SLAC-PUB-1485
September 1974
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SHORT DISTANCE DOMINANCE OF VERTEX FUNCTIONS*

P. Menotti**

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

and

Joseph Henry Laboratories
Princeton University, Princeton, New Jersey 08540

ABSTRACT

In order to calculate high-energy high-momentum transfer processes for particles described by composite fields one needs to know some uniformity properties of the external particle vertex function. We examine such vertex function in a scalar theory in six dimensions in the case in which anomalous dimensions are realized. We start from the result that, in a certain class of renormalizable theories, the vertex function for three fundamental fields is dominated by short distances. This is used to prove that the convolution of the composite particle vertex function with the direct and crossed renormalized skeleton graphs is consistent only if such a vertex function is also dominated by short distances.

(Submitted to Phys. Rev. D.)

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*Work partially supported by the U. S. Atomic Energy Commission under contract AT(11-1) 3072 and AT(04-3)-515.

**On leave of absence from Scuola Normale Superiore Pisa and INFN Sezione di Pisa.

I. INTRODUCTION

A major theoretical problem is whether high-energy high-momentum transfer phenomena are related to short distance behavior of field theory[1] when the external particles are kept on shell. Such a question is related to the concept of dimensional scaling.[2] Some results have been obtained recently with regard to a particular on-shell process — i.e., the form factor — for particles described by fundamental fields with anomalous dimensions.[3] In fact, it has been found[3] that in a class of renormalizable field theories the inhomogeneous term (mass insertion) of the Callan-Symanzik equation for the three-point vertex function is negligible at high momentum transfer even if two of the particles are kept on shell. It follows directly that in such field theories the form factor at high momentum transfer is dominated by short distances — i.e., we have the simple dimensional result (with anomalous dimensions). If the external particles are composite objects (described by composite fields), as probably the hadrons are, the problem appears to be more complicated. The reason is that the structure of the vertex function of the composite field changes abruptly when one goes on shell.[4] This feature compels one to work directly on the pole of the external particle. What is needed is the residue of the vertex function of the composite field decaying into its constituent at the pole of the external particle; once this is known the convolution of such external vertices on a irreducible kernel representing the scattering of fundamental constituents gives the on-shell amplitude. Some characteristics of such vertex function are directly known from dimensional analysis which fixes the strength of the leading high-cone singularity. Some valuable information is also supplied by conformal invariance[5] which appears to be valid on the light-cone in theories with anomalous dimensions,[6] fixing the weight of the leading light-cone singularity. On the other hand such information is insufficient to allow calculations of asymptotic on-shell amplitudes,[7] more stringent uniformity hypotheses being necessary in order to recover, for example, the dimensional scaling results. In the present paper we shall essentially be concerned with these uniformity properties which play such a crucial role in on-shell computations. As one has to work on the composite particle pole it does not seem possible to apply directly perturbation techniques and we shall have to resort to the expansion in terms of skeleton graphs.[8] The model we shall examine here is a field theory with anomalous dimensions; it seems that this situation should be somewhat simpler than the asymptotic freedom case.[9] In order to avoid dealing with spins or with the complicated topology of ϕ^4 we shall work with a scalar field in six dimensions with trilinear coupling.[10] Thus the treatment applied in the attractive domain of an ultraviolet stable point in $(\phi^3)_6$, if it exists, or better should be considered as a guideline to more realistic theories like Yukawa in four dimensions or possibly to asymptotically free field theories.

In Section II, after recalling why uniformity properties are necessary for computing asymptotic behaviors of on-shell amplitudes, we ask ourselves which kind of uniformity properties can be expected in a vertex function. In doing this we examine the conformal contribution to the vertex function and to its discontinuity. This analysis even if not necessary for the treatment of Section III indicates which are the objects related to the vertex function for which simple uniformity properties can be expected; these are the vertex functions (or residues thereof) from which the external renormalized propagators corresponding to elementary fields have been removed.[11] The study of such objects is performed in Section III by using the skeleton graph expansion of the renormalized theory. We start from the result[3] that in a class of field theories the vertex function of three fundamental fields, truncated in all three external propagators, is dominated by the short distances — i.e., has the same power behavior when one, two, or three of the squares of the external momenta tend to infinity. It is shown that such a property imposes restrictions on the spectral function of the parametric representation of the vertex function which is going to play the key role in the subsequent developments. Then we consider the convolution of the composite particle vertex function with the direct exchange and crossed renormalized skeleton graphs. The results are as follows: If one starts from a truncated vertex function of the composite particle dominated by short distances then by using the restrictions on the spectral functions of the vertices appearing in the skeleton graphs one ends up with a consistent result. On the other hand if one starts with a vertex function not dominated by short distances, the convolution both with the direct exchange and the crossed renormalized skeleton graphs turns out to be inconsistent with the initial vertex function, in the sense that the iterated vertex function, as the square of one external momentum goes to infinity (the square of the other momentum being kept constant) decreases faster than the initial vertex function. Expressed in different words, if one should start with a vertex function not dominated by short distances, after a finite number of iterations with the above mentioned skeleton graphs, one would settle down to a vertex function dominated by short distances.

II. GENERAL DISCUSSION OF VERTEX FUNCTIONS

As mentioned in the introduction, a rather detailed knowledge of the vertex function of the external particles is needed in order to calculate asymptotic behaviors of on-shell amplitudes. Due to the relevance of this fact we want to recall it in a simple example. Consider the graph of Fig. 1, which contributes to the elastic form factor with all the internal scalar fields taken as canonical for the sake of illustration. The contribution of such a graph to the asymptotic form factor is an integral of the type[4]

$$\int \frac{g(z,t)\, g(z',t')\, dzdt\, dz'dt'\, d\alpha(1-\alpha^2)^{L+1}}{\left[-q^2(1-\alpha^2)\,(1-z')\,(1-z)+m^2-\frac{M^2}{4}+t(1+\alpha)+t'(1-\alpha)\right]^{L+1}} \tag{2.1}$$

where M and L are the mass and the angular momentum of the external particle and m is the mass of the particle propagated in the loop. g is the Deser-Gilbert-Sudarshan spectral function[12] of the (nontruncated) vertex function of the external particle[4] the dimension of whose field we shall call A. Dilatation invariance tells us that g(z, t) behaves for large t like

$$t^{-\frac{A}{2}+\frac{L}{2}}$$

That is,

$$g(z,t) \sim t^{-\frac{A}{2}+\frac{L}{2}}\, g(z) \tag{2.2}$$

and thus short distances give to (2.1) the contribution

$$\int \frac{d\alpha\, dz\, dz'\, g(z)\, g(z')\, (1-\alpha^2)^{\frac{A}{2}+L}}{\left[-q^2(1-z)\,(1-z')\,(1-\alpha^2)+m^2-\frac{M^2}{4}\right]^{A-1}} \tag{2.3}$$

Conformal invariance, which is realized on the light-cone in theories with anomalous dimensions,[6,13] fixes the asymptotic weight g(z) which in this simple example turns out to be

$$(1-z^2)^{\frac{A}{2}+\frac{L}{2}-1} \quad .$$

Such a restriction[14] gives for the asymptotic behavior of (2.3) $(-q^2)^{1-A}$, which is the dimensional rule. On the other hand if the restriction

$$g(z,t) < (1-z)^{A-2}\, h(t) \tag{2.4}$$

is violated in any finite interval of t the dimensional scaling result is invalidated.

Thus the information given by the conformal invariance even if valuable at the leading light-cone level is insufficient to guarantee the validity of the dimensional scaling rule as a priori it does not give any bound on the conformal breaking terms. Equation (2.4) is the kind of uniformity we were speaking about in the introduction and it would actually follow from the assumption that the vertex function from which the two elementary propagators have been removed has the same power behavior when one or both of the external momenta squared go to infinity.

In order to gain some insight into the structure of the vertex function we shall consider the conformal contribution to the vertex function and to its discontinuity. What we develop here is not strictly necessary for the treatment of Section III but it will be of help in pointing out which are the simple features one can look for in a vertex function. The conformal covariant (nontruncated) vertex function has the form

$$\int \alpha^A \beta^B \gamma^C\, \delta(1-\alpha-\beta-\gamma)\, d\alpha\, d\beta\, d\gamma \left[-q_1^2\beta\gamma - q_2^2\alpha\gamma - q_3^2\alpha\beta\right]^R \tag{2.5}$$

where

$$\begin{aligned} R &= \frac{d_1+d_2+d_3}{2} - D\ , & A &= -\frac{d_2+d_3-d_1}{2} + \frac{D}{2} - 1 \\ B &= -\frac{d_1+d_3-d_2}{2} + \frac{D}{2} - 1\ , & C &= -\frac{d_1+d_2-d_3}{2} + \frac{D}{2} - 1\ . \end{aligned} \tag{2.6}$$

d_1, d_2, d_3 are the dimensions of the fields and D the dimension of space-time.

The noteworthy fact is that if d_1, d_2, d_3 are all larger than $\frac{D}{2}$ then Eq. (2.5) has the asymptotic behavior

$$(-q^2)^R \tag{2.7}$$

irrespective of whether one, two or all three of the arguments $-q_1^2$, $-q_2^2$, $-q_3^2$ go to infinity. This fact is directly checked on (2.5) where one sees that in the above described situation the limit is simply calculated by pulling out the argument, as the left over integral is convergent. On the other hand, in a conformal theory, one can remove one (or more) external propagators by changing in expression (2.5) the related dimension d into the "shadow" dimension[15] D-d. As either d or D-d is bigger than $\frac{D}{2}$ the rule follows: the simple object — i.e., the function which behaves asymptotically with the same power behavior irrespective of the number of external square moments q^2 which are taken to ∞ — is given by the (nontruncated) vertex function from which those external propagators are removed, which corresponds to fields of dimension $< \frac{D}{2}$. Thus the simple object (i.e., with uniform asymptotic behavior) is obtained by truncating the external propagators corresponding to elementary fields. It has to be stressed that this is simply a result of the purely conformal invariant theory, which does not contain any indication about the conformal breaking terms. It appears however that the vertex function truncated in the elementary external fields should be the simplest object to be studied.

The next point about which the conformal covariant vertex function throws light is the following. Let us consider the discontinuity of the vertex function (Eq. (2.5)) in one external leg, e.g., q_3^2. (This has to be considered as the analogue of the residue of the vertex function at the pole corresponding to the external particle.) How does such discontinuity relate to the original vertex function, with particular regard to the large q_1^2 and/or large q_2^2 behavior?

In order to calculate such a discontinuity let us write Eq. (2.5) in the form

$$V = \int_{-1}^{+1} dz \int_0^\infty dy \, (1-z^2)^{R+1+C} \, (1-z^2+4y)^{-2R-3-A-B-C} \, y^{R+1+A+B} \, (1+z)^A \, (1-z)^B$$
$$\left[-q_1^2 \frac{1-z}{2} - q_2^2 \frac{1+z}{2} - q_3^2 \, y\right]^R \tag{2.8}$$

The discontinuity in q_3^2 (clearly for $q_3^2 > 0$) is due to the argument of the square bracket getting negative and is given by

$$\operatorname*{Disc}_{q_3^2} V = \text{const} \int dz \int_{y_0}^\infty dy \, (1-z^2)^{R+C+1} \, (1-z^2+4y)^{-2R-3-A-B-C}$$
$$y^{R+1+A+B} (y-y_0)^R \, (1+z)^A \, (1-z)^B \left(q_3^2\right)^R \tag{2.9}$$

where

$$y_0 = \frac{1}{q_3^2} \left\{-q_1^2 \frac{1-z}{2} - q_2^2 \frac{1+z}{2}\right\} \tag{2.10}$$

If one now examines the behavior of $\operatorname*{Disc}_{q_3^2} V$ for large q_1^2 and q_2^2 one sees that the behavior is

$$\text{const} \left(q_3^2\right)^R \int (y_0)^{-C-1} \, (1-z^2)^{R+C+1} \, (1+z)^A \, (1-z)^B \, dz$$
$$= \text{const} \left(q_3^2\right)^{R+C+1} \int \left\{-q_1^2 \frac{1-z}{2} - q_2^2 \frac{1+z}{2}\right\}^{-C-1} (1-z^2)^{R+C+1} (1+z)^A (1-z)^B \, dz \tag{2.11}$$

This is the same as the light-cone of the simple vertex function if $d_3 < \frac{D}{2}$, in which case we have, defining $\xi = \gamma\left\{-q_1^2\beta - q_2^2\alpha\right\}$

$$\int_0^\infty \xi^C \, d\xi \left\{-q_3^2\,\alpha\beta + \xi\right\}^R \left\{-q_1^2\,\beta - q_2^2\,\alpha\right\}^{-C-1} \delta(1-\alpha-\beta)\, d\alpha\, d\beta$$

$$= \left(-q_3^2\right)^{R+C+1} \int (1-z^2)^{R+C+1} (1+z)^A (1-z)^B \left\{-q_1^2\,\frac{1-z}{2} - q_2^2\,\frac{1+z}{2}\right\}^{-C-1} \qquad (2.12)$$

On the other hand, for $d_3 > \frac{D}{2}$ we have a light cone singularity with exponent R which is completely different from Eq. (2.12). This is the phenomenon of shadow singularities,[16] mentioned in the introduction, which makes the wave function at the pole of a composite particle quite different from the off-shell vertex function.

Going back to Eq. (2.11), one also notices that the right-hand side behaves as $(-q_1^2)^{-C-1}$ when $-q_1^2 \to \infty$ irrespective of whether $-q_2^2$ also goes to ∞ or stays constant, only if

$$R+1+B = d_2 - \frac{D}{2} > 0 \qquad (2.13)$$

That is, the discontinuity of the vertex function may possess a uniform asymptotic behavior only if it is truncated in the external propagators corresponding to elementary fields.

Summing up, while for $d_3 < \frac{D}{2}$ the discontinuity of the vertex function in q_3^2 has the same light-cone as the vertex function itself, for $d_3 > \frac{D}{2}$ (composite external field) even the nature of the light-cone is different. The difficulty outlined above cannot be formally circumvented by considering the vertex function truncated in the external propagator of the composite field as such vertex function changes nature at the value of the physical mass.

As a result, if the external fields are composite and one is interested in on-shell phenomena one has to examine directly the residue at the pole of the vertex function. This is what we shall do in the next section.

III. CONVOLUTION OF THE VERTEX FUNCTION WITH THE RENORMALIZED SKELETON GRAPHS

In this section we shall consider the convolution of the vertex function with the direct exchange skeleton graph (Fig. 2a) and with the crossed skeleton graph (Fig. 2b). As explained in the introduction we shall consider a scalar theory of the type $(\phi^3)_6$ (renormalizable) and we shall treat the situation with anomalous dimensions. The propagators and vertices appearing in Fig. 2 are the renormalized propagators and dressed vertices of the exact theory. Thus the propagators will have the representation

$$\Pi(p^2) = \int_0^\infty \frac{\rho(t)\, dt}{-p^2 + m^2 + t - i\epsilon} \tag{3.1}$$

with

$$\rho(t) \sim t^{d-3} \tag{3.2}$$

for large t. d is the dimension of the field and $d > 2$ as 2 is the canonical value for a scalar field in six dimensions. Following Mack[17] we shall assume the restriction $d < D/2 = 3$ on the dimension of the fundamental field. The lower bubble represents the vertex function of the composite particle, i.e., the residue at the pole of the composite-elementary-elementary vertex function, with the two elementary renormalized propagators removed. Such a vertex function shall be described by the DGS[12, 18] representation

$$\Gamma = \int \frac{G(z,t)\, dz\, dt}{\left[-q_1^2 \frac{1-z}{2} - q_2^2 \frac{1+z}{2} + 4m^2 + t\right]} \tag{3.3}$$

We shall indicate with A the dimension of the composite field and with M the mass of the composite particle, $M < 2m$, and we shall bound ourselves to zero angular momentum. We recall that the dimension A of the composite field is subject to a kinematical restriction[19] which follows from the demand that the vertex with three external composite fields should be convergent. Such a restriction is $A < \frac{2D}{3}$ and thus in our own case we have $A < 4$. These kinematical bounds on the dimensions d and A will play an important role in the following. According to the dimensional rule (3.3) behaves for large q_1^2 and q_2^2 (both large) as

$$(q^2)^{-d-\frac{A}{2}+3}$$

which corresponds to[20]

$$G(z,t) \sim G(z)\, t^{-d-\frac{A}{2}+3} \tag{3.4}$$

for large t.

In the discussion which follows we shall be interested in the case in which $-q_1^2$ becomes very large while $-q_2^2$ is kept constant. The vertex connected with q_1 will be described by the representation[18]

$$\int_0^\infty \frac{f(\alpha',\beta',\gamma',t')\,\delta(1-\alpha'-\beta'-\gamma')\,d\alpha'\,d\beta'\,d\gamma'\,dt'}{\left[-q_3^2\alpha' - q_2^2\beta' - q_1^2\gamma' + t' + L^2\right]} \tag{3.5}$$

Such a representation has been proved by Nakanishi[18] to all orders perturbation theory (as the DGS representation is proved) and will be accepted here.

L is a mass satisfying[18]

$$L^2 > \frac{8}{3}m^2 \tag{3.6}$$

Again the dimensional rule tells us that such a vertex function behaves for q_1^2, q_2^2, q_3^2, all large, as

$$(q^2)^{-\frac{3d}{2}+3}$$

corresponding to

$$f(\alpha', \beta', \gamma', t') \sim t'^{-\frac{3d}{2}+3} f(\alpha', \beta', \gamma') \qquad (3.7)$$

for large t'.

On the other hand the vertex connected with q_2 (where q_2^2 = const) will be described by the representation (A. 15) of Appendix A, which is analogous to the DGS representation and whose spectral function will be called $F(z'', t'')$.

We shall consider first the graph of Fig. 2a. The integral corresponding to it is performed using the Feynman parametrization shown in Fig. 3a, and we get the expression

$$\Big\{-q_1^2\left[\bar{\alpha}\bar{\beta}+\gamma'\eta C\right] - q_2^2\bar{\alpha}\bar{\gamma} - P^2\bar{\beta}\bar{\gamma} + C\Big[t\xi + t_\alpha\alpha + t_\beta\beta + t_\gamma\gamma + t'\eta + t''\zeta + + m^2(\alpha+\beta+\gamma+4\xi) + L^2(\eta+\zeta)\Big]\Big\}^{-3} \qquad (3.8)$$

which has to be integrated over the six Feynman parameters $\alpha\,\beta\,\gamma\,\eta\,\zeta\,\xi$ multiplied by the δ-function $\delta(1-\Sigma)$, Σ being the sum of such parameters, and also has to be integrated over the spectral functions of the three propagators and of the three vertex functions.

C is given by

$$C = \alpha+\beta+\gamma+\xi+\zeta+\eta(\alpha'+\beta') \qquad (3.9)$$

while

$$\bar{\alpha} = \alpha + \eta\alpha' + \frac{1-z''}{2}\,\zeta$$

$$\bar{\beta} = \beta + \eta\beta' + \frac{1-z}{2}\,\xi \tag{3.11}$$

$$\bar{\gamma} = \gamma + \frac{1+z''}{2}\,\zeta + \frac{1+z}{2}\,\xi$$

To examine such integrals consider an expansion of the spectral functions $G(z,t)$, $f(\alpha',\beta',\gamma',t')$, $F(z'',t'')$ in decreasing powers of t (see Appendix A).

$$G(z,t) = t^{-d-\frac{A}{2}+3}\,G(z) + t^{-\theta_1}\,G_1(z) + \ldots$$

$$f(\alpha',\beta',\gamma',t') = t'^{-\frac{3d}{2}+3}\,f(\alpha',\beta',\gamma') + t'^{-\delta_1}\,f_1(\alpha',\beta',\gamma') + \ldots \tag{3.12}$$

$$F(z'',t'') = t''^{-\frac{3d}{2}+3}\,F(z'') + t^{-\delta_1''}\,F_1(z'') + \ldots$$

We now exploit the result[3] that the truncated three-point function corresponding to three elementary fields is dominated by short distances, i.e., has the same dimensional power behavior irrespective of the number of external square moments taken to infinity. It is shown in Appendix A that such a property implies the following restrictions on the spectral functions

$$\int x^{-\frac{3d}{2}+3}\,f(\alpha',\beta',\gamma')\;\delta(1-\alpha'-\beta'-\gamma')\;d\alpha'\,d\beta'\,d\gamma' < \infty \tag{3.13}$$

$$\int (1\pm z'')^{-\frac{3d}{2}+3}\,F(z'')\,dz'' < \infty \tag{3.14}$$

where x stays for either α, β, or γ and similar relations follow for $f_1(\alpha,\beta,\gamma)$ and $F_1(z)$ (see Appendix A). Analogously, the assumption that the vertex function (3.3) be dominated by short distances corresponds to

$$\int_{-1}^{+1} dz\ (1\pm z)^{-d-\frac{A}{2}+3}\ G(z)\ dz < \infty \tag{3.15}$$

and a similar relation for $G_1(z)$.

The contribution to the integral of Fig. 2a of the leading terms in (3.12) and of the leading behavior of the spectral function of the propagator is

$$\int d\alpha\, d\beta\, d\gamma\, d\xi\, d\eta\, d\zeta\ \delta(1-\Sigma)\,(\alpha\beta\gamma)^{2-d}\,(\eta\zeta)^{\frac{3d}{2}-4}\,\xi^{d+\frac{A}{2}-4}\,C^{d+\frac{A}{2}-6}$$
$$\left\{-q_1^2\left[\bar{\alpha}\bar{\beta}+\gamma'\eta C\right]-q_2^2\,\bar{\alpha}\bar{\gamma}\ -P^2\,\bar{\beta}\bar{\gamma}+C\left[m^2(\alpha+\beta+\gamma+4\xi)+L^2(\eta+\zeta)\right]\right\}^{3-d-\frac{A}{2}} \tag{3.16}$$

which has to be integrated over the three functions $G(z)$, $f(\alpha',\beta',\gamma')$, $F(z'')$.

What we shall now prove is that if the three-elementary-field vertex function is dominated by the short distances, i.e., if (3.13) and (3.14) are valid and if we start with a composite-elementary-elementary vertex function (3.3) also dominated by the short distances, i.e., if also (3.15) is true, then the asymptotic behavior of the graph of Fig. 2a for $-q_1^2 \to \infty$, ($-q_2^2 = $ const) is

$$\left(-q_1^2\right)^{3-d-\frac{A}{2}} \tag{3.17}$$

i.e., also the result of the integration is dominated by the short distances. To prove this we have simply to prove that

$$\int d\alpha\, d\beta\, d\gamma\, d\xi\, d\eta\, d\zeta\ \delta(1-\Sigma)\,(\alpha\beta\gamma)^{2-d}\,(\eta\zeta)^{\frac{3d}{2}-4}\,\xi^{d+\frac{A}{2}-4}\,C^{d+\frac{A}{2}-6}$$
$$\left\{\bar{\alpha}\bar{\beta}+\gamma'\eta C\right\}^{3-d-\frac{A}{2}} \tag{3.18}$$

integrated over the spectral functions $G(z)$, $f(\alpha',\beta',\gamma')$, $F(z'')$ is convergent. The proof of this fact is given in Appendix B and it relies on the convergence of integrals (3.13), (3.14), and (3.15) and on the fact that due to the δ-function $\delta(1-\Sigma)$ not all of the six Feynman parameters $\alpha\beta\gamma\eta\zeta\xi$ can go to zero at the same time.

The contributions of nonleading terms are dealt with in a similar way. In fact (3.18) is replaced by an analogous integral in which the powers of the six Feynman parameters are replaced by not lower powers and the remaining is replaced by

$$C^{B-3}\left\{-q_1^2(\bar{\alpha}\beta+\gamma'\eta C)-q_2^2\bar{\alpha}\bar{\gamma}-P^2\bar{\beta}\bar{\gamma}+C\left[m^2(\alpha+\beta+\gamma+4\xi)+L^2(\eta+\xi)\right]\right\}^{-B} \tag{3.19}$$

with $B > \frac{A}{2}+d-3$. Keeping in mind that $P^2 < 4m^2$, we have

$$C\left[m^2(\alpha+\beta+\gamma+\xi)+L^2(\eta+\xi)\right]-P^2\,\bar{\beta}\bar{\gamma} > \text{const } C$$

from which we can majorize (3.19) with

$$\text{const } C^{d+\frac{A}{2}-6}(\bar{\alpha}\beta+\gamma'\eta C)^{3-d-\frac{A}{2}}\left(-q_1^2\right)^{3-d-\frac{A}{2}} \tag{3.20}$$

which can now be subject to the same treatment as (3.18), keeping in mind inequalities (A.5, A.14).

Thus, starting from the ansatz that the truncated vertex function for the composite particle is dominated by short distances, we have shown that also the iteration through the renormalized skeleton graph Fig. 2a is dominated by short distances.

Actually it is possible to show that an ansatz for the vertex function Γ which is not dominated by the short distances is inconsistent. To prove this let us consider a spectral function $G(z,t)$ such that at some level (leading or not leading) it has a contribution $t^{-\theta}\,G'(z)$ with $G'(z)\sim(1-z^2)^x$ with $-1 < x < d+\frac{A}{2}-4$. It implies through (3.3) an asymptotic behavior for $-q_1^2 \to \infty$, $-q_2^2 = \mathrm{const}$

$$\left(-q_1^2\right)^{-x-1} \tag{3.21}$$

with $-x-1 > 3-d-\frac{A}{2}$. What one proves is that the iteration of such contribution due to $t^{-\theta}\, G'(z)$ through the graph of Fig. 2a gives a vertex function Γ' which for $-q_1^2 \to \infty$ decreases quicker than (3.21), thus proving the inconsistency of the ansatz. The proof goes as follows: Integrating over the spectral parameters we obtain instead of Eq. (3.16)

$$\int d\alpha\, d\beta\, d\gamma\, d\xi\, d\eta\, d\zeta\;\; \delta(1-\Sigma)\;\alpha^{2-d_1}\, d\alpha\;\beta^{2-d_2}\, d\beta\;\gamma^{2-d_3}\, d\gamma\;\eta^{\frac{3d_4}{2}-4}$$

$$d\eta\;\zeta^{\frac{3d_5}{2}-4}\; d\zeta\;\xi^{\frac{d_6}{2}+A-4}\; d\xi\; H$$

where $d_i \leq d$ for $i = 1,2,3$ and $d_j \geq d$ for $j = 4,5,6$.

$$H = C^{B-3}\left\{-q_1^2(\bar{\alpha}\bar{\beta}+\gamma'\eta C) - q_2^2\,\bar{\alpha}\bar{\gamma} - P^2\,\bar{\beta}\bar{\gamma} + C\left[m^2(\alpha+\beta+\gamma+4\xi) + L^2(\eta+\zeta)\right]\right\}^{-B} \tag{3.22}$$

with $B \geq d+\frac{A}{2}-3$. H can be majorized by

$$C^{x+1-3+\Delta}\left\{-q_1^2(\bar{\alpha}\bar{\beta}+\gamma'\eta C)\right\}^{-x-1-\Delta} \tag{3.23}$$

provided $-x-1-\Delta > -B$. (Always true for Δ not too large.)

In Appendix B we prove that if $\Delta < \min\left(\frac{d}{2}, d-\frac{A}{2}\right)$ the replacement of H by (3.23) in (3.22) gives rise to a convergent integral which proves our assertion.

In a completely similar way one deals with the crossed skeleton graph shown in Fig. 2b. The graph is easily evaluated by generalizing Symanzik cutting rules[21] to the situation where dressed vertices are present, described by the representation (3.5).

The Feynman parametrization is shown in Fig. 3b. The vertices, except those connected to P and q_2, are parametrized by parameters bearing the same index as the corresponding parameter η. For example, the vertex corresponding to η_1 is represented by

$$\int \frac{f(\alpha_{11}, \alpha_{21}, \gamma_1, t_1)\ \delta(1-\alpha_{11}-\alpha_{21}-\gamma_1)\ d\alpha_{11}\, d\alpha_{21}\, d\gamma_1\, dt_1}{\left[-q_1^2\alpha_{11} - q^2\alpha_{21} - p^2\gamma_1 + L^2 + t_1\right]} \tag{3.24}$$

Then the graph of Fig. 2b is simply given by

$$\int \prod_{i=1}^{6} d\alpha_i \prod_{j=1}^{3} d\eta_j\ d\xi\ d\zeta\ \delta(1-\Sigma_\alpha-\Sigma_\eta-\xi-\zeta)\ C^2$$

$$\left\{-q_1^2\left[A_1 + \eta_1\gamma_1 C\right] - q_2^2 A_2 - P^2 A_3 \right.$$

$$\left. + C\left[t\xi + \sum_{i=1}^{6} t_i\alpha_i + \sum_{j=1}^{3} t'_j\eta_j + \bar{t}\zeta + (\Sigma_\alpha + 4\xi)\ m^2 + (\Sigma_\eta + \zeta)\ L^2\right]\right\}^{-5} \tag{3.25}$$

which has to be integrated over the spectral functions of the vertex functions and propagators. Generalized Symanzik rules give, for C and for A_1,

$$C = \bar{\alpha}_1(\bar{\alpha}_3+\bar{\alpha}_4+\bar{\alpha}_5+\bar{\alpha}_6) + \bar{\alpha}_2(\bar{\alpha}_4+\bar{\alpha}_5+\bar{\alpha}_6) + \bar{\alpha}_3(\bar{\alpha}_2+\bar{\alpha}_5+\bar{\alpha}_6) + \bar{\alpha}_4(\bar{\alpha}_5+\bar{\alpha}_6)$$

$$A_1 = \bar{\alpha}_1\bar{\alpha}_2(\bar{\alpha}_3+\bar{\alpha}_4+\bar{\alpha}_5+\bar{\alpha}_6) + \bar{\alpha}_2\bar{\alpha}_4\bar{\alpha}_6 + \bar{\alpha}_1\bar{\alpha}_3\bar{\alpha}_5 \tag{3.26}$$

and the $\bar{\alpha}$'s are the combinations directly read from the graph of Fig. 3b. For example,

$$\bar{\alpha}_1 = \alpha_1 + \alpha_{11}\eta_1 + \alpha_{13}\eta_3$$

$$\bar{\alpha}_5 = \alpha_5 + \alpha_{52}\eta_2 + \frac{1-z}{2}\,\xi \tag{3.27}$$

The leading contribution at short distances for example is obtained from (3.25) by replacing the exponent -5 with $3-\frac{A}{2}-d$ and replacing C^2 with

$$C^{d+\frac{A}{2}-6}\left(\prod_{i=1}^{6}\alpha_i\right)^{2-d}\left(\zeta\prod_{j=1}^{3}\eta_j\right)^{\frac{3d}{2}-4}\xi^{\frac{A}{2}+d-4} \tag{3.28}$$

The assertion that the graph of Fig. 2b for $-q_1^2\to\infty$, $-q_2^2=\text{const}$, behaves like

$$\left(-q_1^2\right)^{3-d-\frac{A}{2}}$$

amounts, as far as this contribution is concerned, to proving the convergence of

$$\int\left(\prod_{i=1}^{6}\alpha_i^{2-d}\,d\alpha_i\right)\left(\prod_{j=1}^{3}\eta_j^{\frac{3d}{2}-4}\,d\eta_j\right)\zeta^{\frac{3d}{2}-4}\,d\zeta\,\xi^{\frac{A}{2}+d-4}\,d\xi$$

$$\delta(1-\Sigma_\alpha-\Sigma_\eta-\xi-\zeta)\;C^{d+\frac{A}{2}-6}\,(A_1+\eta_1\gamma_1C)^{3-d-\frac{A}{2}} \tag{3.29}$$

once integrated over the spectral functions of the five vertices. The proof is given again in Appendix B and is quite similar to the treatment of the simpler skeleton graph of Fig. 1. Similarly, one can prove the inconsistency of an initial vertex function not dominated by short distances.

Summing up, in this section we have shown, starting from the short distance dominance of the vertex function for three elementary fields, that a composite particle vertex function is consistent with the convolution over the direct and the crossed renormalized skeleton graphs only if it is also dominated by short distances.

IV. CONCLUSIONS

In this paper we have examined a uniformity property of the vertex function of composite particles which is essential for computing large momentum transfer on-shell phenomena. What one needs to prove is that the asymptotic behavior of such vertex function truncated in the external propagators corresponding to elementary fields, has the same asymptotic behavior when one or both of the square of the external momenta are taken to infinity. We have studied the case in which anomalous dimensions are realized, which appears simpler to tackle than the asymptotic freedom case. To avoid complication with spins we have examined a scalar structure in six dimensions and trilinear coupling. We start from the result of Shei that for a class of renormalizable field theories (excluding gluon and gauge theories) the truncated vertex function of three elementary fields is actually dominated by short distances. This result implies restrictions on the spectral functions of vertex of three elementary fields. Using such restrictions we prove that the iteration of the composite particle vertex function with the direct exchange and crossed renormalized skeleton graphs are consistent with the initial vertex function only if such a vertex function is dominated by short distances, i.e., if it has the same asymptotic behavior when one or two of the external momenta square are taken to infinity. This is exactly the property needed in on-shell calculation.

ACKNOWLEDGMENTS

The author is grateful to Stanley Brodsky for very useful discussions. He is also grateful to Professor S. D. Drell and Professor M. L. Goldberger for their hospitality at Stanford Linear Accelerator Center and at the Joseph Henry Laboratories, Princeton University.

APPENDIX A

In this appendix we shall examine the implication on the spectral functions of the dominance of the short distances in the vertex functions. We start with the DGS representation of the truncated vertex function of the composite particle

$$\int_0^{\infty}\int_{-1}^{+1} \frac{G(z,t)\ dt\ dz}{\left[-q_1^2\frac{1-z}{2}-q_2^2\frac{1+z}{2}+4m^2+t\right]} \tag{A. 1}$$

Developing G(z, t) in decreasing powers of t

$$G(z,t) = G(z)\ t^{-\theta}+G_1(z)\ t^{-\theta_1}+\dots \tag{A. 2}$$

with $\theta < \theta_1 \dots$, we have for the leading contribution

$$\int_{-1}^{+1} dz\ G(z)\left[-q_1^2\frac{1-z}{2}-q_2^2\frac{1+z}{2}+4m^2\right]^{-\theta}\ dz \tag{A. 3}$$

If we impose that for $-q_1^2 \to \infty$, $q_2^2 = \text{const}$, (A. 3) behaves like $\left(-q_1^2\right)^{-\theta}$ we must have

$$\int_{-1}^{+1} G(z)\ (1-z)^{-\theta}\ dz < \infty \tag{A. 4}$$

For the lower contributions obtained by replacing in (A. 3) G(z) with $G_1(z)$ and θ with θ_1, the requirement of an asymptotic behavior less or equal to $\left(-q_1^2\right)^{-\theta}$ gives

$$G_1(z) \leq \text{const}\ (1-z)^{\theta-1} \tag{A.5}$$

In the case considered in the text $\theta = d+\frac{A}{2}-3$.

In the expansion (A. 2) the power of t with exponent less than -1 should be interpreted as distribution, i.e., defined through partial integration in (A. 1), or more rigorously after extracting from G(z, t) the powers corresponding to the scaling behaviors higher than -1 one should work with the remainder of the spectral function $G_I(z,t)$ which satisfies

$$\int G_I(z,t)\, dz\, dt < \infty . \tag{A.6}$$

The requirement of the short distance dominance now imposes that the integral of $G_I(z,t)$ multiplied by $(1-z)^{-\delta+0}$ should be finite as can be seen by expanding $G_I(z,t)$ in increasing powers of (1-z), i.e., $\sum_\alpha (1-z)^\alpha f_\alpha(t)$. A similar remark applies to the ensuing treatment of the off-shell vertex function.

We consider now the off-shell truncated vertex function of three elementary fields Eq. (3.5)

$$\int_0^\infty \frac{f(\alpha,\beta,\gamma,t)\ \delta(1-\alpha-\beta-\gamma)\ d\alpha\, d\beta\, d\gamma\, dt}{\left[\left(-q_1^2+m^2\right)\alpha + \left(-q_2^2+m^2\right)\beta + \left(-q_3^2+m^2\right)\gamma + L^2 - m^2 + t\right]} \tag{A.7}$$

We develop again $f(\alpha,\beta,\gamma,t)$ in decreasing powers of t

$$f(\alpha,\beta,\gamma,t) = f(\alpha,\beta,\gamma)\ t^{-\delta} + f_1(\alpha,\beta,\gamma)\ t^{-\delta_1} + \ldots \tag{A.8}$$

The fact that

$$\int f(\alpha,\beta,\gamma)\ \delta(1-\alpha-\beta-\gamma)\ d\alpha\, d\beta\, d\gamma \Big[\left(-q_1^2+m^2\right)\alpha + \left(-q_2^2+m^2\right)\beta + \left(-q_3^2+m^2\right)\gamma + L^2 - m^2\Big]^{-\delta} \tag{A.9}$$

for $-q_1^2 \to \infty$ and $q_2^2 = m^2$, $q_3^2 = m^2$, behaves like $\left(-q_1^2\right)^{-\delta}$ imposes

$$\int f(\alpha,\beta,\gamma)\ \alpha^{-\delta}\ \delta(1-\alpha-\beta-\gamma)\ d\alpha\, d\beta\, d\gamma < \infty \tag{A.10}$$

and similar relations with α replaced by β or γ. In the case considered in the text

$$\delta = \frac{3d}{2} - 3 \quad .$$

For the lower singularities we must have that

$$\int F(\alpha)\, d\alpha \left[\left(-q_1^2+m^2\right)\alpha + L^2 - m^2\right]^{-\delta'} \tag{A. 11}$$

has an asymptotic behavior less or equal to $\left(-q_1^2\right)^{-\delta}$ which gives

$$F(\alpha) \leq \text{const}\ \alpha^{\delta-1} \tag{A. 12}$$

where

$$F(\alpha) = \int f_1(\alpha,\beta,\gamma)\ \delta(1-\alpha-\beta-\gamma)\ d\beta\, d\gamma \quad . \tag{A. 13}$$

Thus

$$\int \alpha^{-\delta+0} f_1(\alpha,\beta,\gamma)\ \delta(1-\alpha-\beta-\gamma)\ d\alpha\, d\beta\, d\gamma < \infty \tag{A. 14}$$

It is seen through direct majorization that Eqs. (A. 10), (A. 14), impose also the short distance dominance of the vertex function for $-q_1^2 \to \infty$, q_2^2 and q_3^2 constant less than m^2.

If we keep $q_2^2 = \text{const} \leq m^2$, as we shall use in one vertex in the text we can transform (A. 7) into

$$\int \frac{F(z'', t'')\ dz''\, dt''}{\left[\left(-q_1^2+m^2\right)\frac{1-z''}{2} + \left(-q_3^2+m^2\right)\frac{1+z''}{2} + L^2 - m^2 + t''\right]} \tag{A. 15}$$

which is obtained through the substitution $\frac{1-z}{2} = \frac{\alpha}{\alpha+\gamma}$ and $t'' = \left[t+\gamma\left(m^2-q_2^2\right)\right](\alpha+\gamma)^{-1}$. We know from the previous treatment that (A. 15) behaves for $-q_1^2 \to \infty$ (or $-q_1^2$ and $-q_3^2 \to \infty$) like $\left(-q_1^2\right)^{-\delta}$ and such a representation can be subject to the same treatment as the DGS representation (A. 1).

APPENDIX B

We shall prove first the convergence of (3.16) integrated over the spectral functions $G(z)\ f(\alpha',\beta',\gamma')\ F(z'')$ subject to the restrictions (A.4, A.5, A.10, A.14). We have to exploit the fact that not all of the Feynman parameters $\alpha\,\beta\,\gamma\,\xi\,\eta\,\zeta$, can go to zero at the same time.

$$C = \alpha+\beta+\gamma+\xi+\zeta+\eta\ (\alpha'+\beta') = 1-\eta\gamma' \tag{B.1}$$

1. For $\eta\,\gamma' < \frac{1}{2}$ we can majorize as follows

$$C^{d+\frac{A}{2}-6}\left[\bar{\alpha}\bar{\beta}+\gamma'\eta C\right]^{3-\frac{A}{2}-d} \leq \text{const}\ (\gamma'\eta)^{-\frac{3d}{2}+3+\epsilon}$$
$$\left[\left(\alpha+\xi\frac{1-z''}{2}\right)\left(\beta+\xi\frac{1-z}{2}\right)\right]^{-\frac{A}{2}+\frac{d}{2}-\epsilon} \tag{B.2}$$

2. For $\eta\,\gamma' > \frac{1}{2}$ $\left(\text{which implies } \eta > \frac{1}{2}\right)$ we can majorize

$$C^{d+\frac{A}{2}-6}\left[\bar{\alpha}\bar{\beta}+\gamma'\eta C\right]^{3-\frac{A}{2}-d} \leq \text{const}\left(\alpha+\beta+\gamma+\xi+\frac{\alpha'}{2}+\zeta\right)^{-3} \tag{B.3}$$

The convergence of the integral containing (B.2) is proved by using the inequality (a, b, c, d all > 0)

$$(a+b)^{-c-d} < a^{-c}\, b^{-d} \tag{B.4}$$

on the two terms appearing inside the square bracket. $\alpha^{2-d}\,d\alpha$ (and $\beta^{2-d}\,d\beta$) carry a power 3-d, $\xi^{\frac{3d}{2}-4}\,d\xi\ F(z'')\,dz''$ a power $\frac{3d}{2}-3$ and $\xi^{\frac{A}{2}+d-4}\,d\xi\ g(z)\,dz$ a power $\frac{A}{2}+d-3$.

We have for the $\alpha+\xi\frac{1-z''}{2}$ term:

$$-\frac{A}{2}+\frac{d}{2}+3-d+\frac{3d}{2}-3=d-\frac{A}{2}>0$$

as $d>2$ and $A<4$ due to the kinematical restrictions. For the term $\beta+\xi\frac{1-z}{2}$ we have instead:

$$-\frac{A}{2}+\frac{d}{2}+3-d+\frac{A}{2}+d-3=\frac{d}{2}>0 \quad .$$

The integral of (B.3) is convergent as

$$-3+3(3-d)+\frac{A}{2}+d-3+2\left(\frac{3}{2}d-3\right)=-3+d+\frac{A}{2}>0 \quad .$$

We examine now the situation in which the input spectral function $G(z,t)$ has a $G(z)$ or $G_1(z)$ behaving like

$$G(z) \sim (1-z^2)^x \qquad \text{for} \quad |z| \to 1 \tag{B.5}$$

with $-1<x<d+\frac{A}{2}-4$. The left-hand side of (B.2) is replaced now by Eq. (3.23), i.e.,

$$C^{x+1-3+\Delta}\left\{-q_1^2\,(\bar{\alpha}\bar{\beta}+\gamma'\eta C)\right\}^{-x-1-\Delta} \tag{B.6}$$

Following the previous treatment we have

1. For $\eta\gamma'<\frac{1}{2}$

$$C^{x+1-3+\Delta}\left\{\bar{\alpha}\bar{\beta}+\gamma'\eta C\right\}^{-x-1-\Delta}$$

$$\leq \text{const}\,(\gamma'\eta)^{-\frac{3d}{2}+3+\epsilon}\left[\left(\alpha+\xi\,\frac{1-z''}{2}\right)\left(\beta+\xi\,\frac{1-z}{2}\right)\right]^{-x-1-\Delta+\frac{3d}{2}-3-\epsilon} \quad .$$

$\beta^{2-d'}\,d\beta$ carries at least the power $3-d$, while $\xi\frac{1-z}{2}$ now only $x+1$. Thus

$$-x-1-\Delta+\frac{3d}{2}-3+3-d+x+1=-\Delta+\frac{d}{2}>0 \qquad \text{if}\ \Delta<\frac{d}{2}$$

and for $\alpha + \xi \frac{1-z''}{2}$ we have

$$-x-1-\Delta + \frac{3d}{2} - 3 + 3-d + \frac{3d}{2} - 3 = 2d - 3 - x-1-\Delta$$

which is positive for

$$2d - 3 - x-1-\Delta > d - \frac{A}{2} - \Delta > 0 ,$$

i.e., for $\Delta < d - \frac{A}{2}$.

2. For $\eta\gamma' > \frac{1}{2}$ we majorize

$$C^{x+1-3+\Delta} \{\bar{\alpha}\bar{\beta} + \gamma'\eta C\}^{-x-1-\Delta} \leq \text{const} \left(\alpha+\beta+\gamma+\xi+\frac{\alpha'}{2}+\xi\right)^{-3}$$

whose convergence has been already discussed.

We consider now the proof of the convergence of (3.29) once integrated over the five reduced spectral functions $G(z)\,F(z'')f(\alpha_{11}, \alpha_{21}, \gamma_1)$, $f(\alpha_{22}, \alpha_{32}, \alpha_{52})$, $f(\alpha_{43}, \alpha_{13}, \alpha_{63})$. Again we exploit the fact that not all Feynman parameters can vanish at the same time. For example, for $\alpha_3 >$ const we majorize

$$C^{d+\frac{A}{2}-6} \left(A_1 + \eta_1\gamma_1 C\right)^{3-d-\frac{A}{2}}$$

$$\leq \text{const} \left(\eta_1\gamma_1\right)^{-\frac{3d}{2}+3+\epsilon} \left(\bar{\alpha}_1 + \bar{\alpha}_2 + \bar{\alpha}_5 + \bar{\alpha}_6\right)^{-3+\frac{A}{2}-\frac{d}{2}+\epsilon}$$

$$\left(\bar{\alpha}_1\bar{\alpha}_2 + \bar{\alpha}_1\bar{\alpha}_5\right)^{-\frac{A}{2}+\frac{d}{2}-\epsilon}$$

$$\leq \left(\eta_1\gamma_1\right)^{-\frac{3d}{2}+3+\epsilon} \left(\alpha_1 + \alpha_{13}\eta_3\right)^{-\frac{A}{2}+\frac{d}{2}-\epsilon} \left(\alpha_6 + \xi\,\frac{1+z}{2}\right)^{-\frac{A}{2}+\epsilon}$$

$$\left(\alpha_2 + \alpha_{22}\eta_2 + \alpha_5\right)^{-3+\frac{A}{2}-\epsilon}$$

which through relation (B.4) gives rise to a convergent integral.

One deals similarly with the other α's. With the parameters $\eta_2 \eta_3 \xi \zeta$ one notices that having, e.g., $\eta_2 >$ const implies (through the definitions of the $\bar{\alpha}_i$ and keeping in mind that $\alpha_{22}+\alpha_{32}+\alpha_{52}=1$) that either $\bar{\alpha}_2$, $\bar{\alpha}_3$ or $\bar{\alpha}_5$ are > const, which reduces the treatment to the previous cases. For $\eta_1 >$ const we have for $\gamma_1 > 1/2$

$$C^{d+\frac{A}{2}-6}\left\{A_1+\eta_1\gamma_1 C\right\}^{3-d-\frac{A}{2}} \leq C^{-3}$$

which using the same technique as above is proved to give rise to a convergent integral. For $\gamma_1 \leq 1/2$ we must have either $\alpha_{21} > 1/4$ or $\alpha_{11} > 1/4$ and we are back to either $\bar{\alpha}_1 >$ const or $\bar{\alpha}_2 >$ const.

REFERENCES

1. M. Gell-Mann and F. E. Low, Phys. Rev. 95, 1300 (1954);
C. G. Callan, Phys. Rev. D 2, 1541 (1970);
K. Symanzik, Comm. Math. Phys. 18, 227 (1970).

2. S. J. Brodsky and G. R. Farrar, Phys. Rev. Letters 31, 1153 (1973) and CALT 68-441 preprint;
V. Matveev, R. Muradyan and A. Tavkhelidze, Lettere Nuovo Cimento 5, 907 (1972) and 7, 719 (1973);
R. Blankenbecler, S. Brodsky and J. Gunion, Phys. Letters 39B, 649 (1972) and Phys. Rev. D 8, 287 (1973);
P. V. Landshoff and J. D. Polkinghorne, Phys. Rev. D 8, 927 (1973).
For a review see S. J. Brodsky in Fifth International Conference on High Energy Collisions at Stony Brook, N. Y., August 1973.

3. S-S. Shei, Rockefeller University preprint (July 1974). The class of theories considered by Shei include all renormalizable field theories except gluon theories and gauge theories. See also M. Creutz and L-L. Wang, BNL-19078 preprint (July 1974). For previous results on particular classes of graphs in $(\phi^3)_6$ and Yukawa theory see G. C. Marques, Phys. Rev. D 9, 386 (1974).

4. P. Menotti, Phys. Rev. D 9, 2767 (1974).

5. G. Mack and A. Salam, Ann. Phys. (N.Y.) 53, 174 (1969).
S. Ferrara, R. Gatto and A. F. Grillo, Phys. Letters 36B, 124 (1971); Phys. Rev. D 5, 3102 (1972); Springer Tracts in Modern Physics 67 (1973) and references therein.
G. Mack and I. Todorov, Phys. Rev. D 8, 1764 (1973).

6. C. G. Callan and D. Gross, Princeton University preprint (1972);

S. Ferrara, A. F. Grillo and G. Parisi, Nucl. Phys. B54, 553 (1973);

B. Schroer, Lettere Nuovo Cimento 2, 867 (1971).

7. M. Ciafaloni and P. Menotti, Phys. Rev. 173, 1575 (1968);

T. Appelquist and E. Poggio, Harvard University preprint (1974).

8. G. Mack and I. T. Todorov, Ref. 5;

G. Mack and K. Symanzik, Comm. Math. Phys. 27, 247 (1972).

9. D. Gross and F. Wilczek, Phys. Rev. Letters 30, 1343 (1973);

H. D. Politzer, Phys. Rev. Letters 30, 1346 (1973).

10. K. Symanzik, Lectures on High Energy Physics, Zagreb, 1961 (Gordon and Breach, New York, 1965);

G. Mack in Strong Interaction Physics (Springer-Verlag New York, Inc., New York, 1973);

T. Appelquist and G. Poggio, Ref. 7 in the context of asymptotic freedom.

11. We shall assume, following Mack (Ref. 17) the restriction $d < D/2$ (D being the dimension of space time) for the dimensions of the fundamental fields appearing in the Lagrangian. A field with dimension less than D/2 will be called elementary.

12. S. Deser, W. Gilbert and E.C.G. Sudarshan, Phys. Rev. 115, 731 (1959). This function $g(z,t)$ is not to be confused with the function $G(z,t)$ of Section III which refers to the truncated vertex function.

13. S. Ferrara, R. Gatto and A. F. Grillo, Ref. 5.

14. We recall that in this model $A < 2+L$. See Ref. 4.

15. S. Ferrara and G. Parisi, Nucl. Phys. B42, 281 (1972);
S. Ferrara, A. F. Grillo, G. Parisi and R. Gatto, Lettere Nuovo Cimento 4, 115 (1972) and Nuovo Cimento 19A, 667 (1974);
L. Bonora, L. Sartori and M. Tonin, Nuovo Cimento 10A, 667 (1972).

16. S. Ferrara, A. F. Grillo and G. Parisi, Lettere Nuovo Cimento 5, 147 (1972).

17. G. Mack, University of Bern preprint (1973).

18. N. Nakanishi, Graph Theory and Feynman Integrals (Gordon and Breach, New York, 1971).

19. See G. Mack and I. T. Todorov, Ref. 5.

20. Representation (3.3) holds for $d+\frac{A}{2}-3 < 1$. Otherwise one has to use a representation with a higher integer power in the denominator, or equivalently consider the spectral function $G(z,t)$ as a distribution. A similar remark holds for Eq. (3.5).

21. K. Symanzik, Progr. Theor. Phys. 20, 690 (1958);
R. J. Eden, P. V. Landshoff, D. I. Olive and J. C. Polkinghorne, The Analytic S-Matrix (Cambridge University Press, New York, 1966).

FIGURE CAPTIONS

1. Elementary example showing the relevance of the different asymptotic regions in the vertex function.
2. Iteration of the vertex function through the direct exchange and crossed renormalized skeleton graphs.
3. Feynman parametrization of the graphs of Fig. 2.

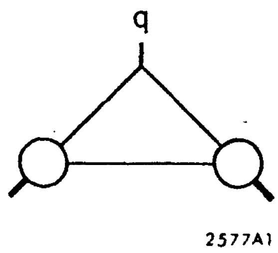

Fig. 1

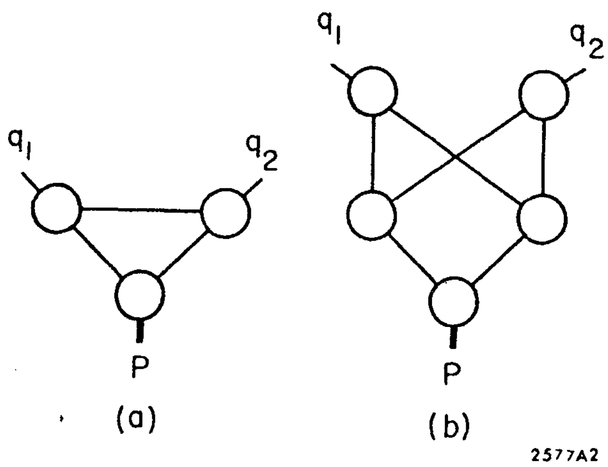

Fig. 2

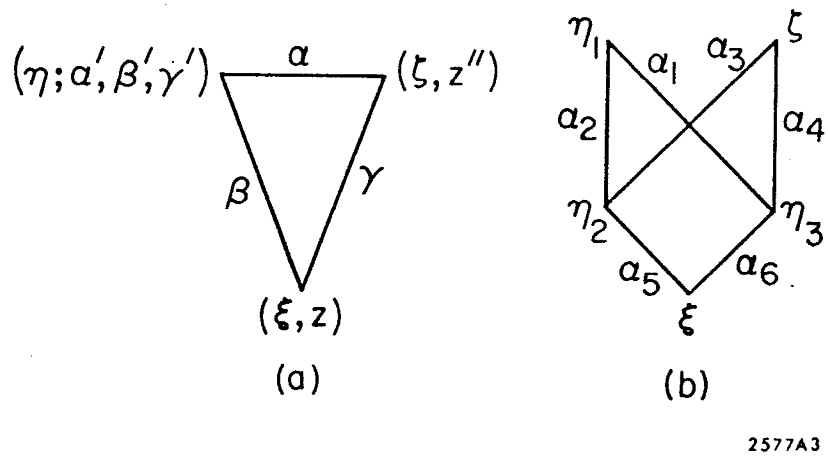

Fig. 3