the energy densities are always positive and decrease with increasing cosmic time in the model. The evolutionary trend of the energy density is not changed by a variation of λ , rather an increase in λ simply decreases the value of ρ at a given time.

Figure 4 describes the behaviour of pressure versus cosmic time t . It shows that the pressure p of the universe is an increasing function of cosmic time t , which begins from a large negative value and tends to zero at present epoch. As per the observation, the negative pressure is due to dark energy in the context of accelerated expansion of the universe. Hence, the behavior of pressure in our model is agreed with

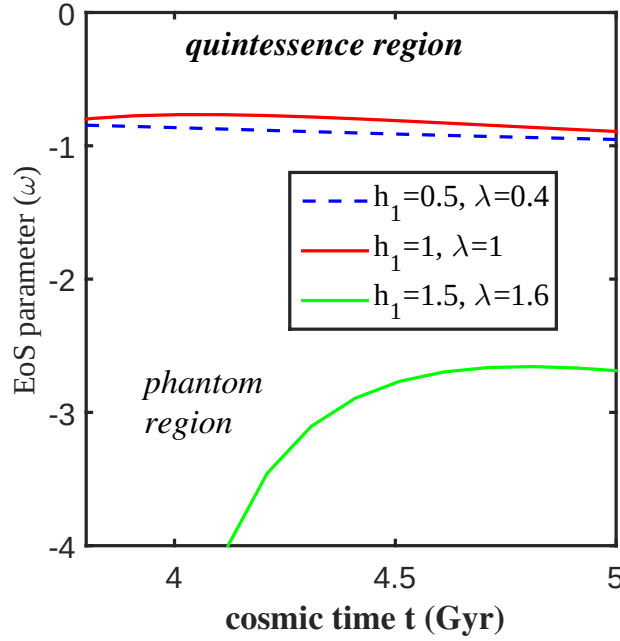


Figure 6: Plot of equation of state parameter (ω) versus cosmic time t for $k_1 = 0.5$ and $c_1 = 1$.

this observation. Here, pressure is a negative quantity at the present epoch in a given model. The choice of the parameter h_1 and λ has some effects on the evolutionary trend. In general, lower value of λ results in a pressure curve that lies to more negative values.

The equation of state (EoS) parameter can be obtained in a straightforward manner from Eqs. (44) and (45) as

$$\omega = \frac{p}{\rho} = \frac{-3k_1^2(1+2(1+3\lambda)(1-h_1))X^{-2}Y^4 - 6k_1^2h_1(1+3\lambda)Y^2 - 4c_1^2h_1^2(1+4\lambda)X^{\frac{-6}{h_1}}}{3k_1^2(3+2\lambda(3+h_1))X^{-2}Y^4 - 6k_1^2\lambda h_1Y^2 - 4h_1^2c_1^2(1+4\lambda)X^{\frac{-6}{h_1}}}, \quad (48)$$

where $X = \tan\left(\frac{1}{2}k_1t\right)$ and $Y = \sec\left(\frac{1}{2}k_1t\right)$.

The behavior of EoS parameter (ω) versus cosmic time t for our model is depicted in figure 6 for the chosen constant parameters h_1 and λ . It may be observed that the model starts in quintessence regions for $h_1 = 0.5, 1$, and $\lambda = 1.4, 1$ which varies in the same region. In this case, the EoS parameter remains within the quintessence region with a value close to Λ CDM model in late times. However, the model for parameters $h_1 = 1.5$ and $\lambda = 1.6$ starts in high phantom region and lies in the same region which a dark energy-driven accelerated phase ($\omega < -1$) which is consistent with the current observational data of the universe [50]. It may be noted that the equation of state parameter became influenced by the parameters of h_1 and λ . Moreover, the EoS parameter exhibits an oscillatory behavior in both regions. One interesting feature of the equation of state parameter is that, it does not acquire any singular values during the cosmic cycle within time frame. Since the periodic varying deceleration parameter does not have singularity, the same thing also occurs in the EoS parameter. In these constructed model, the EoS evolves with cosmic time which is more evident in the reconstruction history of the dynamical dark energy based on recent data sets [51, 52].

We can obtain the density parameter (Ω) for the present model as

$$\Omega = \frac{\rho}{3H^2} = \frac{(3+2\lambda(3+h_1))}{3[(1+3\lambda)^2-\lambda^2]} - \frac{2\lambda h_1 \sin^2(\frac{1}{2}k_1 t)}{3[(1+3\lambda)^2-\lambda^2]} - \frac{(1+4\lambda)c_1^2 h_1^2 (\tan(\frac{1}{2}k_1 t))^{\frac{-6}{h_1}} \sin^2(k_1 t)}{9k_1^2 [(1+3\lambda)^2-\lambda^2]}. \quad (49)$$

It can be expressed as a function of redshift (z) as

$$\Omega = \frac{(3+2\lambda(3+h_1))}{3[(1+3\lambda)^2-\lambda^2]} - \frac{2\lambda h_1 [(z+1)^{2h_1}+1]^{-1}}{3[(1+3\lambda)^2-\lambda^2]} - \frac{4(1+4\lambda)c_1^2 h_1^2 (z+1)^{6+2h_1}}{9k_1^2 (1+(z+1)^{2h_1})^2 [(1+3\lambda)^2-\lambda^2]}. \quad (50)$$

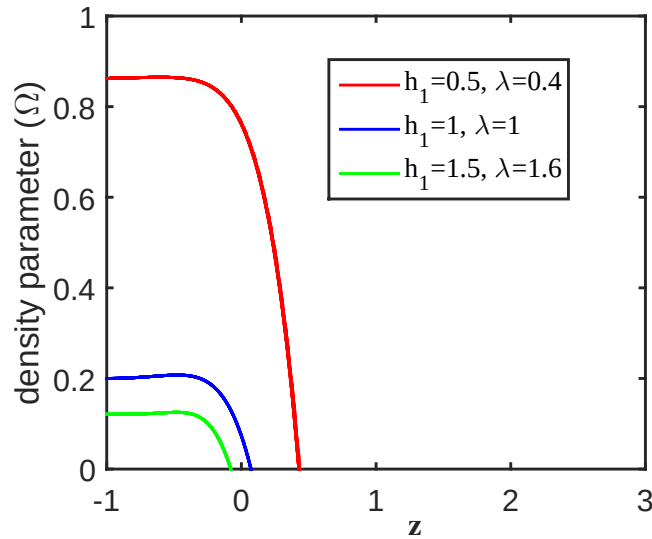


Figure 7: Plot of density parameter (Ω) versus redshift (z) for $k_1 = 0.5$ and $c_1 = 1$.

Figure 7 represents the behavior of Ω versus redshift z . It can be seen that it increases as the universe evolves. Here we have chosen the constant values ($c_1 = 1, k_1 = 0.5, h_1 = 0.5, 1, 1.5$ and $\lambda = 0.4, 1, 1.6$) such that we arrive at Ω approaching 1 for small values of h_1 and λ . Hence, the density parameter shows that in agreement with the observational data of the universe. Moreover, with the cosmic evolution, Ω decreases with cosmic time. The density parameter, at a given redshift, is observed to have lower value for higher values of λ .

Model of the universe close to Λ CDM can be described using the cosmic jerk parameter j , a dimensionless third derivative of the scale factor with respect to the cosmic time [53]. The value of the jerk parameter is constant for a flat Λ CDM model. The jerk parameter j which shows the deviation of a model from the Λ CDM in our case is given by

$$j = \frac{\ddot{a}}{aH^3} = q(1+2q) - \frac{\dot{q}}{H} = h_1^2 (\cos^2(k_1 t) + 1) - 3h_1 \cos(k_1 t) + 1. \quad (51)$$

For small value(s) parameters h_1 near to zero, then the jerk parameter becomes approach to 1 which explains the time evolution of the Λ CDM model.

6. Conclusions

In this paper, we investigated a spatially homogeneous locally rotationally symmetric Bianchi type-I space-time in the presence of perfect fluid cosmological model within the framework of $f(R, T)$ theory of gravity by following the works of [3], choosing $f(R, T) = R + 2f(T)$, where $f(T) = \lambda T$. To obtain an exact solution of the model, we assumed periodically varying deceleration parameter. We examined the model by looking at cosmological parameters as the followings: the deceleration parameter is assumed to be periodically varying declaration parameter and its graphical representation with respect to cosmic time is shown in figure 1. It can be observed from figure 2 that for $h_1 = 1.5$ the model describes a smooth transition from early deceleration to the present accelerated phase of the universe. It may also be seen that the model exhibits transition at $z \approx 0.71$ which is quite in accordance with recent cosmological observations [54–56]. But from the figure 2, it can be observed that the universe completely lays in the accelerating phase for $h_1 = 0.5$ and 1. The energy density (ρ) of universe is positive decreasing functions of cosmic time (figure 3). The energy density of the model positive throughout the evolution of the Universe and approaches to zero for large values of cosmic time t . The evolution of the universe in model for the pressure p versus cosmic time t is shown in figure 4 with different values of parameters h_1 and λ , with constants k_1 and c_1 . The pressure has negative values for the model which shows that the universe is accelerated expanding for late times. Similarly figure 5 shows the cosmological parameter Λ versus cosmic time becomes approaching to zero at late times. Hence our model is in excellent agreement with observational constraints providing that the present value of Λ is chosen [57]. The dynamics of the universe is studied through the equation of state parameter. As it is shown in figure 6, for the values of $h_1 = 0.5, 1$ and $\lambda = 0.4, 1$ the equation of state parameter ω lies in quintessence region which an EoS parameter that more likely approaches to Λ CDM model at late times, while for $h_1 = 1.5$ and $\lambda = 1.6$ larger values of the phantom behavior is attained in the near future. An interesting consequence of the present model is that it allows both quintessence and the phantom like behaviour for free parameters. From figure 7, we can conclude that for smaller values of the parameters $h_1 = 0.5$ and $\lambda = 0.4$, the density parameter approaches to 1 which describes the flatness of universe which confirms the present cosmological data of the universe. It can be seen that the density parameter Ω increases with respect to redshift as the universe evolves. We have seen that for small value(s) of h_1 approaching to zero, the jerk parameter becomes approximately equal to 1 which indicates a flat Λ CDM model. Therefore, it is concluded that the findings support the current accelerating expansion of the universe.

7. References

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