

NEW COSMOLOGICAL PROBES FOR OLD FUNDAMENTAL QUESTIONS

by

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Abstract

To a good approximation our Universe is flat and homogeneous, and possesses perturbations which, albeit once upon a time small, have seeded the large-scale structure that we observe around us. We believe this to be result of inflation, a period of extremely rapid expansion in the very-early Universe. During this period, it is assumed that the inflaton—a scalar field driving the expansion—receives small quantum-mechanical perturbations, described by a power spectrum. It is interesting to study to what extent this assumption is correct. More than one active field during inflation can generate non gaussianities, which can be observed with the 21-cm line prior to the formation of the first stars. Additionally, passive fields during inflation give rise to isocurvature, making different species cluster differently, which can be observed with the cosmic microwave background. Finally, the power spectrum might not be entirely scale invariant, in which case its dependence on scales holds information about the duration of inflation. Moreover, the last few decades have turned cosmology into a precision science, from which we have learned that baryonic

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matter, constitutes only a small part of the total matter of the Universe, the rest of it being “dark. We do not know the composition of this dark matter. If it is a particle it may interact with baryons, which might be observable in the 21-cm line aforementioned. However, a significant part of dark matter could be made of compact objects, such as primordial black holes. This would have signatures, both in the form of gravitational-wave events and as gravitational lenses of fast radio bursts. These signatures will allow us to detect any compact component of the dark matter down to one part in a hundred, if it is more massive than 20 solar masses. These studies will shed light into the nature and distribution of the matter in our Universe.

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$\mathcal{O}(1)$ people on Earth that will lay eyes on this document.

Dedication

A mi madre, por haberme hecho quien soy hoy.

A mi padre, por enseñarme el valor del conocimiento.

A mi tío, por siempre estar ahí cuando te necesité.

Espero que os sintáis orgullosos.

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*“The most exciting phrase to hear in science
is not ‘Eureka!’ but ‘That’s funny...’”*

— Isaac Asimov

Chapter 1

Introduction

We live in the golden era of cosmology. During the last few decades a plethora of cosmological experiments have probed our Universe to astounding precision. This has greatly increased our understanding of many physical processes, from the inflationary phase at the birth of the Universe, all the way to the dark-energy domination era we observe today. In this thesis I present how to use different cosmological probes to study our Universe, with the goal of attempting to answer fundamental questions.

Our Universe seems to be well described by a simple model, Λ CDM, which posits that dark matter, an mysterious and seemingly invisible form of matter, outweighs regular baryonic matter five to one.^{1,2} This dark matter ought to be cold, in order to collapse and seed the large scale structure we observe today.^{3,4} Moreover, the energy budget of the Universe is dominated by an even more prevalent dark energy, of unknown origin.^{5,6} It is our goal, as cosmologists, to find the nature of dark matter and dark energy.

In order to do that we will employ different cosmological probes. The reason for doing that is twofold. First, different observables suffer from entirely different systematics, which is of great help to disentangle the primary signal (physics) from secondaries. Second, each cosmological measurement has a rich phenomenology, which I enjoy unveiling and understanding.

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In this thesis I will study four cosmological observables... The first probe I will describe is the cosmic microwave background, or CMB. When the primordial baryon-photon plasma reached a temperature low enough for hydrogen to be stable, at $z \approx 1100$, the Universe first became transparent, and the radiation in equilibrium got emitted.^{7,8} In it are imprinted the overdensities at the time of decoupling, and by observing them we have learned a great deal about our Universe.

Before jumping on to the properties of the CMB, in Chapter 2 I discuss how the inflationary regime gives rise to the fluctuations in all cosmological scales. I focus on the reheating era at the end of inflation, where the inflaton degree of freedom, along with any other fields, decay into standard-model particles and dark matter.^{9,10} This is a complicated process, and due to the high energies at which it occurs it is unlikely we will ever be able to probe it. However, parametrizing reheating through its equation of state, which is bound to have some reasonable range of values, allows us to make predictions about inflationary parameters better than the naive estimate in the literature.¹¹

The next question I try to answer is whether baryons and dark matter are distributed the same way. If there was only one degree of freedom during inflation that is exactly what you would expect, and we denote that as adiabatic initial conditions.^{12,13} However, a curvaton active during inflation can give rise to isocurvature perturbations. In particular, I focus on compensated isocurvature perturbations, or CIPs, for which baryons and dark matter have overdensities such that the total matter is unperturbed.^{14,15} These are complicated to find, and only appear at higher orders in perturbation theory. In Chapter 3 I discuss how the CMB power spectra are sensitive to CIPs, and find the strongest constraints to date. For this I employ a linear-estimator formalism, for which it is computationally efficient to evaluate likelihoods,¹⁶ and an improved estimator using the full Planck likelihood, as well as lensing data.¹⁷

The duration of inflation is also a mystery. A longer period of quasi de Sitter domination would leave its imprint as a flatter power spectrum of perturbations.^{18,19} The departures from pure scale-invariance, and therefore from pure de Sitter, are parametrized through a few numbers. From our current observations²⁰ we detect a tilt of the power-spectrum index at $5\text{-}\sigma$. Under the most basic assumptions this would imply a next-order tilt, known as running, a factor of ~ 5 too small to be detected by our current experiments. Nonetheless,

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as the next generation of cosmological probes will greatly surpass in sensitivity the current one,²¹ I have forecasted how well they can detect this quantity.²²

The dynamics of inflation, whether with one degree of freedom or more, can be observed through the statistics of the cosmic perturbations. Not only do we have access to the power spectrum (or two-point function) of these perturbations, but also to higher-order point functions. In particular, one expects nongaussianity, manifested through a nonzero three-point function (or bispectrum) for interacting fields during inflation. In the minimal inflation model of a single field slowly rolling down a potential, however, this nongaussianity would be extremely small. The CMB has already been used to search for nongaussianities, and yielded null results within its precision. Therefore, more futuristic probes are needed, such as the 21-cm line of hydrogen.

In Chapter 4 I review the physics of the 21-cm line during the dark ages. The dark ages are the cosmic period after decoupling, and prior to the formation of the first stars, where the baryonic gas cooled down adiabatically.²³ Due to this cooling, and the frequent interactions between atoms, the spin temperature of the hydrogen was lower than that of the CMB, which made it absorb photons. This will allow us to observe the Universe like never before, given that there are almost no fundamental limitations on the resolution.²⁴ I propose using this 21-cm line to study nongaussianities. As opposed to the CMB, where the perturbations are linear, here secondary nongaussianities arise due to the nonlinear evolution of the Universe. I model and propose how to subtract these secondaries.²⁵

The standard inflationary paradigm predicts that our Universe is statistically isotropic, so there is no preferred direction in the Universe. However, vectorial degrees of freedom during inflation can change this picture. The 21-cm line is a fantastic probe of this scenario, and can constrain statistical isotropy to great precision.²⁶

One of the most fundamental questions of cosmology is the nature of dark matter. This puzzling substance permeates our Universe, and through its gravitational effects pulls baryons into galaxies and clusters. Despite the realm of observations to which we have access, little is known about the specific structure of dark matter. It might be a yet-to-discover fundamental particle, or a bigger object, with stellar mass.

If the dark matter is a particle, it is important to study whether it can interact with

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baryons, since that might determine its freeze-out history. Besides collider-based bounds,²⁷ cosmology can provide interesting constraints on baryon-dark matter interactions. These interactions would tend to thermalize both dark matter and baryons, causing the latter to cool down, and the former to heat up. However, we noticed that given the relative velocity between both fluids, and the need to reach mechanical equilibrium, additional heating is produced as the velocity is damped.²⁸ This causes additional fluctuations in the 21-cm line, proportional to that of the relative velocity.²⁹

There exists the possibility, however, that dark matter only interacts gravitationally with baryons. This would be the case if the dark matter was composed of primordial black holes (PBHs),^{30,31} or any other kind of massive compact halo object (MACHO). The possibilities for detection, then, would be extremely limited. Nonetheless, the recent developments in gravitational-wave astronomy have paved the way for this tool to be able to detect PBHs.

In Chapter 5 I discuss how to use gravitational waves to search for primordial black holes. First, we realized that if all dark matter was composed of PBHs the rate of formation of binaries, through close encounters, would be comparable to the rate inferred from LIGO.^{32,33} To distinguish these black holes from regular astrophysical ones, however, remains a challenge. The events produced in PBH binaries would be less clustered, as they would tend to form in small-mass haloes.³⁴ Moreover, PBH binaries are formed in close encounters, leading to short inspirals and quick mergers, so there might remain some eccentricity in them by the time they are observed by LIGO.³⁵

The most direct way to probe dark matter is gravitational lensing, which I discuss in Chapter 6. This would not require the dark matter to interact with baryons in any direct way. Using microlensing of nearby stars, the MACHO collaboration was able to constrain compact-object dark matter, lighter than 30 solar masses, in the Milky-Way halo.³⁶ However, this does not rule out the PBHs suggested in Ref.³² to be the dark matter. I developed a way to use fast radio bursts (FRBs) as lensing targets to constraint dark matter.³⁷ FRBs are extragalactic emissions of radio, of unknown origin, of millisecond duration.³⁸ If they are lensed by a PBH—or any other compact object—in their way to Earth, they would be echoed, leading to a double-peak FRB.

To sum up, this thesis treats about a handful of cosmological probes: the CMB, the

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21-cm line, gravitational waves, and FRBs, and what we can learn about the Universe from them.

“Your theory is crazy, but it’s not crazy enough to be true.”

— Neils Bohr

Chapter 2

Reheating after Inflation

Models of inflation that rely on the slow rolling of a single scalar field have become the canonical family of models for inflation.^{39–41} These models are specified by a potential-energy density $V(\phi)$ given as a function of the inflaton field ϕ . As long as the slow-roll conditions, which require that the slope and curvature of $V(\phi)$ are sufficiently small, are satisfied, the Universe inflates. Inflation then ends and is followed by a period of reheating (see Ref.¹⁰ for a review) that converts the energy density in the inflaton to the thermal bath, at a reheating temperature T_{re} , that fills the Universe at the beginning of the standard radiation-dominated epoch.

In the canonical reheating scenario,¹² oscillations of the inflaton around the minimum of its potential correspond to massive inflaton particles, and these particles then decay to the plasma of Standard Model particles that compose the radiation-dominated Universe. However, the physics of reheating may be far more complicated. For example, different rates for different types of decays into different Standard Model particles may yield different clocks for starting the usual radiation-dominated epoch. There may be a preheating stage,⁴² where there is a resonant production of particles,⁴³ which can enhance the inflaton decay via scattering,⁹ or where inhomogeneous modes may be excited.⁴⁴ Turbulence may also play a role.⁴⁵ It is generally assumed that the reheat temperature is above the electroweak transition (presumably so that weak-scale dark matter can be produced). More

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conservatively, though, the reheat temperature must be above an MeV, the temperature of big-bang nucleosynthesis, the earliest time for which we have clear empirical relics. The theoretical uncertainty in reheating is often taken into account, in the consideration of experimental constraints to inflation models, by surmising some reasonable range—e.g., $N_k = 46$ to $N_k = 60$ —for the number N_k of e -folds of inflation between the time that our observable horizon exited the horizon during inflation and the end of inflation. The upper limit to this range arises if inflaton oscillations reheat the Universe instantaneously to a grand unified theory-scale temperature, and the lower limit arises if reheating is closer to the electroweak scale.

Here we consider an alternative approach where we parametrize the cosmic fluid during reheating by an effective equation-of-state parameter w_{re} , that tells us how its energy density ($\rho \propto a^{-3(1+w_{\text{re}})}$) decays during this epoch. In the canonical-reheating scenario $w_{\text{re}} = 0$, but numerical studies of thermalization indicate a possibly broader range of values $0 \lesssim w_{\text{re}} \lesssim 0.25$.⁴⁶ By demanding that the equation-of-state parameter fall within this range, we infer slightly better constraints to inflation models than in the usual approach wherein some overly permissive range of N_k is assumed. The approach we use here was discussed in Refs.^{47–51} and applied post-Planck to power-law potentials in Ref.⁵² In this work we explore this approach and show its general validity for single field inflation models. As an example, we apply it to study constraints to the parameter space for natural inflation^{53,54} and Higgs-like inflation models.⁵⁵ We show in particular that the lower limit to the tensor-to-scalar ratio r inferred from current measurements of n_s should be a bit higher (by about 25%) if we restrict the value of w_{re} to the range suggested by reheating theory.

The structure of this section is as follows. First, we will study how to parametrize reheating through its equation of state, and derive constraints to different inflationary models. We will, then, take a brief detour to study new models of inflation, such as transplanckian inflation and DBI. Finally, we will briefly discuss the consequences of this work. We note that this section draws heavily from Ref.,¹¹ on which it is based.

2.1 The Equation of State of Reheating

Constraints to the parameters of inflation models are often derived assuming some plausible range for the number—e.g., $N_k = 46$ to $N_k = 60$ —of e -folds of inflation that occurred between the time that our current observable Universe exited the horizon and the end of inflation. However, that number is, for any specific inflaton potential, related to an effective equation-of-state parameter w_{re} and temperature T_{re} , for reheating. Although the physics of reheating is highly uncertain, there is a finite range of reasonable values for w_{re} . Here we show that by restricting w_{re} to this range, more stringent constraints to inflation-model parameters can be derived than those obtained from the usual procedure. To do so, we focus in this work in particular on natural inflation and inflation with a Higgs-like potential, and on power law models as limiting cases of those. As one example, we show that the lower limit to the tensor-to-scalar ratio r , derived from current measurements of the scalar spectral index, is about 20%-25% higher (depending on the model) with this procedure than with the usual approach.

2.1.1 Formalism

Fig. 2.1 shows the comoving Hubble parameter aH with time.⁸ It grows for N_k e -folds during inflation with a time dependence that is fixed given a specific inflaton potential $V(\phi)$. It then decreases for N_{re} e -folds of expansion during which the energy in the inflaton potential is dissipated into a radiation bath. The standard radiation-dominated era then proceeds for N_{RD} e -folds before the advent of matter domination (and then dark-energy domination). It is clear from the Figure that the number of e -folds of expansion between the time that a given scale exits the horizon and the end of inflation is related to the number of e -folds since the end of inflation until that scale re-enters the horizon during matter/radiation-domination. The expansion history also determines the evolution of the energy density, and a second relation can be obtained from a given expansion history by demanding the proper relation between the energy density during inflation and the energy density today.

A consistent model for inflation must have an inflaton potential $V(\phi)$ that at some point steepens so that the slow-roll condition $\epsilon < 1$ (where $\epsilon = (V'/V)^2/2M_{\text{pl}}^2$ is the slow-roll

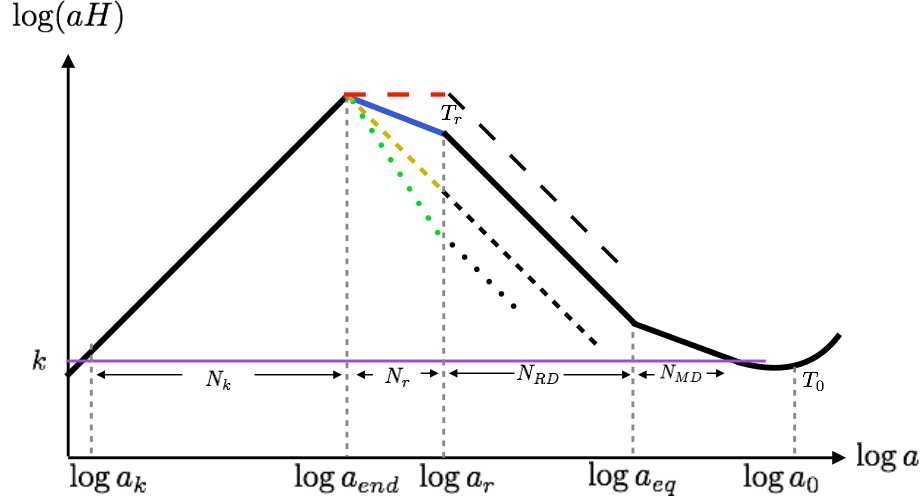


Figure 2.1: Comoving Hubble parameter aH versus scale factor $\log a$. A comoving mode with wavenumber k exits the horizon during inflation when $k = aH$ and then reenters during matter domination. Different equations of state for reheating are plotted: canonical reheating ($w_{\text{re}} = 0$) in blue (solid); $w_{\text{re}} = -1/3$ in red (long dash); $w_{\text{re}} = 1/3$ in brown (short dash); and the limiting case $w_{\text{re}} = 1$ in green (dotted).

parameter and M_{pl} is the reduced Planck mass) breaks down, at which point inflation ends. The number of e -folds between the time that a comoving scale k exits the horizon and the end of inflation is

$$N_k = \int_{\phi_k}^{\phi_{\text{end}}} \frac{H d\phi}{\dot{\phi}}, \quad (2.1)$$

where ϕ_k is the inflaton value when k exits the horizon, $H(\phi)$ is the Hubble parameter, and the dot denotes a derivative with respect to time t . The Hubble parameter can then be written in terms of the inflaton potential using the Friedmann equation, $H^2 \simeq V/(3M_{\text{pl}}^2)$, and $\dot{\phi}$ is evaluated through the slow-roll equation, $3H\dot{\phi} + V'(\phi) \simeq 0$, where the prime denotes derivative with respect to ϕ . The values of the scalar spectral index n_s and tensor-to-scalar ratio r can be obtained as a function of N_k . Given the relation between N_k and the number of post-inflation e -folds of expansion, the value of N_k relevant for cosmic microwave background measurements is a fixed function of n_s once a given reheating history (specified by w_{re} and the reheat temperature T_{re}) is assumed. Below we will use the fairly well-determined value of n_s to infer, for a given reheat scenario, the inflaton-potential parameters and from them the allowable values of r .

Let us consider the pivot scale $k = 0.05 \text{ Mpc}^{-1}$ at which Planck determines n_s .²⁰ The

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comoving Hubble scale $a_k H_k = k$ when this mode exited the horizon is related to that, $a_0 H_0$, of the present time by,

$$\frac{k}{a_0 H_0} = \frac{a_k}{a_{\text{end}}} \frac{a_{\text{end}}}{a_{\text{re}}} \frac{a_{\text{re}}}{a_{\text{eq}}} \frac{a_{\text{eq}} H_{\text{eq}}}{a_0 H_0} \frac{H_k}{H_{\text{eq}}}, \quad (2.2)$$

where quantities with subscript k are evaluated at horizon exit. The other subscripts refer to the end of inflation (end), reheating (re), radiation-matter equality (eq), and the present time (0). Using $e^{N_k} = a_{\text{end}}/a_k$, $e^{N_{\text{re}}} = a_{\text{re}}/a_{\text{end}}$ and $e^{N_{\text{RD}}} = a_{\text{eq}}/a_{\text{re}}$, we obtain the constraint,

$$\ln \frac{k}{a_0 H_0} = -N_k - N_{\text{re}} - N_{\text{RD}} + \ln \frac{a_{\text{eq}} H_{\text{eq}}}{a_0 H_0} + \ln \frac{H_k}{H_{\text{eq}}}, \quad (2.3)$$

on the total expansion.⁵⁶ The Hubble parameter during inflation is given by $H_k = \pi M_{\text{pl}} (r A_s)^{1/2} / \sqrt{2}$, with the primordial scalar amplitude $\ln(10^{10} A_s) = 3.089^{+0.024}_{-0.027}$ from Planck.²⁰

The energy density ρ_{end} at the end of inflation is related to the energy density ρ_{re} at the end of reheating by the equation-of-state parameter w_{re} during reheating via

$$\frac{\rho_{\text{re}}}{\rho_{\text{end}}} = \exp[-3N_{\text{re}}(1 + w_{\text{re}})], \quad (2.4)$$

where N_{re} is the number of e -folds of expansion during reheating.

The energy density at the end of inflation is obtained from

$$\rho_{\text{end}} = (1 + \lambda)V_{\text{end}}, \quad (2.5)$$

where the ratio λ of kinetic to potential energies at the end of inflation is

$$\lambda = \frac{1}{3/\epsilon - 1}. \quad (2.6)$$

When inflation ends ($\epsilon \approx 1$), we have $\lambda \approx 1/2$.

We next calculate the energy density at reheating. Assuming conservation of entropy,

$$g_{\text{s, re}} T_{\text{re}}^3 = \left(\frac{a_0}{a_{\text{re}}} \right)^3 \left(2T_0^3 + \frac{21}{4} T_{\nu, 0}^3 \right), \quad (2.7)$$

where $g_{\text{s, re}}$ is the effective number of relativistic degrees of freedom at reheating, and $T_{\nu, 0} =$

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$(4/11)^{1/3}T_0$ is the current neutrino temperature. Thus,

$$\frac{T_{\text{re}}}{T_0} = \left(\frac{43}{11g_{\text{s, re}}} \right)^{1/3} \frac{a_0}{a_{\text{eq}}} \frac{a_{\text{eq}}}{a_{\text{re}}}. \quad (2.8)$$

Since the energy density at reheating is $\rho_{\text{re}} = (\pi^2 g_{\text{re}}/30)T_{\text{re}}^4$, we plug Eq. (2.8) into Eq. (2.4) to get the number N_{re} of e -folds during reheating as a function of the number N_{RD} of e -folds during radiation domination. Plugging that into Eq. (2.3) we obtain finally,

$$\begin{aligned} N_{\text{re}} = & \frac{4}{1-3w_{\text{re}}} \left[-N_k - \log\left(\frac{k}{a_0 T_0}\right) - \frac{1}{4} \log\left(\frac{30}{g_{\text{re}} \pi^2}\right) \right. \\ & - \frac{1}{3} \log\left(\frac{11g_{\text{s, re}}}{43}\right) - \frac{1}{4} \log(V_{\text{end}}) - \\ & \left. - \frac{1}{4} \log(1+\lambda) + \frac{1}{2} \log\left(\frac{\pi^2 r A_s}{2}\right) \right], \end{aligned} \quad (2.9)$$

where g_{re} and $g_{\text{s, re}}$ can be both taken to be ≈ 100 and we will use $k = 0.05 \text{ Mpc}^{-1}$ throughout this work, albeit keeping the subindex k in N_k to avoid confusion. Then using Eq. (2.4), the reheating temperature is,

$$T_{\text{re}} = \exp\left[-\frac{3}{4}(1+w_{\text{re}})N_{\text{re}}\right] \left(\frac{3}{10\pi^2}\right)^{1/4} (1+\lambda)^{1/4} V_{\text{end}}^{1/4}. \quad (2.10)$$

2.1.2 Inflaton potentials

We now discuss the two classes of inflation models that we consider in this work.

Natural Inflation

This model, first proposed in Ref.⁵³ appears when a global $U(1)$ symmetry is spontaneously broken. The inflaton is then the pseudo-Nambu-Goldstone boson. The shift symmetry protects the flatness of the potential. The inflaton potentials we consider are,

$$V(\phi) = \frac{2\Lambda^4}{2^m} (1 + \cos \phi/f)^m, \quad (2.11)$$

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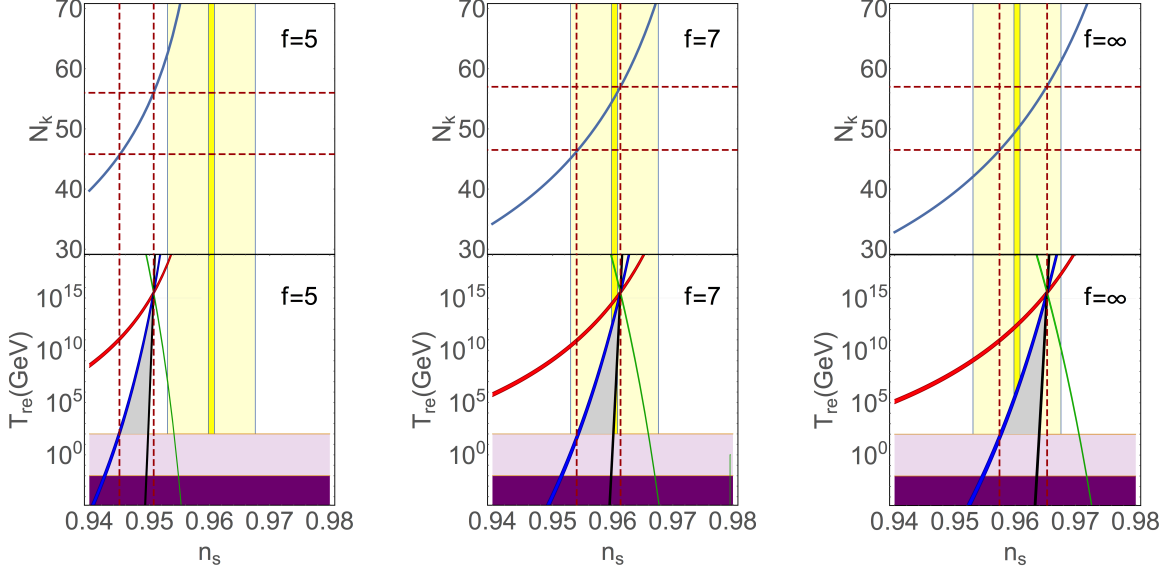


Figure 2.2: In the lower panels we plot the reheat temperature T_{re} for natural inflation as determined by matching the number of e -folds during and after inflation. Results are shown for decay constants $f = 5 M_{\text{pl}}$, $7 M_{\text{pl}}$, and ∞ , where the latter corresponds to the $m^2\phi^2$ limit. Four different effective equation-of-state parameters w_{re} for reheating are considered in each case: from left to right in their intersection with the bottom of the plots they are $w_{\text{re}} = -1/3$ (red), $w_{\text{re}} = 0$ (blue), $w_{\text{re}} = 0.25$ (black), and $w_{\text{re}} = 1$ (green). The values $w_{\text{re}} = -1/3$ and $w_{\text{re}} = 1$ bracket the very most conservative allowed range of values for w_{re} , while $w_{\text{re}} = 0$ and $w_{\text{re}} = 0.25$ bracket the range suggested by the literature on reheating. All curves intersect at the point where reheating occurs instantaneously, and the $w_{\text{re}} = 1/3$ curve would be vertical. Values of the termination condition in the range $0.1 \lesssim \epsilon \lesssim 1$ give rise to variations that are narrower than the widths of the curves. The light purple regions are below the electroweak scale $T_{\text{EW}} \sim 100$ GeV. The dark purple regions, below 10 MeV, would ruin the predictions of big bang nucleosynthesis (BBN). Temperatures above the intersection point are unphysical as they correspond to $N_{\text{re}} < 0$. The gray shaded triangles indicate the parameter space allowed if $0 < w_{\text{re}} < 0.25$. The light yellow band indicates the 1σ range in $n_s - 1 = -0.0397 \pm 0.0073$ from Planck,²⁰ and the dark yellow band assumes a projected uncertainty of 10^{-341} for $n_s - 1$ as expected from future experiments (assuming the central value remains unchanged). The top panels plot the number N_k of e -folds of inflation as a function of n_s . The vertical dashed red lines demarcate the allowed range of n_s , inferred from the lower panel, and the horizontal dashed red lines in the upper panels indicate the allowed range of values of N_k .

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where the energy density Λ^4 and decay constant f are the parameters of the model. We generalize the usual natural-inflation potential, which has $m = 1$, to other values of m to broaden slightly the class of models we consider. The slow-roll parameters for this model are

$$\epsilon = m^2 \frac{e^{-x}}{2f^2(1 - e^{-x}) + m}, \quad \text{where } x = \frac{mN_k}{f^2}, \quad (2.12)$$

and

$$\eta = \eta_V - \epsilon = \frac{-m}{2f^2} \frac{2f^2(1 - me^{-x}) + m}{2f^2(1 - e^{-x}) + m}. \quad (2.13)$$

These lead to the observables r and $n_s - 1$, which are

$$r = 8m^2 \frac{e^{-x}}{2f^2(1 - e^{-x}) + m}, \quad (2.14)$$

and

$$n_s - 1 = -\frac{m}{f^2} - \frac{2m(m+1)e^{-x}}{2f^2(1 - e^{-x}) + m}. \quad (2.15)$$

We will also need to calculate the number N_k of e -folds that happen after a mode with wavenumber k exits the horizon, which is found to be

$$N_k = \frac{f^2}{m} \log \left[\frac{1}{1 + m/(2f^2)} \frac{(n_s - 1)f^2 - m^2}{(n_s - 1)f^2 + m} \right]. \quad (2.16)$$

Even though the model has two parameters (Λ and f) only one of them is free, since they are related through the amplitude of the scalar power spectrum. From the value of the potential V_k at horizon exit we find Λ to be,

$$\Lambda = \left(\frac{3}{4} \pi^2 r A_s \left[\frac{2f^2 + n}{2f^2(1 - e^{-mN_k/f^2}) + m} \right]^m \right)^{1/4}. \quad (2.17)$$

In the $f \rightarrow \infty$ limit these potentials behave like pure power laws; i.e.,

$$V(\phi) \sim M^{4-2m} \phi^{2m} \quad \text{when } f \rightarrow \infty, \quad (2.18)$$

where M is an energy scale that plays the role of Λ and is also fixed.

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Higgs-like Inflation

The potentials we consider for Higgs-like inflation are,

$$V(\phi) = \Lambda^4 [1 - (\phi/\mu)^2]^n, \quad (2.19)$$

with slow-roll parameters,

$$\epsilon = \frac{2n^2 y}{\mu^2(1-y)^2}, \quad (2.20)$$

and

$$\eta = \eta_V - \epsilon = \frac{2n[-1 + (n-1)y]}{\mu^2(1-y)^2}. \quad (2.21)$$

The variable y is defined as,

$$y(\mu) \equiv \phi_0^2/\mu^2 = -W\left(-g(\mu) \exp\left[-g(\mu) - \frac{8N_k}{\mu^2}\right]\right), \quad (2.22)$$

where $W(z)$ is the Lambert W function, and

$$g(\mu) \equiv (\phi_{\text{end}}/\mu)^2 = 1 + \frac{n^2}{\mu^2} - \frac{\sqrt{n^4 + 2\mu^2 n^2}}{\mu^2} < 1. \quad (2.23)$$

Again, we generalize the usual case ($n = 2$) to explore a broader class of models. In the general case the tensor-to-scalar ratio and scalar spectral index are,

$$r = \frac{16n^2 y}{\mu^2(1-y)^2}, \quad (2.24)$$

and

$$n_s - 1 = -\frac{4n}{f^2} \frac{[1 + (n+1)y]}{(1-y)^2}. \quad (2.25)$$

We will again need the number,

$$N_k = \frac{\mu^2}{4n} \left[-\log\left(\frac{y}{g}\right) + y - g \right], \quad (2.26)$$

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of e -folds of inflation, and once again we can express the amplitude Λ of the potential in terms of the scalar power-spectrum amplitude A_s and the decay constant μ ,

$$\Lambda = \left[\frac{3}{2} \pi^2 r A_s (1-y)^{-n} \right]^{1/4}. \quad (2.27)$$

This model also behaves as a power law in the $\mu \rightarrow \infty$ limit, the exponent being in this case n ,

$$V(\phi) \sim M^{4-n} \phi^n \quad \text{when } \mu \rightarrow \infty. \quad (2.28)$$

2.1.3 Results

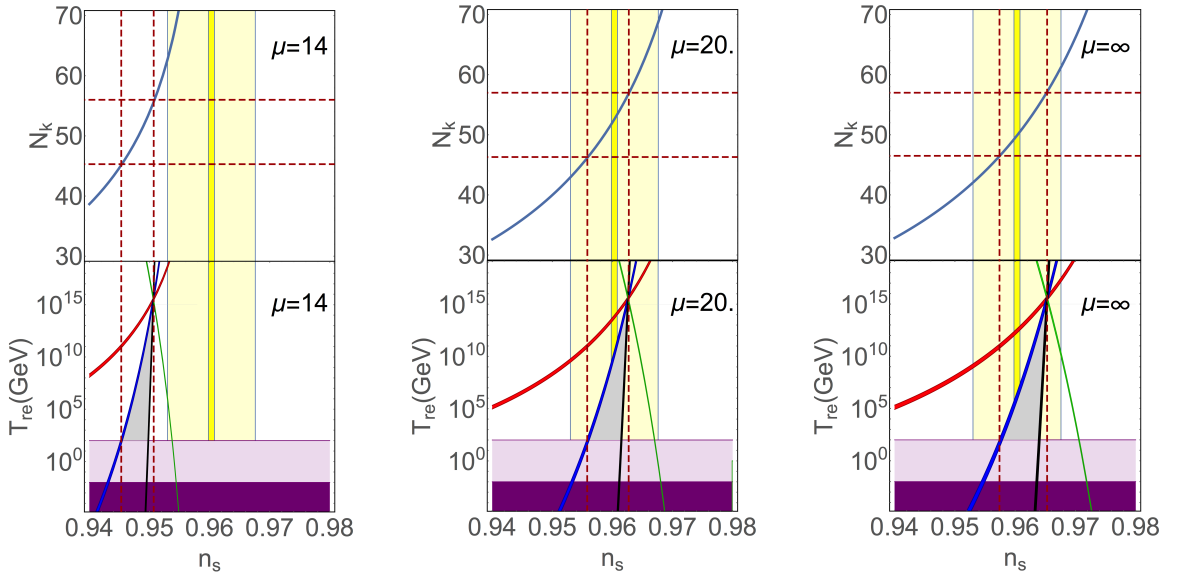


Figure 2.3: Same as Fig. 2.2 but for Higgs-like inflation with parameter values $\mu = 14 M_{\text{pl}}$, $20 M_{\text{pl}}$, and ∞ .

The results of the calculation are shown for usual natural inflation in Fig. 2.2 and for usual Higgs-like inflation in Fig. 2.3. The reheat temperature T_{re} determined by matching the number of e -folds during and after inflation is shown in the lower panels of each Figure. We show results for four different reheating effective equation-of-state parameters w_{re} . The value $w_{\text{re}} = -1/3$ indicates the smallest possible value of w_{re} required for inflation to end. The value $w_{\text{re}} = 1$ provides the most conservative upper limit which comes simply from causality. The values $w_{\text{re}} = 0$ and $w_{\text{re}} = 0.25$ bracket the range of values of w_{re} in detailed

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models of reheating. The curves for all values of w_{re} intersect at the point where reheating is instantaneous, and the $w_{\text{re}} = 1/3$ curve would be vertical and intersect this point. The gray shaded triangles indicate the region allowed if the reheating equation-of-state parameter lies in the range $0 < w_{\text{re}} < 0.25$.

The top panels of Figs. 2.2 and 2.3 plot the number N_k of e -folds during inflation for each model and value of f (for natural inflation) or μ (for Higgs-like inflation). It can be seen, in particular, that the limit to the allowable range of values of n_s imposed by reheating considerations thus restricts the allowed range of values of N_k . The range of values of N_k is generally smaller than the range $N_k \simeq 46 - 60$ often assumed, being replaced (at our pivot scale $k = 0.05 \text{ Mpc}^{-1}$) by $N_k \simeq 47 - 57$ for the large f, μ limit, and slightly smaller values for lower f, μ .

It is also important to note that the tightness of the constraint to the n_s parameter space for fixed f (for natural inflation) or μ (for Higgs-like inflation) is determined not by the precision of current measurements, but by the self consistency of the inflationary-plus-reheating model. For the $m^2\phi^2$ case the new range of possible n_s for inflation is (0.958,0.965).

We also show results in Fig. 2.4 as plots of the r - n_s parameter space for natural inflation and for Higgs-like inflation. It is seen here that even after considering the complete range of values of f (for natural inflation) or μ (for Higgs-like inflation), the parameter space allowed by restricting the reheating equation-of-state parameter to physically plausible values is more constrained than that assumed simply taking a range $N_k = 46 - 60$ for the number of e -folds of inflation. In particular, we see that the smallest tensor-to-scalar ratio r allowed by the current 1σ range of values for n_s is a bit larger with our approach than that obtained with the less restrictive analysis. The black (dashed) curves correspond to the maximum reheating possible with equation-of-state parameter $w_{\text{re}} = 0$. Increasing the value of w_{re} would only shift the black curves to the right.

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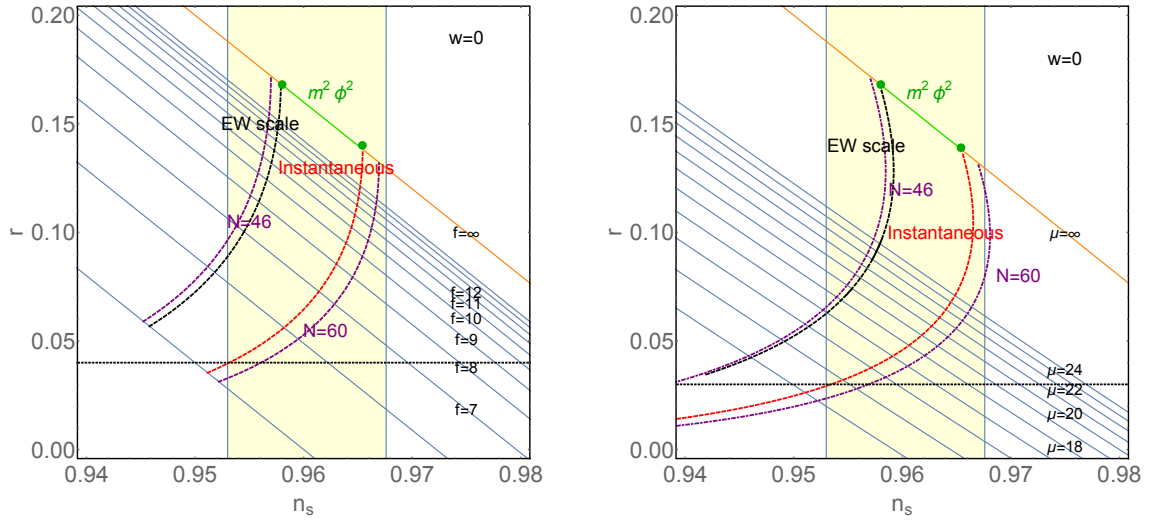


Figure 2.4: The n_s - r parameter space for (left) natural inflation and (right) Higgs-like inflation. Curves that indicate instantaneous reheating (red) and reheating at the electroweak scale (black) are shown as well as curves that show $N_k = 46$ and $N_k = 60$ e -folds of reheating (purple). Diagonal blue lines indicate different values of the decay constants f or μ , where the orange line is the power-law limit. The horizontal dotted lines indicate the smallest tensor-to-scalar ratio r consistent with the 1σ range of values of the scalar spectral index n_s , obtained by restricting the reheating equation-of-state parameter to physically plausible values, which are higher by about 25% than those obtained by simply taking a range $N_k = 46 - 60$ for the number of e -folds of reheating.

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Model	r_{min} old	r_{min} new
Higgs $n = 1$	0.020	0.025
Higgs $n = 2$	0.024	0.030
Higgs $n = 3$	0.035	0.050
Higgs $n = 4$	0.055	0.070
Natural $m = 1$	0.033	0.040
Natural $m = 3/2$	0.055	0.070
Natural $m = 2$	0.10	not allowed
$m^2\phi^2$	0.13	0.14

Table 2.1: Minimum value of the tensor-to-scalar ratio r at the pivot scale $k = 0.05 \text{ Mpc}^{-1}$ allowed by reheating considerations and the Planck 1σ range of values of the scalar spectral index n_s for each of the models studied. In the central column we show the minimum r from the usual analysis in which a range of N_k is allowed, and in the right column the new minimum obtained by constraining the reheating equation-of-state.

2.2 Expanding the Inflationary Paradigm

In general inflation generates primordial B-modes, albeit with an unknown amplitude. If it generates them within our observable range ($r \gtrsim 10^{-3}$), it is very likely the field excursion is transplanckian. This might be hard to accommodate within our usual QFT formalism. We will study one particular model, based on a complex scalar field, in which transplanckian inflation might be realized.

Given the null detection of B-modes by the Planck+BICEP team,⁵⁷ current data suggests that inflation is small-scale. One of the small-scale inflationary models is DBI inflation.⁵⁸ We will study how reheating affects this model, and whether it can agree with current CMB data from the Planck satellite.

2.2.1 Stability of the Inflationary Era

In Ref.⁵⁹ a novel model for inflation was presented. In this model inflation would be driven by a complex scalar field, which remains subplanckian during inflation but possesses transplanckian motion, preserving the Lyth bound on r . This is achieved by creating a potential with a minimum along a spiraling line into the origin of the 2D Φ -plane, so the total path length of the field would be orders of magnitude longer than the extent of the potential.

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The potential in Ref.⁵⁹ is

$$V(\phi, \theta) = \lambda \phi^n \frac{1}{\sqrt{2}^n} M_{\text{pl}}^{4-n} (1 + A/2 \sin(\left(\frac{\phi}{\sqrt{2}\Lambda}\right)^m + \theta)). \quad (2.29)$$

This potential has a minimum for a certain $\phi(\theta)$, such that the sin is -1 , i.e.,

$$\left(\frac{\phi}{\sqrt{2}\Lambda}\right)^m = -\theta + 2k\pi - \pi/2. \quad (2.30)$$

Therefore, the length transversed in the complex plane is given by,

$$da = \sqrt{d\phi^2 + \phi^2 d\theta^2} \approx \phi d\theta, \quad (2.31)$$

where we have approximated $\phi^m \gg \Lambda^m$, which we will justify later. Under that hypothesis we can calculate a to be

$$a = \int da = \int \phi d\theta = \int \phi \frac{d\theta}{d\phi} d\phi = \frac{m}{m+1} \frac{1}{(\sqrt{2}\Lambda)^m} (\phi_0^{m+1} - \phi_f^{m+1}). \quad (2.32)$$

As in regular power-law inflation the final value of the field will be very small compared to the initial, so we can neglect it on the formula above. This parameter a tells us the length of the path of the field, so inflation starts at $a = 0$ and finishes at some a which satisfies $\epsilon = 1$. What we would like is a canonical field that starts at $b = b_0$ and stops rolling at some smaller value $b_f \ll b_0$, so we reparametrize,

$$b(\phi) = a(\phi = 0) - a(\phi), \quad (2.33)$$

having then

$$b(\phi) = \frac{m}{m+1} \frac{\phi^{m+1}}{(\sqrt{2}\Lambda)^m}. \quad (2.34)$$

We can invert this relation to find the potential along the minimum (the path along which the field is rolling down), that would give us,

$$V(b) = \frac{\lambda M_{\text{pl}}^{4-n}}{\sqrt{2}^n} \left(1 - \frac{A}{2}\right) \left(\frac{m}{m+1}\right)^{\frac{n}{m+1}} (\sqrt{2}\Lambda)^{\frac{nm}{m+1}} b^{\frac{n}{m+1}}. \quad (2.35)$$

This shows that the field rolling down feels a simple power-law potential (ϕ^α), with

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$\alpha \equiv n/(m+1)$. Therefore, we can do a standard slow-roll analysis. First, the number of e-folds of inflation would be

$$N \approx d\phi \frac{V'}{V} = \frac{\phi_0^2}{2\alpha}, \quad (2.36)$$

where the two slow-roll parameters are

$$\epsilon = \frac{1}{2} \frac{V'}{V} \approx \frac{\alpha}{4N} = \frac{n}{4(m+1)} \frac{1}{N}, \quad (2.37)$$

and

$$\eta = \frac{V''}{V} \approx \frac{\alpha - 1}{2N} = \frac{n - m - 1}{2(m+1)} \frac{1}{N}. \quad (2.38)$$

Moreover, these give rise to a tensor-to-scalar ratio

$$r = 16\epsilon = \frac{4n}{(m+1)} \frac{1}{N}, \quad (2.39)$$

and a scalar tilt

$$ns - 1 = 2\eta_V - 6\epsilon_V = -\frac{n + 2m + 2}{2(m+1)} \frac{1}{N}. \quad (2.40)$$

Finally, we can reverse the relation for the number of e-folds and describe the field in terms of it as

$$b_0 = \sqrt{\frac{2n}{m+1}} N M_{\text{pl}}, \quad (2.41)$$

or equivalently,

$$b_0 = \left(\sqrt{\frac{2n}{m+1}} N \right)^{1/(m+1)} (\sqrt{2}\Lambda)^{m/(m+1)} \left(\frac{m+1}{m} \right)^{1/(m+1)} M_{\text{pl}}^{\frac{1}{m+1}}. \quad (2.42)$$

Now, we can use the measured amplitude of the scalar spectrum to bound Λ , since we know that

$$\Delta_s^2 = \frac{V}{24\pi^2 M_{\text{pl}}^4 \epsilon} \approx 2.2 \cdot 10^{-9}. \quad (2.43)$$

Solving for Λ we find

$$\Lambda = (\sqrt{2})^{\frac{m-1}{m}} \left(\frac{6\pi^2 A_s n}{N(m+1)} \right)^{\frac{m+1}{nm}} \left(\frac{m}{m+1} \right)^{\frac{1}{m}} \left(\frac{m+1}{2nN} \right)^{\frac{1}{2m}} [\lambda(1 - A/2)]^{\frac{-m-1}{nm}}. \quad (2.44)$$

For $n=4, m=1$ the authors claim that $\Lambda \approx 3.3 \cdot 10^{-6} \lambda^{-1/2} M_{\text{pl}} \ll M_{\text{pl}}$. This shows that the

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assumption of $\phi \gg \Lambda$ is justified, since by the end of inflation $b \approx M_{\text{pl}}$, so

$$\phi > \phi_f = (\Lambda^m b_f)^{1/(m+1)} \approx (\Lambda^m M_{\text{pl}})^{1/(m+1)}, \quad (2.45)$$

and

$$(\phi/\Lambda)^m > (M_{\text{pl}}/\Lambda)^{m/(m+1)} \gg 1. \quad (2.46)$$

Stability Analysis

We will now study whether this solution is stable. Remember the general form of the potential,

$$V(\phi, \theta) = \lambda \phi^n \frac{1}{\sqrt{2}^n} M_{\text{pl}}^{4-n} (1 + A/2 \sin(\left(\frac{\phi}{\sqrt{2}\Lambda}\right)^m + \theta)), \quad (2.47)$$

where we have only considered motion along the curve that minimizes the potential as the field rolls down, and therefore ignored perturbations perpendicular to that motion. First of all, the vector of the direction of the motion of the rolling field would be (on (v_ϕ, v_θ) notation)

$$v_{\parallel} = \left(1, \frac{-m}{(\sqrt{2}\Lambda)^m} \phi^{m-1}\right) \frac{1}{|v_{\parallel}|} \approx (0, 1), \quad (2.48)$$

and the direction perpendicular to the motion is

$$v_{\perp} = \left(1, \frac{(\sqrt{2}\Lambda)^m}{m} \phi^{-m-1}\right) \frac{1}{|v_{\perp}|} \approx (1, 0), \quad (2.49)$$

where the approximations are to leading order on Λ/ϕ , which is of order $(\Lambda/M_{\text{pl}})^{1/(m+1)} \ll 1$.

Then, the effective mass of the perpendicular modes would be

$$m_{\perp}^2 = \frac{\partial^2 V}{\partial w_{\perp}^2} \approx \frac{\partial^2 V}{\partial \phi^2}, \quad (2.50)$$

which we find to be

$$m_{\phi}^2 = \frac{\partial^2 V}{\partial \phi^2} = \frac{\lambda M_{\text{pl}}^{4-n}}{\sqrt{2}^n} \phi^{n-2} \left(n(n-1)(1 - A/2) + \frac{A}{4} m^2 \left(\frac{\phi}{\sqrt{2}\Lambda} \right)^{2m} \right). \quad (2.51)$$

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For $m > 0$ the second term in the parenthesis dominates, so

$$m_\phi^2 \gtrsim \frac{A}{4} m^2 \frac{\phi^{n+2m-2} M_{\text{pl}}^{4-n}}{\Lambda^{2m} 2^{n/2+m}}, \quad (2.52)$$

and, within an order of magnitude, we find

$$\frac{m_\phi^2}{M_{\text{pl}}^2} \sim \lambda A \left(\frac{\Lambda}{M_{\text{pl}}} \right)^{m(n-4)/(m+1)}. \quad (2.53)$$

During inflation $H^2 \sim \lambda \left(\frac{\Lambda}{M_{\text{pl}}} \right)^{mn/(m+1)} M_{\text{pl}}^2$, so we can rewrite Eq. (2.53) as

$$\frac{m_\phi^2}{H^2} \sim A \left(\frac{M_{\text{pl}}}{\Lambda} \right)^{4m/(m+1)} \gg 1. \quad (2.54)$$

This shows that the effective mass of the perpendicular motion is much bigger than the Hubble scale (unless A is extremely small), so we can treat the effective theory as we have done so far and ignore the perturbations on the perpendicular direction to the spiraling curve. However, this might still create nongaussianities, as well as isocurvature. We will explore this in future work.

Jeans length

From Ref.,⁶⁰ Eq. (10), the Jeans wavenumber for a highly rotating field—such as this one—is

$$k_J^2 = \frac{1}{2} \left(\frac{V'}{\sqrt{2}\phi} - V'' + \sqrt{\left(\frac{V'}{\sqrt{2}\phi} - V'' \right)^2 + 4V'^2 \frac{\phi^2}{M_{\text{pl}}^2}} \right), \quad (2.55)$$

which, considering a power law potential, and since along the minimum $V \sim (1 - A/2)\phi^n$, yields

$$k_J^2 = \frac{1}{4} \frac{V}{\phi^2} \left(n(2 - n) + \sqrt{(n(2 - n))^2 + 4\phi^2/M_{\text{pl}}^2} \right). \quad (2.56)$$

We now neglect the ϕ/M_{pl} factor, as it is suppressed, to find

$$k_J^2 \approx \frac{1}{4} \frac{V}{\phi^2} (n(2 - n) + |n(2 - n)|), \quad (2.57)$$

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so for $n > 2$ and $n < 0$ we don't have instabilities, since this leading term would vanish and then $k_J \sim \phi^2/M_{\text{pl}}^2$. However, for $0 < n < 2$, we see that

$$\frac{k_J^2}{H^2} \approx \frac{1}{2H^2} \frac{V}{\phi^2} n(2-n) = \frac{3}{2} n(2-n) \left(\frac{M_{\text{pl}}}{\phi} \right)^2 \gg 1, \quad (2.58)$$

so instabilities may arise.

Quantum tunneling

The minima of the potential for a fixed θ are separated by a radial direction that we will call $\Delta\phi$, if this separation (and the height of the potential) are small enough there is a chance of quantum tunneling from one minimum to the one right below (instead of turning the 2π). A representation of how the potential looks for a fixed θ for $m = n = 1$ is on figure 2.5.

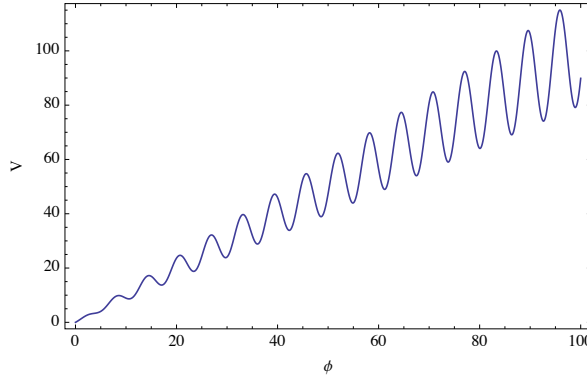


Figure 2.5: Inflationary potential $V(\phi)$ for a fixed θ direction, both in arbitrary units.

Following Ref.,⁶¹ the rate of formation of bubbles per unit time and volume is given by

$$\text{Rate} = \Gamma/V = K e^{-S_0}, \quad (2.59)$$

where K is the ratio of determinants times $\sqrt{S_0/2\pi}$, and S_0 is the action. We calculate the volume of the bubbles by finding the radii R of the bubbles that minimizes the action. The final result is that,

$$S_0 = \frac{27\pi^2 S_1^4}{2\epsilon^3}, \quad (2.60)$$

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where S_1 is the action of the instanton,

$$S_1 = \int_{\phi_0}^{\phi_f} \sqrt{2V} d\phi \leq \Delta\phi \sqrt{2[V_{max} - V(\phi_0)]}, \quad (2.61)$$

and ϵ is the difference in potentials between between the false vacuum and the real one (or in this case between the potential in one step and the lower step). That difference in this case is, to first order in $\Delta\phi$,

$$\begin{aligned} \epsilon &= V(\phi_n) - V(\phi_{n+1}) \approx V'_{min}(\phi) \Delta\phi = \\ &= \lambda(1 - A/2) 1/\sqrt{2}^n M_{\text{pl}}^{4-n} n \phi^{n-1} \Delta\phi, \end{aligned} \quad (2.62)$$

then,

$$S_0 = \frac{27\pi^2 S_1^4}{2\epsilon^3} \approx \frac{54\pi^2 V^2 \Delta\phi^4}{V'^3 \Delta\phi^3} = \frac{54\pi^2 V^2 \Delta\phi}{V'^3}. \quad (2.63)$$

The value of the change on the field is,

$$\Delta\phi \approx \frac{2\pi}{m} \left(\frac{\sqrt{2}\Lambda}{\phi} \right)^{m-1} \sqrt{2}\Lambda. \quad (2.64)$$

So,

$$\begin{aligned} S_0 &\approx \frac{54\pi^2 A^2 \lambda^2 \phi^{2n} M_{\text{pl}}^{8-2n} \frac{2\pi}{m} \left(\frac{\sqrt{2}\Lambda}{\phi} \right)^{m-1} \sqrt{2}\Lambda}{\left(\lambda(1 - A/2) 1/\sqrt{2}^n M_{\text{pl}}^{4-n} n \phi^{n-1} \right)^3} = \\ &= \frac{108\pi^3 A^2 \sqrt{2}^{n+m}}{\lambda(1 - A/2) n^3} \left(\frac{\phi}{M_{\text{pl}}} \right)^{4-n} \left(\frac{\Lambda}{\phi} \right)^m, \end{aligned} \quad (2.65)$$

where the prefactor is roughly of order 10^3 (depending on the model parameters), and since generically we saw that $\Lambda/M_{\text{pl}} \sim 10^{-6}$,

$$S_0 \approx 10^3 \cdot 10^{-6m} \left(\frac{\phi}{M_{\text{pl}}} \right)^{4-n-m}, \quad (2.66)$$

remember that $\Gamma/V = \sqrt{\frac{S_0}{2\pi}} e^{-S_0}$, so only for values of n and m such that the exponent is zero that quantity is non negligible, since for high values the exponential dominates, and for small values the polynomial does.

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In this model the field is always subplanckian, and inflation ends for $b_f \approx M_{\text{pl}}$, which translates into a minimum value of ϕ of $\phi_f \approx (\Lambda^m M_{\text{pl}})^{1/(m+1)}$, therefore,

$$\begin{aligned} S_0 &\gtrsim 10^{3-6m} \left(\frac{\Lambda}{M_{\text{pl}}} \right)^{(4-n-m)\frac{m}{(m+1)}} \approx \\ &\approx 10^{3-6m-6(4-n-m)\frac{m}{(m+1)}} = 10^{3-6m-24\frac{m}{m+1}+6\frac{m(n+m)}{m+1}}. \end{aligned} \quad (2.67)$$

To make the exponent vanish we need, $0 = 3(m+1) - 6m(m+1) - 24m + 6(m(n+m))$, or in other words $m = \frac{1}{9-2n}$, and remembering that the power in the equivalent power law inflation is $\alpha = \frac{n}{m+1}$, that means that for each power law index α that we want there is a value of m such that tunneling is the most likely, given by,

$$\alpha = \frac{9/2 - 1/(2m)}{m+1} \quad (2.68)$$

and in that case the maximum rate is $\Gamma/V = 0.17$.

Given a certain Γ/V its meaning is the rate at which critical bubbles of the real vacuum form, this critical bubbles are those that would make the universe decay into the real vacuum (instead of self collapsing due to surface effects). The total rate of the universe tunneling into a lower vacuum then is

$$\Gamma \approx \Gamma/V H^{-3}. \quad (2.69)$$

The field takes a certain time in circulating one step down in the potential, which can be roughly calculated by assuming the angle changes by 2π , For quasi-circular motion $T = \frac{2\pi}{\dot{\theta}}$, and the angular frequency is roughly $\dot{\theta} \sim \sqrt{\frac{E_{\text{kin}}}{\phi^2}}$. We know that the field is slow rolling, so then $E_{\text{kin}} = \frac{1}{3/\epsilon - 1} H^2 \approx \frac{\epsilon}{3} H^2 M_{\text{pl}}^2$, and as we have seen $\phi \sim (\Lambda^m M_{\text{pl}})^{1/(m+1)}$, so then

$$\dot{\theta} \sim \sqrt{\epsilon} H (M_{\text{pl}}/\Lambda)^{2m/(m+1)} \sim 10^{1+\frac{12m}{m+1}} H, \quad (2.70)$$

which shows that the field moves fast angularly. This shows that, even if a bubble of true vacuum formed, the slow-rolling potential would have already moved to a lower vacuum before the bubble grows to be a Hubble horizon. So it is not necessary to worry about quantum tunneling.

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To sum up, we have shown that the potential of Ref.⁵⁹ is not prone to quantum tunneling, and may only generate non-gaussianities and isocurvature at a small level. However, it is unclear whether it is stable for all values of n and m , which should be addressed.

2.2.2 Reheating for DBI inflation

Most models of inflation rely on a very flat potential during the slow-roll phase to produce inflation in a scale-invariant way. The same slow rolling can be produced by the inclusion of non-standard kinetic terms in the Lagrangian. We will study a model in which the slowness of the roll is achieved via a "scalar speed limit" mechanism, similar to the Lorentz factor of a relativistic particle, called DBI (Dirac-Born-Infeld) inflation.⁵⁸

The model

We start with the action⁶²

$$S_\phi = \int d^4x \sqrt{-g} \left[1/f(\phi) \sqrt{1 + f(\phi) g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi} - 1/f(\phi) + V(\phi) \right], \quad (2.71)$$

where $f(\phi)$ is a warping factor, given by the geometry of the background, and commonly taken to be $f(\phi) = \lambda/\phi^4$. For a homogeneous solution $\phi(t)$, we can define $\gamma = \frac{1}{\sqrt{1 - f(\phi)\dot{\phi}^2}} \geq 1$, which is equivalent to the Lorentz factor in special relativity. This points to the fact that there is a maximum velocity, $\dot{\phi} \leq \dot{\phi}_{\max} = 1/\sqrt{f(\phi)} = \phi^2/\sqrt{\lambda}$. Thus, from the inverse of the γ factor we can compute a speed of sound $c_s = \sqrt{1 - f(\phi)\dot{\phi}^2}$.

From the action in Eq. (2.71) we can extract the energy density and the pressure of the inflaton with this kinetic term, finding

$$\begin{aligned} \rho &= V(\phi) - 1/f(\phi) + \gamma/f(\phi) \\ p &= -V(\phi) + 1/f(\phi) + 1/(\gamma f(\phi)), \end{aligned} \quad (2.72)$$

which in turn gives us the equation of state parameter ($w = p/\rho$) as

$$w = \frac{-(Vf - 1) - 1/\gamma}{(Vf - 1) + \gamma}. \quad (2.73)$$

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It is easy to check that at the onset of inflation, when ϕ starts rolling and $\gamma \approx 1$ we have $w = -1$, and when inflation ends, and $\gamma \rightarrow \infty$, we get $w \rightarrow 0$, meaning that we have a canonical reheating era. Inflation would end when we cross $w = -1/3$, we can solve at which γ that would happen, to find,

$$\gamma_{end} = Vf - 1 + \sqrt{(Vf - 1)^2 + 3} \approx 2Vf. \quad (2.74)$$

There are two ways to achieve inflation on DBI, either coming from large values of ϕ and moving towards the origin (UV DBI), or starting near the origin and rolling down a potential (IR DBI). The main difference arises because of the ϕ dependent warp factor f . The UV model presents a more scale invariant power spectrum $n_s = 1 - O(\epsilon^2)$, ruled out by Planck data, so we will focus on the IR model. In particular, we take a potential

$$V = V_0 - m^2 \phi^2 / 2. \quad (2.75)$$

In the IR-DBI model there are 2 parameters that can be tuned, which will be c_s and m , since during inflation $V \approx V_0$. We define $R = m/M_{\text{pl}} 10^{-10}$. We require that $\phi_0 < M_{\text{pl}}$, which will translate into $R \lesssim 100$. The observable parameters are found to be

$$n_s - 1 = -4/N, \quad (2.76)$$

and

$$r = \frac{8m^2 c_s}{3\sqrt{\lambda} M_{\text{pl}}^2} N^3 (4\pi^2 A_s)^{-3/2}, \quad (2.77)$$

so this model has a highly suppressed value of r , and therefore does not produce observable primordial B-modes. Moreover, from the initial value of the energy density we can calculate the power spectrum, which gives us,

$$\lambda = \frac{N^4}{4\pi^2 A_s}. \quad (2.78)$$

Before moving forward we note that this model can present significant nongaussianities for low sound speeds. Current Planck data constraints the sound speed via the formula,

$$f_{NL}^c \approx 0.32 \, cs^{-2} \quad \rightarrow \quad f_{NL}^{c, \text{Planck}} = 11 \pm 69, \quad (2.79)$$

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from where we find

$$c_s > .063, \quad (2.80)$$

which is not extremely constraining. Furthermore, it is worth mentioning that the usual relation to find the energy scale of inflation (H_k) holds for this model as well,

$$H_k^2 = \frac{r A_s \pi^2}{2}, \quad (2.81)$$

so small r will give rise to small-scale inflation, as usual.

Reheating

Now we move on to find how this model reheats. We know that the equation of state for DBI inflation ranges from $w = -1/3$ to 0. We will approximate it as a constant.

In order to get n_s in the 1σ range from Planck we need to get between 85 and 125 e-folds on inflation, which is a value too high for all reheating conditions. We will see, however, that the curves of instantaneous reheating and $N = 60$ e-folds do not coincide, which seems to point at the fact that the usual analysis breaks down for this model. The plots can be seen on Fig. 2.6.

From our results it is clear that DBI inflation does not work with current measurements of n_s , even though it can predict an extremely small r . It remains unclear whether a negative equation-of-state during reheating, or a preheating phase, could remedy the situation. Future work will address these issues.

2.3 Conclusions

We have explored a new technique to find constraints to inflationary models by studying their reheating period. Instead of focusing on the physics of the reheating phase itself, or assuming an overly ample parameter space by constraining the number of e -folds of inflation, we characterize the whole reheating era by a single equation-of-state parameter w_{re} , that we constrain to have physically reasonable values. This leads to more precise constraints to

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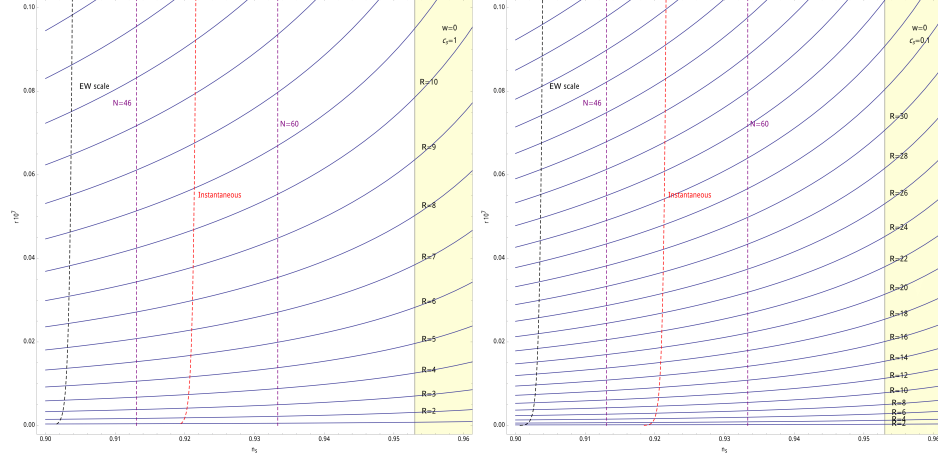


Figure 2.6: n_s vs r for IR DBI inflation with $c_s = 1$ (left) and $c_s = 0.1$ (right). Note that the vertical axis r has been multiplied by a factor of 10^7 to compensate for its small value.

the inflationary observables.

We have applied this formalism to two families of potentials (natural inflation and Higgs-like inflation), finding better lower bounds for the tensor-to-scalar ratio r , as can be seen in Table 2.1 (where the usual $m = 1$, $n = 2$ potentials are in bold face). It is important to notice that these results are robust to changes in the equation-of-state parameter as long as it is kept under $w_{\text{re}} = 1/3$, as suggested by previous work.

The results derived for the potentials studied also apply, taking the limiting cases f or $\mu \rightarrow \infty$, to power-law models and, as we show in Figure 2.4, the allowed region for the power-law case (green line) is more constrained using our method than with the usual analysis in which the range for the numbers of e -folds is fixed. For comparison, the right-hand plots in Figures 2.2 and 2.3 correspond to the plot made on⁵² for $m^2\phi^2$ potential, showing in the upper panel N_k instead of N_{re} .

The most interesting feature of this technique is its general validity. It was considered for power-law potentials in Refs.,^{52,63} and we have generalized here to natural and Higgs-like potentials. Still, the approach can be similarly applied to any single-field inflation model and will generically lead to slightly more restrictive bounds to the inflationary parameter space, including the range of values of the tensor-to-scalar ratio r . As a result, upper bounds to r , for example, will generally be slightly more restrictive to inflationary models than they would otherwise be.

*“Science may be described as the art of
systematic over-simplification.”*

— Karl Popper

Chapter 3

Cosmic Microwave Background

When the temperature of the universe first became low enough for hydrogen atoms to form, photons became able to travel without scattering with free electrons, and thus the universe became transparent. This created a cosmic microwave background (CMB), which covers the entire sky with an almost-homogeneous temperature of ~ 3 K. However, if we looked at a region with slightly more matter than the average, the photons would be redshifted due to the gravitational potential created by said matter, so we would measure the CMB temperature of that region to be colder. Likewise, an underdense region would appear hotter. This, plus a plethora of additional effects, creates anisotropies in the CMB temperature on the order of one in a hundred thousand. First *COBE*,⁶⁴ then *WMAP*,⁶⁵ and now *Planck*⁶⁶ have measured these CMB anisotropies to an increasingly exquisite precision. This has allowed us to map the underlying matter perturbations at the time of recombination, and to establish the cosmological standard model, Λ CDM, which can explain very well all the CMB data, as well as the rest of cosmological observations.

Part of this model is that CMB anisotropies are adiabatic, so a region with more baryons also contains more dark matter, neutrinos, and photons. This need not be the case if there is more than one degree of freedom creating the CMB anisotropies, like a curvaton. In Ref.¹⁶ I performed a Gaussian analysis of the latest Planck data release, to find constraints to compensated isocurvature. Additionally, in Ref.¹⁷ we extended that analysis to include

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the lensing effects of the CMB, finding the strongest constraint to compensated isocurvature to date.

There are a number of tensions in Planck's CMB data. Among the most important ones is that there seems to be more gravitational lensing due to substructure than expected. This is a $2-\sigma$ tension with both the result from a trispectrum analysis, as well as with the prediction from Λ CDM. Furthermore, the second running, which describes how the power spectrum behaves at small scales, seems larger than expected, also at $2-\sigma$. We will explore what are the consequences of these tensions and how well we will be able to resolve them with next-generation CMB experiments.

The Planck experiment has finished acquiring data, and the last data release by the Planck team is expected in the next year. The future of CMB observations lies in the so-called Stage-4 CMB experiment, a series of Earth-based observatories aiming to measure CMB anisotropies to unprecedented precision.²¹ As part of the S4 forecasting team, I have developed code to find the precision that the S4 CMB experiment will reach in measuring different parameters.

I will start this chapter by reviewing the basics of CMB physics. Then, I will move on to compensated isocurvature perturbations, before briefly mentioning Planck's lensing anomaly. Finally, I devote the last part of the chapter to how well we can measure the shape of the primordial power spectrum from the CMB, in particular by measuring its scalar runnings.

3.1 Basic Formalism

3.1.1 Linear estimators for Planck

Codes like CosmoMC⁶⁷ and Python Monte Carlo⁶⁸ are commonly used for parameter analysis. It is, however, a computationally costly procedure. We already have a best fit for the six Λ CDM model parameters in the absence of any additional physics,¹ so we can perturb the model around this best fit by adding a new parameter, and increasing the number N_p of parameters accordingly. In that case the new best-fit parameters will not be

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too far away in parameter space from the old ones, so we can perform a linear analysis. In reality

We construct a linear estimator of the parameters near their current best-fit values.¹ To do so we parametrize the power spectra as,

$$C_\ell^{X,\text{obs}} - C_\ell^{X,\text{best-fit}} = \sum_{i=1}^{N_p} \delta A_i^X g_i^X(\ell), \quad (3.1)$$

where $C_\ell^{X,\text{best-fit}}$ is the best-fit (lensed) power spectrum, with $X = \{TT, TE, EE\}$. We have left out other CMB observables, such as B-mode polarization, due to the absence of sufficiently sensitive and foreground-free CMB polarization data. These could, however, potentially have significant constraining power.⁶⁹

We define the first six original amplitudes to be the Λ CDM parameters as $A_i = \{\omega_b, \omega_c, n_s - 1, A_s, \tau, H_0\}$, where $\omega_b = \Omega_b h^2$ and $\omega_c = \Omega_c h^2$ are the baryon and cold-dark-matter physical densities, n_s is the tilt of the scalar power spectrum, and A_s its amplitude. Here, τ is the optical depth of reionization and H_0 is the Hubble parameter. We define the deviations of these parameters from their best-fit values to be δA_i .

The basis functions $g_i^X(\ell)$ for $i = 1-6$ are constructed as

$$g_i^X(\ell) \equiv \frac{\partial C_\ell^X}{\partial A_i}, \quad (3.2)$$

where the derivatives are taken by fitting in CAMB⁷⁰ near the best-fit values of the six Λ CDM parameters.

We show all the derivatives with respect to the Λ CDM parameters in Figures 3.1, 3.2, and 3.3. There are well-known correlations between the high- ℓ effects of changing the dark-matter density ω_c and the Hubble parameter H_0 . Similarly, increasing A_s and decreasing τ produce very similar changes in the power spectra, except at the lowest ℓ s.

Notice that in those plots we are also showing the derivative with respect to the lensing amplitude as an eighth parameter. The basis functions for CIPs and lensing are very similar. This could help resolve the tension between the observed level of CMB lensing in Planck power spectra and expectations from the Λ CDM model. We will explore this topic later.

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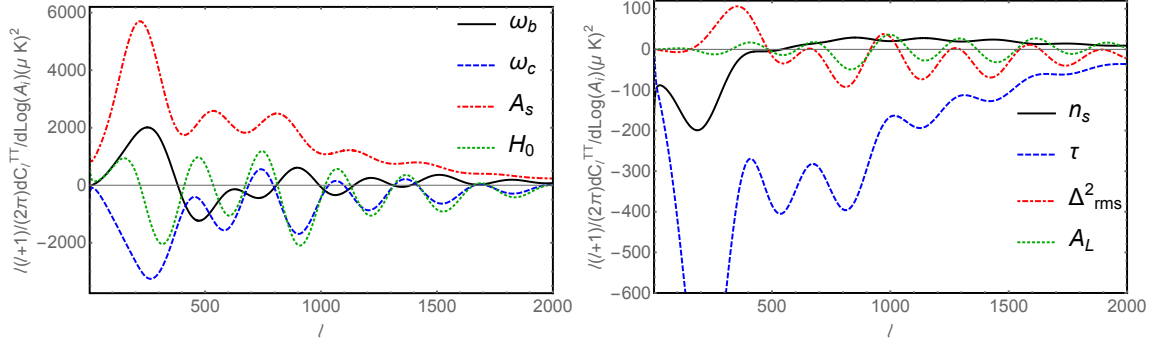


Figure 3.1: Derivatives of the CMB TT power spectrum at the current best-fit values. We employ derivatives with respect to the logarithm of each amplitude A_i to account for their different orders of magnitude. Consequently, there is a factor of A_i different to translate to the g_i s in the text. In the top panel we show the derivatives with respect to ω_b (in solid-black), ω_c (in dashed-blue), A_s (in red-dot-dashed), and H_0 (in dotted-green). In the lower panel we plot the derivatives with respect to n_s (in solid-black), τ (in dashed-blue), the CIP variance Δ_{rms}^2 (in red-dot-dashed), and the lensing amplitude A_L (in dotted-green). For visual purposes we have chosen an arbitrary CIP normalization in these plots.

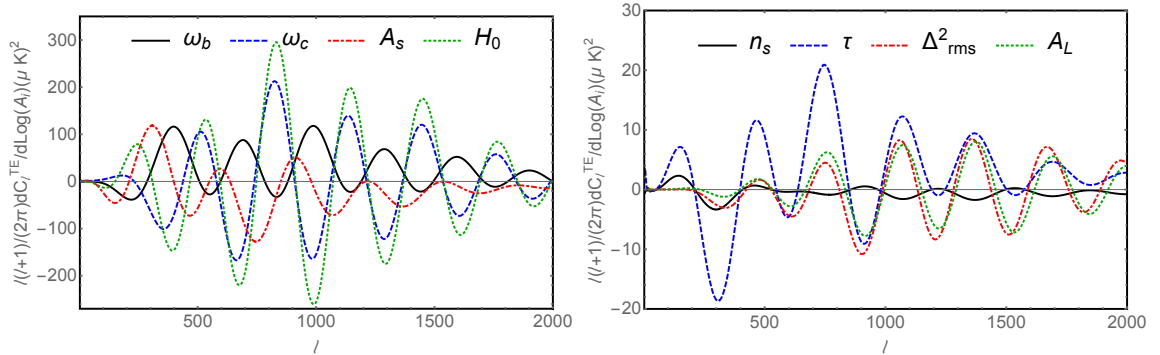


Figure 3.2: Derivatives of the CMB TE power spectrum at the current best-fit values. We use the same conventions as in Figure 3.1.

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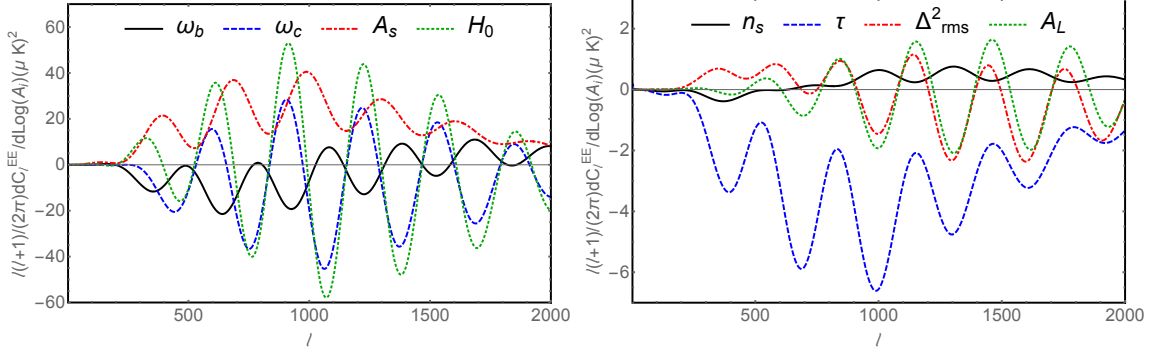


Figure 3.3: Derivatives of the CMB EE power spectrum at the current best-fit values. We use the same conventions as in Figure 3.1.

3.1.2 Fisher Matrix

We now study the detectability of the different δA_i simultaneously through a Fisher analysis. We employ the usual definition of the Fisher matrix,^{71,72} with components

$$F_{ij} = \langle g_i, g_j \rangle, \quad (3.3)$$

where the inner product $\langle \cdot, \cdot \rangle$ is defined as

$$\langle g_i, g_j \rangle \equiv \sum_{X,Y} \sum_{\ell} g_i^X(\ell) \mathcal{C}_{XY}^{-1} g_j^Y(\ell). \quad (3.4)$$

The covariance matrix $\mathcal{C}_{\ell}$ is given by^{73,74}

$$(\mathcal{C}_{\ell})_{XY} = \frac{2}{2\ell+1} \frac{1}{f_{\text{sky}}} \times \begin{pmatrix} \left(\tilde{\mathcal{C}}_{\ell}^{TT}\right)^2 & \left(\tilde{\mathcal{C}}_{\ell}^{TE}\right)^2 & \tilde{\mathcal{C}}_{\ell}^{TT} \tilde{\mathcal{C}}_{\ell}^{TE} \\ \left(\tilde{\mathcal{C}}_{\ell}^{TE}\right)^2 & \left(\tilde{\mathcal{C}}_{\ell}^{EE}\right)^2 & \tilde{\mathcal{C}}_{\ell}^{EE} \tilde{\mathcal{C}}_{\ell}^{TE} \\ \tilde{\mathcal{C}}_{\ell}^{TT} \tilde{\mathcal{C}}_{\ell}^{TE} & \tilde{\mathcal{C}}_{\ell}^{EE} \tilde{\mathcal{C}}_{\ell}^{TE} & \left[\left(\tilde{\mathcal{C}}_{\ell}^{TE}\right)^2 + \tilde{\mathcal{C}}_{\ell}^{TT} \tilde{\mathcal{C}}_{\ell}^{EE} \right] \frac{1}{2} \end{pmatrix}, \quad (3.5)$$

where we have defined

$$\begin{aligned} \tilde{\mathcal{C}}_{\ell}^{TT} &\equiv C_{\ell}^{TT} + N_{\ell}^{TT}, \\ \tilde{\mathcal{C}}_{\ell}^{TE} &\equiv C_{\ell}^{TE}, \\ \tilde{\mathcal{C}}_{\ell}^{EE} &\equiv C_{\ell}^{EE} + N_{\ell}^{EE}, \end{aligned} \quad (3.6)$$

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and the N_ℓ^X are the instrumental noises, for which we use the Planck tabulated noise for the Planck analysis and zero in the cosmic-variance-limited case. These are equivalent to the inverse-variance weighted sum over all channels i :

$$C_\ell^{XX,N} = \left(\sum_i w_{X,i}^{-2} e^{-\ell^2 \sigma_{b,i}^2} \right)^{-1}, \quad (3.7)$$

with $\sigma_{b,i} \equiv \theta_i / \sqrt{8 \ln 2}$, θ_i is the full-width-half-maximum, and w_i are the weights per solid angle for each channel. The two main Planck channels, with 143 and 217 GHz frequencies, have the properties listed in Table 3.1.

Channel	θ (arcmin)	w_T ($\mu\text{K arcmin}$)	w_P ($\mu\text{K arcmin}$)
Planck 143 GHz	7	30	60
Planck 217 GHz	5	40	95
Stage-4 CMB	3	1	1.4

Table 3.1: Planck and S4 sensitivities to temperature and polarization at the frequencies used to estimate the lensing potential. For Planck we take the two main frequencies that contribute to cosmology, and for S4 we take one effective band.

3.1.3 Lensing

The CMB is lensed by large-scale structure along the line of sight, let us now briefly review this effect. Lensing creates coupling between modes. Given a lensing multipole ϕ_{LM} the coupling between the $a_{lm}^{(1)}$ and $a_{lm}^{(2)}$ temperature anisotropies (where $a^{(i)}$ ranges over $\{T, E, B\}$) is given by

$$\langle a_{\ell m}^{(1)} a_{\ell' m'}^{(2)} \rangle = C_\ell^\alpha \delta_{\ell\ell'} \delta_{mm'} (-1)^m + \sum_{LM} (-1)^M \begin{pmatrix} \ell & \ell' & L \\ m & m' & -M \end{pmatrix} \phi_{LM} f_{\ell\ell'L}^\alpha, \quad (3.8)$$

where the coupling function $f_{\ell\ell'L}^\alpha$ is tabulated in Okamoto and Hu,⁷⁵ and $\alpha \equiv (12)$ is an index running over combinations of modes.

The way Planck measures lensing is using the formalism in Okamoto and Hu, which

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establishes an estimator for $d_{LM} = \sqrt{L(L+1)}\phi_{LM}$ as

$$\hat{d}_{LM}^\alpha = A_L^\alpha \sum_{\ell m, \ell' m'} a_{\ell m}^{(1)} a_{\ell' m'}^{(2)} \begin{pmatrix} \ell & \ell' & L \\ m & m' & -M \end{pmatrix} g_{\ell \ell' L}^\alpha, \quad (3.9)$$

where A_L^α and $g_{\ell \ell' L}^\alpha$ are selected to obtain

$$\langle d_{LM}^\alpha \rangle = \sqrt{L(L+1)}\phi_{LM}. \quad (3.10)$$

while minimizing the variance of the estimator,

$$\langle d_{LM}^\alpha d_{L'M'}^\alpha \rangle|_{\text{Gaussian}} = \delta_{LL'} \delta_{MM'} (C_L^{dd} + N_L^{\alpha\alpha}), \quad (3.11)$$

where the subindex Gaussian means that we take $\phi_{LM} = 0$, or only the disconnected part of the four point function. Using Eq. (3.9) we can obtain

$$N_L^{\alpha\beta} = \frac{A_L^\alpha A_L^\beta}{L(L+1)(2L+1)} \sum_{\ell \ell'} (g_{\ell \ell' L}^\alpha)^* \left[C_\ell^{13} C_{\ell'}^{24} g_{\ell \ell' L}^\beta + (-1)^{\ell+\ell'+L} C_\ell^{14} C_{\ell'}^{23} g_{\ell \ell' L}^\beta \right], \quad (3.12)$$

where here C_ℓ^α includes instrumental noise, and those cross terms arise because of the four-point function

$$\left\langle a_{\ell_1 m_1}^{(1)} a_{\ell_2 m_2}^{(2)} a_{\ell_3 m_3}^{(3)} a_{\ell_4 m_4}^{(4)} \right\rangle|_{\text{Gaussian}} = C_{\ell_1}^{13} C_{\ell_2}^{24} \delta_{\ell_1 \ell_3} \delta_{\ell_2 \ell_4} + C_{\ell_1}^{14} C_{\ell_2}^{23} \delta_{\ell_1 \ell_4} \delta_{\ell_2 \ell_3}, \quad (3.13)$$

where the (12)-(34) term gives δ_{L0} , so we ignore it.

This clearly shows that there are two ways to detect lensing. The first one is to search for the off-diagonal correlations induced by lensing, as in the last term of Eq. (3.8), via the four-point function (or trispectrum). This allows to probe each lensing mode independently, as each L produces different correlations. The second method consists on using the integrated effect of lensing on the CMB power spectra. For that we would need to expand Eq. (3.8) to second order in ϕ , and average over ϕ as well, to obtain an integrated measure of the lensing power spectrum C_L^{dd} . The main effects of the lensing on the CMB power spectra are to add power at small scales and to smooth the acoustic peaks.^{76,77} The amount of lensing inferred from CMB TT measurements seems, however, to be higher (by about two standard deviations) than the predicted value, and the result obtained from the trispectrum.

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This difference is parametrized through the lensing amplitude A_L ,^{78,79} which is fixed to be $A_L = 1$ in Λ CDM, but letting it vary can better fit the data. An analysis of the Planck measurements of the TT power spectrum found a best-fit value of $A_L = 1.22 \pm 0.10$.¹ We will see how this result is degenerate with a compensated isocurvature perturbation.

3.1.4 Stage-4 CMB

The specifications we use for the S4 CMB experiment follow those of Ref.,²¹ which has a sensitivity $\Delta_T = 1 \mu\text{K-arcmin}$, with a resolution of $\theta_{\text{FWHM}} = 3 \text{ arcmin}$, over 40% of the sky. To that we add Planck over an additional 20% of the sky, and a prior on the optical depth of reionization of $\tau = 0.06 \pm 0.01$. The S4 experiment is expected to observe the ℓ range between 30 and 5000 for polarization, although the highest modes will be noise-dominated; and between $\ell = 30$ and 3000 for temperature, as higher multipoles would be contaminated by foregrounds. For Planck we take two bands, corresponding to frequencies of 143 and 217 GHz, respectively, with noises $\Delta_T = \{43, 66\} \mu\text{K-arcmin}$, and $\Delta_E = \{81, 134\} \mu\text{K-arcmin}$, a resolution of $\theta_{\text{FWHM}} = \{7, 5\} \text{ arcmin}$ and we do not include lensing data.

The CMB power spectra can be written as

$$C_\ell^{XY} = (4\pi)^2 \int dk k^2 \mathcal{T}_\ell^X(k) \mathcal{T}_\ell^Y(k) P_\zeta(k), \quad (3.14)$$

where the indices $X, Y = \{T, E, \text{ or } d\}$ stand for temperature, E-mode polarization, and lensing potential respectively, and $\mathcal{T}_\ell^X$ are their transfer functions.^{73,80–82} These $\mathcal{T}_\ell^X$ do not depend on the primordial power spectrum, so the runnings only affect the C_ℓ through the change in $P_\zeta(k)$.

To forecast the errors in a set of parameters θ_i we define the Fisher matrix as^{83,84}

$$F_{ij} = \sum_\ell \frac{2\ell + 1}{2} f_{\text{sky}} \text{Tr} \left[\mathbf{C}_\ell^{-1} \frac{\partial \mathbf{C}_\ell}{\partial \theta_i} \mathbf{C}_\ell^{-1} \frac{\partial \mathbf{C}_\ell}{\partial \theta_j} \right], \quad (3.15)$$

where f_{sky} is the sky-fraction covered, and the covariance matrix, ignoring $E-d$ correlations

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is given by

$$\mathbf{C}_\ell = \begin{pmatrix} \tilde{C}_\ell^{TT} & C_\ell^{TE} & C_\ell^{Td} \\ C_\ell^{TE} & \tilde{C}_\ell^{EE} & 0 \\ C_\ell^{Td} & 0 & \tilde{C}_\ell^{dd} \end{pmatrix}, \quad (3.16)$$

and we have defined again

$$\begin{aligned} \tilde{C}_\ell^{TT} &\equiv C_\ell^{TT} + N_\ell^{TT}, \\ \tilde{C}_\ell^{EE} &\equiv C_\ell^{EE} + N_\ell^{EE}, \\ \tilde{C}_\ell^{dd} &\equiv C_\ell^{dd} + N_\ell^{dd}, \end{aligned} \quad (3.17)$$

where N_ℓ^{XX} are the noise power spectra, given by

$$\begin{aligned} N_\ell^{TT} &= \Delta_T^2 e^{\ell(\ell+1)\sigma_b^2}, \\ N_\ell^{EE} &= 2 \times N_\ell^{TT}, \end{aligned} \quad (3.18)$$

where Δ_T is the temperature sensitivity as defined above, and $\sigma_b = \theta_{\text{FWHM}}/\sqrt{8\log 2}$, with the full-width-half-maximum θ_{FWHM}^2 given in radians. For the lensing noise N_ℓ^{dd} we follow the approach in Ref.,⁸⁵ where the E- and B-mode data is used to reconstruct the lensing power spectrum, C_ℓ^{dd} , whose effect is then subtracted from the B-mode data. This allows us to iteratively compute the maximum delensing possible given the S4 polarization noise, and thus forecast the sensitivity to lensing modes.^{75, 86–88} For reference, the S4 lensing noise is predicted to be smaller than the signal for $\ell \lesssim 1000$.

We note that the Fisher matrix in Eq. (3.15) is equivalent to the one in Eq. (3.5), albeit looking very different. The reason for using two different formalisms is that only with the Fisher matrix in Eq. (3.5) one can look for residuals, as we will do for Planck data; whereas the matrix in Eq. (3.15) is clearly of lower dimensionality, which is useful when adding lensing dataset, as we do for the S4 CMB experiment.

3.2 Compensated Isocurvature Perturbations

Cosmological density perturbations are thought to have their origin during inflation.^{89–91} These perturbations seed the large-scale inhomogeneities that later grow to become galaxies and clusters.⁹² From structure formation and cosmic-microwave-background (CMB) observations, we know that primordial density perturbations have amplitude $\delta \sim 10^{-5}$.¹

Primordial perturbations can be classified into two groups depending on their initial conditions. Adiabatic perturbations are perturbations to the total energy density that leave the ratios of the different constituents of matter everywhere the same. Isocurvature perturbations involve perturbations to the relative number densities of different components of matter.^{13,19,93,94} The simplest inflationary models have purely adiabatic fluctuations, while isocurvature fluctuations usually signal the presence of a second field during inflation, as in curvaton models.^{95,96}

Isocurvature perturbations between photons and a single other species are in general well constrained.^{97–99} If, however, there is a baryon-density perturbation that is compensated by a dark-matter perturbation in such a way that the total matter density remains constant, then there are no pressure or gravitational-potential perturbations above the baryonic Jeans scale. These compensated isocurvature perturbations (CIPs) thus have no observable effect on the CMB at linear order in the CIP amplitude.^{100,101} There are constraints from other observables, but these are a factor of $\sim 10^4$ weaker than the adiabatic component.^{14,15,69,102}

In the standard scenario, the CMB power spectrum is determined given fixed values of the baryon and dark-matter densities Ω_b and Ω_m , respectively, in units of the critical density. CIPs, however, introduce variations to Ω_m and Ω_b between different patches of the CMB sky. They thus induce a variation in the power spectrum from one patch of sky to another.

The *mean* power spectrum—that obtained by measurements over the entire sky—remains unaltered, to linear order in the CIP amplitude. The variations show up, however, in two different ways. First of all, the spatial modulation of the power spectrum is characterized by a departure from gaussianity, a specific nontrivial four-point function, or

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trispectrum. A search for such a trispectrum was performed in Ref.¹⁵ The second consequence, however, is a change to the power spectrum that arises to quadratic order in the CIP amplitude, which can be understood heuristically as a smoothing of features in the CMB power spectrum when different power spectra are averaged.

Here we seek the effect of CIPs on the CMB power spectra obtained by Planck. We parametrize the magnitude of the effect of CIPs in terms of an rms CIP amplitude Δ_{rms} . We find from a temperature-only analysis a constraint of $\Delta_{\text{rms}}^2 \leq 7.1 \times 10^{-3}$, which is competitive with, and complements, that obtained from the complete trispectrum, although with a far simpler analysis. That figure improves to $\Delta_{\text{rms}}^2 \leq 5.0 \times 10^{-3}$ if Planck polarization data are included. We then make CIP sensitivity forecasts for future experiments. We also show that CIPs have a very similar effect on the power spectrum to changing the lensing amplitude A_L . They can thus alleviate the tension between the lensing amplitude obtained from the Planck spectrum ($A_L = 1.22 \pm 0.1$) and that expected from theory ($A_L = 1$).¹

CIPs change the baryon and dark-matter densities in such a way that the total matter energy density, $\Omega_m = \Omega_b + \Omega_c$, remains unaltered. We parametrize their effect as

$$\begin{aligned}\Omega_b &= \bar{\Omega}_b[1 + \Delta(\hat{n})], \quad \text{and} \\ \Omega_c &= \bar{\Omega}_c - \bar{\Omega}_b\Delta(\hat{n}),\end{aligned}\tag{3.19}$$

where Ω_b and Ω_c are the baryon and dark-matter energy densities respectively, the overbar represents their unperturbed values, and $\Delta(\hat{n})$ is the amplitude of the CIP in the specific direction $\hat{n}$ at recombination. This expression is accurate for CIPs of sufficiently large angular scale, where they can be treated as a modulation of background parameters.¹⁰³

The linear-order effects of CIPs are on scales at which the baryons behave differently from dark matter, corresponding to angular scales $\ell \gtrsim 10^{5-6}$,⁶⁹ which makes them unobservable in the CMB, although potentially detectable using cosmological 21-cm absorption measurements at high redshift.¹⁰¹ CIPs will also have consequences for the CMB fluctuations induced by adiabatic perturbations. In a region of high $\Delta(\hat{n})$ (high baryon density), decoupling will be longer, thereby smoothing the peak structure in the CMB power spectrum. The mean-free path of CMB photons would be reduced by the higher electron density, leading to less damping of CMB fluctuations on small angular scales. A higher baryon

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density also decreases the plasma sound speed and hence decreases the sound horizon at recombination.⁹³

3.2.1 Angular distribution

We expand the amplitude of the compensated isocurvature perturbations in spherical harmonics as

$$\Delta(\hat{n}) = \sum_{LM} Y_L^M(\hat{n}) \Delta_{LM}, \quad (3.20)$$

where the spherical-harmonic coefficients Δ_{LM} are statistically independent and have a variance given by

$$\langle \Delta_{LM} \Delta_{L'M'}^* \rangle = \delta_{LL'} \delta_{MM'} C_L^\Delta. \quad (3.21)$$

We take the *ansatz* of a scale-invariant power spectrum for Δ in k -space. For $L \lesssim 800$, this creates a scale-invariant angular power spectrum $C_L = A L^{-2}$ when projected onto the last-scattering surface (LSS), where A is a dimensionless amplitude.⁶⁹ The simple picture of CIPs as a modulation of background parameters corresponds to a separate universe approximation, which was shown in Ref.¹⁰³ to only be valid for $L \lesssim 100$, as the imprint of CIPs are washed for at smaller CIP angular scales. We thus restrict our analysis to $L \leq 100$.

We assume that the CIP amplitude $\Delta(\hat{n})$ is a Gaussian random variable with zero average and variance $\Delta_{\text{rms}}^2 \equiv \langle \Delta^2 \rangle$. Instead of finding an estimator for each Δ_{LM} we will directly measure its variance, which can be expressed in terms of the CIP angular power spectrum C_L^Δ as

$$\Delta_{\text{rms}}^2 = \sum_{L=1}^{100} \frac{(2L+1)}{4\pi} C_L^\Delta, \quad (3.22)$$

which means that our constraints will be on the total power in CIPs and not on each individual C_L^Δ . By using Eq. (3.22) we can relate the CIP variance Δ_{rms}^2 to the amplitude A of the power spectrum as,

$$\Delta_{\text{rms}}^2 \approx 0.96A. \quad (3.23)$$

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3.2.2 Previous constraints

As CIPs do not change CMB power spectra at linear order, past work has relied on other observables to constrain their amplitude. For example, measurements of galaxy-cluster baryon fractions (obtained through X-ray observations) were used in Ref.¹⁰² to search for CIPs, imposing the constraint $\Delta_{\text{rms}}^2 \lesssim 5 \times 10^{-3}$. This technique, however, relies on clusters being fair samples of the baryon density in the universe, as well as being kinematically relaxed.

In Ref.,⁶⁹ the off-diagonal correlations in the CMB created by CIPs were computed, and a forecast was made of the sensitivities that could be reached by studying them with different instrumental setups. Data from the WMAP mission¹⁰⁴ were analyzed in Ref.¹⁵ to constrain the amplitude A_{CIP} of the CIP power spectrum C_L to be smaller than 5.5×10^{-3} at 68% C.L., which translates to a constraint on the CIP variance of $\Delta_{\text{rms}}^2 \lesssim 4 \times 10^{-3}$, where the $L = 1$ mode has been ignored due to reconstruction uncertainties.

3.2.3 Averaged-Sky method

CIP effects on the power spectrum

In our picture we treat the CIP amplitude as a Gaussian random variable. This allows us to calculate the observed CMB angular power spectrum C_ℓ^{obs} by averaging over the CIP amplitudes,

$$C_\ell^{\text{obs}} = \frac{1}{\sqrt{2\pi\Delta_{\text{rms}}^2}} \int d\Delta e^{-\Delta^2/(2\Delta_{\text{rms}}^2)} C_\ell(\Delta), \quad (3.24)$$

which to first non-zero order in Δ_{rms} is given by

$$C_\ell^{\text{obs}} \approx C_\ell|_{\Delta=0} + \frac{1}{2} \frac{d^2 C_\ell}{d\Delta^2} \Delta_{\text{rms}}^2. \quad (3.25)$$

We calculate the second derivative by fitting $C_\ell|_{\Omega_b, \Omega_c}$ near $\{\bar{\Omega}_b, \bar{\Omega}_c\}$ as a function of Δ . We have checked that terms that are higher order in Δ_{rms}^2 are negligible for the upper limits to Δ_{rms}^2 that we infer. This causes a change in the power spectrum, from where we

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can extract a seventh basis function

$$g_7^X(\ell) \equiv \frac{1}{2} \frac{d^2 C_\ell^X}{d\Delta^2}, \quad (3.26)$$

where the derivative is calculated in the separate-universe approximation, and has associated amplitude $\delta A_7^X = \Delta_{\text{rms}}^2$. We show this base function in Figures 3.1, 3.2, and 3.3 for each CMB dataset.

We have found that the CIP-induced corrections to CMB power spectra from Eq. (3.25) numerically agree with those computed with the full mode-coupling formalism of Ref.,⁶⁹ but are simpler to evaluate.

We have shown expressions for how CIPs alter CMB power spectra. Now we consider how to estimate Δ_{rms}^2 with measurements of the three main CMB power spectra, C_ℓ^{TT} , C_ℓ^{TE} , and C_ℓ^{EE} . For that we use a Fisher-matrix analysis to fully capture the correlations between the CIP variance, Δ_{rms}^2 , and the usual cosmological parameters.

Now we are ready to find estimates and errors for the six standard cosmological parameters, as well as the CIP amplitude Δ_{rms}^2 .

We consider two cases. First, for Planck, we not only obtain estimators for the CIP variance, but also apply them to the data to obtain actual limits to CIPs. We will take a small detour to study the viability of CIPs as a solution for the lensing tension in the Planck CMB power spectra. Second, we will study a cosmic-variance limited (CVL) experiment.

Planck constraint

Let us begin by considering the Planck 2015 power spectra ($C_\ell^{X,\text{Planck}}$), obtained from the Planck Legacy Archive (<http://pla.esac.esa.int/pla/>). To diminish the effects of correlations between different ℓ s, we used binned data for $\ell \geq 30$, with width $\Delta\ell = 30$. The minimum-variance unbiased estimators for these seven amplitudes δA_i are

$$\widehat{\delta A}_i = \sum_j (F^{-1})_{ij} \langle R(\ell), g_j(\ell) \rangle, \quad (3.27)$$

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where $(F^{-1})_{ij}$ is the inverse of the Fisher matrix, and $R(\ell)$ is the residual after subtracting the best fit from the data, $R^X(\ell) = C_\ell^{X,\text{Planck}} - C_\ell^{X,\text{best-fit}}$.

With the current data in the Planck Legacy Archive, however, it is hard to disentangle the optical depth τ and the scalar amplitude A_s , since the effect of changing either is highly degenerate.¹⁰⁵ The main difference between A_s and τ is the reionization bump, caused by τ , that appears at low ℓ in polarization measurements.^{106–108} Our linear analysis underestimates the errors when using low- ℓ polarization data, so in lieu of them we will add a prior $\tau = 0.068 \pm 0.019$ to the optical depth for robustness. We choose the final ℓ ranges to be $\ell = 30 - 2500$ for TT, and $\ell = 30 - 1995$ for TE and EE power spectra, where the maximum ℓ is that available in the Planck Legacy Archive. Later on, when considering lensing, we will add the full low- ℓ data to the analysis.

We show the best fits derived with this analysis in Table 3.2. The best fit to the CIP amplitude with TT-data only is $\Delta_{\text{rms}}^2 = (5.8 \pm 7.1) \times 10^{-3}$, and with the combined data set is $\Delta_{\text{rms}}^2 = (0.9 \pm 5.0) \times 10^{-3}$. There is thus no evidence for the existence of CIPs, and the constraint is of the same order of magnitude as the trispectrum constraint of Ref.¹⁵ Notice that we have not required Δ_{rms}^2 to be positive. Imposing a prior $\Delta_{\text{rms}}^2 \geq 0$ would change the 68%C.L. constraints to $\Delta_{\text{rms}}^2 \leq 0.011$ for TT, $\Delta_{\text{rms}}^2 \leq 0.012$ for TE, $\Delta_{\text{rms}}^2 \leq 0.052$ for EE, and $\Delta_{\text{rms}}^2 \leq 0.0054$ for the combined data set. Notice that these limits have become more constraining in the case of the TE data set, due to the negative best-fit value for Δ_{rms}^2 , whereas the opposite is true for the TT and EE data sets.

Parameter	TT	TE	EE	Combined
$\omega_b \dots\dots\dots$	0.02238 ± 0.00028	0.02175 ± 0.00052	0.0251 ± 0.0015	0.02243 ± 0.00017
$\omega_c \dots\dots\dots$	0.1193 ± 0.0026	0.1217 ± 0.0033	0.1112 ± 0.0055	0.1194 ± 0.0015
$n_s \dots\dots\dots$	0.9653 ± 0.0081	0.939 ± 0.023	0.986 ± 0.018	0.9620 ± 0.0049
$\log(10^{10} A_s)$	3.097 ± 0.036	3.08 ± 0.040	3.11 ± 0.040	3.11 ± 0.034
$\tau \dots\dots\dots$	0.082 ± 0.018	0.080 ± 0.019	0.078 ± 0.019	0.088 ± 0.017
$H_0 \dots\dots\dots$	67.7 ± 1.3	66.0 ± 1.7	72.1 ± 3.1	67.4 ± 0.71
$\Delta_{\text{rms}}^2 \dots\dots\dots$	0.0058 ± 0.0071	-0.023 ± 0.020	0.040 ± 0.023	0.0009 ± 0.0050

Table 3.2: Best-fit values and standard deviations for cosmological parameters with the three different Planck data sets (TT, TE and EE polarizations for $\ell > 30$), as well as combining them. We have used a prior in τ in addition to all the data sets to break the degeneracy between τ and A_s .

We show the confidence ellipses for the Planck experiment on Figures 3.4 and 3.5,

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where it is clear that the CIP contribution to the CMB power spectrum is highly correlated with most of the rest of parameters. The correlation coefficients, defined as $r_{ij} \equiv (F^{-1})_{ij} / \sqrt{(F^{-1})_{ii}(F^{-1})_{jj}}$, are found to be $r_{\omega_b, \Delta_{\text{rms}}^2} = 0.73$, $r_{\omega_c, \Delta_{\text{rms}}^2} = -0.57$, $r_{n_s, \Delta_{\text{rms}}^2} = 0.76$, $r_{A_s, \Delta_{\text{rms}}^2} = -0.46$, and $r_{H_0, \Delta_{\text{rms}}^2} = 0.69$.

We do not show the covariance between Δ_{rms}^2 and τ , since the prior applied to τ renders the correlation coefficients meaningless. Even though this high- ℓ analysis shows no strong evidence for the existence of CIPs, they have the potential to resolve the lensing tension mentioned above, when including low- ℓ data. We now explore this possibility.

Solving the Lensing Tension

The CMB is lensed by large-scale structure along the line of sight. The main effects of the lensing on the CMB power spectra are to add power at small scales and to smooth the acoustic peaks.^{76,77} The amount of lensing inferred from CMB TT measurements seems, however, to be higher (by about two standard deviations) than the predicted value. This difference is parametrized through the lensing amplitude A_L ,^{78,79} which is fixed to be $A_L = 1$ in Λ CDM, but letting it vary can better fit the data. An analysis of the Planck measurements of the TT power spectrum found a best-fit value of $A_L = 1.22 \pm 0.10$.¹

Adding a new parameter to the likelihood analysis changes the best-fit A_L if the new parameter is correlated with it.^{109,110} The effects on the CMB of increasing A_L are very similar to adding CIPs, as can be seen from Figures 3.1, 3.2, and 3.3. Then, we can compute the offset induced in A_L due a non-zero CIP variance Δ_{rms}^2 as

$$\delta A_L = (F^{-1})_{A_L, \Delta} F_{\Delta, \Delta} \Delta_{\text{rms}}^2. \quad (3.28)$$

In the Planck TT case, the product $(F^{-1})_{A_L, \Delta} \times F_{\Delta, \Delta}$ evaluates to be ≈ -150 . This means that a CIP variance of $\Delta_{\text{rms}}^2 \approx 10^{-3}$ would induce a bias in the lensing amplitude of $\delta A_L \approx -0.2$, completely eliminating the tension between the Λ CDM value of $A_L = 1$ and the observed value. This value of Δ_{rms}^2 is allowed by the current constraints on CIPs, being a factor of ~ 7 smaller than our TT-only bound.

Of course this is only an approximate analysis ignoring the rest of the cosmological

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parameters. To include all correlations we use a Fisher-matrix analysis as above, adding $\delta A_8^X \equiv A_L - 1$ as an eighth parameter in our analysis. Its associated basis function, g_8 , is defined as in Eq. (3.2).

We fit for the value of A_L from the Planck data, first without CIPs (to show the tension) and then with CIPs. To follow more closely the analysis carried out by Planck,¹ we will use the low- ℓ polarization data instead of setting a prior for τ . These data are available as part of the Planck likelihood package.¹

The results are displayed in Table 3.3. We show the fit for the six original Λ CDM parameters + A_L first, where it is clear that the best-fit lensing amplitude deviates ~ 2 standard deviations from the Λ CDM value of $A_L = 1$ for the TT, EE, and the combined data set.

In Figure 3.6 we plot the likelihoods for A_L , when marginalizing over the rest of parameters, before and after including CIPs.² This Figure shows a significant widening of the likelihood curves, which added to the bias from Eq. (3.28) is responsible for the decrease in the tension of the fit.

In Table 3.3 we also show the standard deviations (and new best-fit values) when including the six Λ CDM parameters + $\Delta_{\text{rms}}^2 + A_L$ (so $N_p = 8$). In that case the tension in the TT data set vanishes, due to the correlations between A_L and Δ_{rms}^2 .

A χ^2 analysis of the TT power spectrum shows a preference for a non-standard lensing amplitude. The change in χ^2 from the standard Λ CDM model (with $A_L = 1$) to an A_L -varying model (usually denoted Λ CDM+ A_L) is $\Delta\chi^2 = -4.1$, giving rise to a p -value of 0.043, which makes it a significantly better fit.

Adding CIPs to this Λ CDM+ A_L model changes χ^2 by $\Delta\chi^2 = -0.3$, with a p -value of 0.58. This implies that Λ CDM+ A_L +CIPs does not fit the TT power spectrum better than Λ CDM+ A_L .

Interestingly, CIPs alone can do as well as A_L alone improving the χ^2 statistic. The change in χ^2 from the Λ CDM model to Λ CDM+CIPs is $\Delta\chi^2 = -3.9$, with a p -value of

¹<http://wiki.cosmos.esa.int/>

²Note that, since we are using a linear Fisher-matrix analysis, these likelihoods are Gaussian by construction.

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0.048 (to be compared with 0.043 when adding a varying A_L to Λ CDM). Notice, though, that the best-fit CIP variance in that case would be $\Delta_{\text{rms}}^2 = (12.9 \pm 6.4) \times 10^{-3}$, which is in tension with both the trispectrum bound,¹⁵ and the galaxy-cluster bound.¹⁰²

This shows that adding either a varying A_L or CIPs to a standard Λ CDM model provides a better fit for the TT Planck power spectrum, by a comparable amount. Adding both, however, is not supported by the data. There are, however, a few systematic effects in the analysis that could bias the result. The most important example is that our treatment of the low- ℓ data is too simplistic. As a result, the uncertainties in τ and A_L in Table 3.3 are small when compared to the Planck 2015 result.¹ This indicates that our Fisher-matrix analysis is too optimistic when inferring the optical depth from the low- ℓ polarization data, which could be due to the non-gaussian nature of the low- ℓ likelihoods, to the mode coupling, or to the linear approximation breaking down. A full likelihood analysis could show that CIPs absorb more of the lensing tension than indicated in this simple analysis.

Summarizing, we conclude that CIPs are unlikely to solve the lensing tension with current Planck data. Nonetheless, they remain one of the simplest prospective solutions, due to their high correlation with the lensing amplitude ($r_{A_L, \Delta_{\text{rms}}^2} = -0.82$). High-quality low- ℓ polarization data will be publicly available in the next few years,¹¹¹ so a reanalysis using the full Planck likelihoods, perhaps also including higher- ℓ multipoles from SPTpol,^{112, 113} will resolve the matter definitively.

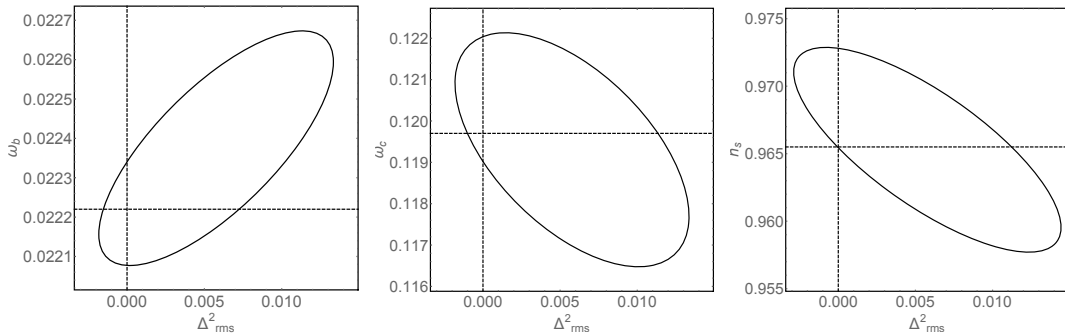


Figure 3.4: 1σ (68%) confidence ellipses for the Planck TT data set. From left to right we show ω_b , ω_c , and n_s vs Δ_{rms}^2 . The unperturbed (Planck) best-fit values are shown as dashed lines.

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Parameter	TT	TE	EE	Combined
ω_b	0.02235 ± 0.00020	0.02257 ± 0.00032	0.0245 ± 0.0013	0.02228 ± 0.00014
ω_c	0.1180 ± 0.0023	0.1168 ± 0.0023	0.1095 ± 0.0053	0.1185 ± 0.0014
n_s	0.9660 ± 0.0054	0.983 ± 0.015	0.992 ± 0.014	0.9641 ± 0.0037
$\log(10^{10} A_s)$	3.038 ± 0.031	3.065 ± 0.035	3.07 ± 0.034	3.038 ± 0.031
τ	0.056 ± 0.016	0.062 ± 0.016	0.062 ± 0.016	0.055 ± 0.015
H_0	68.1 ± 1.0	68.6 ± 1.1	72.4 ± 2.9	67.75 ± 0.64
A_L	1.13 ± 0.064	1.17 ± 0.17	1.46 ± 0.23	1.108 ± 0.054
Parameter	TT	TE	EE	Combined
ω_b	0.02248 ± 0.00029	0.0223 ± 0.0046	0.02593 ± 0.00017	0.02222 ± 0.00018
ω_c	0.1176 ± 0.0025	0.1183 ± 0.0029	0.1102 ± 0.0053	0.1187 ± 0.0015
n_s	0.9699 ± 0.0086	0.977 ± 0.017	1.004 ± 0.016	0.9622 ± 0.0052
$\log(10^{10} A_s)$	3.041 ± 0.031	3.05 ± 0.035	3.081 ± 0.036	3.037 ± 0.031
τ	0.057 ± 0.016	0.058 ± 0.016	0.065 ± 0.016	0.054 ± 0.015
H_0	68.5 ± 1.2	67.7 ± 1.5	73.0 ± 3.0	67.60 ± 0.70
A_L	1.07 ± 0.11	1.43 ± 0.35	-0.39 ± 0.65	1.142 ± 0.085
Δ_{rms}^2	0.007 ± 0.011	-0.028 ± 0.033	0.088 ± 0.064	-0.0038 ± 0.0074

Table 3.3: Best-fit values and standard deviations for cosmological parameters with the three different Planck data sets (TT, TE and EE polarizations for $\ell > 30$), as well as combining them. In the top part we have fitted for the original six parameters and the lensing amplitude A_L . In the bottom part we have also added a CIP amplitude Δ_{rms}^2 . Instead of a prior in τ we have used the low- ℓ polarization data ($\ell < 30$) from Planck in addition to all the data sets to disentangle τ and A_s .

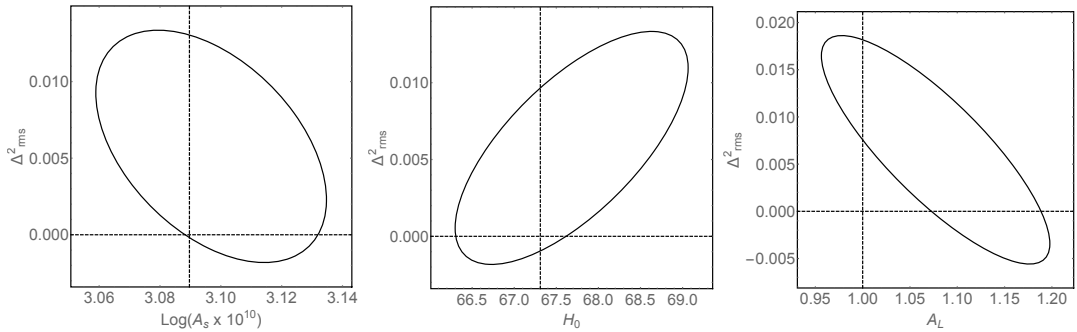


Figure 3.5: 1σ (68%) confidence ellipses for the Planck TT data set. From left to right we show Δ_{rms}^2 vs A_s , H_0 , and A_L , using low- ℓ polarization data instead of a prior on τ for the latter. The unperturbed (Planck) best-fit values are shown as dashed lines.

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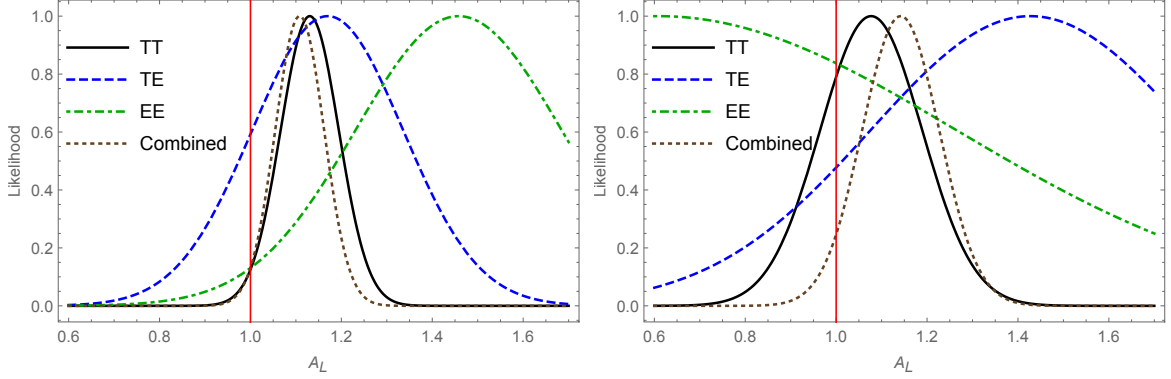


Figure 3.6: Normalized likelihoods for the lensing amplitude for all data sets. In the top panel without considering CIPs, and in the lower panel adding them as well.

Cosmic-variance limit

We now find the minimum Δ_{rms}^2 observable in the cosmic-variance limited case for different data sets.

We consider an experiment with no instrumental noise N_ℓ (i.e. $N_\ell = 0$), full sky coverage ($f_{\text{sky}} = 1$) and range of observation from $\ell = 2$ to 2500. In reality the lowest multipoles should be treated with care, due to possible Galactic-foreground subtraction,¹¹⁴ which we ignore here. We show the results for the uncertainties of such an experiment in Table 3.4.

The best CVL constraints to Δ_{rms}^2 arise from the polarization power spectra (EE especially) instead of the TT power spectrum, as holds true for the six original parameters.⁷²

The minimum CIP variance observable in the CVL is $\Delta_{\text{rms}}^2 = 9 \times 10^{-4}$, a factor of ~ 5 better than the current trispectrum constraint.¹⁵ This result pales in comparison to the sensitivity of a CVL trispectrum experiment, as described in Ref.,¹⁰³ which would be able to measure $\Delta_{\text{rms}}^2 \leq 3 \times 10^{-6}$.

Here τ is free, unlike the Planck case, where we included a prior. This leads to higher correlations of the CIP amplitude Δ_{rms}^2 with the optical depth τ , and the scalar amplitude A_s . We find the correlation coefficients in the CVL case to be $r_{\omega_b, \Delta_{\text{rms}}^2} = 0.30$, $r_{\omega_c, \Delta_{\text{rms}}^2} = -0.02$, $r_{n_s, \Delta_{\text{rms}}^2} = 0.47$, $r_{A_s, \Delta_{\text{rms}}^2} = -0.71$, $r_{\tau, \Delta_{\text{rms}}^2} = -0.66$, and $r_{H_0, \Delta_{\text{rms}}^2} = 0.15$. When including a lensing amplitude, we find $r_{A_L, \Delta_{\text{rms}}^2} = -0.27$.

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Data	ω_b	ω_c	n_s	A_s	τ	H_0	Δ_{rms}^2
TT	1.6×10^{-4}	1.7×10^{-3}	5.0×10^{-3}	8.1×10^{-11}	0.019	0.80	4.8×10^{-3}
TE	1.1×10^{-4}	1.0×10^{-3}	4.7×10^{-3}	3.6×10^{-11}	8.3×10^{-3}	0.45	2.5×10^{-3}
EE	7.4×10^{-5}	7.6×10^{-4}	2.8×10^{-3}	9.9×10^{-12}	2.4×10^{-3}	0.33	1.5×10^{-3}
Combined	2.8×10^{-5}	4.6×10^{-4}	1.8×10^{-3}	8.0×10^{-12}	1.9×10^{-3}	0.19	9.0×10^{-4}

Table 3.4: Standard deviations forecast for a CVL experiment measuring from $\ell = 2$ to $\ell = 2500$ and with $f_{\text{sky}} = 1$. We consider the six Λ CDM parameters + Δ_{rms}^2 being fitted at the same time.

3.2.4 Modification to Lensing Estimators

Here we will also use the CIP contribution to the standard lensing potential power spectrum estimator and use the Planck estimates of weak lensing to further constrain CIPs. We find that from the CMB power spectrum (temperature and polarization) the CIP amplitude can be constrained to $\Delta_{\text{rms}}^2 < 0.014$ at 95% confidence level (corresponding to a constraint on the amplitude of a scale-invariant CIP power spectrum $A_{\text{CIP}} < 0.056$). When including the Planck estimates of the lensing potential the constraint improves by more than a factor of four to $\Delta_{\text{rms}}^2 < 0.0034$ ($A_{\text{CIP}} < 0.013$). This upper limit improves upon the previous best upper limit which uses estimates of the baryon fraction in galaxy clusters. Using the approach we present here, estimates of the large-scale lensing potential power spectrum from the CMB can constrain the CIP amplitude to $\Delta_{\text{rms}}^2 < 8 \times 10^{-5}$ ($A_{\text{CIP}} < 1.2 \times 10^{-4}$).

Both weak gravitational lensing and compensated isocurvature perturbations can be thought of as a modulation of a ‘background’ CMB anisotropy. In this subsection we consider these effects under the flat-sky approximation and focus on the temperature anisotropy (related expressions which include polarization are straight forward to generalize).

In the presence of lensing plus a CIP we have

$$\begin{aligned}
 T(\hat{n}) &= T\left[\hat{n} + \vec{\nabla}\phi(\hat{n}), \Delta(\hat{n})\right], \\
 &\simeq T(\hat{n}) + \nabla_i \phi \nabla^i T + \Delta(\hat{n}) \frac{\partial T}{\partial \Delta} \Big|_{\Delta=0} + \frac{1}{2} \left(\nabla_i \phi \nabla_j \phi \nabla^i \nabla^j T + \Delta(\hat{n})^2 \frac{\partial^2 T}{\partial \Delta^2} \Big|_{\Delta=0} \right) + \dots
 \end{aligned} \tag{3.29}$$

In addition to this, the finite experimental sensitivity of any observation adds a noise term so that the total observed temperature at each point on the sky can be written $T^t(\hat{n}) = T(\hat{n}) + T^N(\hat{n})$ where we will assume that we are using beam-deconvolved maps. This leads

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to a power spectrum of the observed map of the form

$$C_\ell^{TT,t} = C_\ell^{TT} + C_\ell^{TT,N}, \quad (3.30)$$

where the superscript N denotes noise. Then, the observed power spectrum can be written through $\delta^{(2)}(\vec{\ell} + \vec{\ell}') C^{TT,\text{obs}}(\ell) \equiv \langle T(\vec{\ell}) T^*(\vec{\ell}') \rangle / (2\pi)^2$. In the flat-sky approximation the observed power spectrum is given by

$$\begin{aligned} C_\ell^{TT,\text{obs}} &= C_\ell^{TT} \left[1 - \int \frac{d^2 L}{(2\pi)^2} C_L^{\phi\phi} (\vec{L} \cdot \vec{\ell})^2 \right] + \int \frac{d^2 L}{(2\pi)^2} C_{|\vec{\ell}-\vec{L}|}^{TT} C_L^{\phi\phi} [(\vec{\ell} - \vec{L}) \cdot \vec{L}]^2 \\ &+ \int \frac{d^2 L}{(2\pi)^2} C_{|\vec{\ell}-\vec{L}|}^{dT,dT} C_L^{\Delta\Delta} + C_\ell^{T,d^2T} \int \frac{d^2 L}{(2\pi)^2} C_L^{\Delta\Delta}. \end{aligned} \quad (3.31)$$

The CIP modulation also produces a contribution to higher order correlations. In particular, Refs.^{14,15,69,103} construct an optimal estimator for Δ_{LM} from the connected part of the CMB trispectrum. The analysis of the CMB trispectrum is far from trivial, so here we utilize the fact that estimates of the lensing potential power spectrum, ϕ , are also built out of the connected part of the CMB trispectrum.^{75,77,115} In the presence of CIPs, the estimator used to reconstruct the lensing potential power spectrum becomes biased.

In the case of Planck, the lensing estimator is made using maps constructed from the 143 GHz and 217 GHz channels, as in Table 3.1. We furthermore note that the Planck analysis uses a bandpass filter and harmonic space to restrict the power spectrum multipoles to $100 \leq \ell \leq 2048$ and use a fiducial cosmology that is spatially flat with parameters: $\Omega_b h^2 = 0.0222$, $\Omega_c h^2 = 0.1203$, $\Omega_\nu h^2 = 0.00064$ (corresponding to two massless neutrinos and one massive neutrino with $m = 0.06$ eV), $H_0 = 67.12 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $A_s = 2.09 \times 10^{-9}$, $n_s = 0.96$, and $\tau = 0.065$.

The minimum-variance temperature estimator derived in Ref.⁷⁵ is

$$\hat{d}_{TT}(\vec{L}) \equiv \frac{i\vec{L} A_{TT}(L)}{L^2} \int \frac{d^2 \ell_1}{(2\pi)^2} T^t(\vec{\ell}_1) T^t(\vec{\ell}_2) F_{TT}(\vec{\ell}_1, \vec{\ell}_2), \quad (3.32)$$

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which can be shown to give yield to a lensing power spectrum¹⁷

$$\begin{aligned} \langle \hat{d}_{TT}^*(\vec{L}) \hat{d}_{TT}(\vec{L}') \rangle &= (2\pi)^2 \delta^{(2)}(\vec{L} - \vec{L}') \left\{ L^2 C_L^{\phi\phi} + L^2 C_L^{\Delta\Delta} \left[\frac{\int \frac{d^2 \ell_1}{(2\pi)^2} h_{TT}(\vec{\ell}_1, \vec{\ell}_2) F_{TT}(\vec{\ell}_1, \vec{\ell}_2)}{\int \frac{d^2 \ell_1}{(2\pi)^2} f_{TT}(\vec{\ell}_1, \vec{\ell}_2) F_{TT}(\vec{\ell}_1, \vec{\ell}_2)} \right]^2 \right. \\ &\quad \left. + N_{TT,TT}^{(0)}(L) + N_{TT,TT}^{(1),\phi\phi}(L) + N_{TT,TT}^{(1),\Delta\Delta}(L) \right\}, \end{aligned} \quad (3.33)$$

where $N_{TT,TT}^{(0)}(L) = A_{TT}(L)$ is the Gaussian bias produced by the unconnected part of the trispectrum, $N_{TT,TT}^{(1),X}(L)$ is the non-Gaussian (connected) part of the trispectrum due to the lensing potential ($X = \phi\phi$) and the CIP modulation ($X = \Delta\Delta$). In practice, the Planck lensing analysis uses a combination of observed and simulated CMB maps to subtract the Gaussian bias.⁷⁷ Since the simulated maps do not include a CIP contribution, some fraction of the CIP contribution to the Gaussian bias may not be fully subtracted.

We showed in Ref.¹⁷ that the only contribution which is significant is

$$Q_L \equiv \frac{\int \frac{d^2 \ell_1}{(2\pi)^2} h_{TT}(\vec{\ell}_1, \vec{\ell}_2) F_{TT}(\vec{\ell}_1, \vec{\ell}_2)}{\int \frac{d^2 \ell_1}{(2\pi)^2} f_{TT}(\vec{\ell}_1, \vec{\ell}_2) F_{TT}(\vec{\ell}_1, \vec{\ell}_2)} \quad (3.34)$$

we find that for Planck

$$Q_L \simeq \frac{0.12}{L^2}. \quad (3.35)$$

This scaling is a result of the optimal weight, $f_{TT} \sim \ell^2 C_\ell^{TT}$, used in the standard lensing potential estimator, compared to the CIP weight $h_{TT} \sim C_\ell^{T,dT}$. We can now see that constraints to Δ_{rms}^2 when including estimates of the lensing potential power spectrum are dominated by the lower values of L . The CIP contribution to a given estimator depends on the noise properties of the experiment, so in Fig. 3.7 we show the result for Planck and S4 CMB.

The final result from this analysis is that $\Delta_{\text{rms}}^2 \leq 4.3 \times 10^{-3}$ at 95% C.L., a factor of three smaller than the result without this lensing contribution.^{22,116}

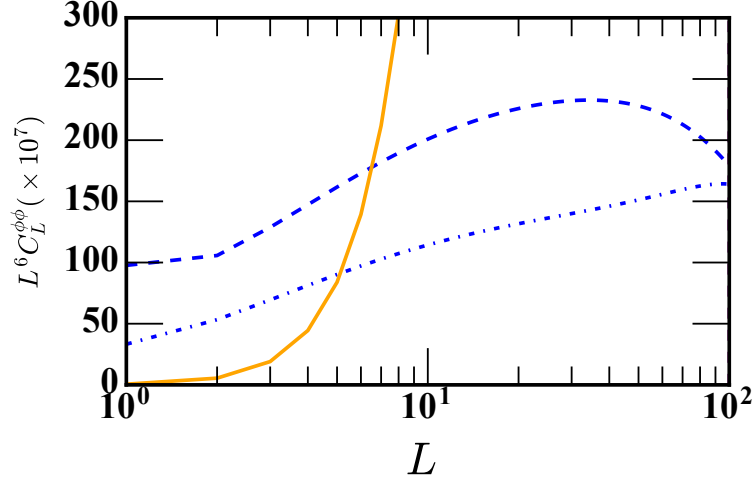


Figure 3.7: The CIP contribution to the estimator for the lensing potential power spectrum for both Planck (dashed blue) and CMB S4 (dot-dashed blue) for $\Delta_{\text{rms}}^2 = 2.5 \times 10^{-3}$. For comparison lensing potential power spectrum is shown in the solid orange curve.

Forecasts

We now assess the sensitivity of future CMB experiments to CIPs. As for lensing, the improvements will come primarily from small-scales (in particular, polarization), and so we focus on the proposed S4 CMB experiment, as described in Ref.²¹

We model the S4 CMB as a single channel experiment, as described in Table 3.1, and follow the Fisher formalism from Eq. (3.15). Since the CIP contribution to the lensing-potential power-spectrum estimator roughly decreases as L^{-2} , the sensitivity of future estimates to the scale-invariant CIP amplitude, A_{CIP} , is highly dependent on the minimum observable L -value, L_{min} , and therefore highly dependent on the sky coverage. Assuming that the non-lensing biases contributing to the lensing potential power spectrum estimator can be robustly subtracted on large angular scales, the minimum multipole which can be estimated is approximately given by $L_{\text{min}} \sim f_{\text{sky}}^{-1/2}$. Unfortunately, galactic foregrounds¹¹⁷ and temperature/polarization leakage²¹ could degrade the largest-scale measurements ($L < 30$).

We defer analysis of these complications to future work and obtain forecasts as a function of the minimum L_{min} detectable by the S4 CMB. Using information from the lensing estimator and power-spectrum smoothing, we obtain the sensitivity to $\Delta_{\text{rms}}^2(R_{\text{CMB}})$ for S4

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CMB experiment as a function of $L_{\min}$ (shown in Fig. 3.8). For $L_{\min} \geq 10$ the majority of the constraint comes from the CIP modulation of the CMB power spectrum, whereas for $L_{\min} < 10$ the CIP contribution to the lensing potential estimator dominates. We note that the S4 CMB lensing noise is very close to the cosmic-variance limit, and therefore the results should be the same for any other nearly CVL experiment. In particular, a full-sky CVL measurement of the lensing potential power spectrum has the potential to constrain $\Delta_{\text{rms}}^2(R_{\text{CMB}}) \lesssim 10^{-5}$, and therefore $A_{\text{CIP}} \lesssim 4 \times 10^{-5}$.

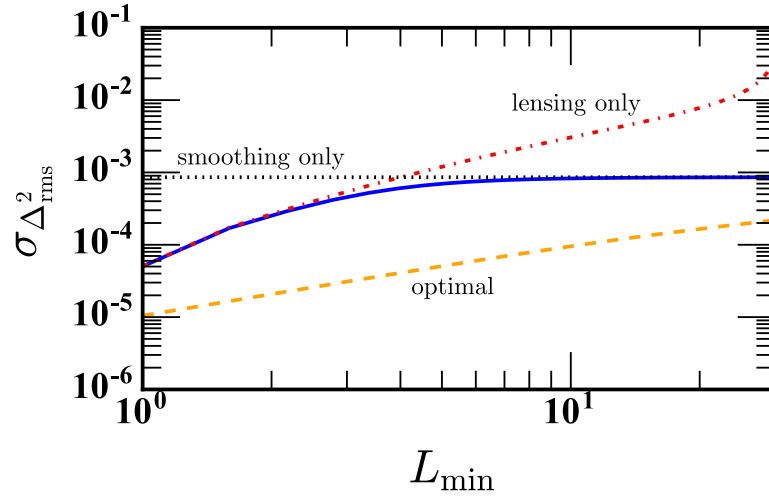


Figure 3.8: Projected sensitivity to σ_{rms}^2 for the S4 CMB. The overall sensitivity is shown in the solid blue curve. The sensitivity can be divided into a contribution from the smoothing of the CMB multipoles (dotted black) and from estimates of the lensing potential power spectrum (dot-dashed red). The lensing potential contribution is only important if future experiments can probe $L_{\min} \lesssim 5$.

Conclusions

Compensated isocurvature perturbations leave no imprint on the observable CMB to linear order, so their amplitude can be considerably larger than the $\sim 10^{-5}$ amplitude of primordial adiabatic perturbations. Currently the best constraints arise from analyzing the four-point function of the CMB, from where one can probe the first $L \sim 20$ multipoles of a CIP power spectrum, corresponding to scales larger than 10 degrees in the sky.

We use a different method to search for CIPs, based on studying the CMB power

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spectrum that arises to second order in the CIP-perturbation amplitude. We find a simple form for the contribution to the CMB power spectrum, proportional to the CIP variance Δ_{rms}^2 , which has the advantage of being easier to analyze than the trispectrum.

The amplitude Δ_{rms}^2 of this new contribution to the power spectrum can be expressed in terms of a sum over all the modes of a scale-invariant CIP power spectrum, although only the first $L \sim 100$ modes are important in CMB studies. This allows us to probe the CIPs down to angular scales of ~ 2 degrees in the sky.

We show that CIPs can alleviate the 2σ discrepancy in the lensing amplitude A_L , between that inferred from the Planck TT power spectrum and the Λ CDM expectation ($A_L = 1$). Adding CIPs to a standard Λ CDM model can improve the fit of the TT power spectrum as much as adding a varying A_L , making it unnecessary to have $A_L \neq 1$. The best-fit value for Δ_{rms}^2 in that case, however, would be three standard deviations above the current bounds. A full MCMC analysis would precisely determine whether CIPs provide a viable solution to the lensing tension.

We find a 1σ constraint on the CIP variance of $\Delta_{\text{rms}}^2 \leq 7.1 \times 10^{-3}$ using Planck temperature data alone, which improves to $\Delta_{\text{rms}}^2 \leq 5.0 \times 10^{-3}$ if polarization data are included. Moreover, once the lensing data from Planck is included the bound becomes $\Delta_{\text{rms}}^2 \leq 2.2 \times 10^{-3}$ at 1σ , a factor of 3 more stringent than with temperature+polarization alone.

3.3 Tilt and Running

Single-field slow-roll inflation predicts a nearly scale-free power spectrum of perturbations, as observed at the scales accessible to current cosmological experiments. This spectrum is slightly red, showing a tilt $(1 - n_s) \sim 0.04$. A direct consequence of this tilt are nonvanishing runnings $\alpha_s = dn_s/d\log k$, and $\beta_s = d\alpha_s/d\log k$, which in the minimal inflationary scenario should reach absolute values of 10^{-3} and 10^{-5} , respectively. In this section we calculate how well the upcoming S4 CMB experiment can measure these two runnings.

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3.3.1 Inflationary Power Spectrum

During inflation, quantum fluctuations generate an almost-scale-invariant power spectrum of fluctuations. The scalar perturbations $\zeta_{\mathbf{k}}$ thus have a two-point function given by

$$\langle \zeta_{\mathbf{k}} \zeta_{\mathbf{k}'}^* \rangle = P_\zeta(k) (2\pi)^3 \delta_D(\mathbf{k} + \mathbf{k}'), \quad (3.36)$$

where $P_\zeta(k)$ is the scalar power spectrum, for which we can define an amplitude as

$$\begin{aligned} \log \Delta_s^2(k) \equiv \log \left[\frac{k^3}{2\pi^2} P_\zeta(k) \right] &= \log A_s + (n_s - 1) \log \left(\frac{k}{k_*} \right) \\ &+ \frac{1}{2} \alpha_s \log^2 \left(\frac{k}{k_*} \right) + \frac{1}{6} \beta_s \log^3 \left(\frac{k}{k_*} \right), \end{aligned} \quad (3.37)$$

where A_s is the scalar amplitude, n_s is the scalar tilt, and α_s and β_s are the running and the second running, respectively. At the pivot scale of $k_* = 0.05 \text{ Mpc}^{-1}$, Planck has measured a scalar amplitude $A_s = 2.196 \times 10^{-9}$, with tilt $n_s = 0.9655$.¹¹⁸ We will take these values, with $\alpha_s = \beta_s = 0$, as our baseline. We will also assume for the rest of this section the fiducial Λ CDM parameters to be: $\omega_b = 0.02222$, $\omega_c = 0.1197$, $\tau = 0.06$, and $H_0 = 67.5 \text{ km/s}$.

We show the logarithmic derivative of the matter power spectrum with respect to the tilt n_s , the running α_s , and the second running β_s , at our fiducial values in Fig. 3.9. From this Figure it is clear that scales away from the pivot scale $k_* = 0.05 \text{ Mpc}^{-1}$ change the most when higher-order runnings are introduced. Note that the logarithmic derivative with respect to the scalar amplitude A_s would just be a horizontal line in this plot. We also show the regular matter power spectrum P_δ for comparison, both without any runnings and with $\beta_s = 0.03$, as argued in Ref.¹¹⁹ to be enough to produce primordial black holes (PBHs).

3.3.2 Single-Field Slow-Roll Predictions

Let us now briefly review the dynamics of single-field slow-roll (SFSR) inflation, and how the runnings change it. In the case of inflation being driven by a single field ϕ under a potential $V(\phi)$, the amplitude A_s and tilt n_s of the scalar power spectrum are determined by a combination of $V'(\phi_*)$ and $V''(\phi_*)$,¹¹⁸ where ϕ_* is the value of the field at the pivot

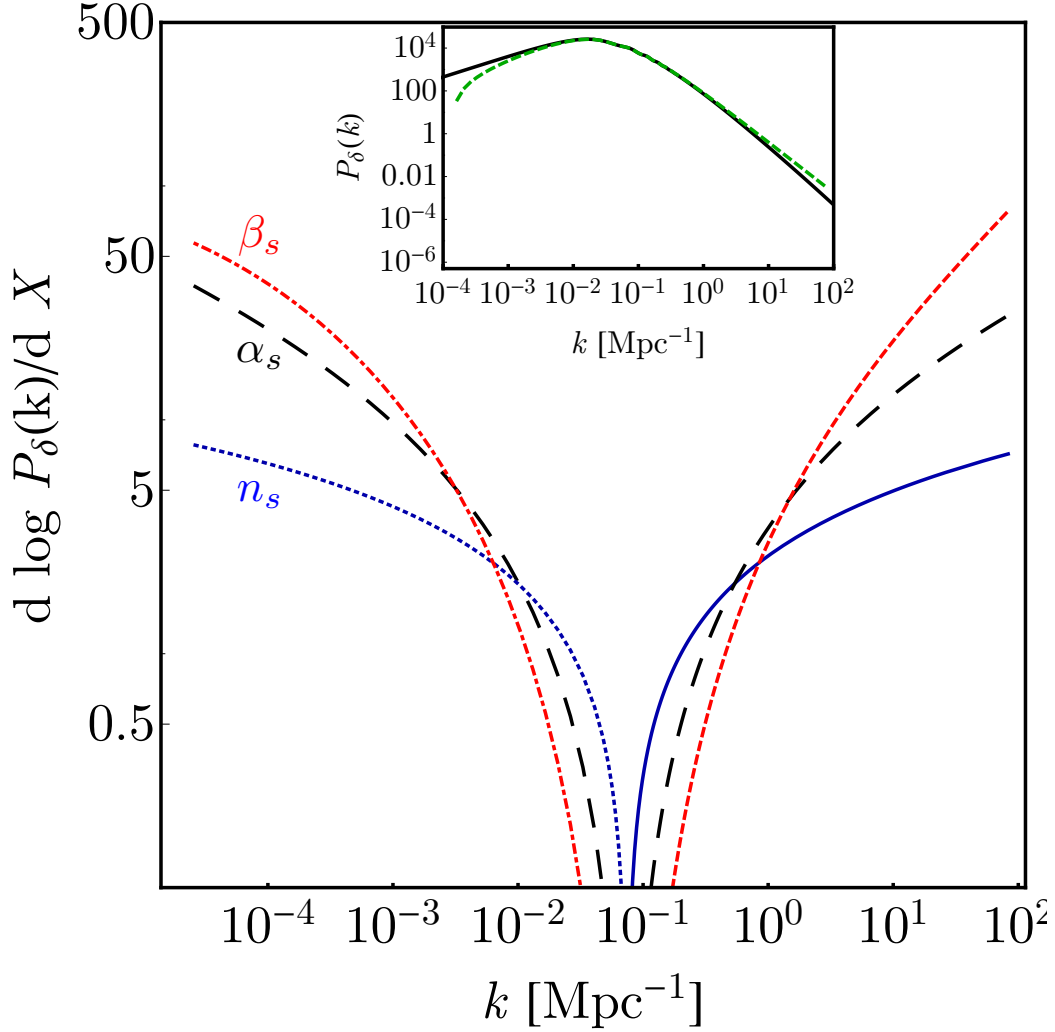


Figure 3.9: Logarithmic derivatives of the power spectrum $P_\delta(k)$, as a function of the wavenumber k in Mpc^{-1} , with respect to the scalar tilt n_s in blue line (solid for positive and dotted for negative values), to the running α_s in black long-dashed line, and the running of the running β_s in red line (dashed for positive and dash-dotted for negative values), at the Λ CDM best-fit values. The matter power spectrum $P_\delta(k)$ is shown in the top center for comparison, where the case with no runnings corresponds to the solid black line, and the case with $\beta_s = 0.03$ —the highest running allowed by Planck at 68% C.L.—to the green-dashed line.

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scale, and $'$ denotes a derivative with respect to ϕ . The absolute magnitude of $V(\phi_*)$ is related to the tensor-perturbation amplitude, or alternatively to its ratio r to the scalar amplitude.¹²⁰ The uncertainty in the reheating phase at the end of inflation, however, hampers a unique determination of the shape of the inflaton potential from A_s , n_s , and r .^{10, 11, 42, 52} The inclusion of an additional observable, such as the scalar running α_s , may help alleviate these uncertainties, as it provides information about the $V'''(\phi_*)$ term.¹²¹ A similar argument applies to the second running β_s with the fourth derivative of the potential. Unfortunately, in single-field slow-roll inflation these runnings are expected to be rather small. To illustrate why, let us define the slow-roll parameters

$$\begin{aligned}\epsilon &= \frac{M_{\text{pl}}^2}{2} \left(\frac{V'}{V} \right)^2, \\ \eta &= M_{\text{pl}}^2 \frac{V''}{V}, \\ \xi^2 &= M_{\text{pl}}^4 \frac{V'V'''}{V^2}, \\ \sigma^3 &= M_{\text{pl}}^6 \frac{V'^2 V^{(4)}}{V^3},\end{aligned}\tag{3.38}$$

where $M_{\text{pl}} = (8\pi G)^{-1/2} \approx 2.4 \times 10^{18} \text{ GeV}/c^2$ is the reduced Planck mass, and we have defined the third- and fourth-order slow-roll parameters, ξ and σ respectively, to be of the same order as ϵ and η in SFSR inflation. In this case, both the scalar and tensor indices (denoted by n_t , α_t , and β_t) can be found to first non-vanishing order in the slow-roll parameters as^{122, 123}

$$\begin{aligned}r &= 16\epsilon, \\ 1 - n_s &= 2\eta - 6\epsilon, & n_t &= -2\epsilon, \\ \alpha_s &= -2\xi^2 + 16\eta\epsilon - 24\epsilon^2, & \alpha_t &= 4\eta\epsilon - 8\epsilon^2, \\ \beta_s &= 2\sigma^3 + 2\xi^2(\eta - 12\epsilon) - 32\epsilon(\eta^2 - 6\eta\epsilon + 6\epsilon^2), & \beta_t &= -4\epsilon\xi^2 - 8\epsilon(\eta^2 - 7\eta\epsilon + 8\epsilon^2),\end{aligned}\tag{3.39}$$

so for $\sigma \sim \xi \sim \eta$ the prediction of single-field slow-roll inflation is $\alpha_s = O[(n_s - 1)^2] \sim 10^{-3}$, and $\beta_s = O[(n_s - 1)^3] \sim 10^{-5}$, as long as the potential does not experience a sudden change near CMB scales.^{124–126} In the same way that large local non-gaussianities would rule out single-field slow-roll inflation,^{127, 128} a running α_s much larger than $\sim 10^{-3}$ would also imply

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a more complex model of inflation. A similar argument holds with β_s , although with a 10^{-5} amplitude. Furthermore, it has been argued that very-large positive values of $\beta_s \gtrsim 10^{-2}$ are not possible within any SFSR model, as they would force the inflationary era to end before the largest observable scales exited the horizon.^{125,129,130} Thus, PBH production with a value of the second running $\beta_s = 0.03$ requires additional degrees of freedom during inflation. In general, reaching a precision of $\sigma(\beta_s) \lesssim 10^{-2}$ will provide a useful test of the single-field inflationary paradigm.

3.3.3 Forecasts

The proposed S4 CMB experiment, which we model as in Section 3.1.4, will be able to map modes up to $\ell \sim 5000$, both in temperature and polarization. We will study the level of precision that this S4 CMB experiment can reach for the running α_s , and the second running β_s .

The parameters θ_i that we will forecast in this section are the six Λ CDM parameters (ω_b , ω_c , n_s , A_s , τ , and H_0), plus the running α_s , and the second running β_s .

Results

We show in Fig. 3.10 the confidence ellipses between α_s , β_s , and the six Λ CDM parameters. Both runnings are mainly degenerate with ω_b , n_s , and A_s . Moreover, α_s and β_s are also correlated, which is to be expected, since both α_s and β_s increase the power at $k > k_*$, or $\ell \gtrsim 500$, where a great part of the CMB information comes from. We show the forecast uncertainties for these parameters in Table 3.5. In this table we show the minimum α_s that could be measured by a S4 CMB experiment (marginalizing over Λ CDM parameters but setting $\beta_s = 0$), which is $\sigma(\alpha_s) = 0.0025$, enough to detect significant departures from slow-roll single-field inflation, albeit not sufficient to detect the slow-roll prediction $\alpha_s \approx 10^{-3}$. Meanwhile, the $1-\sigma$ C.L. on β_s (marginalizing over Λ CDM+ α_s) will be $\sigma(\beta_s) = 0.0045$. This will shed light on the claimed detection of a non-zero second running $\beta_s = 0.02 \pm 0.01$,¹³⁰ and will have the power to test whether primordial black holes with tens of solar masses are formed from a positive second running. From these results we

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find that the S4 experiment alone will not suffice to determine the dynamics of inflation. Similar results have been forecasted for the proposed COrE satellite.¹³¹

As an additional test we have studied the correlation between both the running α_s , and the second running β_s , and the neutrino mass m_ν for the S4 experiment. We find a correlation, defined as $r_{ij} \equiv C_{ij}/\sqrt{C_{ii}C_{jj}}$, with $C \equiv F^{-1}$, of $r_{\alpha_s, m_\nu} = -0.24$ between the neutrino mass and the running, and $r_{\beta_s, m_\nu} = -0.23$ between the neutrino mass and the second running. Marginalizing over m_ν in addition to the other Λ CDM parameters worsens the sensitivity of the S4 CMB experiment to the runnings at percent level, which justifies neglecting the effects of the mass of the neutrinos in our forecasts.

Model	$\sigma(\omega_b)$	$\sigma(\omega_c)$	$\sigma(n_s)$	$\sigma(A_s)$	$\sigma(\tau)$	$\sigma(H_0)$	$\sigma(\alpha_s)$	$\sigma(\beta_s)$
Λ CDM+ α_s	3.4×10^{-5}	6.0×10^{-4}	2.2×10^{-3}	2.1×10^{-11}	0.0055	0.23	0.0025	—
Λ CDM+ α_s + β_s	3.5×10^{-5}	7.0×10^{-4}	2.7×10^{-3}	2.1×10^{-11}	0.0056	0.27	0.0026	0.0045

Table 3.5: $1 - \sigma$ C.L. forecast for the S4 CMB experiment. We consider the six Λ CDM parameters and α_s in the first row, and we add the second running β_s in the last row.

Combined constraints

The strength of the Fisher-matrix approach we follow is that we can easily add the information from different experiments by summing their Fisher matrices. We take as a our baseline case the proposed S4 CMB experiment, and in Table 3.6 we show the minimum α_s and β_s observable at $1 - \sigma$ when adding different combinations of experiments to the S4 CMB, from Ref.²² The results for CMB and galaxy surveys marginalize over all Λ CDM parameters and the bias b for α_s , and we include α_s in the marginalization for β_s . For details of galaxy surveys see.²² These results show that only very-futuristic experiments will be able to probe the dynamics of inflation, with the S4+SKA reaching a sensitivity of $\sigma(\alpha_s) \approx 10^{-3} \lesssim (1 - n_s)^2$ Interestingly, in all cases $\sigma(\beta_s) \sim 2\sigma(\alpha_s)$, which attests to the difficulty of determining the second running.

In Fig. 3.11 we plot the $1 - \sigma$ confidence ellipses in the $\alpha_s - \beta_s$ plane for the S4 CMB experiment, as well as the combination of S4+DESI, and S4+SKA. A representation of the current Planck 68% C.L. region is also shown (from Ref.¹³⁰), which displays a slight preference for a non-zero β_s . We draw a line in the $\alpha_s - \beta_s$ plane above which PBHs

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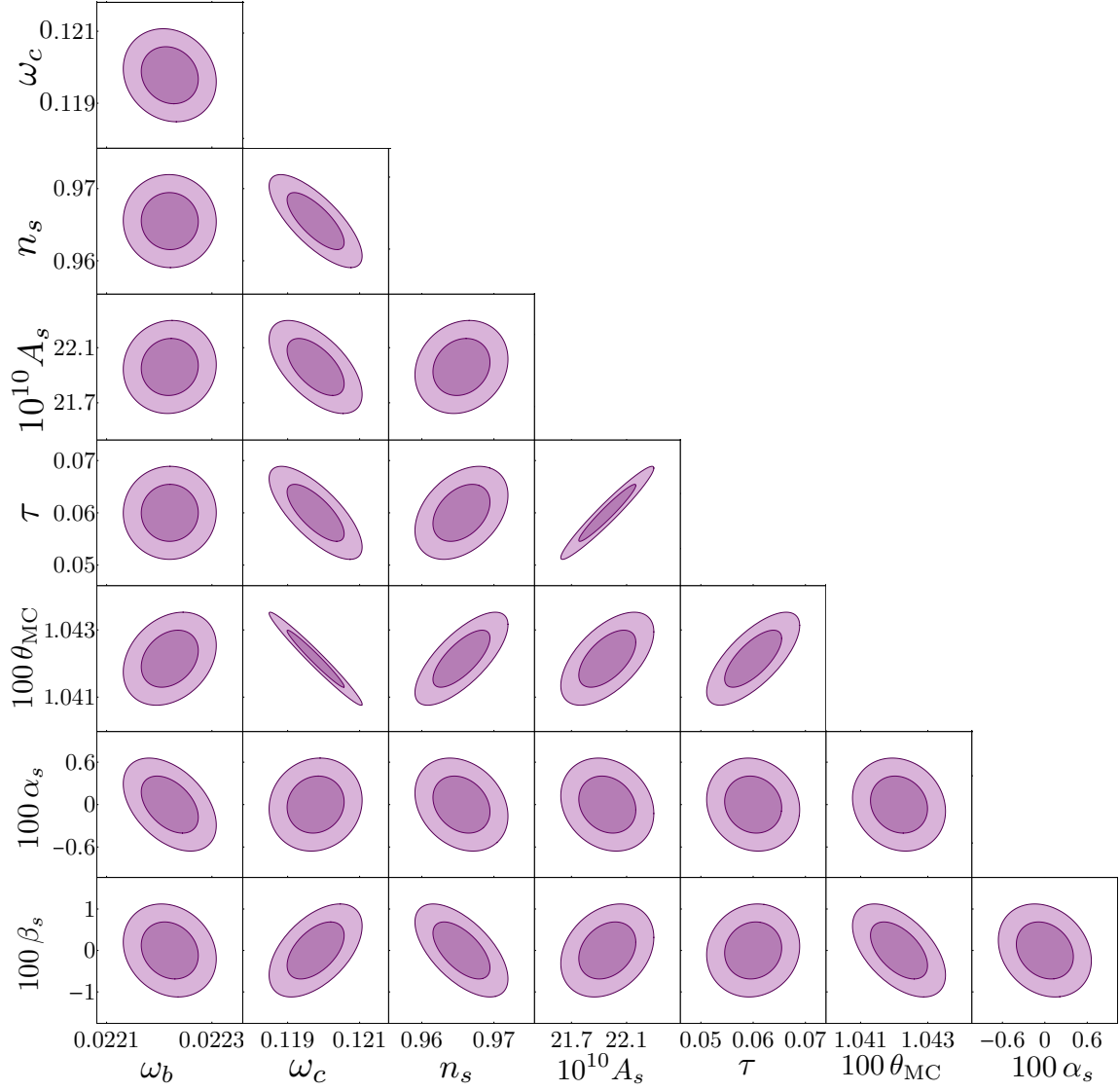


Figure 3.10: Confidence ellipses for the Λ CDM parameters and α_s and β_s , for the S4 CMB experiment. In darker purple we show the 68% C.L. region, and in lighter purple the 95% C.L. region.

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Experiment	$\sigma(\alpha_s)$	$\sigma(\beta_s)$
S4 CMB	0.0025	0.0045
S4+WFIRST	0.0020	0.0035
S4+DESI	0.0018	0.0034
S4+SKA	8.5×10^{-4}	0.0019

Table 3.6: $1 - \sigma$ uncertainties on the running α_s and the second running β_s for the S4 CMB experiment plus different proposed galaxy surveys. We marginalize over the six Λ CDM parameters (plus the bias amplitude b for the galaxy surveys) when computing $\sigma(\alpha_s)$ and over α_s as well for $\sigma(\beta_s)$.

with masses larger than 10^{15} gr could be formed, calculated as in Sec. 5.2. Comparing the ellipses in Fig. 3.11 with the predictions from slow-roll inflation it is clear that departures from slow-roll behavior should be detectable in α_s , although not in β_s unless they are very drastic.

Conclusions

In this section we have studied how well the S4 CMB experiment will be able to measure the runnings of the scalar power spectrum. A summary of our results is in Table 3.6, where we show that the S4 CMB will be able to measure the Λ CDM parameters to astounding precision (see Table 3.5). This S4 CMB experiment, however, will probe the runnings to a precision $\sigma(\alpha_s) = 0.0025$ and $\sigma(\beta_s) = 0.0045$, insufficient to detect the single-field slow-roll inflation prediction, although enough to measure significant departures from it.

We added to the S4 CMB results the information from upcoming galaxy surveys, such as WFIRST and DESI. In Ref.²² we found that these surveys will marginally improve the S4 measurements, reducing the error bars by at most 30%. However, more futuristic surveys, such as a billion-object SKA, will add enough information to half the S4 CMB uncertainties, reaching enough sensitivity to detect $\alpha_s \sim 10^{-3}$, as predicted by slow-roll inflation. Additionally, these measurements will allow us to falsify the model for PBH production proposed in Ref.¹¹⁹ with the improved β_s accuracy.

To summarize, within the next few decades the uncertainties in the runnings α_s and β_s will decrease by a significant factor, as new cosmological experiments are developed and their data is analyzed. This will allow us to very-precisely characterize the dynamics of

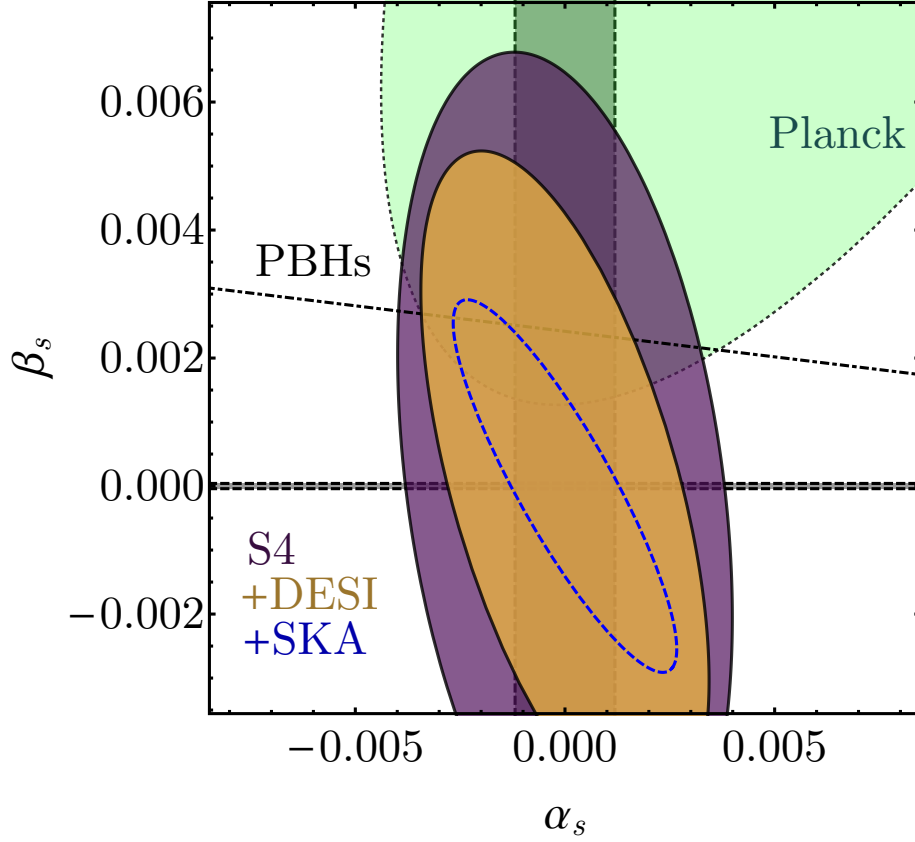


Figure 3.11: 68% confidence ellipses in the $\alpha_s - \beta_s$ plane for the S4 CMB experiment (purple), S4+DESI (yellow), and S4+SKA (blue). We show the current Planck ellipse from Ref.¹³⁰ in green. In gray we plot the range predicted by slow-roll single-field inflation. The region above the dash-dotted black line could produce PBHs with masses $M_{\text{pbh}} > 10^{15}$ gr, if extrapolated to the smallest scales.

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inflation, and test deviations from the standard slow-roll scenario. Such a measurement will be invaluable for characterizing the inflationary potential beyond the first-order slow-roll approximation, opening a window into the first moments of the Universe.

*“It’s more fun to experience things when you
don’t know what’s going to happen.”*

— Louis C.K.

Chapter 4

The 21-cm Hydrogen Line

Increasingly precise cosmic microwave background (CMB)^{104,118} and large-scale structure (LSS)¹³² measurements have zeroed in on a rather simple model of the cosmos, requiring only a handful of parameters. In particular, initial fluctuations seem to be mostly scalar and highly Gaussian.^{57,133,134} They are well described by a simple power-law spectrum, whose slope is consistent with a single scalar field driving inflation while slowly rolling down a very flat potential.^{8,39}

Neither the CMB nor LSS will reach enough number of modes to probe the ultimate predictions of slow-roll inflation. However, fluctuations in the brightness temperature of the 21-cm line of neutral hydrogen have the potential to open a new window on the high-redshift universe. This observable can in principle allow us to probe a fantastic number of modes, largely surpassing those available from CMB observations alone. First, fluctuations are undamped down to the baryon Jeans scale (with wavenumber $k \sim 300 \text{ Mpc}^{-1}$) three orders of magnitude smaller than the photon diffusion scale ($k \sim 0.2 \text{ Mpc}^{-1}$). In addition, whereas CMB anisotropies probe a single surface, a line such as the 21-cm transition makes it possible to observe the early universe in tomography, and to coadd the information from each independent redshift slice. While the 21-cm line can in principle be observed all the way to cosmological reionization, at $z \sim 10$, the signal is cleaner at higher redshifts $z \gtrsim 30$, the dark ages preceding the formation of the first luminous objects. We focus on this

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redshift range in this work.

The technical challenges to observe high-redshift 21-cm fluctuations are daunting, and will most likely require a telescope array on the far side of the Moon,¹³⁵ as well as foreground-removal of the Galactic synchrotron radiation to an exquisite accuracy. We will not tackle these problems in the present work, but we will study what can be learned from the dark ages about cosmology once foregrounds are subtracted.

While the baryon-photon fluctuations are highly linear at the epoch of last-scattering $z \sim 1100$, the perturbations in the cold dark matter (CDM) and baryon fluids have significantly grown by $z \sim 50$. Even if they remain small enough that no bound structure has formed yet, gravitational growth leads to a non-linear dependence of the density field on initial conditions. In addition, the 21-cm brightness temperature depends non-linearly on the local baryon density, velocity gradient, and temperature. Unless treated appropriately, all this can jeopardize the usefulness of 21-cm fluctuations to measure cosmology.

4.1 Formalism

The nuclear spin of the hydrogen atom makes its triplet ground state have a slightly higher energy than its singlet counterpart, giving rise to a transition with characteristic wavelength $\lambda \approx 21$ cm in the radio spectrum. Its very long wavelength makes it a good probe of the early Universe, being easily identifiable. We start by reviewing the physics of the 21-cm line during the dark ages and its angular power spectrum.

4.1.1 Global signal

The ratio of the populations of the triplet and singlet Hydrogen states defines a temperature, which we denote as spin temperature T_s . During the dark ages CMB photons stimulate radiative transitions between the singlet and the triplet Hydrogen states.¹³⁶ Collisions between different Hydrogen atoms will also create upwards and downwards transitions. The timescale of both these effects is much smaller than the evolution of the universe,¹³⁷

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so we can use the quasi-steady-state approximation,

$$n_0(C_{01} + R_{01}) = n_1(C_{10} + R_{10}), \quad (4.1)$$

where n_1 and n_0 are the densities of triplet- and singlet-state Hydrogen atoms, C_{ij} are the collisional transition rates, and R_{ij} are the rates of radiative transition due to the CMB blackbody photons. This allows us to define the spin temperature, which we can approximate very well by

$$T_s = T_\gamma + \frac{C_{10}}{C_{10} + A_{10} \frac{T_H}{T_*}} (T_H - T_\gamma), \quad (4.2)$$

where T_H is the temperature of the neutral hydrogen, T_γ that of the CMB, $T_* = 0.068$ K $= 5.9 \mu\text{eV}$ is the characteristic temperature of the 21-cm transition, and A_{10} is the Einstein spontaneous-emission coefficient of the 21-cm transition.

During the redshift period of interest collisions dominate over radiative transitions, which couples the spin temperature to that of the Hydrogen. This enables Hydrogen atoms to resonantly absorb CMB photons with a rest wavelength of 21 cm, which from Earth results in a decrease in the brightness temperature of the CMB at the corresponding redshifted wavelength. We ignore here low-redshift effects, such as the Wouthuysen-Field effect,^{138–140} heating of the Hydrogen gas due to miniquasars,^{141, 142} or early stellar formation.^{143, 144}

Let us define the 21-cm line temperature for small optical depths τ as

$$T_{21} = \frac{T_s - T_\gamma}{1 + z} \tau, \quad (4.3)$$

corresponding to the contrast with the CMB temperature redshifted to today. This parameter is given by

$$\tau = \frac{3}{32\pi} \frac{T_*}{T_s} n_H \lambda_*^3 \frac{A_{10}}{H(z) + (1 + z) \partial_r v_r}, \quad (4.4)$$

where $\lambda_* \approx 21$ cm, n_H is the number density of neutral Hydrogen, and $\partial_r v_r$ is the proper gradient of the peculiar velocity along the line of sight.

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4.1.2 Fluctuations

The optical depth and the spin temperature of a Hydrogen clump depend on its density and velocity divergence. The small anisotropies in these two quantities create fluctuations on the 21-cm temperature T_{21} . Let us define $\delta_v \equiv -(1+z)\partial_r v_r/H(z)$. Then, at linear order, the 21-cm fluctuations can be expressed as^{25,145}

$$\delta T_{21}(\mathbf{x}) = \alpha(z)\delta_b(\mathbf{x}) + \bar{T}_{21}(z)\delta_v(\mathbf{x}), \quad (4.5)$$

with $\bar{T}_{21}(z)$ being the spatially-averaged 21-cm brightness temperature, and $\alpha(z) = dT_b/dn_H$, including gas-temperature fluctuations as in Ref.²⁵ The observed fluctuation of this quantity in a direction $\hat{n}$ of the sky and at a certain frequency ν is given by

$$\delta T_{21}(\hat{n}, \nu) = \int_0^\infty dx W_\nu(x) \delta T_{21}(\mathbf{x}), \quad (4.6)$$

where $W_\nu(x)$ is the window function selecting the information at a certain frequency band centered in ν . In Fourier space the primordial curvature perturbation $\zeta_{\mathbf{k}}$ is related to the baryonic anisotropies by $\delta_b(\mathbf{k}, z) = M_\zeta(k, z)\zeta_{\mathbf{k}}$, and $\delta_v(\mathbf{k}, z) = \mu^2\delta_b(\mathbf{k}, z)$, with $\mu = (\hat{k} \cdot \hat{n})$. We can, therefore, define the transfer function of the 21-cm temperature fluctuations as,

$$\mathcal{T}_\ell(k, \nu) = \int_0^\infty dx W_\nu(x) M_\zeta[k, z(x)] [\bar{T}_{21}(z) J_\ell(kx) + \alpha(z) j_\ell(kx)], \quad (4.7)$$

where j_ℓ is the spherical bessel function with index ℓ , and we have defined $J_\ell(kx) \equiv -\partial^2 j_\ell(kx)/(\partial kx)^2$, which can be written in terms of j_ℓ , and $j_{\ell\pm 2}$.¹⁴⁶ Given this, we can easily compute the 21-cm line angular power spectrum at a certain frequency ν as

$$C_\ell = \frac{2}{\pi} \int_0^\infty k^2 dk P_{\delta\delta}(k) \mathcal{T}_\ell^2(k, \nu), \quad (4.8)$$

where $P_{\delta\delta}$ is the (isotropic) primordial curvature power spectrum, given in $\langle \zeta_{\mathbf{k}}^{\text{iso}} \zeta_{\mathbf{k}'}^{\text{iso}} \rangle = (2\pi)^3 P_{\delta\delta}(k) \delta^{(3)}(\mathbf{k} + \mathbf{k}')$, with $\zeta_{\mathbf{k}}^{\text{iso}}$ being the isotropic part of the curvature perturbations, as defined in Chapter 3.

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4.1.3 Instrumental Noise

In the cosmic-variance limit (CVL) the only source of noise is the variance given by having a finite number of measurements of the power spectrum C_ℓ itself. If one considers, however, an interferometer looking at the dark ages at a certain frequency ν , there is an additional noise power spectrum given by^{23, 86, 87, 147}

$$\ell^2 C_\ell^N = \frac{(2\pi)^3 T_{\text{sys}}^2(\nu)}{\Delta\nu t_o f_{\text{cover}}^2} \left(\frac{\ell}{\ell_{\text{cover}}(\nu)} \right)^2, \quad (4.9)$$

where $\Delta\nu$ is the bandwidth of the survey, t_o is the total time of observation, $\ell_{\text{cover}}(\nu) \equiv 2\pi D_{\text{base}}/\lambda$ is the maximum multipole observable, with D_{base} being the largest baseline of the interferometer. The amplitude of this noise is given by the system temperature T_{sys} , which we take to be the synchrotron temperature of the observed sky,

$$T_{\text{sys}}(\nu) = 180 \left(\frac{\nu}{180 \text{ MHz}} \right)^{-2.6} \text{ K}, \quad (4.10)$$

found from extrapolating to lower frequencies the results in Ref.¹⁴⁸

We will use different baselines and coverage fractions for different applications, so we defer assigning specific values to each subsection. We also show the CVL results, as the ultimate limit attainable by 21-cm measurements.

4.2 Dark-matter Interactions with Baryons

4.2.1 Motivation

The standard picture of cold dark matter (CDM)¹⁴⁹ seems to fit very well with our current observational constraints.¹ There are, however, a few puzzles that would require dark matter to have non-zero interactions.^{150–152} Moreover, several models for the dark-matter (DM) particle predict some level of weak non-gravitational interaction with standard-model baryons.^{153, 154} Here we will study these interactions during the dark ages.

The simplest way to observe these interactions would be through direct detection experiments, such as DarkSide,^{155, 156} LUX¹⁵⁷ and XENON100.¹⁵⁸ These experiments are very

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sensitive to large dark-matter masses but cannot constrain interactions for DM masses below ~ 10 GeV due to the small recoil of the nuclei in any given interaction. A different probe would consist of indirect early-time effects of these interactions. One example would be the modification of the small-scale power spectrum, due to the drag induced in the DM by the interactions,¹⁵⁹ which would be observable in the cosmic microwave background (CMB), as well as in Ly- α forest measurements. Another example is CMB spectral distortions, which would be created by the indirect coupling, through baryons, of dark matter and photons in the very early universe.¹⁶⁰ These last two probes require interactions to be relevant at early times, so they are not sensitive to all velocity dependences. Some models for dark-matter–baryon interactions may elude constraints because interactions get stronger at later times. We will focus on one of those models, in which the interaction cross section is parametrized by $\sigma = \sigma_0 v^{-4}$, one realization of which would be dark-matter milicharge.¹⁵³ To constrain interactions at later times, a useful probe is the 21-cm line during the dark ages.

Interactions between baryons and dark matter can be detected through their effect on the brightness temperature of the 21-cm line. This brightness temperature is proportional to the difference between the spin temperature of the neutral Hydrogen and the CMB temperature. In the standard scenario the spin temperature is coupled to the baryon temperature during the redshift range $z \sim 30 - 200$. This creates a departure between spin temperature and CMB temperature. As shown in Ref.,¹⁶¹ if the baryons are cooled down (by interacting with a colder fluid, like the dark matter) the spin temperature will be lower, modifying the overall brightness temperature.

We emphasize that these interactions do not cause just cooling of the baryons, but also heating. In the usual picture of interaction between two fluids, the warmer fluid will lose energy toward heating up the colder one, while there will be no energy transfer if both fluids have the same temperature. However, if there is a relative velocity between the two fluids –dark matter and baryons in our case– there will be an additional friction term that will tend to damp this relative velocity. The kinetic energy lost in this manner will induce heating in both fluids. The magnitude of this effect depends on the initial relative velocity, which is given by a Gaussian variable with a (3D) variance of ~ 29 km/s at kinematic decoupling ($z \approx 1010$).^{29,145}

The brightness temperature will then acquire an additional spatial dependence, through

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the local variation of the relative velocities. Quantifying this effect, we find that during the dark ages it creates an additional contribution to the power spectrum of 21-cm temperature fluctuations, which can be more than an order of magnitude bigger at large scales than the standard one, even for values of the cross section allowed by current CMB studies.¹⁵⁹ We study the detectability of this new signal with an SKA-inspired interferometer¹ and with a more futuristic proposed experiment. We also study how the global signal changes due to interactions and discuss the prospects for experiments such as NenuFAR².

4.2.2 Evolution of interacting dark matter and baryon fluids

In this section we will study how the interactions between DM and baryons change their temperatures. To do that we will have to calculate the drag on the relative velocity due to interactions with baryons, as well as the heating effect on both fluids. Our results will rely on the current understanding of relative velocities, so let us start with a brief review.

Velocities

In the standard cosmological evolution, dark matter starts collapsing as soon as matter-radiation equality is reached. Baryons, however, cannot cluster due to radiation pressure, until they decouple from the photon background. This difference in their evolution history generates a relative velocity between the two components. After the baryons and photons kinematically decouple, at redshift $z \approx 1010$, this velocity redshifts away, since the baryons experience infall into the DM gravitational wells. Ref.²⁹ first pointed out that relative velocities affect the formation of small-scale structure. Their effect on the standard power spectrum of 21-cm fluctuations in the dark ages was studied in Ref.¹⁴⁵

At kinematic decoupling, the relative velocities $\mathbf{V}_{\chi b} \equiv \mathbf{V}_{\chi} - \mathbf{V}_b$ follow a Gaussian distribution, where $\mathbf{V}_{\chi}$ and $\mathbf{V}_b$ are the DM and baryon bulk velocities. Then the differential probability of having an initial relative velocity $\mathbf{V}_{\chi b,0}$ is given by

$$\mathcal{P}(\mathbf{V}_{\chi b,0}) = \frac{e^{-3\mathbf{V}_{\chi b,0}^2/(2V_{\text{rms}}^2)}}{(\frac{2\pi}{3}V_{\text{rms}}^2)^{3/2}}, \quad (4.11)$$

¹<https://www.skatelescope.org/>.

²<http://nenufar.obs-nancay.fr>.

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where the value of the (3D) width of this distribution is $V_{\text{rms}} = 29 \text{ km/s} \sim 10^{-4} c$ at kinematic decoupling ($z = 1010$).¹⁴⁵ This rms value as well as the full power spectrum of $V_{\chi b,0}$ can be simply extracted from standard linear Boltzmann codes.^{137,162}

Elastic interactions between fluids with a relative velocity will have two different effects. First, they will tend to decrease the relative velocity and achieve mechanical equilibrium, which in our scenario will manifest itself as a drag on the relative velocity.¹⁵⁹ Second, they will thermally couple the fluids, tending to equilibrate their temperatures.

We start by calculating the drag on the relative velocity.

Drag term

Throughout the text we consider cross sections parametrized as $\sigma = \sigma_0 v^{-4}$. First we analyze the velocity change due to the collision with a baryon with velocity $\mathbf{v}_b$. In the center-of-mass (CM) frame the initial velocity of the DM particle will be

$$\mathbf{v}_\chi^{(\text{CM}),0} = (\mathbf{v}_\chi - \mathbf{v}_b) \frac{m_b}{m_b + m_\chi}, \quad (4.12)$$

and in an elastic collision the final velocity can be parametrized by the angle toward which it is scattered, so the final velocity of the dark matter particle is

$$\mathbf{v}_\chi^{(\text{CM}),f} = v_\chi^{(\text{CM}),0} \hat{n}, \quad (4.13)$$

where $\hat{n}$ is a unit vector. The change in velocity in a single collision (which is Galilean invariant, and hence frame independent) is

$$\Delta \mathbf{v}_\chi = \frac{m_b}{m_b + m_\chi} |\mathbf{v}_\chi - \mathbf{v}_b| \left(\hat{n} - \frac{\mathbf{v}_\chi - \mathbf{v}_b}{|\mathbf{v}_\chi - \mathbf{v}_b|} \right). \quad (4.14)$$

To calculate the full effect of the interactions we need to include the rate at which interactions happen, and average over the velocities of the fluid elements. The rate of interactions in a particular direction $d\hat{n}$ is $d\sigma/d\hat{n} |\mathbf{v}_\chi - \mathbf{v}_b| n_b$, where $\sigma(|\mathbf{v}_\chi - \mathbf{v}_b|)$ is the cross section as a function of the relative velocity, and n_b is the number density of baryons

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(targets). The time derivative of the DM bulk velocity will then be

$$\frac{d\mathbf{V}_\chi}{dt} = n_b \int d^3v_\chi f_\chi \int d^3v_b f_b |\mathbf{v}_\chi - \mathbf{v}_b| \int d\hat{n} \frac{d\sigma}{d\hat{n}} \Delta\mathbf{v}_\chi, \quad (4.15)$$

and we can perform the inner integral, by plugging Eq. (4.14) into Eq. (4.15) and realizing it has to be proportional to the only direction $(\mathbf{v}_\chi - \mathbf{v}_b)$ inside the integral, to find

$$\frac{d\mathbf{V}_\chi}{dt} = -\frac{\rho_b}{m_b + m_\chi} \int d^3v_\chi f_\chi \int d^3v_b f_b (\mathbf{v}_\chi - \mathbf{v}_b) |\mathbf{v}_\chi - \mathbf{v}_b| \bar{\sigma}, \quad (4.16)$$

where we have defined the momentum-transfer cross section as

$$\bar{\sigma}(|\mathbf{v}_\chi - \mathbf{v}_b|) \equiv \int d(\cos\theta) \frac{d\sigma}{d\cos\theta} (1 - \cos\theta). \quad (4.17)$$

Alternatively, we could have calculated the drag on the baryon velocity, which is given by exchanging $\chi \leftrightarrow b$ in Eq. (4.16), so that $d\mathbf{V}_b/dt = -(\rho_\chi/\rho_b)d\mathbf{V}_\chi/dt$. The relative velocity between the two fluids will then evolve as

$$\frac{d\mathbf{V}_{\chi b}}{dt} = -\frac{\rho_m}{m_b + m_\chi} \int d^3v_\chi f_\chi \int d^3v_b f_b (\mathbf{v}_\chi - \mathbf{v}_b) |\mathbf{v}_\chi - \mathbf{v}_b| \bar{\sigma}, \quad (4.18)$$

where we have defined $\rho_m \equiv \rho_b + \rho_\chi$.

To calculate the two integrals over velocities we define two new variables $\mathbf{v}_m$ and $\mathbf{v}_{th}$, as

$$\mathbf{v}_m \equiv \frac{\frac{m_\chi}{T_\chi} \mathbf{v}_\chi + \frac{m_b}{T_b} \mathbf{v}_b}{\frac{m_\chi}{T_\chi} + \frac{m_b}{T_b}}, \quad \text{and} \quad (4.19)$$

$$\mathbf{v}_{th} \equiv \mathbf{v}_\chi - \mathbf{v}_b, \quad (4.20)$$

so that the velocity distributions f factorize

$$\int d^3v_\chi f_\chi \int d^3v_b f_b = \int d^3v_{th} f_{th} \int d^3v_m f_m. \quad (4.21)$$

Nothing will depend on $\mathbf{v}_m$, so we can just integrate it out, leaving then only the integral of the relative velocity $\mathbf{v}_{th}$. The distribution function f_{th} of this velocity is a Gaussian displaced from the origin by $\mathbf{V}_{\chi b}$ and with thermal width given by the sums of the baryon

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and DM widths, $T_\chi/m_\chi + T_b/m_b$. The integral to calculate hence reduces to

$$\frac{d\mathbf{V}_{\chi b}}{dt} = -\frac{\rho_m}{m_b + m_\chi} \int d^3v_{\text{th}} f_{\text{th}} \mathbf{v}_{\text{th}} v_{\text{th}} \bar{\sigma}(v_{\text{th}}), \quad (4.22)$$

Focusing on the case in which the interaction cross section is parametrized as $\bar{\sigma} = \sigma_0 v^{-4}$, the drag term is given by

$$D(V_{\chi b}) \equiv -\frac{dV_{\chi b}}{dt} = \frac{\rho_m \sigma_0}{m_b + m_\chi} \frac{1}{V_{\chi b}^2} F(r), \quad (4.23)$$

where we have defined $r \equiv V_{\chi b}/u_{\text{th}}$, and $u_{\text{th}}^2 \equiv T_b/m_b + T_\chi/m_\chi$, which is the variance of the thermal relative motion of the two fluids. The function $F(r)$ is determined as

$$F(r) \equiv \text{erf}\left(\frac{r}{\sqrt{2}}\right) - \sqrt{\frac{2}{\pi}} e^{-r^2/2} r, \quad (4.24)$$

which grows with r from zero at $r = 0$ to one at $r \rightarrow \infty$.

Heating

We now study the second effect that interactions have on the dark-matter and baryon fluids, namely heating. Interactions between two fluids (1 and 2) with different temperatures will tend to heat up the colder fluid (in our case the cold dark matter) at the expense of the energy of the warmer fluid, tending to equalize their temperatures. The heating rate is usually proportional to the temperature difference ($T_1 - T_2$). We will show here that, if there is a relative velocity between the two fluids, the heating rate will also include a friction term that will heat up both fluids, independently of their temperature difference.

There is an intuitive reason to expect a heating term even for equal-temperature fluids, if two fluids with the same temperature collide with a relative velocity, and then equilibrate, this final relative velocity should vanish. The kinetic energy would hence get transformed into a higher final temperature for both fluids, due to conservation of energy.

Let us calculate the heating rate $\dot{Q}_b$ of the baryons in their instantaneous rest frame, where the change in energy will directly give us the heat instead of having to add bulk

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motions. A baryon changes its energy in a collision by $\Delta E_b = m_b \mathbf{v}_{\text{CM}} \cdot \Delta \mathbf{v}_b = -m_\chi \mathbf{v}_{\text{CM}} \cdot \Delta \mathbf{v}_\chi$,¹⁵⁹ where $\mathbf{v}_{\text{CM}} = (m_b \mathbf{v}_b + m_\chi \mathbf{v}_\chi)/(m_b + m_\chi)$. The heating of the baryonic fluid per unit time is

$$\begin{aligned} \frac{dQ_b}{dt} = & \frac{m_b \rho_\chi}{(m_\chi + m_b)} \int d^3 v_b f_b \int d^3 v_\chi f_\chi(v_\chi) \\ & \times \bar{\sigma}(|\mathbf{v}_\chi - \mathbf{v}_b|) |\mathbf{v}_\chi - \mathbf{v}_b| [\mathbf{v}_{\text{CM}} \cdot (\mathbf{v}_b - \mathbf{v}_\chi)], \end{aligned} \quad (4.25)$$

where we have already integrated over outgoing angles $d\hat{n}$ using Eqs. (4.14) and (4.17).

We perform this integral and find

$$\frac{dQ_b}{dt} = \frac{2m_b \rho_\chi \sigma_0 e^{-\frac{r^2}{2}} (T_\chi - T_b)}{(m_\chi + m_b)^2 \sqrt{2\pi} u_{\text{th}}^3} + \frac{\rho_\chi}{\rho_m} \frac{m_\chi m_b}{m_\chi + m_b} V_{\chi b} D(V_{\chi b}). \quad (4.26)$$

The first term, in the $r \rightarrow 0$ limit, was derived in,^{159,161} but here we also find the second term, which is non-zero for $r \neq 0$.

By symmetry, $\dot{Q}_\chi$ is obtained by simply substituting $b \leftrightarrow \chi$ in Eq. (4.26). We see that these expressions, with the drag $D(V_{\chi b})$ in Eq. (4.23), conserve the total kinetic energy density in the baryon-DM fluid, i.e.

$$n_\chi \frac{dQ_\chi}{dt} + n_b \frac{dQ_b}{dt} - \frac{\rho_\chi \rho_b}{\rho_m} D(V_{\chi b}) V_{\chi b} = 0. \quad (4.27)$$

Now that we know how the interactions change the energy of the baryons and DM at any given time, let us find how their temperatures are modified.

4.2.3 Temperature evolution

Using the expressions for the drag $D(V_{\chi b})$, in Eq. (4.23), and the heating rates $\dot{Q}_b$ and $\dot{Q}_\chi$, in Eq. (4.26), we can write the equations of the temperature evolution,^{159,161} In our analysis we also evolve the relative velocity $V_{\chi b}$. The set of equations we will have to solve

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is then

$$\frac{dT_\chi}{da} = -2\frac{T_\chi}{a} + \frac{2\dot{Q}_\chi}{3aH}, \quad (4.28)$$

$$\frac{dT_b}{da} = -2\frac{T_b}{a} + \frac{\Gamma_C}{aH}(T_\gamma - T_b) + \frac{2\dot{Q}_b}{3aH}, \quad (4.29)$$

$$\frac{dV_{\chi b}}{da} = -\frac{V_{\chi b}}{a} - \frac{D(V_{\chi b})}{aH}, \quad (4.30)$$

where we have assumed the photon temperature T_γ is unaltered, H is the Hubble parameter and Γ_C is the Compton interaction rate, which depends on the free-electron density n_e . Since the free-electron abundance also depends on the baryon temperature through the recombination rate, we must solve for Eqs. (4.28)-(4.30) simultaneously with the free-electron fraction $x_e = n_e/n_H$

$$\frac{dx_e}{da} = -\frac{C}{aH} \left(n_H \mathcal{A}_B x_e^2 - 4(1 - x_e) \mathcal{B}_B e^{3E_0/(4T_\gamma)} \right), \quad (4.31)$$

where C is the Peebles factor,¹⁶³ E_0 is the ground energy of Hydrogen, and $\mathcal{A}_B(T_b, T_\gamma)$ and $\mathcal{B}_B(T_\gamma)$ are the effective recombination coefficient and the effective photoionization rate to and from the excited state respectively.^{7, 164}

For convenience, we parametrize the results in terms of a dimensionless cross section σ_{41} , defined as

$$\sigma_{41} \equiv \frac{\sigma_0}{10^{-41} \text{cm}^2}, \quad (4.32)$$

so that $\sigma_{41} \leq 3.2(m_\chi/\text{GeV})$ is the 95% C.L. constraint from CMB-analysis,¹⁵⁹ valid only for $m_\chi \gg m_b$.

Limiting cases

To gain understanding of the implications of Eq. (4.26) it is enlightening to study the extreme cases of very-heavy and very-light dark matter.

- For very massive dark matter ($m_\chi \gg m_b \approx 1 \text{ GeV}$), the first term in Eq. (4.26) is small and the second one dominates, which means that the new effect we have calculated is more relevant than the previously-known result. In this limit we then have $\dot{Q}_b = (\rho_\chi/\rho_m)m_b V_{\chi b} D(V_{\chi b})[1 + O(m_b/m_\chi)]$, which means $\dot{Q}_b \propto \sigma_0/m_\chi$. Equivalently, the

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DM heating term will be given by $\dot{Q}_\chi = (\rho_b/\rho_m)m_b V_{\chi b} D(V_{\chi b})[1 + O(m_b/m_\chi)]$, so that $\dot{Q}_\chi \propto \sigma_0/m_\chi$ as well, so for $m_\chi \gg m_b$ the constraints we will find will behave as $\sigma_0 \propto m_\chi$.

- In the opposite limit, in which $m_\chi \ll m_b$, we find that the temperature-independent heating term (second term in Eq. (4.26)) is linear in m_χ and hence subdominant. The first term is roughly constant. Although u_{th} depends on T_χ/m_χ , T_χ starts as zero and does not change unless there are interactions. This leads to a net mass-independent cooling $\dot{Q}_b < 0$, whereas the dark matter decouples, since $\dot{Q}_\chi \propto m_\chi \rightarrow 0$.

Let us now briefly discuss the two limiting cases where either thermal or relative velocities dominate,

- When $V_{\chi b} \ll u_{\text{th}} \equiv \sqrt{T_\chi/m_\chi + T_b/m_b}$ (thermal velocity dominates), we recover the results of Ref.,¹⁶¹ where baryons get cooled down and tend to thermalize with the dark matter fluid. This is shown in Fig. 4.1 as the “ $V_{\chi b,0} = 0$ ” case.

- In the limit where $V_{\chi b}$ is much bigger than u_{th} , the second term in Eq. (4.26) dominates, which causes a net heating of the baryon fluid. However, the overall rate of interactions (and hence net heating or cooling) is suppressed for large velocities, due to the fact that the cross section is proportional to v^{-4} .

Numerical results

We solve the system Eqs. (4.28)-(4.31) for different values of σ_{41} and m_χ , starting at $z = 1010$ with the baryons tightly coupled to the photon fluid ($T_b = T_\gamma$) and with perfectly cold dark matter ($T_\chi = 0$), although we tested that having slightly warm dark matter at recombination does not change our results significantly. We use cosmological parameters consistent with their current best-fit values.¹ We have also checked that, for the values of σ_{41} considered in our analysis, the system is not already tightly coupled at $z = 1010$, which would require us to start evolving the system at an earlier redshift.

As for the initial conditions for $V_{\chi b}$, we will solve the system for an array of values from zero initial velocity to three times the width of its Gaussian distribution. For purposes of illustration we will plot two different cases, one in which $V_{\chi b,0} = V_{\text{rms}} = 29$ km/s at initial redshift, and another in which $V_{\chi b,0} = 0$, to show how the relative velocity affects the

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results. In the case with $V_{\chi b,0} \neq 0$, higher values of m_χ imply a more significant heating of the baryons.

In Fig. 4.1 we show how the baryon temperature changes with the strength of the interactions. In the central and bottom panels we have $m_\chi \geq m_p$. In those two figures it is explicit that having $V_{\chi b,0} \neq 0$ (red lines) induces extra heat in the system as a result of the damping of the relative velocity, which increases the temperature of both baryons and dark matter. However, when considering the case with $V_{\chi b,0} = 0$ (blue lines), the interactions cool down the baryons and only heat up the dark matter. In the upper panel of Fig. 4.1 we have set $m_\chi = 0.1$ GeV. In this case it is clear that introducing interactions can only cool down the baryons, albeit with a more pronounced temperature drop in the $V_{\chi b,0} = 0$ case.

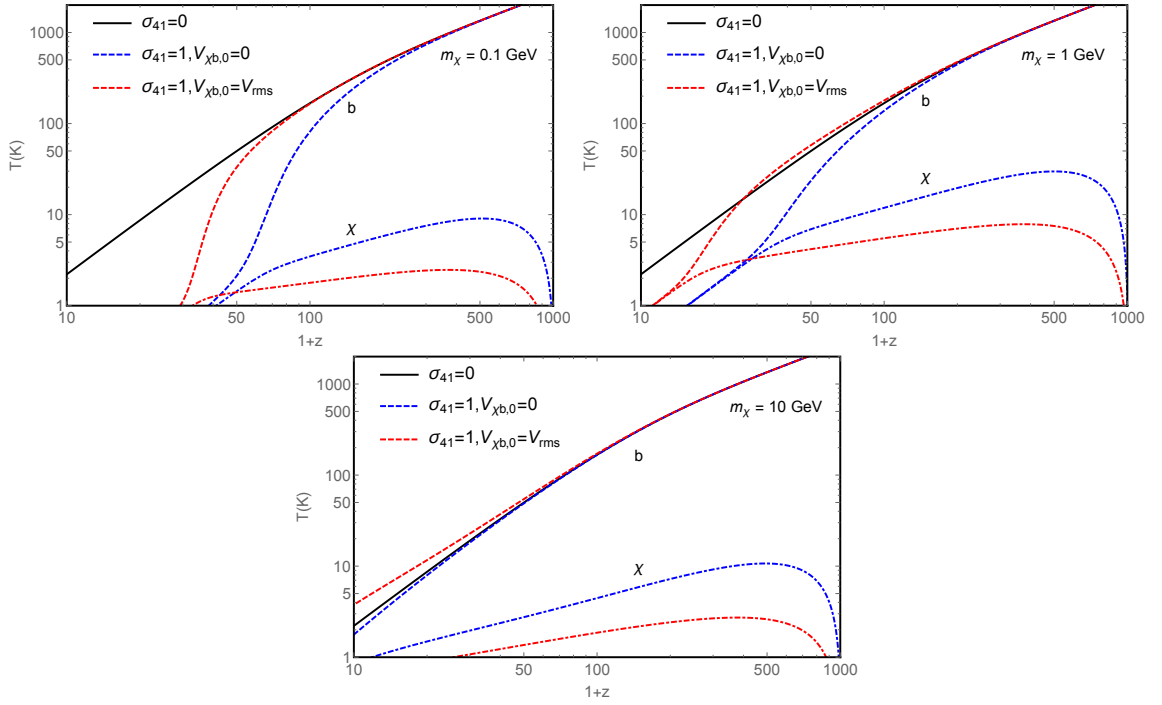


Figure 4.1: Baryon temperatures (three upper curves) without interactions (solid curve) and when adding interactions with $\sigma_{41} = 1$ (dashed-blue curve for the case where $V_{\chi b,0} = 0$ and red curve for $V_{\chi b,0} = V_{\text{rms}}$), as well as dark-matter temperatures (two lower curves, dash—dotted-blue curve for the case where $V_{\chi b,0} = 0$ and red curve for $V_{\chi b,0} = V_{\text{rms}}$). From top to bottom we show the results for $m_\chi = 0.1, 1$, and 10 GeV.

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4.2.4 Effects on the 21-cm dark ages signal

We have seen how the baryon and dark matter temperatures change when adding interactions. Now we will study how this modified baryon temperature gives rise to a different spin temperature for the gas during the dark ages, which in turn modifies the 21-cm brightness temperature we would observe.

21-cm brightness temperature

In the usual scenario the spin temperature follows the gas temperature as of decoupling and until $z \sim 30$, which makes it different from the CMB temperature in the redshift range $z \sim 30$ to 200. This creates a non-zero 21-cm line temperature T_{21} in this range. As we have shown, dark-matter–baryon interactions can either cool down or heat up the baryons, thus changing the spin temperature.

We show this effect in Fig. 4.2, where we plot for reference the CMB temperature, as well as the usual non-interacting gas and spin temperatures. We also plot the gas and spin temperature for interacting cases with either $V_{\chi b,0} = 0$ or $V_{\chi b,0} = V_{\text{rms}}$. The deviation of the spin temperature in the interacting cases is apparent, even for a cross section of $\sigma_{41} = 1$, compatible with CMB bounds.

If there is more heating than cooling of the baryons, the 21-cm brightness temperature decreases in magnitude, since the spin temperature is closer to the CMB temperature during the dark ages. Cooling of the baryons increases the brightness temperature, as long as the spin temperature stays coupled to the baryons. In Fig. 4.3 we plot the 21-cm brightness temperature, from Eq. (4.55), for different values of the relative velocity and DM mass. It is interesting to note that the heating increases with the mass of the dark matter, as predicted, so that the average brightness temperature $\bar{T}_{21}$ during the dark ages is higher when including interactions.

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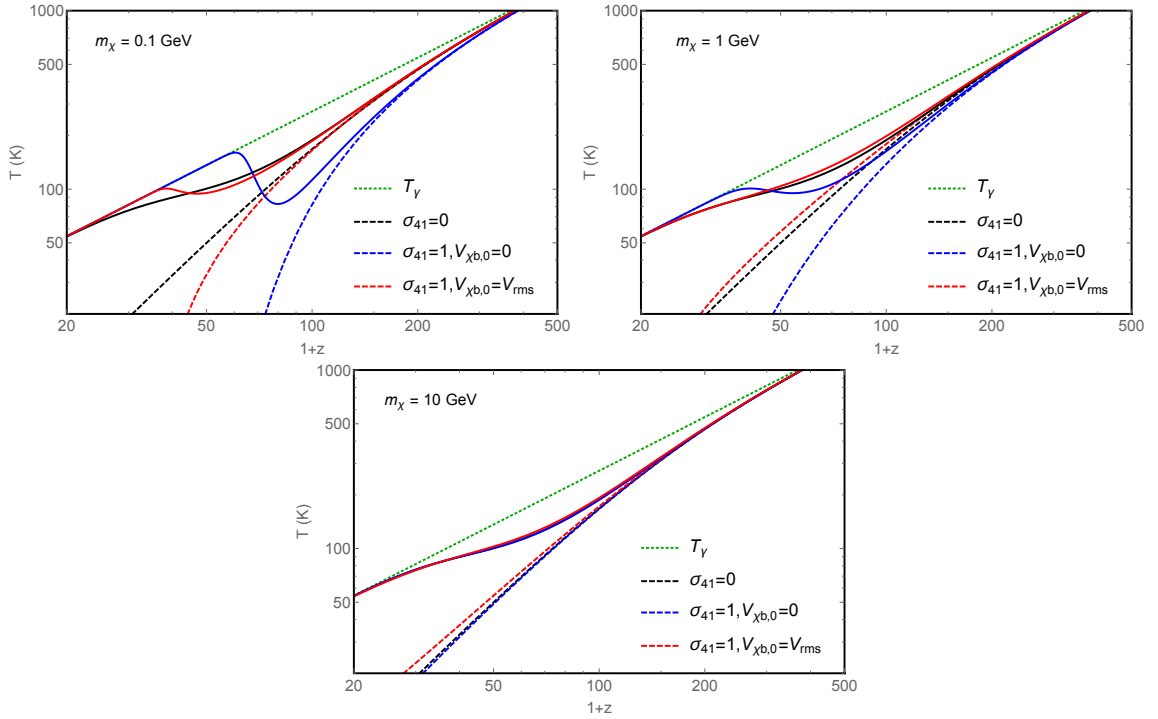


Figure 4.2: Values of the spin temperature (dashed curves) and the gas temperature (solid curves) for the collisionless case (black curve) and when including collisions (blue curve for $V_{\chi b,0} = 0$ and red curve for $V_{\chi b,0} = V_{\text{rms}}$), as well as the CMB temperature in the dashed-green curve. From top to bottom we show the results for $m_\chi = 0.1, 1$, and 10 GeV.

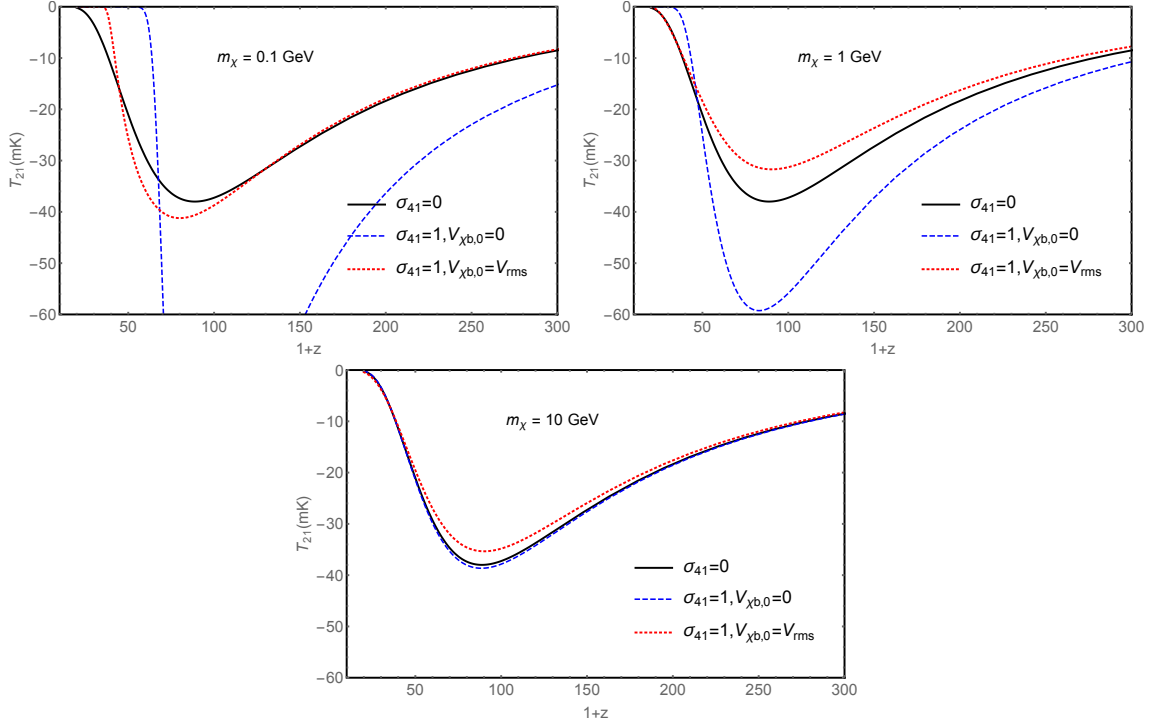


Figure 4.3: Values of the average brightness temperature of the 21-cm line for the collisionless case (solid-black curve), the case with interactions (blue-dashed curve for $V_{\chi b,0} = 0$, purple-dot-dashed curve for $V_{\chi b,0} = V_{rms}$), and the average over initial velocities in the red-dotted curve. From top to bottom we show the results for $m_{\chi} = 0.1, 1$, and 10 GeV .

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Global signal

Let us define $\bar{T}_{21}(V_{\chi b})$ as the brightness temperature in the absence of density perturbations. In the standard scenario this quantity is spatially homogeneous and is termed the global 21-cm signal. Once DM-baryon interactions are included, $\bar{T}_{21}(V_{\chi b})$ is still a function of the initial relative velocities. We calculate its average over said initial velocities as

$$\langle \bar{T}_{21} \rangle = \int d^3V_{\chi b,0} \bar{T}_{21}(V_{\chi b,0}) \mathcal{P}(V_{\chi b,0}), \quad (4.33)$$

with the probability distribution $\mathcal{P}(V_{\chi b,0})$ given by Eq. (4.11). We show this quantity in Fig. 4.4 for the interacting case and for three different DM masses.

21-cm fluctuations

As we have shown, the brightness temperature T_{21} of the 21-cm line is modified by the inclusion of interactions, and this modification depends on the initial relative velocity. The large-scale fluctuations of the relative velocity will therefore be imprinted on the brightness temperature, since two regions with different initial relative velocities will appear with different brightness temperatures (compare blue and red lines in Fig. 4.3), which will actually generate an additional contribution to the power spectrum of the 21-cm fluctuations. Let us calculate it.

The standard deviation of T_{21} as a function of $V_{\chi b,0}$ is

$$T_{21,\text{rms}} \equiv \sqrt{\langle T_{21}^2 \rangle - \langle T_{21} \rangle^2}. \quad (4.34)$$

Even if T_{21} had no explicit spatial dependence, it would fluctuate because relative velocities are not homogeneous. In principle, to compute the power spectrum of T_{21} , one should first compute its two-point correlation function. This is obtained by integrating over the six-dimensional joint probability distribution of the relative velocities at two different points (see Ref.¹⁴⁵). To simplify matters we shall make the following approximation

$$T_{21}(V_{\chi b,0}) \approx \langle T_{21} \rangle + T_{21,\text{rms}} \sqrt{\frac{2}{3}} \left(1 - \frac{V_{\chi b}^2}{V_{\text{rms}}^2} \right), \quad (4.35)$$

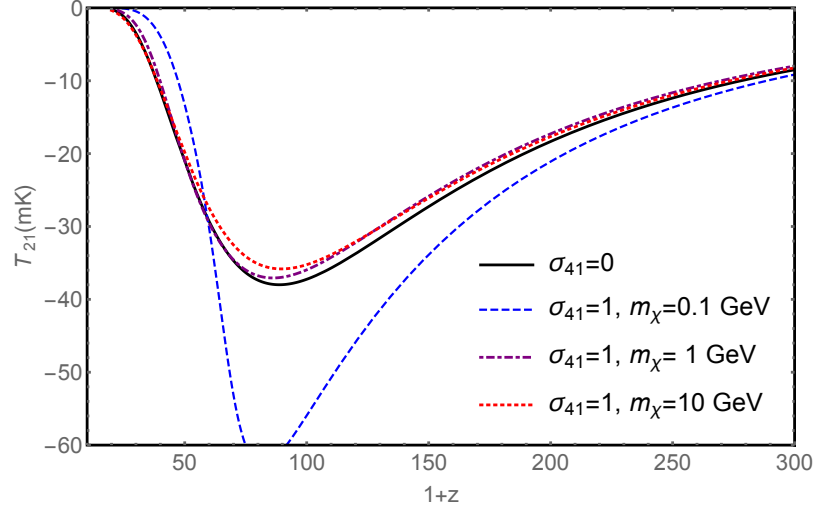


Figure 4.4: Values of the brightness temperature of the 21-cm line for the collisionless case in solid-black curve, and three with interactions ($\sigma_{41} = 1$), in dashed-blue curve $m_\chi = 0.1$ GeV, in dot-dashed—purple $m_\chi = 1$ GeV, and in dotted-red curve $m_\chi = 10$ GeV.

which has the advantage of resulting in simple analytic expressions¹⁶⁵ while still reproducing adequately the variance of T_{21} . For illustration we show T_{21} as a function of $V_{\chi b,0}$ for the $m_\chi = 1$ GeV case in Fig. 4.5. We calculate the power spectrum of $T_{21}(V_{\chi b,0})$ in this approximation to be

$$\langle T_{21}(\mathbf{k})T_{21}^*(\mathbf{k}') \rangle = T_{21,\text{rms}}^2 P_{V_{\chi b}^2}(k) (2\pi)^3 \delta_D(\mathbf{k} + \mathbf{k}'), \quad (4.36)$$

where $P_{V_{\chi b}^2}$ is the power spectrum of $\sqrt{2/3}(1 - V_{\chi b,0}^2/V_{\text{rms}}^2)$. We plot $P_{V_{\chi b}^2}(k)$ in Fig. 4.6.

Our observable, the brightness temperature of the 21-cm line, varies in space through its dependence on the baryon density n_b , as well as on the initial relative velocities $V_{\chi b,0}$. To linear order in density perturbations the temperature of the 21-cm line, Eq. (4.55), will be given by²⁵

$$T_{21} = \bar{T}_{21}(V_{\chi b,0}) + \frac{dT_{21}}{d\delta} \delta, \quad (4.37)$$

where $\delta \equiv (n_b - \bar{n}_b)/\bar{n}_b$, $dT_{21}/d\delta$ is a well-known function of redshift for $V_{\chi b,0} = 0$, and $\bar{T}_{21} = \bar{\tau}(\bar{T}_s - T_\gamma)/(1+z)$ is the unperturbed value of the brightness temperature. Both $\bar{T}_{21}$ and $dT_{21}/d\delta$ depend on the initial relative velocities. The average over initial velocities of

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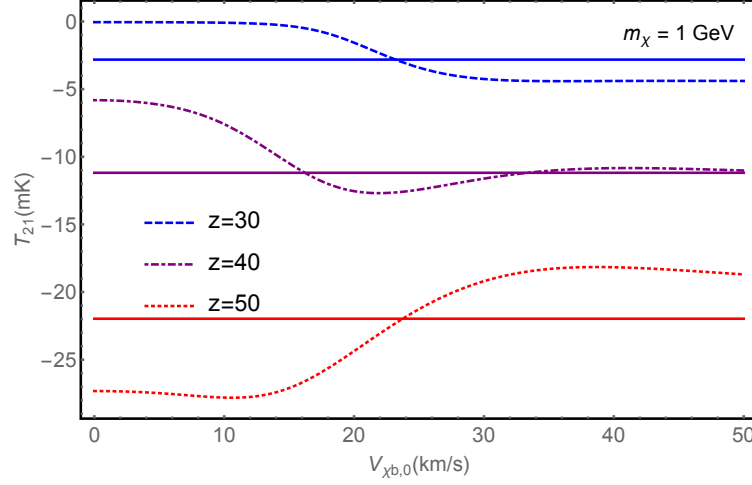


Figure 4.5: Brightness temperature $\bar{T}_{21}$ of the 21-cm line for $m_\chi = 1$ GeV and $\sigma_{41} = 1$ at redshifts $z = 30$ in dashed-blue curve, $z = 40$ in dash-dotted-purple curve and $z = 50$ in dotted-red. We also show the average over velocities for each redshift, as defined in Eq. (4.33), in solid curves and their corresponding colors.

T_{21} is then

$$\langle T_{21} \rangle = \langle \bar{T}_{21} \rangle + \left\langle \frac{dT_{21}}{d\delta} \right\rangle \delta. \quad (4.38)$$

We can, however, approximate $\langle dT_{21}/d\delta \rangle \approx dT_{21}/d\delta(V_{\chi b,0} = 0)$, since the error made in the 21-cm temperature would be of order $\delta T_{21}(k) \sim T_{21,\text{rms}}\delta$, which is subdominant. We can calculate the variance of T_{21} over both initial relative velocities and overdensities (as in the usual power spectrum) to find

$$P_{T_{21}}(k) = \bar{T}_{21,\text{rms}}^2 P_{V_{\chi b}}(k) + \left(\alpha(z) + \bar{T}_{21} \frac{k_{\parallel}^2}{k^2} \right)^2 P_b(k, z), \quad (4.39)$$

where $k_{\parallel}$ is the magnitude of k in the line-of-sight direction, $\alpha(z)$ as defined in Ref.²⁵ and P_b is the usual baryon power spectrum.

We can convert easily from k -space to ℓ -space by using a harmonic transform,¹⁶⁶ which is exact in the case of the flat-sky limit and still a very good approximation for $\ell \geq 10$, which should be good enough for our order-of-magnitude estimates. We define the angular

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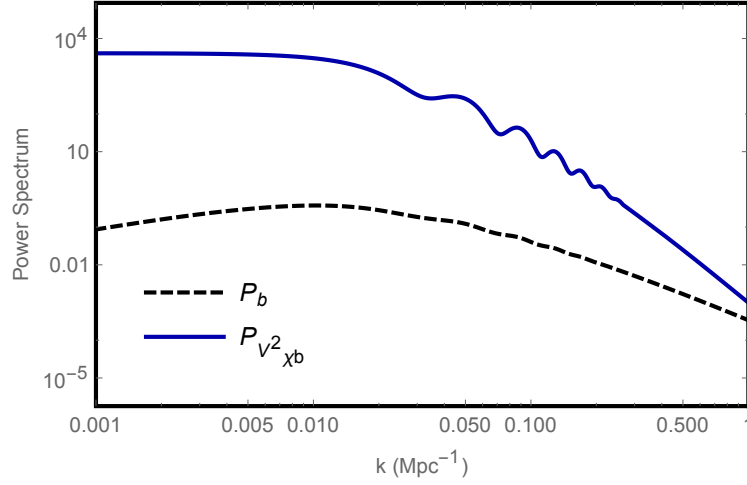


Figure 4.6: Power spectra as a function of k . In solid-blue curve we show the power spectrum of $V_{\chi b}^2$, as defined in Eq. (4.36), and in dashed-black curve the usual power spectrum of baryon overdensities at redshift $z = 30$.

power spectrum for the usual fluctuations as

$$C_\ell^{\text{usual}} = \frac{1}{r^2} \int \frac{dk_{||}}{2\pi} \left| \tilde{W}(k_{||}) \right|^2 \left(\alpha + \bar{T}_{21} \frac{k_{||}^2}{k^2} \right)^2 P_b(k), \quad (4.40)$$

where $k \equiv \sqrt{\ell^2/r^2 + k_{||}^2}$, and $\tilde{W}(k_{||})$ is the window function. The new angular power spectrum ($V_{\chi b}$), due to interactions, will be

$$C_\ell^{V_{\chi b}} = \frac{\bar{T}_{21,\text{rms}}^2}{r^2} \int \frac{dk_{||}}{2\pi} \left| \tilde{W}(k_{||}) \right|^2 P_{V_{\chi b}^2}(k). \quad (4.41)$$

Before going into a full-scale analysis one might be interested in what would happen at a single ℓ , and at different redshifts. We show in Fig. 4.7 the value of the square root of the velocity power spectrum $(C_\ell^{V_{\chi b}})^{1/2}$ for different values of the cross section. For illustration purposes we also show the usual power spectrum $(C_\ell^{\text{usual}})^{1/2}$, from Eq. (4.40), where for simplicity we have taken $\alpha(z)/\alpha(z_0)(C_\ell^{\text{usual}})^{1/2}(z_0)$ as a proxy for $(C_\ell^{\text{usual}})^{1/2}(z)$ as well as a redshift-independent bandwidth of $\Delta\nu/\nu = 0.02$ to avoid recalculating the integral in Eq. (4.40) for each redshift in this plot. We show the cases of $\ell = 30$ and $\ell = 1000$ in the upper and lower panels, respectively.

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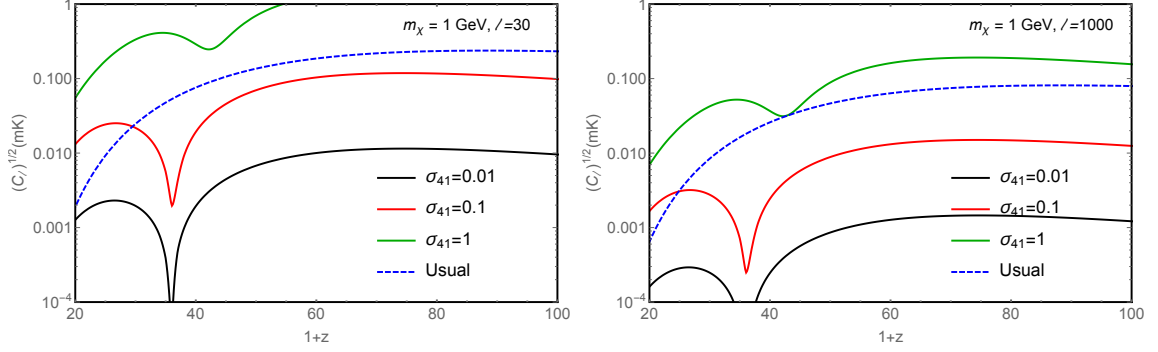


Figure 4.7: Amplitude of the brightness-temperature fluctuations induced by relative velocity fluctuations for three different cross sections (solid-black curve for $\sigma_{41} = 0.01$, red curve for $\sigma_{41} = 0.1$, and green curve for $\sigma_{41} = 1$), as well as for the usual baryon density perturbations. We calculate at two different scales: in the first panel we show the results for $\ell = 30$ and in the second one for $\ell = 1000$.

4.2.5 Detectability

So far we have shown that DM-baryons interactions modify the baryon temperature, raising it or lowering it, depending on the initial relative velocity. Varying the baryon temperature will change the spin temperature and hence the brightness temperature of the 21-cm line. This quantity, also known as the “global signal”, is the main observable during the dark ages. We will study how to detect interactions with a global-signal experiment.

Moreover, since the temperatures depend on initial velocities, and these have a spatial-dependence, we have argued that there will be a new contribution to the power spectrum, which, at large scales, can overcome the usual one for values of the cross section of $\sigma_{41} \gtrsim 0.1$. We will study the detectability of this signal with interferometry later in this section.

Global signal

Let us start by analyzing the most direct effect of DM-baryon interactions, the change in the global signal during the dark ages. Next-generation experiments, such as NenuFAR, will survey the 21-cm line brightness temperature down to frequencies possibly as low as $\nu \sim 10$ MHz, which corresponds to a redshift $z > 100$.

We have seen in Fig. 4.3 how the brightness temperature changes when adding inter-

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actions. We will use the amplitude of the brightness temperature at its peak as a proxy for the detectability of the signal, even though its very high redshift ($z \sim 90$) may make it unobservable.

Let us first find the signal-to-noise ratio to detect interactions having a cross section $\sigma_{41} = 1$. If we could determine the brightness temperature $\bar{T}_{21}$ at its peak with 5% precision, we would be able to detect interactions with $\sigma_{41} = 1$ at a signal-to-noise ratio $S/N \sim 10$ for $m_\chi = 0.1$ GeV, $S/N \sim 0.5$ for $m_\chi = 1$ GeV, and $S/N \gtrsim 1$ for $m_\chi = 10$ GeV.

More interestingly, if we were able to improve the error by a factor of 5, reaching 1% precision of peak-temperature determination, we would be able to detect cross sections as small as $\sigma_{41} \lesssim 0.04$ for $m_\chi = 0.1$ GeV, $\sigma_{41} \lesssim 0.1$ for $m_\chi = 1$ GeV, and $\sigma_{41} \lesssim 0.2$ for $m_\chi = 10$ GeV, all of which are beyond what can be achieved by current CMB analysis.¹⁵⁹

Fluctuations

We now turn our focus to the measurement of the 21-cm power spectrum, Eq. (4.39). In a maximum-likelihood analysis, the Fisher forecast for the error in the measurement of the amplitude A of a power spectrum C_ℓ is given by¹⁴⁷³,

$$\frac{1}{\sigma_A^2} = \sum_\ell \left(\frac{\partial C_\ell}{\partial A} \right)^2 \frac{1}{\sigma_\ell^2}. \quad (4.42)$$

For a given sky coverage f_{sky} , the error for an individual ℓ in the estimated value $\hat{A}$ is^{86,87,147}

$$\sigma_{\hat{A}} = \sqrt{\frac{2}{f_{\text{sky}}(2\ell + 1)}} \left(C_\ell^{\text{usual}} + C_\ell^N \right), \quad (4.43)$$

where C_ℓ^N is the instrumental noise power spectrum, defined in Eq. (4.9), and C_ℓ^{usual} is the usual power spectrum of 21-cm fluctuations (under the null hypothesis of no DM-baryon interactions), Eq. (4.39).

³We assume that the likelihood function is Gaussian in the vicinity of its maximum.¹⁶⁷

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The minimum detectable amplitude $\hat{A}$ at $1\text{-}\sigma$ significance is thus

$$\sigma^{\hat{A}} = \left[\frac{f_{\text{sky}}}{2} \sum_{\ell_{\text{min}}}^{\ell_{\text{max}}} \frac{(2\ell + 1)(\tilde{C}_{\ell}^{V_{\text{x}b}})^2}{(C_{\ell}^{\text{usual}} + C_{\ell}^N)^2} \right]^{-\frac{1}{2}}, \quad (4.44)$$

where $\tilde{C}_{\ell}^{V_{\text{x}b}} = C_{\ell}^{V_{\text{x}b}}/A$ encodes the ℓ dependence of the velocity power spectrum from DM-baryon interactions, and $\ell_{\text{min}} = 180/\theta$ is the largest scale accessible by an experiment with sky coverage $f_{\text{sky}} = \theta^2$. Because of the use of the harmonic transform in Eq. (4.40), we take $\ell_{\text{min}} = 15$. This should not affect the results significantly since there are very few modes at lower ℓ .

We will consider two different scenarios, first a realistic experiment modeled after SKA that could be taking data within the next few years and second a more idealized experiment whose noise level will be low enough to detect the primordial power spectrum at redshift $z = 30$ (but still not cosmic-variance limited, since the usual primordial power spectrum vanishes for smaller redshifts but the noise will not).

We will study the redshift range $z = 20$ to 30 , at the very end of the dark ages. This range is chosen to avoid complex astrophysical processes at low redshift as well as to still be observable from Earth. We may not be fully free of contamination, however, since the epoch of the formation of the first stars is unknown, and the X-rays generated during star formation may start to heat up the gas at $z \lesssim 25$.¹⁶⁸ Moreover, accreting intermediate-mass black holes (sometimes termed miniquasars) may also be an important source of X-rays during this era.^{141, 142, 144} Once data of the gas temperature during the dark ages are acquired, a careful analysis should take these processes into account along with the heating produced by DM-baryon interactions, and by studying their different redshift behaviors and angular structures, disentangle them. We motivate future work to address this issue.

We find the angular noise power spectrum of an interferometer from Eq. (4.9). Inspired by design plans for the Square Kilometer Array, we first consider a future ground-based interferometer with access to the final stages of the dark ages, $z \sim 20 - 30$, with a baseline of $D = 6$ km [corresponding to a maximum angular scale $\ell_{\text{cover}}(\nu) \sim 5800$ at redshift $z = 30$], with $f_{\text{cover}} = 0.02$, surveying a sky fraction $f_{\text{sky}} = 0.75$ for a total of five whole years. As for the bandwidth, we surveyed a range between $\Delta\nu = 0.1$ MHz and 10 MHz

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and found that $\Delta\nu \sim 1$ MHz is the optimum value (for smaller bandwidths the noise C_ℓ s dominate over the signal and for larger ones the number of redshift slices is too small).

For more optimistic constraints, we set $D = 50$ km, $f_{\text{cover}} = 0.1$, and assume ten whole years of observations. In order to get a result closer to the cosmic-variance limit we could perform the analysis from $z = 20$, going up to the beginning of the dark ages, $z = 200$. However, we find that it does not improve the results significantly, due to the rise of synchrotron radiation at low frequencies, which grows much more rapidly than the signal. We consider then the same redshift range as before, z from 20 to 30.

One of the great advantages of 21-cm as a probe is the ability to analyze the tomography of the signal, enabling us to coadd information from different redshift slices. Summing over redshift slices, the signal-to-noise ratio is given by

$$(S/N) = \left[\sum_z \frac{f_{\text{sky}}}{2} \sum_{\ell_{\text{min}}}^{\ell_{\text{max}}} \frac{(2\ell + 1) \left(C_\ell^{V_{\chi^b}}(z) \right)^2}{\left(C_\ell^{\text{usual}}(z) + C_\ell^N(z) \right)^2} \right]^{\frac{1}{2}}, \quad (4.45)$$

In Fig. 4.8 we show the C_ℓ s for the usual primordial perturbations (C_ℓ^{usual}), for the instrumental noise (C_ℓ^N , both with next-generation and futuristic parameters), and for the new contribution due to interactions ($C_\ell^{V_{\chi^b}}$), all of them at redshift $z = 30$.

Results

Let us start by considering the realistic noise case (that corresponds to the experimental parameters of SKA) and find what the signal-to-noise ratio would be for detecting $\sigma_{41} = 1$. We calculate the signal-to-noise ratio for $\sigma_{41} = 1$ in each redshift bin between $z = 20$ and $z = 30$ with Eq. (4.45). We find the total signal-to-noise ratio to be $S/N \sim 3$ for the case of $m_\chi = 0.1$ GeV, $S/N \sim 9$ for $m_\chi = 1$ GeV, and $S/N \sim 0.2$ for $m_\chi = 10$ GeV. We could alternatively express the results in terms of the smallest σ_{41} that would still give us a signal-to-noise ratio of 1, taken to be approximately $\sigma_{41, \text{min}} = 1/\sqrt{S/N}$. We show the minimum detectable cross sections in Tab. 4.1.

Let us now move on to trying to find the smallest possible σ_{41} detectable at $S/N = 1$

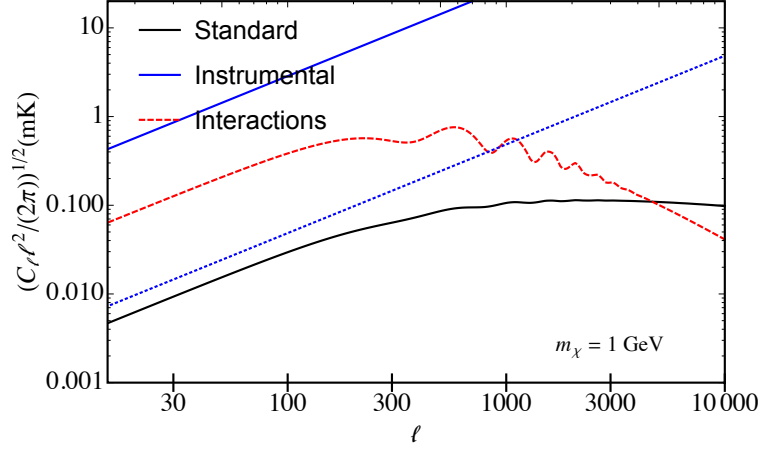


Figure 4.8: Angular power spectra at redshift $z = 30$ with bandwidth $\Delta\nu = 1$ MHz. In solid-black curve we show the usual primordial perturbations, in solid- and dotted-blue curves the instrumental noises for the realistic and optimistic cases [see Eq. (4.9) and discussion below] and in dashed-red curve the new piece due to interactions for $\sigma_{41} = 1$ and $m_\chi = 1$ GeV.

in the more optimistic case. In principle the amplitude A of $C_\ell^{V_\chi^b}$, equal to $\bar{T}_{21,\text{rms}}^2$, is a non-trivial function of redshift and σ_{41} . However, we find that for small values of σ_{41} ($\sigma_{41} \lesssim 0.1$), the quantity $f(z) \equiv \bar{T}_{21,\text{rms}}/\sigma_{41}$ is approximately independent of σ_{41} (although it does depend on m_χ). Then we can construct an estimator for σ_{41} for each redshift slice,

$$\left(\widehat{\sigma_{41}^2}\right)_z = \frac{(\hat{A})_z}{f^2(z)}, \quad (4.46)$$

with variance given by $\sigma_{(\sigma_{41}^2)_z}^2 = \sigma_A^2(z)/f^4(z)$. We can then combine all the estimators into a minimum-variance one, finding the variance of the final redshift-independent estimator,

$$\frac{1}{\sigma_{\sigma_{41}^2}^2} = \sum_z \frac{f^4(z)}{\sigma_A^2(z)}. \quad (4.47)$$

With the optimistic experimental parameters defined above we find that the minimum σ_{41} observable at 68% C.L. (1σ) is $\sigma_{41} \lesssim 1.7 \times 10^{-3}$ for $m_\chi = 0.1$ GeV, $\sigma_{41} \lesssim 4.3 \times 10^{-3}$ for $m_\chi = 1$ GeV, and $\sigma_{41} \lesssim 3.6 \times 10^{-2}$ for $m_\chi = 10$ GeV. These results are about 2 orders of magnitude better than the CMB constraints found in,¹⁵⁹ where $\sigma_{41} \lesssim 16(m_\chi/10 \text{ GeV})$.

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m_χ [GeV]	1/10	1	10
Fluctuations (realistic)	6×10^{-42}	3×10^{-42}	2×10^{-41}
Fluctuations (optimistic)	2×10^{-44}	4×10^{-44}	4×10^{-43}
Global signal (1% error)	4×10^{-43}	1×10^{-42}	2×10^{-42}

Table 4.1: Minimum σ_0 (in cm^2 , corresponding to $\sigma_{41} \times 10^{41}$) detectable with both realistic and optimistic interferometer parameters at 68% C.L., as well as with global-signal analysis with 1% accuracy for three different dark-matter masses m_χ (in GeV).

4.2.6 Discussion and Conclusions

Let us close this section with some comments.

- As we have shown, interactions between dark matter and baryons give rise to a new heating term, which can increase the temperature of the baryons significantly. We only used said heating to study dark-ages physics but this result may have applications beyond our analysis, for example in the epoch of reionization.¹⁶⁹

- In this work we have focused only on the case where $\sigma \sim v^n$ with $n = -4$, but one may wonder whether the dark ages can potentially provide new information not contained in the CMB analysis for other values of n . Since the dark ages occur more recently than decoupling, we have only been interested in interactions that increase at later times. Ref.¹⁵⁹ showed that the interaction rate grows for $n \leq -3$, so all results that we could forecast for $n > -3$ would be worse than those obtained with CMB studies. That still leaves $n = -3$ as a potential interaction to study, for example.

- It is also worth mentioning that if we wanted to translate these results to a constraint specific to a dark-matter milicharge model,¹⁵³ the ionization fraction of the baryons would cause a suppression of $x_e \sim 10^{-4}$.

- We have also found a decrease in the bulk relative velocity of baryons and dark matter characterized by a drag, Eq. (4.23). In Fig. 4.9 we show the unperturbed relative velocity $V_{\chi b}$, found by solving Eqs. (4.28)-(4.31) with initial relative velocity $V_{\chi b,0} = V_{\text{rms}}$, and baryon speed of sound $c_s \equiv \sqrt{3T_b/m_b}$. We also plot the same two velocities for an interacting case. All velocities are divided by a factor of $1/(z+1)$ to eliminate a fiducial redshift dependence.

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In the standard case the speed of sound is always below the bulk one, which creates supersonic flow of the baryons.^{29,145} Including collisions can both raise the thermal velocity as well as decrease the relative one, so it reduces the Mach number $\mathcal{M}$ to be lower than 1 at lower redshifts, which could affect the formation of small-scale structure.¹⁷⁰

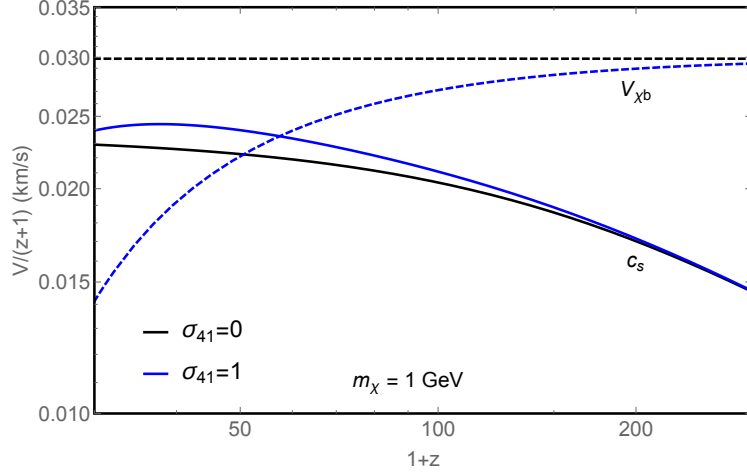


Figure 4.9: Values of the relative velocity (solid curves) and the thermal speed of sound (dashed curves) divided by $(1+z)$. We show the collisionless case (black curves) and the case with $\sigma_{41} = 1$ (blue curves), for $m_\chi = 1$ GeV.

- Finally, throughout the text we have quoted results for $m_\chi = 0.1, 1$, and 10 GeV. For lower masses, the result is independent of mass, and for higher masses it depends on σ_0/m_χ . We show a larger range of dark-matter masses in Fig. 4.10, where we plot the minimum σ_0 one could detect at a signal-to-noise ratio of 1, as a function of the dark-matter mass m_χ . We show how the result asymptotes for very high and very low m_χ , and we also compare with the CMB+Ly α analysis in Ref.,¹⁵⁹ shown in dotted-green curve, which is only valid for large m_χ .

To conclude, interactions between dark matter and baryons create a new contribution to the 21-cm power spectrum. We found that this will allow to detect interactions more than 2 orders of magnitude better than can be achieved by CMB+Ly α analysis, and with a broader mass range.

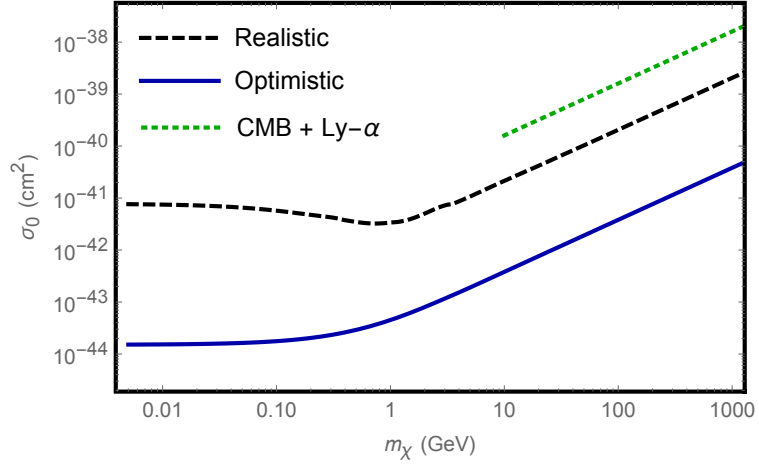


Figure 4.10: Minimum σ_0 (in cm^2) detectable as a function of m_χ in GeV. In black curve we show the results for the case with realistic parameters and in blue curve the one with optimistic parameters. In dotted-green curve we display the current CMB constraint (only valid for $m_\chi \gg \text{GeV}$).

4.3 Non Gaussianities

While single-field inflation has the merit of simplicity, a plethora of alternative models remain consistent with current data.^{171,172} The main characteristic that differentiates them from the simplest inflationary scenario is that they can generate significant primordial non-gaussianities (PNGs). The simplest form of PNG is a non-vanishing three-point function for the primordial curvature perturbation ζ , parametrized by a dimensionless amplitude $f_{\text{NL}} \sim \langle \zeta^3 \rangle / \langle \zeta^2 \rangle^2$. Single-field inflation leads to a small three-point function, corresponding to $f_{\text{NL}} \sim 10^{-2}$.^{127,128} Alternative models typically generate $f_{\text{NL}} \sim 1$, as a result of interactions with other fields, higher-derivative terms in the Lagrangian,¹⁷³ or other mechanisms.¹⁷¹ Measuring $f_{\text{NL}} \lesssim 1$ is therefore a natural target for future experiments to start significantly constraining the physics of inflation.¹⁷⁴

The best constraints on f_{NL} to date are obtained from CMB studies,¹⁷⁵ and are consistent with zero, though with a large uncertainty, $\sigma_{f_{\text{NL}}} \sim 5 - 40$ depending on the shape considered. CMB measurements are now cosmic-variance limited in temperature down to the photon diffusion scale corresponding to multipole $\ell \sim 2000$. The anticipated improvement in polarization measurements is expected to only marginally tighten the constraints on f_{NL} . Reaching the $f_{\text{NL}} \sim 1$ frontier will therefore most likely require other data sets.

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A prime dataset to be considered are 21-cm fluctuations. These, however, suffer from a large source of contamination: the intrinsic non-gaussian nature of 21-cm fluctuations, even for perfectly gaussian initial conditions. Two previous studies have partially addressed this issue. Ref.¹⁷⁶ computed the bispectrum of 21-cm fluctuations resulting from non-linear gravitational growth, but treated it approximately as a confusion noise rather than a bias. Ref.¹⁷⁷ computed all contributions of the secondary bispectrum, but did not account for it in their final forecasts. In addition they only computed the bispectrum for specific triangle configurations. These two groups moreover get significantly different final results.

In this work we compute the primary and secondary bispectra using the flat-sky formalism. This accurately reproduces the full-sky calculation with a much lower computational cost, and greatly simplifies the analysis. We show that the shapes of the primary and secondary bispectra overlap significantly. Unsubtracted, the secondary bispectrum would lead to a bias $\Delta f_{\text{NL}} \sim 10^3$. Even percent-level residuals after subtraction would lead to a non-zero non gaussianity of order $\Delta f_{\text{NL}} \sim 10$. This warrants a Fisher analysis, fitting simultaneously for the amplitude of PNG and for nuisance parameters characterizing the residual secondary bispectrum after a best-estimate is subtracted. For a single redshift slice, we find that the uncertainty in f_{NL} after marginalizing over the nuisance parameters is increased by a factor of $\sim 3 - 6$ in comparison to an ideal case without secondaries. Finally, we optimally combine redshift slices accounting for the smoothness of the secondary bispectrum as a function of redshift. Our forecasts for a cosmic-variance-limited experiment targeting $30 \leq z \leq 100$ with a bandwidth of 0.1 MHz and angular resolution of 0.1 arcminute are: $\sigma_{f_{\text{NL}}^{\text{local}}} \sim 0.03$, $\sigma_{f_{\text{NL}}^{\text{equil}}} \sim 0.04$, and $\sigma_{f_{\text{NL}}^{\text{ortho}}} \sim 0.03$. For the same angular resolution but a bandwidth of 1 MHz our forecast is $\sigma_{f_{\text{NL}}^{\text{local}}} \sim 0.12$, $\sigma_{f_{\text{NL}}^{\text{equil}}} \sim 0.39$, and $\sigma_{f_{\text{NL}}^{\text{ortho}}} \sim 0.29$.

4.3.1 Secondaries

We can expand the 21-cm temperature T_{21} , from Eq. (4.5) to second order in perturbations as¹⁴⁵

$$T_{21} = \bar{T}_{21}(1 + \delta_v + \delta_v^2) + (\mathcal{T}_b \delta_b + \mathcal{T}_T \delta_{T_{\text{gas}}})(1 + \delta_v) + \mathcal{T}_{bb} \delta_b^2 + \mathcal{T}_{bT} \delta_b \delta_{T_{\text{gas}}} + \mathcal{T}_{TT} \delta_{T_{\text{gas}}}^2, \quad (4.48)$$

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where $\delta_b \equiv \delta n_b / \bar{n}_b$ is the fractional fluctuation of the baryon density and δT_{gas} is the fractional fluctuation of the gas temperature, which affect T_{21} through the collision rates. This equation neglects fluctuations of the ionization fraction $x_e \sim 10^{-4}$ at the redshifts of interest, as they lead to negligible fluctuations of T_{21} which is proportional to $(1 - x_e)$. We compute the coefficients in the above equation as described in Ref.¹⁴⁵ They ought to be used for detailed prediction when actual data is available. For this study, however, we shall make simplifying assumptions regarding the gas temperature fluctuations in order to keep calculations tractable. We now describe our approximations.

The evolution of the gas temperature can be obtained from the first law of thermodynamics. Neglecting fluctuations of the CMB temperature and the effect of gravitational potentials, the full non-linear equation is¹⁴⁵

$$\dot{\delta}_{T_{\text{gas}}} - \frac{2}{3} \dot{\delta}_b \frac{1 + \delta_{T_{\text{gas}}}}{1 + \delta_b} = \Gamma_C \left[\frac{\bar{T}_{\text{cmb}} - \bar{T}_{\text{gas}}}{\bar{T}_{\text{gas}}} \delta_{x_e} - \left(\frac{\bar{T}_{\text{cmb}}}{\bar{T}_{\text{gas}}} + \delta_{x_e} \right) \delta_{T_{\text{gas}}} \right], \quad (4.49)$$

where $\Gamma_C \times (T_{\text{cmb}} - T_{\text{gas}})$ is the rate at which Thomson scattering of CMB photons by free electrons heats up the gas. Since $\Gamma_C \propto T_{\text{cmb}}^4 x_e$, the fluctuations of the gas temperature are coupled to those of the free-electron fraction δ_{x_e} . In principle this equation should be solved simultaneously with the evolution of δ_{x_e} , obtained by perturbing the recombination rate.¹⁴⁵ We find that neglecting δ_{x_e} leads to errors of order $\sim 10\%$ for the linear evolution and we shall set $\delta_{x_e} \rightarrow 0$ for simplicity. With this simplification, the equation for $\delta_{T_{\text{gas}}}$ to second order is

$$\dot{\delta}_{T_{\text{gas}}} - \frac{2}{3} \dot{\delta}_b (1 - \delta_b + \delta_{T_{\text{gas}}}) + \frac{\bar{T}_{\text{cmb}}}{\bar{T}_{\text{gas}}} \Gamma_C \delta_{T_{\text{gas}}} = 0. \quad (4.50)$$

We shall consider scales larger than the baryonic Jeans scale: $k \ll k_J \sim 300 \text{ Mpc}^{-1}$. On these scales baryons behave just like CDM, so their evolution equation does not depend on T_{gas} . Given δ_b , we can therefore solve for the gas-temperature fluctuations. We decompose the baryon-density fluctuation into a piece linear in the initial conditions $\delta_b^{(1)}$ and a quadratic piece $\delta_b^{(2)}$ resulting from non-linear gravitational collapse. We can then solve for the linear

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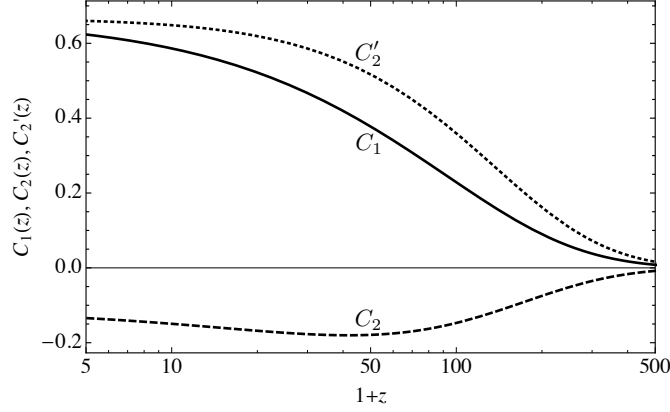


Figure 4.11: Coefficients of the approximate decomposition of the gas-temperature fluctuations as a quadratic function of baryon density fluctuations: $\delta_{T_{\text{gas}}}(\mathbf{x}, z) \approx C_1(z)\delta_b^{(1)}(\mathbf{x}, z) + C_2(z)[\delta_b^{(1)}(\mathbf{x}, z)]^2 + C_2'(z)\delta_b^{(2)}(\mathbf{x}, z)$. At high redshift, Compton heating is efficient and maintains $T_{\text{gas}} = T_{\text{cmb}}$, with negligible fluctuations, so $C_1 \approx C_2 \approx C_2' \approx 0$. At low redshift, the gas decouples thermally from the CMB and starts cooling down adiabatically, asymptoting towards $T_{\text{gas}} \propto n_b^{2/3}$, which implies $C_1 \approx C_2' \rightarrow 2/3$ and $C_2 \rightarrow -1/9$.

and quadratic parts of $\delta_{T_{\text{gas}}}$:

$$\dot{\delta}_{T_{\text{gas}}}^{(1)} + \frac{\bar{T}_{\text{cmb}}}{\bar{T}_{\text{gas}}} \Gamma_{\text{C}} \delta_{T_{\text{gas}}}^{(1)} = \frac{2}{3} \dot{\delta}_b^{(1)}, \quad (4.51)$$

$$\dot{\delta}_{T_{\text{gas}}}^{(2)} + \frac{\bar{T}_{\text{cmb}}}{\bar{T}_{\text{gas}}} \Gamma_{\text{C}} \delta_{T_{\text{gas}}}^{(2)} = \frac{2}{3} \dot{\delta}_b^{(2)} + \frac{2}{3} \dot{\delta}_b^{(1)} (\delta_{T_{\text{gas}}}^{(1)} - \delta_b^{(1)}). \quad (4.52)$$

Our final approximation is to assume that $\delta_b^{(1)}$ is uniformly proportional to the scale factor a , i.e. $\delta_b^{(1)}(\mathbf{x}, a') = (a'/a)\delta_b^{(1)}(\mathbf{x}, a)$, independently of the position $\mathbf{x}$, and similarly that $\delta_b^{(2)} \propto a^2$. We then solve Eqs. (4.51) and (4.52) starting at $z = 1000$ with vanishing initial conditions. The mean free-electron fraction x_e required for Γ_{C} and mean gas temperature $\bar{T}_{\text{gas}}$ are obtained from HYREC.^{7,164} This allows us to obtain three coefficients $C_1(z)$, $C_2(z)$ and $C_2'(z)$ such that

$$\delta_{T_{\text{gas}}}^{(1)}(\mathbf{x}, z) = C_1(z)\delta_b^{(1)}(\mathbf{x}, z), \quad (4.53)$$

$$\delta_{T_{\text{gas}}}^{(2)}(\mathbf{x}, z) = C_2(z)[\delta_b^{(1)}(\mathbf{x}, z)]^2 + C_2'(z)\delta_b^{(2)}(\mathbf{x}, z), \quad (4.54)$$

which we show in Fig. 4.11.

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The assumption that $\delta_b^{(1)} \propto a$ and $\delta_b^{(2)} \propto a^2$ is not quite correct. Indeed this assumes that baryons behave exactly like CDM. In reality, they start with different “initial” conditions at $z \approx 1000$, after they decouple from the photon fluid shortly after cosmological recombination: their overdensity is typically significantly smaller than that of the CDM on sub-horizon scales, and their velocity field, though comparable to that of the CDM in magnitude, has a very different scale dependence (hence leading to the relative-velocity effect²⁹). Baryons therefore take some time to “catch up” to the CDM, and their growth rate at early times differs from $\delta_b \propto a$, and is scale-dependent. Given that Thomson scattering maintains $T_{\text{gas}} = T_{\text{cmb}}$ at $z \gtrsim 200$, regardless of the exact value of δ_b , this should not be a major issue, but should be properly accounted for in a detailed analysis.

With these caveats in mind, we substitute our approximation $\delta_{T_{\text{gas}}} = C_1 \delta_b^{(1)} + C_2 [\delta_b^{(1)}]^2 + C'_2 \delta_b^{(2)}$ into equation (4.48) and obtain the following simpler expression for the 21-cm brightness temperature fluctuations to second order, with which we shall work for the rest of this work:

$$\begin{aligned} \delta T_{21} \approx \bar{T}_{21} (\delta_v^{(1)} + \delta_v^{(2)} + [\delta_v^{(1)}]^2) \\ + \alpha(z) \delta_b^{(1)} (1 + \delta_v^{(1)}) + \beta(z) [\delta_b^{(1)}]^2 + \gamma(z) \delta_b^{(2)}. \end{aligned} \quad (4.55)$$

The effective coefficients α , β , and γ are straightforwardly obtained from the coefficients of Eq. (4.48) and C_1, C_2, C'_2 and are shown in Figure 4.12.

Neglected sources of fluctuations

The above analysis is only valid on subhorizon scales, and does not account for several relativistic effects. First, the gas is not at rest with respect to comoving observers. We have already accounted for the resulting perturbation to the local Hubble expansion rate due to the velocity gradient. In addition, a local velocity leads to (i) a difference between the proper time in the baryon rest frame and the comoving frame, (ii) a dipolar anisotropy of the CMB intensity in the baryon rest frame, and (iii) an additional redshifting of the observed frequency. Gravitational potentials also affect the observed brightness temperature through: (i) a time dilation, (ii) a perturbation to the local expansion rate, (iii) the Sachs-Wolfe and integrated Sachs-Wolfe effects, and (iv) lensing by intervening structure, as is

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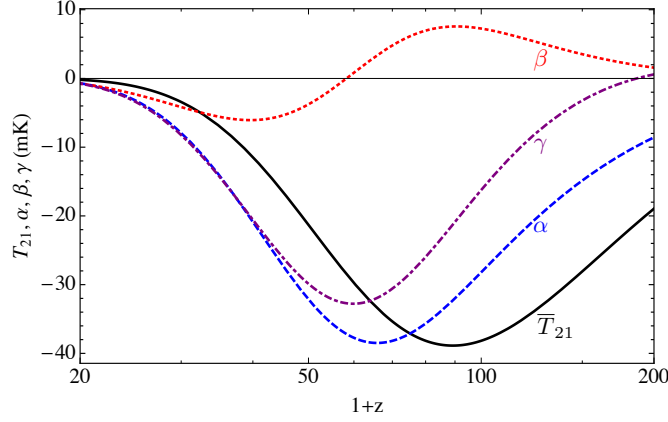


Figure 4.12: Coefficients of the approximate decomposition of the 21-cm brightness temperature given in Eq. (4.55), in mK. In solid black we plot $\bar{T}_{21}$, in blue dashed $\alpha = \partial T_{21} / \partial \delta_b^{(1)}$, in red dotted $\beta = \frac{1}{2} \partial^2 T_{21} / \partial \delta_b^2$, and in purple dot-dashed $\gamma = \partial T_{21} / \partial \delta_b^{(2)}$.

familiar from CMB studies. All these relativistic corrections are rigorously accounted for using the relativistic Boltzmann equation in Ref.¹³⁷ They lead to fluctuations on scales comparable to the horizon at the redshift of absorption, i.e. $k \lesssim 10^{-3} \text{ Mpc}^{-1}$.¹³⁷ We will neglect them in this study, which is justified as we shall see that most of the signal-to-noise for PNGs comes from small scales, with $k \gg 10^{-3} \text{ Mpc}^{-1}$.

Redshift-space distortions are an additional source of non-linear fluctuations. The observer has only access to the total redshift $z_{\text{obs}} \equiv \lambda_{\text{obs}} / \lambda_{21} - 1$, and will compute the angular power spectrum on slices of fixed z_{obs} . The observed redshift is the sum of the cosmological redshift z and the redshift due to the relative peculiar velocity $v_{\parallel}$ along the line of sight: $z_{\text{obs}} = z + v_{\parallel}/c$. The observed brightness temperature at wavelength λ_{obs} is therefore

$$T_{21}^{\text{obs}}(\lambda_{\text{obs}}, \hat{n}) = \frac{T_{21}^{\text{loc}}(z, \hat{n})}{1 + z_{\text{obs}}}, \quad (4.56)$$

where the true redshift $z \equiv z_{\text{obs}} - v_{\parallel}(z, \hat{n})/c$ depends implicitly on the unknown local velocity. The angular power spectrum at fixed z_{obs} therefore has additional non-linear terms.¹³⁷ We shall not account for those in this study but they should of course be modeled accurately when actual data is available.

Finally, Ref.¹⁴⁵ showed that the non-linear dependence of the 21-cm fluctuation on the local baryon density and temperature leads to enhanced large-scale fluctuations due to the

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relative velocity effect.²⁹ The magnitude of the enhanced fluctuations is $\delta T_{21} \sim \beta \Delta \langle \delta_s^2 \rangle$, where β is the coefficient of quadratic terms in the brightness-temperature fluctuations, and $\Delta \langle \delta_s^2 \rangle$ is the large-scale fluctuation of small-scale power due to the relative velocity effect. These enhanced fluctuations are most important for scales $k \lesssim 0.1 \text{ Mpc}^{-1}$, and we will not account for them in this study, where we focus mostly on smaller scales. The relative-velocity effect also leads to a suppression of the average small-scale power, but this takes place at scales $k \gtrsim 100 \text{ Mpc}^{-1}$, which we do not consider.

Flat-sky formalism

We consider a small patch on the sky, across which we can assume that the line of sight $\hat{n}$ is a constant direction. We then define the Fourier transform of the brightness temperature as

$$\delta T(\mathbf{k}) \equiv \int dr d^2 x_{\perp} e^{-i\mathbf{k} \cdot \mathbf{x}} \delta T(r\hat{n}, \mathbf{x}_{\perp}). \quad (4.57)$$

Assuming matter domination and that the baryons have caught up to the dark matter so that $\delta_b \propto a$, at linear order the peculiar velocity term is $\delta_v(\mathbf{k}) = (\hat{k} \cdot \hat{n})^2 \delta_b(\mathbf{k})$. The linear terms of Eq. (4.55) therefore contribute a Fourier transform

$$\delta T^{\text{lin}}(\mathbf{k}) = [\alpha + \bar{T}_{21} (\hat{k} \cdot \hat{n})^2] \delta_b(\mathbf{k}). \quad (4.58)$$

The power spectrum of 21-cm fluctuations is therefore anisotropic: to lowest order, and defining $k_{\parallel} \equiv \mathbf{k} \cdot \hat{n}$,

$$P_{\delta T}(\mathbf{k}) = \left(\alpha + \bar{T}_{21} k_{\parallel}^2 / k^2 \right)^2 P_{\delta_b}(k). \quad (4.59)$$

Similarly, the bispectrum $B_{\delta T}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$ is anisotropic, and depends on the orientation of the wavenumbers with respect to the line of sight. It is defined as usual through

$$\langle \delta T_{21}(\mathbf{k}_1) \delta T_{21}(\mathbf{k}_2) \delta T_{21}(\mathbf{k}_3) \rangle = (2\pi)^3 \delta_D(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3) \times B_{\delta T}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3). \quad (4.60)$$

Since we focus on small angular scales, we adopt a flat-sky formalism.¹⁶⁶ We assume that the 21-cm temperature is observed with a finite window function W in frequency. The observed temperature is therefore the convolution of the underlying temperature with W ,

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which we shall denote by $W * \delta T$. We define the flat-sky harmonic transform

$$\delta T(r, \boldsymbol{\ell}) \equiv \int_{\mathcal{A}} \frac{d^2 x_{\perp}}{r^2} e^{-i\boldsymbol{\ell} \cdot \mathbf{x}_{\perp}/r} (W * \delta T)(r\hat{n}, \mathbf{x}_{\perp}), \quad (4.61)$$

where $\hat{n}$ is the line of sight, assumed constant over the small survey area $\mathcal{A}$, and $\mathbf{x}_{\perp}$ is perpendicular to the line of sight. In terms of the Fourier modes of δT , this gives

$$\delta T(\boldsymbol{\ell}) = \int \frac{d^3 k}{(2\pi)^3} e^{irk_{\parallel}} \tilde{W}(k_{\parallel}) \delta T(\mathbf{k}) (2\pi)^2 \tilde{\delta}_{\text{D}}(r\mathbf{k}_{\perp} - \boldsymbol{\ell}), \quad (4.62)$$

where $\tilde{W}(k_{\parallel})$ is the Fourier transform of the window function and we have defined

$$\tilde{\delta}_{\text{D}}(\boldsymbol{\ell}) \equiv \frac{1}{(2\pi)^2} \int_{\mathcal{A}} \frac{d^2 x_{\perp}}{r^2} e^{i\mathbf{x}_{\perp} \cdot \boldsymbol{\ell}/r}. \quad (4.63)$$

The function $\tilde{\delta}_{\text{D}}$ peaks at the origin, with value $\tilde{\delta}_{\text{D}}(\mathbf{0}) = f_{\text{sky}}/\pi$, where f_{sky} is the fraction of sky subtended by the survey. It has a characteristic width $\Delta\ell \sim (f_{\text{sky}})^{-1/2}$ and integrates to unity. Finally, a convolution of $\tilde{\delta}_{\text{D}}$ with itself gives $\tilde{\delta}_{\text{D}}$ back.

The covariance of $\delta T(\boldsymbol{\ell})$ at equal r is given by

$$\begin{aligned} \langle \delta T(\boldsymbol{\ell}) \delta T^*(\boldsymbol{\ell}') \rangle &= \int d^2 k_{\perp} (2\pi)^2 \tilde{\delta}_{\text{D}}(r\mathbf{k}_{\perp} - \boldsymbol{\ell}) \tilde{\delta}_{\text{D}}^*(r\mathbf{k}_{\perp} - \boldsymbol{\ell}') \\ &\times \int \frac{dk_{\parallel}}{2\pi} |\tilde{W}|^2(k_{\parallel}) P_{\delta T}(k_{\parallel}, \mathbf{k}_{\perp}). \end{aligned} \quad (4.64)$$

For $\ell \gg (f_{\text{sky}})^{-1/2}$, we may approximate $\mathbf{k}_{\perp} \approx \boldsymbol{\ell}/r$ in the inner integral. Carrying out the outer integral, we arrive at¹⁷⁸

$$\langle \delta T(\boldsymbol{\ell}) \delta T^*(\boldsymbol{\ell}') \rangle \approx (2\pi)^2 \tilde{\delta}_{\text{D}}(\boldsymbol{\ell}' - \boldsymbol{\ell}) C_{\ell}, \quad (4.65)$$

where, as in Eq. (4.40),

$$C_{\ell} \equiv \frac{1}{r^2} \int \frac{dk_{\parallel}}{2\pi} |\tilde{W}|^2(k_{\parallel}) P_{\delta T}(k_{\parallel}, \boldsymbol{\ell}/r). \quad (4.66)$$

We show the flat-sky power spectrum C_{ℓ} computed with different widths of the window function and for several redshift slices in Fig. 4.13.

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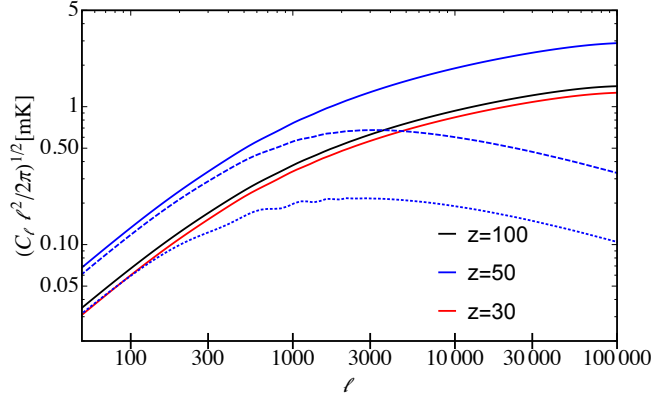


Figure 4.13: Flat-sky power spectrum C_ℓ in the limit of infinitely narrow window function, for redshifts (top to bottom) $z = 50$ (blue), $z = 100$ (black) and $z = 30$ (red). We also show the C_ℓ at redshift $z = 50$ for a Gaussian window function of width 0.1 MHz (blue dashed) and width of 1 MHz (blue dotted).

Similarly, the three-point function of $\delta T(\ell)$ defines our flat-sky bispectrum:

$$\langle \delta T(\ell_1) \delta T(\ell_2) \delta T(\ell_3) \rangle = (2\pi)^2 \tilde{\delta}_D(\ell_1 + \ell_2 + \ell_3) B_{\ell_1 \ell_2 \ell_3}, \quad (4.67)$$

with

$$B_{\ell_1 \ell_2 \ell_3} \equiv \int \frac{dk_{1\parallel} dk_{2\parallel}}{(2\pi)^2 r^4} \tilde{W}(k_{1\parallel}) \tilde{W}(k_{2\parallel}) \tilde{W}(-k_{1\parallel} - k_{2\parallel}) B_{\delta T}(k_{1\parallel}, \ell_1/r; k_{2\parallel}, \ell_2/r), \quad (4.68)$$

where we have dropped the dependence on $\mathbf{k}_3$ in the k -space bispectrum since it is fixed by the triangle condition given $\mathbf{k}_1 = k_{1\parallel} \hat{n} + \ell_1/r$ and $\mathbf{k}_2 = k_{2\parallel} \hat{n} + \ell_2/r$. Note that we do not use the Limber approximation and perform the full integrals over $k_{\parallel}$'s.

We now describe and compute the different contributions to $B_{\ell_1 \ell_2 \ell_3}$.

4.3.2 Bispectrum of 21-cm fluctuations

The bispectrum gets contributions from primordial non-gaussianities, which we would like to extract from the data, but also from secondary non-gaussianities, arising from the non-linear relation between the observable and the initial conditions, even if the latter are perfectly gaussian.

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We only consider multipoles $\ell \gtrsim 100$, which correspond to wavenumbers $k \gtrsim 0.01 \text{ Mpc}^{-1}$, as most of the signal-to-noise ratio for bispectrum measurements is expected to come from small-scale modes. We therefore neglect the contributions of relativistic terms to the bispectrum, in particular the ISW-lensing bispectrum, which is the dominant secondary bispectrum for CMB anisotropies.^{179, 180}

Primordial non-gaussianities

The contribution of PNG to the bispectrum of 21-cm fluctuations can be obtained to lowest order by only considering the linear terms in Eq. (4.55), and assuming that they are linearly related to the primordial curvature fluctuations. The Fourier transform of the linear terms is given in Eq. (4.58). We define $M(k, z) \equiv \delta_b(\mathbf{k}, z)/\Phi(\mathbf{k})$, where $\Phi = (3/5)\zeta$ is Bardeen's gravitational potential. The bispectrum of brightness-temperature fluctuations gets a contribution

$$B_{\delta T}^{\text{prim}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = \prod_{i=1}^3 (\alpha + \bar{T}_{21} \mu_i^2) M(k_i) \times B_{\Phi}(k_1, k_2, k_3) \quad (4.69)$$

from primordial non-gaussianities, where $\mu_i \equiv (\mathbf{k}_i \cdot \hat{n})/k_i$.

We now review the different shapes of the initial potential bispectrum $B_{\Phi}(k_1, k_2, k_3)$ that we will consider in this work (see e. g. Ref.¹⁷⁵ for a larger variety of shapes).

Local The simplest form of PNG is of the local type, where the primordial potential Φ is a local non-linear function of a gaussian field ϕ :

$$\Phi(\mathbf{x}) = \phi(\mathbf{x}) + f_{\text{NL}}^{\text{local}} \left(\phi^2(\mathbf{x}) - \langle \phi \rangle^2 \right). \quad (4.70)$$

This implies a non-vanishing bispectrum for Φ , given to lowest order by

$$B_{\Phi}^{\text{local}}(k_1, k_2, k_3) = 2f_{\text{NL}}^{\text{local}} [P_{\Phi}(k_1)P_{\Phi}(k_2) + 2 \text{ perm.}]. \quad (4.71)$$

This form of the bispectrum peaks in the squeezed configuration ($k_1 \ll k_2 \sim k_3$ and permutations).

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Local-type PNG typically arises in multi-field inflation models, such as the curvaton model or modulated reheating.

Equilateral PNGs of the equilateral type arise when there are non-standard kinetic terms in the inflation Lagrangian, which are included in the so-called $P(X)$ models of inflation, concrete examples of which are k-inflation^{173,181} and Dirac-Born-Infeld inflation.¹⁸² In these models the effective sound speed c_s can be very different from the speed of light ($c = 1$), and the non-gaussianity parameter is related to this departure via $f_{\text{NL}}^{\text{equil}} = -(35/108)(c_s^{-2} - 1)$.

This shape peaks when the three modes cross the horizon at the same time, and hence $k_1 \sim k_2 \sim k_3$. A good template for it is,¹⁸³

$$B_{\Phi}^{\text{equil}}(k_1, k_2, k_3) = 6f_{\text{NL}}^{\text{equil}} A_{\Phi}^2 \left\{ - \left[\frac{1}{(k_1 k_2)^{4-n_s}} + 2 \text{ perm.} \right] - \frac{2}{(k_1 k_2 k_3)^{2\frac{4-n_s}{3}}} + \left[\frac{1}{(k_1 k_2^2 k_3^3)^{\frac{4-n_s}{3}}} + 5 \text{ perm.} \right] \right\}, \quad (4.72)$$

where A_{Φ} is the normalization of the power spectrum of Φ : $P_{\Phi}(k) = A_{\Phi}/k^{4-n_s}$.

Orthogonal The “orthogonal” shape of PNG was defined in Ref.¹⁸⁴ to be orthogonal to the equilateral shape for the scalar product $B^a \cdot B^b \equiv \sum_{k_1, k_2, k_3} B_{k_1, k_2, k_3}^a B_{k_1, k_2, k_3}^b / [P_{\Phi}(k_1)P_{\Phi}(k_2)P_{\Phi}(k_3)]$. Its form is

$$B_{\Phi}^{\text{ortho}}(k_1, k_2, k_3) = 6f_{\text{NL}}^{\text{ortho}} A_{\Phi}^2 \left\{ \left[\frac{-3}{(k_1 k_2)^{4-n_s}} + 2 \text{ perm.} \right] - \frac{8}{(k_1 k_2 k_3)^{2\frac{4-n_s}{3}}} + \left[\frac{3}{(k_1 k_2^2 k_3^3)^{\frac{4-n_s}{3}}} + 5 \text{ perm.} \right] \right\}. \quad (4.73)$$

The models of Galileon inflation¹⁸⁵ and ghost inflation¹⁸⁶ predict very high values of $f_{\text{NL}}^{\text{ortho}}$. In general, in terms of the Lagrangian for the Goldstone boson π during inflation, both equilateral and orthogonal shapes arise from cubic kinetic interactions, and the f_{NL} s are linearly related to the coefficients of the $\dot{\pi}^3$ and $\dot{\pi}(\partial\pi)^2$ terms.

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It is interesting to also mention the folded form of non-gaussianity, where the shape of the bispectrum peaks at flattened (folded) triangles ($k_1 = k_2 = k_3/2$ and permutations). Initial conditions different from the standard Bunch-Davies vacuum would give rise to this kind of PNG. It can be expressed as a combination of the two above, as $B^{\text{folded}} = (B^{\text{equil}} - B^{\text{ortho}})/2$.

Directional dependence In some models where inflation is driven by a gauge vector field or in solid inflation¹⁸⁷ there is an additional form of PNG, that induces an extra dependence in the angle between the $\mathbf{k}_i$ vectors. In this case the bispectrum can be decomposed in Legendre polynomials,¹⁸⁸ where each component would be

$$B_{\Phi}^{(J)}(k_1, k_2, k_3) = f_{\text{NL}}^{(J)} \left[P_{\Phi}(k_1) P_{\Phi}(k_2) \mathcal{P}_J(\cos \theta_{12}) + 2 \text{ perm.} \right], \quad (4.74)$$

where $\mathcal{P}_J$ is the Legendre polynomial of order J , and θ_{12} is the angle between $\mathbf{k}_1$ and $\mathbf{k}_2$, whose cosine can be expressed as $\cos \theta_{12} = (k_3^2 - k_1^2 - k_2^2)/2k_1k_2$. We consider $J = 1, 2$ and 3.

Secondary non-gaussianities

Non-linear gravitational collapse The growth of overdensities by gravitational collapse is a fundamentally non-linear process, leading to a non-vanishing 3-point function, even when starting from perfectly gaussian initial conditions. The resulting bispectrum can be computed from second-order perturbation theory (see e.g. Ref.¹⁷⁸). The correlation of two linear perturbations with a second-order density perturbation or normalized velocity divergence ($\theta \equiv -\nabla \cdot \mathbf{v}/H$) takes the form

$$\langle \delta^{(1)}(\mathbf{k}_1) \delta^{(1)}(\mathbf{k}_2) \delta^{(2)}(\mathbf{k}_3) \rangle' = 2F(\mathbf{k}_1, \mathbf{k}_2) P_1 P_2, \quad (4.75)$$

$$\langle \delta^{(1)}(\mathbf{k}_1) \delta^{(1)}(\mathbf{k}_2) \theta^{(2)}(\mathbf{k}_3) \rangle' = 2G(\mathbf{k}_1, \mathbf{k}_2) P_1 P_2, \quad (4.76)$$

where $\langle \dots \rangle'$ is the three-point function divided by $(2\pi)^3 \delta_{\text{D}}(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)$, and $P_i \equiv P_{\delta}(\mathbf{k}_i)$ is the power spectrum of the linear overdensity. The mode-coupling kernels $F(\mathbf{k}_1, \mathbf{k}_2)$ and

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$G(\mathbf{k}_1, \mathbf{k}_2)$ are both of the form¹⁷⁸⁴

$$K(\mathbf{k}_1, \mathbf{k}_2) = c_1 + c_2 \hat{k}_1 \cdot \hat{k}_2 \left(\frac{k_1}{k_2} + \frac{k_2}{k_1} \right) + c_3 (\hat{k}_1 \cdot \hat{k}_2)^2. \quad (4.77)$$

For a CDM-only universe, $(c_1, c_2, c_3) = (\frac{5}{7}, \frac{1}{2}, \frac{2}{7})$ for F and $(\frac{3}{7}, \frac{1}{2}, \frac{4}{7})$ for G . In reality, however, baryons start clustering after recombination, while the CDM overdensities have already been growing since their scales entered the horizon. Their density and velocity fields at recombination are therefore very different and the subsequent growth factor of matter fluctuations is therefore not just $D(a) \propto a$. It moreover has a scale dependence, as baryons, though they start with effectively zero overdensity at recombination ($\delta_b \ll \delta_c$ on sub-horizon scales), have a velocity comparable to that of the CDM, but with a different scale dependence. Different wavenumbers therefore grow at slightly different rates. The coefficients c_i in Eq. (4.77) are therefore in reality weakly dependent on redshift and, perhaps to a lesser extent, on scale. We shall ignore these complications here and take their standard values.

Assuming $\delta_b = \delta$ and using $\delta_v(\mathbf{k}) = \mu^2 \theta(\mathbf{k})$ and $\delta_v^{(1)}(\mathbf{k}) = \mu^2 \delta_b^{(1)}(\mathbf{k})$ [note that this last relation only holds for the first-order perturbations], the bispectrum of 21-cm fluctuations due to gravitational collapse is straightforwardly obtained from Eq. (4.55):

$$B_{\delta T}^{\text{grav}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = 2(\alpha + \bar{T}_{21} \mu_1^2)(\alpha + \bar{T}_{21} \mu_2^2) P_1 P_2 \\ \times [\gamma F(\mathbf{k}_1, \mathbf{k}_2) + \bar{T}_{21} (\mu_1 + \mu_2)^2 G(\mathbf{k}_1, \mathbf{k}_2)] + 2 \text{ perm.} \quad (4.78)$$

Non-linear relation between brightness temperature and baryon density The relationship between the 21-cm brightness temperature and the underlying density and velocity field is fundamentally non-linear, due to (i) the non-linear dependence of the optical depth on the local peculiar velocity gradient ($\tau \propto 1/(1 - \delta_v)$), (ii) the non-linear dependence of the spin temperature on the baryon density and temperature, and (iii) the non-linear dependence of the gas temperature on the baryon density. Therefore even for a perfectly gaussian underlying density field, this non-linear mapping leads to a non-vanishing bispectrum.

⁴These coupling kernels are derived in the sub-horizon limit. Since we are mostly interested in small scales we shall not concern ourselves with subtle issues regarding the squeezed limit of the gravitational bispectrum on horizon scales.

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This contribution to the bispectrum can be obtained from the following three-point functions:

$$\langle \delta_b(\mathbf{k}_1) \delta_b(\mathbf{k}_2) [\delta_b^2](\mathbf{k}_3) \rangle' = 2P_1 P_2, \quad (4.79)$$

$$\langle \delta_b(\mathbf{k}_1) \delta_b(\mathbf{k}_2) [\delta_b \delta_v](\mathbf{k}_3) \rangle' = (\mu_1^2 + \mu_2^2) P_1 P_2, \quad (4.80)$$

$$\langle \delta_b(\mathbf{k}_1) \delta_b(\mathbf{k}_2) [\delta_v^2](\mathbf{k}_3) \rangle' = 2\mu_1^2 \mu_2^2 P_1 P_2, \quad (4.81)$$

where the superscript (1) is implicit in all the fluctuations. Using Eq. (4.55), the explicit expression for the bispectrum arising from the non-linearity of δT_{21} as a tracer is then

$$\begin{aligned} B_{\delta T}^{\text{nl}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3) = & (\alpha + \bar{T}_{21} \mu_1^2)(\alpha + \bar{T}_{21} \mu_2^2) \times (2\beta + \alpha(\mu_1^2 + \mu_2^2) + 2\bar{T}_{21} \mu_1^2 \mu_2^2) P_1 P_2 \\ & + 2 \text{ perm.} \end{aligned} \quad (4.82)$$

The total secondary bispectrum is obtained by summing Eqs. (4.78) and (4.82). Note that the bispectrum arising from Eq. (4.78) requires the kernels F and G to be non-zero, whereas the bispectrum from (4.82) does not.

Numerical evaluation and comparison

Inserting the Fourier-space primordial bispectra Eq. (4.69) and secondary bispectra Eqs. (4.78) and (4.82) into Eq. (4.68), we obtain the harmonic-space bispectra in the flat-sky limit.

We show the total secondary bispectrum in Fig. 4.14, along with the bispectra resulting from local, equilateral and orthogonal PNGs. As found by previous authors,¹⁷⁷ we find that the secondary bispectrum is typically at least two orders of magnitude larger than the bispectrum due to PNGs for $f_{\text{NL}} = 1$. This order-of-magnitude difference can be understood quite simply: the ratio of secondary to primary bispectra is of order

$$\frac{B^{\text{sec}}}{B^{\text{prim}}} \sim \frac{\langle \delta \delta \delta \delta \rangle}{\langle \delta \delta \delta f_{\text{NL}} \Phi \rangle} \sim \frac{\delta(z)}{f_{\text{NL}} \Phi}. \quad (4.83)$$

We know that $\delta(z=0) \sim 1$ at the non-linear scale $k_{\text{NL}}(z=0) \approx 0.1 \text{ Mpc}^{-1}$. Scaling back to $z=100$ gives $\delta(z=100) \sim 10^{-2}$ at $k \sim 0.1$, with an amplitude increasing logarithmically

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with wavenumber. The primordial gravitational potential is nearly scale-independent and of order $\Phi \sim 3 \times 10^{-5}$. We therefore obtain $B^{\text{sec}}/B^{\text{prim}} \sim \text{few} \times 100$ for $f_{\text{NL}} = 1$, consistent with our more detailed calculation.

Note that our estimates for both the primary and the secondary bispectra neglected higher-order terms, for example terms of order $\langle \delta^2 \delta^1 \delta^3 \rangle$ in the secondary bispectrum. These terms are suppressed by an additional factor of order $\delta^2 \sim 10^{-4}$ at $z \approx 100$, and are therefore comparable to the primary bispectrum only if $f_{\text{NL}} \sim \text{few times } 10^{-2}$. We will not consider them in this study, but they should be accounted for in a final data analysis aiming for a few percent uncertainty in f_{NL} .

In practice, we carry out the integrals up to some maximum multipole ℓ_{max} corresponding to the resolution of the observations.

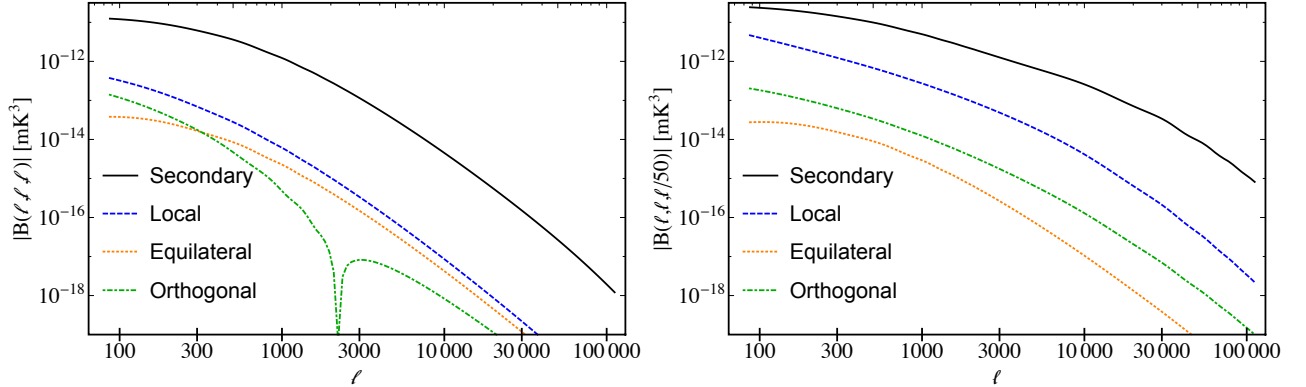


Figure 4.14: Bispectra of 21-cm brightness-temperature fluctuations resulting from secondary non-gaussianities and different shapes of primordial non-gaussianity, with $f_{\text{NL}} = 1$, at $z = 50$. The top panel shows the bispectra for equilateral triangles ($\ell \equiv \ell_1 = \ell_2 = \ell_3$). The bottom panel shows the bispectra for squeezed triangles ($\ell \equiv \ell_1 = \ell_2 \gg \ell_3 = \ell/50$). In dashed blue we plot local, in dotted orange equilateral and in dash-dotted green orthogonal non-gaussianity. In solid black we plot the secondary bispectrum. The bispectra are computed in the flat-sky approximation for an infinitesimally narrow redshift slice.

4.3.3 Fisher analysis

Bias due to secondary non-gaussianities

Assuming a single type of primordial non-gaussianity with bispectrum $B_{\ell_1\ell_2\ell_3} = f_{\text{NL}} b_{\ell_1\ell_2\ell_3}^{\text{prim}}$, if secondary non-gaussianities were negligible the minimum-variance cubic estimator for f_{NL} from a single redshift z would be¹⁸⁹

$$\hat{f}_{\text{NL}} = \frac{(b^{\text{prim}}, B^{\text{obs}})_z}{(b^{\text{prim}}, b^{\text{prim}})_z}, \quad (4.84)$$

where we defined

$$B_{\ell_1, \ell_2, \ell_3}^{\text{obs}} \equiv \frac{1}{4\pi f_{\text{sky}}} \delta T(\ell_1) \delta T(\ell_2) \delta T(\ell_3), \quad (4.85)$$

and the scalar product $(\ , \)_z$ is constructed as

$$(B^i, B^j)_z \equiv 4\pi f_{\text{sky}} \iiint_{\ell_1 \geq \ell_2 \geq \ell_3} \frac{d^2\ell_1 d^2\ell_2 d^2\ell_3}{(2\pi)^4} \delta_{\text{D}}(\ell_1 + \ell_2 + \ell_3) \times \frac{B_{\ell_1\ell_2\ell_3}^i B_{\ell_1\ell_2\ell_3}^j}{C_{\ell_1}^{\text{tot}} C_{\ell_2}^{\text{tot}} C_{\ell_3}^{\text{tot}}}. \quad (4.86)$$

In this equation C_{ℓ}^{tot} is the total variance of $\delta T(\ell)$, due to cosmic variance and all other sources of noise, including the instrument and foregrounds. We assume throughout that the noise can be approximately computed neglecting non-gaussian contributions to $\delta T(\ell)$. In practice, all our results will be quoted in the cosmic-variance limit, i.e. for $C_{\ell}^{\text{tot}} = C_{\ell}$ given in Eq. (4.66)⁵.

We saw in the previous Section that the bispectrum resulting from secondary non-gaussianities is much larger than the one arising from PNGs, typically by two to three orders of magnitude for $f_{\text{NL}} = 1$. Using the estimator, Eq. (4.84), would therefore lead to a bias

$$\Delta f_{\text{NL}} = \frac{(b^{\text{prim}}, B^{\text{sec}})_z}{(b^{\text{prim}}, b^{\text{prim}})_z} \equiv c_{\text{prim,sec}} \sqrt{\frac{(B^{\text{sec}}, B^{\text{sec}})_z}{(b^{\text{prim}}, b^{\text{prim}})_z}}, \quad (4.87)$$

where $c_{\text{prim,sec}} \in [-1, 1]$ quantifies the shape overlap or degeneracy of the primordial and secondary bispectra [geometrically, $c_{\text{prim,sec}}$ is the cosine of the angle between the two bispec-

⁵Note that in order to have tenth-of-arcminute resolution at redshift $z = 100$ one would need a baseline $D \gtrsim 350$ km. In order to reach cosmic variance limit at $z = 50$ and for resolution of one arcminute, the parameters of the interferometer would have to be really optimistic, with complete coverage $f_{\text{cover}} = 1$, a baseline of order the diameter of the moon $D = 3500$ km, and a time of observation of 2 years.

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tra, for the scalar product (4.86)]. The shapes of primordial and secondary non-gaussianity being different, one may hope that their overlap is small¹⁷⁶. However, assuming a cosmic-variance-limited experiment with an infinitely narrow window function and a resolution of $\sim 0.1'$ (corresponding to $\ell_{\max} = 10^5$), we find that $c_{\text{loc,sec}} = 0.89$, $c_{\text{equi,sec}} = 0.79$, and $c_{\text{ortho,sec}} = -0.83$. The unsubtracted secondary bispectrum would therefore lead to large biases $\Delta f_{\text{NL}}^{\text{loc}} = 870$, $\Delta f_{\text{NL}}^{\text{equi}} = 3900$, and $\Delta f_{\text{NL}}^{\text{ortho}} = -3900$. For maximum resolution of $1'$ (corresponding now to $\ell_{\max} = 10^4$), the values of the degeneracy coefficients would be $c_{\text{loc,sec}} = 0.80$, $c_{\text{equi,sec}} = 0.89$, and $c_{\text{ortho,sec}} = -0.88$, which in turn would make the biases $\Delta f_{\text{NL}}^{\text{loc}} = 420$, $\Delta f_{\text{NL}}^{\text{equi}} = 2400$, and $\Delta f_{\text{NL}}^{\text{ortho}} = -2100$.

Such a strong degeneracy may seem surprising at first, given the large number of triangles on which the scalar product depends. However, because the bispectra are essentially smooth featureless functions of ℓ for small angular scales, they can have significant overlap in the sense defined in Eq. (4.87)⁷. The equilateral and orthogonal-type bispectra have more complex shapes in k and ℓ -space than the local type, which is why their overlap with the secondary bispectrum decreases with increasing $\ell_{\max}$ while it increases for the latter.

In the next section we describe how to deal with these degeneracies.

Estimators for a single redshift slice

One could in principle try and model the secondary bispectrum from first principles and subtract the resulting bias Δf_{NL} from the estimated PNG amplitude. This strategy is the one adopted for the bispectrum of CMB anisotropies, where the main contaminant is the ISW-lensing bispectrum. Given the now well-measured cosmological parameters, the latter can indeed be modeled to sufficient accuracy, i.e. with an error smaller than the statistical uncertainty in f_{NL} .^{166,175} In the case of 21-cm fluctuations, however, even percent-level residuals in the modeled secondary bispectrum would lead to biases of order $\Delta f_{\text{NL}} \sim 10$, significantly larger than the statistical errors one may hope to achieve. Reaching sub-percent accuracy would require, first, a very careful treatment of subtle microphysical

⁶Ref.¹⁷⁶ treat the secondary non-gaussianity as a source of noise instead of a bias, which is inappropriate.

⁷Consider for instance the 1-dimensional scalar product $\langle F.G \rangle = \int_{\ell_{\min}}^{\ell_{\max}} F(\ell)G(\ell)d\ell$, with $\ell_{\min} \ll \ell_{\max}$. If $F(\ell) \propto \ell^\alpha$ and $G(\ell) \propto \ell^\beta$, then their degeneracy coefficient is approximately $c = \sqrt{1 + 2x}/(1 + x)$, where $x \equiv (\beta - \alpha)/(2\alpha + 1)$. This is greater than 0.4 for $x \leq 10$.

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processes affecting the population of the hyperfine levels. In addition, it will be limited by the accuracy of cosmological parameters.

The amplitudes of all the secondary bispectra depend on the coefficients c_i in Eq. (4.77) for the kernels F and G . Although we use the values derived for a matter-dominated, CDM-only universe in our analysis, we assume that the exact values could be computed exactly should one want to do so. On the other hand, the four parameters $A_i \equiv \bar{T}_{21}, \alpha, \beta, \gamma$ in Eq. (4.55) depend on the detailed microphysics of the hyperfine transition. For now we assume that they can be modeled up to subpercent accuracy and denote their best estimates by A_i^0 .

Our model for the bispectrum is

$$B_{\ell_1 \ell_2 \ell_3} = B_{\ell_1 \ell_2 \ell_3}^{\text{sec},0} + f_{\text{NL}} b_{\ell_1 \ell_2 \ell_3}^{\text{prim}} + \sum_{i=1}^4 f_i b_{\ell_1 \ell_2 \ell_3}^i, \quad (4.88)$$

where $B_{\ell_1 \ell_2 \ell_3}^{\text{sec},0}$ is the best estimate for the secondary bispectrum obtained with the A_i^0 , $f_i \equiv \Delta A_i$ are the unknown residuals of the four coefficients and $b_{\ell_1 \ell_2 \ell_3}^i \equiv \partial B_{\ell_1 \ell_2 \ell_3}^{\text{sec}} / \partial A_i$. To make the notation more compact we denote⁸ $f_0 \equiv f_{\text{NL}}$ and $b_{\ell_1 \ell_2 \ell_3}^0 \equiv b_{\ell_1 \ell_2 \ell_3}^{\text{prim}}$, and recall that we search for one type of PNG at a time. Notice that we disregard higher order correction terms, proportional to $\Delta\alpha^2$ and $\Delta\bar{T}_{21}^2$, since we will be able to model those two parameters to a precision better than 0.3%, as shown in Tab. 4.3, which would mean that the bias in the non-gaussianity amplitude is of order $\Delta f_{\text{NL}} \sim 10^3(3 \times 10^{-3})^2 \lesssim 10^{-2}$.

We fit simultaneously for the amplitude of the PNG and for the residual coefficients of the secondary bispectrum. We treat the latter as nuisance parameters over which we will marginalize. In geometrical terms, we construct an estimator for the PNG by projecting the observed bispectrum on the component of the primordial bispectrum orthogonal to all shapes of secondary non-gaussianity.

The minimum-variance cubic estimators for the parameters f_i are given by

$$\hat{f}_i \equiv \sum_j (F^{-1})_{ij} (b^j, B^{\text{obs}} - B^{\text{sec},0})_z, \quad (4.89)$$

⁸Note that f_{NL} and f_i do not have the same dimensions, but this does not affect the analysis.

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where F^{-1} is the inverse of the Fisher matrix F whose components are

$$F_{ij} \equiv (b^i, b^j)_z. \quad (4.90)$$

The variances of these estimators are

$$\sigma_{\hat{f}_i}^2 = (F^{-1})_{ii}, \quad (4.91)$$

and the signal-to-noise ratio (SNR) for f_{NL} is $f_{\text{NL}}/\sqrt{(F^{-1})_{00}}$.

We show in Fig. 4.15 the forecasted SNR for the local-type PNG, for a single narrow redshift slice around $z = 50$, as a function of the maximum multipole moment ℓ_{max} (with $\ell_{\text{min}} = 100$). We also show for reference the SNR one would obtain if one neglected the secondary non-gaussianities, i.e. when substituting $(F^{-1})_{00} \rightarrow 1/F_{00}$ as in Ref.¹⁷⁷ We see that properly accounting for secondary non-gaussianities and their correlation with the primordial bispectrum reduces the SNR by a factor of ~ 6 .

We also show the SNR integrated starting from $\ell_{\text{max}} = 10^5$ down to a minimum ℓ_{min} , as a function of the latter. It plateaus for $\ell_{\text{min}} \sim 10^3$, so modes with smaller ℓ do not contribute significantly to the signal-to-noise ratio, which justifies our neglect of several contributions to the bispectrum on large scales.

In Fig. 4.16 we show the forecasted SNRs for the other shapes of PNG we considered. Secondary non-gaussianities are less correlated with these shapes than they are with the local type, so the reduction in SNR is not as dramatic (a factor of ~ 3).

We summarize the forecasted SNR in Table 4.2 for a single narrow redshift-slice at $z = 50$, for $\ell_{\text{max}} = 10^4$ (corresponding of an angular resolution of roughly 1 arcmin) and $\ell_{\text{max}} = 10^5$ (0.1 arcmin angular resolution), assuming a cosmic-variance-limited experiment (i.e. taking $C_\ell^{\text{tot}} = C_\ell$, and neglecting additional thermal noise). In particular, we find that values of $f_{\text{NL}}^{\text{loc}} \sim 1.3$ and ~ 0.23 could be reached for $\ell_{\text{max}} = 10^4$ and 10^5 , respectively. The bigger improvement for better resolution for the orthogonal and equilateral shapes with respect to the local one is due to the fact that they become less degenerate with the secondary bispectra as more modes are added in the analysis, as argued in Section 4.3.3.

It is interesting to discuss how well we could probe the four secondary coefficients,

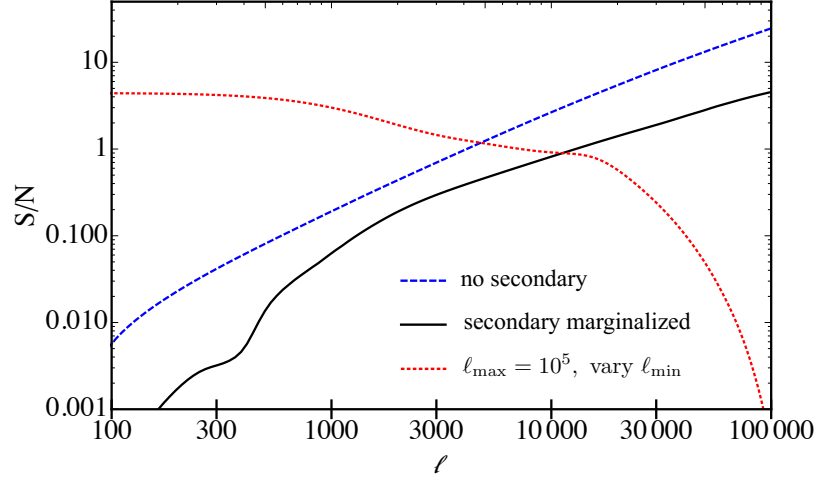


Figure 4.15: Signal-to-noise ratio (SNR) for PNG of the local type with $f_{\text{NL}} = 1$, for a single narrow redshift slice at $z = 50$ and assuming $f_{\text{sky}} = 1$. The blue dashed curve shows $(F_{00})^{1/2}$, the SNR obtained if one neglected secondary non-gaussianity. The black solid and red dotted curves show $[(F^{-1})_{00}]^{-1/2}$, the SNR after marginalization over the unknown residual amplitudes of the secondary bispectrum, as a function of ℓ_{max} (black solid) and as a function of ℓ_{min} at fixed $\ell_{\text{max}} = 10^5$ (red dotted).

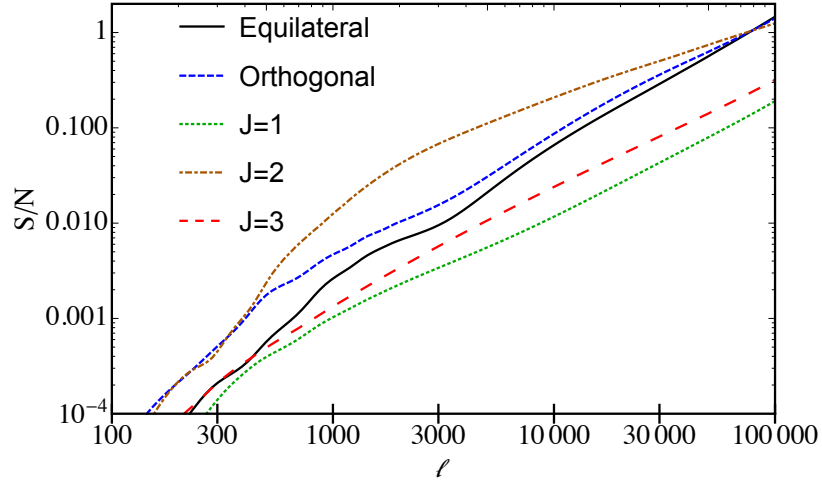


Figure 4.16: SNR for different shapes of PNGs (with $f_{\text{NL}} = 1$), after marginalization over the residual amplitudes of the secondary bispectrum. The different lines correspond to equilateral-type PNG (solid black), orthogonal-type PNG (blue dashed), and the three direction-dependent shapes $J = 1, 2$ and 3 in dotted green, dash-dotted brown, and long-dashed red, respectively.

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PNG type	$\sigma_{f_{\text{NL}}} \text{ (arcmin)}$	$\sigma_{f_{\text{NL}}} \text{ (0.1 arcmin)}$
Local	1.3	0.23
Equilateral	14	0.71
Orthogonal	11	0.71
$J = 1$	83	5.3
$J = 2$	4.5	0.83
$J = 3$	40	3.1

Table 4.2: Detection forecasts for different shapes of PNG for a cosmic-variance-limited experiment observing the full sky at a single narrow redshift slice at $z = 50$. The central column gives the results for $\ell_{\text{max}} = 10^4$ (equivalent to having an experiment with arcminute resolution), and the right column those for $\ell_{\text{max}} = 10^5$ (one tenth of arcminute).

$\overline{T}_{21}$, α , β , and γ . In Tab. 4.3 we show the relative errors reachable for each of them with a 0.1 arcminute resolution, as well as the correlation with the rest of parameters.

	$\overline{T}_{21}$	α	β	γ	f_{NL}
$\overline{T}_{21}$	5.9×10^{-4}				
α	-0.95	2.6×10^{-3}			
β	-0.97	0.91	0.012		
γ	0.41	-0.63	-0.43	7.2×10^{-4}	
f_{NL}	0.89	0.85	-0.92	0.36	0.23

Table 4.3: Fractional error and correlation coefficients of f_{NL} and secondary bispectrum amplitudes. The diagonal elements are the fractional errors for each parameter, calculated as $\sqrt{(F^{-1})_{00}}$ for the case of f_{NL} and $\sqrt{(F^{-1})_{ii}}/A_i^0$ for the rest. The off-diagonal elements are the correlations between parameters, defined as $(F^{-1})_{ij}/\sqrt{(F^{-1})_{ii}(F^{-1})_{jj}}$. For these results we considered local non gaussianity, at redshift $z = 50$ and a resolution of 0.1 arcminutes.

Choice of nuisance parameters

In our analysis we have marginalized over the residuals of the four coefficients $\overline{T}_{21}$, α , β , γ . Here we discuss how different choices would affect our results.

On the optimistic side, if we were able to relate the four secondary coefficients to each other to high precision we could choose to marginalize over a single overall amplitude for the secondary bispectrum.

On the pessimistic side, we may choose to marginalize over all geometrically distinct contributions to the secondary bispectrum. This would account for unknown redshift de-

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pendences in the c_i coefficients. Recalling that the kernels F and G are made of three geometrically distinct pieces, Eq. (4.78) gives 18 different geometric shapes. Equation (4.82) adds three independent shapes. This amounts to a total of 21 distinct geometric shapes, the amplitudes of which we marginalize over.

We show the resulting SNRs in Fig. 4.17, where for reference we also show the SNR in the absence of secondary non-gaussianities, and our main result, which considers 4 nuisance parameters. As expected, our result lies between the optimistic and pessimistic cases, which act as bounds for the SNR when considering additional secondary bispectra.

In particular, in the optimistic approach, the SNR is improved by a factor of ~ 5 : we find detection limits $f_{\text{NL}}^{\text{local}} \sim 0.12$, $f_{\text{NL}}^{\text{equil}} \sim 0.75$, $f_{\text{NL}}^{\text{ortho}} \sim 0.58$, $f_{\text{NL}}^{J=1} \sim 5.7$, $f_{\text{NL}}^{J=2} \sim 0.88$, $f_{\text{NL}}^{J=3} \sim 13$ at arcminute resolution and $f_{\text{NL}}^{\text{local}} \sim 0.0063$, $f_{\text{NL}}^{\text{equil}} \sim 0.032$, $f_{\text{NL}}^{\text{ortho}} \sim 0.030$, $f_{\text{NL}}^{J=1} \sim 0.19$, $f_{\text{NL}}^{J=2} \sim 0.04$, $f_{\text{NL}}^{J=3} \sim 0.40$ at maximum resolution for a single redshift slice at $z = 50$.

In a real experiment, a χ^2 -like test should be carried out to find out whether additional secondary bispectra to the four proposed here need to be considered.

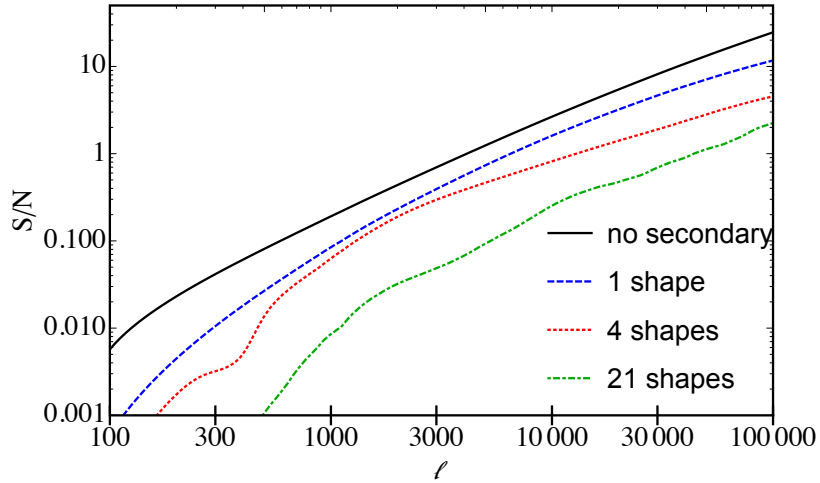


Figure 4.17: SNR for local-type PNG with $f_{\text{NL}} = 1$ as a function of ℓ_{max} , when neglecting secondary non-gaussianities (top, solid black curve), marginalizing over an overall amplitude of the secondary bispectrum (blue dashed), marginalizing over 4 coefficients as we do in the main text (red, dashed), and marginalizing over the amplitudes of the 21 geometrically distinct shapes of secondary bispectra (bottom, green dash-dotted).

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Tomography

So far we have been studying the bispectrum on a single redshift slice, which would correspond to observing the 21-cm line with a single frequency channel. However, one of the great advantages of the 21-cm line is that it enables us to coadd information from different redshifts.

Before thinking of how to add different redshift shells we will study whether they contain the same or different information. Let us construct a measure of the correlation between two slices at a radial distance Δr from each other. We define the correlation length $\xi_r(\ell)$ as the radial separation beyond which the cross-correlation between two redshift-slices is less than 1/2 the power spectrum:

$$C_\ell[\Delta r = \xi(\ell)] = \frac{1}{2} C_\ell[\Delta r = 0], \quad (4.92)$$

where the cross-power spectrum $C_\ell[\Delta r]$ is obtained from

$$C_\ell[\Delta r] \equiv \frac{1}{r^2} \int \frac{dk_{||}}{2\pi} P_{\delta T}(k_{||}, \ell/r) e^{ik_{||}\Delta r}. \quad (4.93)$$

From the correlation length in radial comoving separation ξ_r we obtain the characteristic correlation length in frequency space ξ_ν through

$$\xi_\nu = \frac{d\nu}{dz} \frac{dz}{dr} \xi_r = \nu_0 H_0 \sqrt{\Omega_M} (1+z)^{-1/2} \xi_r \quad (4.94)$$

$$\approx 1 \text{ MHz} \left(\frac{51}{1+z} \right)^{1/2} \left(\frac{\xi_r}{60 \text{ Mpc}} \right), \quad (4.95)$$

where $\nu_0 = 1.4 \text{ GHz}$ is the rest-frame frequency of the 21-cm transition.

We show the function $\xi_\nu(\ell)$ in Figure 4.18. For $\ell \lesssim 100$ (corresponding to $k \lesssim k_{\text{eq}} \sim 0.01 \text{ Mpc}^{-1}$), $P(k_{||}, \ell/r)$ peaks at $k_{||} \sim k_{\text{eq}}$, independently of ℓ , and the cross-correlation $C_\ell[\Delta r]$ has a characteristic length scale $\xi_\nu \approx 0.3 \text{ MHz}$, independent of ℓ . For $\ell \gtrsim 100$, the function $P(k_{||}, \ell/r)$ has a characteristic turnaround scale at $k_{||} \sim \ell/r$, which leads to a correlation length $\xi_\nu(\ell) \propto 1/\ell$.

In order to compare with previous results in the literature^{176,177} we will assume band-

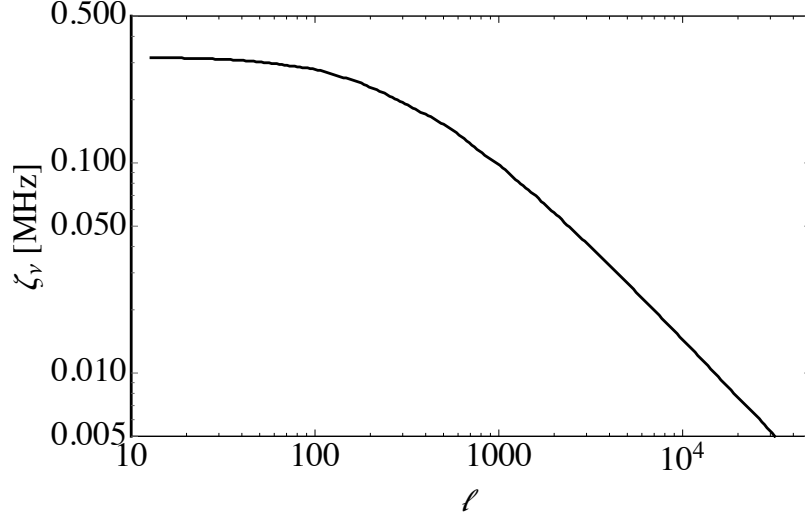


Figure 4.18: Correlation length ξ_ν as a function of ℓ , defined as the separation in frequency beyond which two redshift slices are correlated by less than $1/2$. This curve was calculated with an infinitely narrow bandwidth at $z = 50$, and for each ℓ the correlation would increase to match the value of the bandwidth if it is bigger than the ξ_ν in the plot.

widths $\Delta\nu$ of 1 MHz and 0.1 MHz. As argued above (Fig. 4.15) most of the signal comes from large- ℓ modes ($\ell \gtrsim 1000$), for which the correlation length $\xi_\nu < 0.1$ MHz, so in both cases we may assume that different redshift slices are completely uncorrelated. An observation of 21-cm fluctuations between 14 MHz ($z = 100$) and 45 MHz ($z = 30$) with frequency resolution $\Delta\nu$ would therefore have $N_{\text{slices}} \approx 30 \times 1 \text{ MHz} / \Delta\nu$ independent redshift slices.

The simplest analysis would consist in finding the best-fit f_{NL} for each redshift slice and coadd the estimators with inverse-variance weighting. This procedure is not optimal, however, as the secondary bispectrum (and by extension, the residual after subtraction of the best-estimate $B^{\text{sec},0}$) is a smooth function of redshift. The redshift dependence of the residuals $f_i = \Delta A_i$ can therefore be modeled by a linear combination of a few basis functions and depends on a few coefficients instead of N_{slices} independent amplitudes:

$$f_i(z) = \sum_{j=0}^{N_{\text{bases}}} f_{ij} P_j(z). \quad (4.96)$$

Several choices of basis functions could be made. We found that in the redshift range 30–100 the coefficients $A_i(z)$ could be fit to ~ 1 , 0.1, and 0.01 percent accuracy with third, fifth, or seventh-order polynomials in $\log(z)$, respectively. We assume that this will also

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hold for the residuals. We therefore adopt $P_j(z) = [\log(z/50)]^j$ for $j = 0$ to $N_{\text{bases}} = 3$ or 7 as our basis set. Our full model for the redshift-dependent bispectrum is therefore

$$B_{\ell_1\ell_2\ell_3}(z) = B_{\ell_1\ell_2\ell_3}^{\text{sec},0}(z) + f_{\text{NL}} b_{\ell_1\ell_2\ell_3}^{\text{prim}}(z) + \sum_{i=1}^4 \sum_{j=0}^{N_{\text{bases}}-1} f_{ij} b_{\ell_1\ell_2\ell_3}^{(ij)}(z), \quad (4.97)$$

where $b_{\ell_1\ell_2\ell_3}^{(ij)}(z) \equiv P_j(z) \times b_{\ell_1\ell_2\ell_3}^i(z)$.

We now fit simultaneously for f_{NL} and $4 \times N_{\text{bases}}$ nuisance parameters f_{ij} . Because we assume the redshift slices are uncorrelated (specifically, the noise is uncorrelated in different slices, but the signal is not), the total scalar product between two bispectra is simply obtained by summing the single-redshift scalar product over redshift slices:

$$(b^n, b^m) \equiv \sum_z (b^n, b^m)_z, \quad (4.98)$$

where $n \equiv (ij)$ is a generalized index, and $(b^n, b^m)_z$ is the scalar product of two bispectra at redshift z defined in Eq. (4.86). The usual Fisher analysis leads to $\sigma_{f_{\text{NL}}}^2 = (F^{-1})_{00}$, where the $(1 + 4N_{\text{bases}}) \times (1 + 4N_{\text{bases}})$ Fisher matrix F_{nm} is now defined from the total scalar product (4.98). We find that using seventh-order instead of third-order polynomials degrades the SNR by no more than $\sim 20\%$. The final results we quote are obtained using third-order polynomials.

Our final results are shown in Table 4.4, where we quote the minimum f_{NL} detectable for $f_{\text{sky}} = 1$ and $\ell_{\text{max}} = 10^5$ for two different bandwidths ($\Delta\nu = 1$ and 0.1 MHz). For $f_{\text{sky}} < 1$ all the results scale as $\sigma_{f_{\text{NL}}} \propto f_{\text{sky}}^{-1}$.

In summary, with a bandwidth of 1 MHz we could cross the $f_{\text{NL}} = O(1)$ threshold, enabling us to rule out a big class of models of inflation if no PNG is detected. Increasing the frequency resolution to 0.1 MHz the numbers improve to $f_{\text{NL}} \sim \text{few } 10^{-2}$, which would be close to the ultimate limit of the consistency relation ($f_{\text{NL}} \sim n_s - 1$), and hence should be present even in the simplest model of inflation.

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PNG type	$\sigma_{f_{\text{NL}}} (1 \text{ MHz})$	$\sigma_{f_{\text{NL}}} (0.1 \text{ MHz})$
Local	0.12	0.03
Equilateral	0.39	0.04
Orthogonal	0.29	0.03
$J = 1$	1.1	0.1
$J = 2$	0.33	0.05
$J = 3$	0.85	0.09

Table 4.4: Minimum f_{NL} detectable integrating all redshift slices between $z = 30$ and $z = 100$ for $f_{\text{sky}} = 1$. In the central column we show the result for a bandwidth of $\Delta\nu = 1$ MHz and in the right column for $\Delta\nu = 0.1$ MHz.

4.3.4 Conclusions

Now that the information from the CMB on non-gaussianity has been almost fully mined, it is time to consider other potential data sets. Intensity fluctuations in the 21-cm line during the dark ages offer a window into yet unexplored times and scales, and a promising future probe of PNGs.

The technical challenges that need to be overcome before the required experiments see the light of day are daunting. Because of atmospheric attenuation it would require an observatory on the Moon. Even then, care should be taken with intense Galactic foreground emission. Nevertheless, this is not an impossible task.

An additional issue is that the 21-cm signal is intrinsically highly non-gaussian, due to non-linear gravitational growth, and the non-linear mapping between brightness temperature and the underlying density field. In this work we have, for the first time, addressed this issue with a rigorous Fisher analysis approach, assuming cosmic-variance limited experiments with a finite angular and frequency resolution. We have shown that for a single redshift slice the secondary bispectrum is significantly degenerate with the primordial one, which results in a noticeable decrease of the forecasted signal-to-noise ratio (SNR) for PNGs. This contrasts with the results of previous work, where this degeneracy was either neglected when forecasting the SNR,¹⁷⁷ or where it was claimed to be weak.¹⁷⁶ We then co-added the information of independent redshift slices while enforcing a smooth variation of the secondary bispectrum amplitudes with redshift.

For a full-sky experiment with $\Delta\nu = 0.1$ MHz and 0.1-arcminute resolution, we forecast

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a sensitivity $\sigma_{f_{\text{NL}}^{\text{local}}} \approx 0.03$, which would enable us to check the famous inflationary consistency relation. We also forecast $\sigma_{f_{\text{equil}}} \approx 0.04$, $\sigma_{f_{\text{ortho}}} \approx 0.04$, $f^{J=1} \approx 0.1$, $f^{J=2} \approx 0.05$, and $f^{J=3} \approx 0.09$. Measurements of 21-cm fluctuations therefore have the potential to significantly improve upon cosmic-variance-limited CMB bounds.

4.3.5 Extending to the Cosmic Collider

In realistic inflation models, besides one effectively-massless scalar field driving inflation, there is a vast landscape of heavy fields. Although classically these heavy fields are not very important (except when excited by sharp features in the model), their quantum fluctuations leave distinctive signatures in the density perturbations. These fluctuations are most appreciable when the masses of the fields are of the same order as the Hubble scale H of the inflationary background or less. For this reason, this class of models is called Quasi-single-field (QSF) inflation models.^{190–204} Fields with these masses may already be present in the spectrum of a UV-completed unification theory, such as the KK spectrum and stringy states in string theory;¹⁹⁶ they may arise from fields that are originally light, but their mass gets uplifted by loop corrections in the inflationary background^{205,206} or by coupling to the background curvature. The presence of supersymmetry can also provide a natural mechanism to stabilize scalar fields with mass of order H .¹⁹² In addition, it has been suggested that particles with mass somewhat heavier than $\mathcal{O}(H)$ may be used as the “primordial standard clocks”^{207–209} to track the evolution of the scale factor $a(t)$ of any time-dependent background, providing direct evidence for either inflation or alternative scenarios. As such, these heavy fields could provide a wealth of information about fundamental physics and our primordial Universe.

Interestingly, heavy fields imprint potentially observable distinctive signatures in the primordial non-Gaussianities^{190–193,195,196} that are not captured in the low-energy effective theories of single-field inflation models. In particular, Arkani-Hamed and Maldacena derived a general result¹⁹⁶ showing that the entire particle spectrum, including the mass and spin, is reflected in the scaling behavior of various soft limits of primordial non-Gaussianities. Moreover, the spin of particles coupled to the inflaton can be geometrically disentangled with 3D surveys, by studying the trispectrum.^{202,210–212} It is likely that these are the

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highest mass scales directly observable⁹ in nature and provides a strong motivation to fully explore the observability of such signatures in the data. In this sense inflation has been referred to as a “cosmological collider experiment”.

In this subsection I will study what constraints can be placed on the presence of heavy particles during inflation, considering future cosmological surveys of the 21-cm field. For a large class of models, one can infer the mass of the heavy field from the power of the momentum ratio between the long and short mode of the three-point function. For light fields in the mass spectrum this power is a real number, with the power-laws between those of the local- and equilateral-type non-Gaussianities, behaving as “intermediate non-Gaussianities”. For heavier fields in the spectrum, however, the power becomes a complex number, which results in oscillatory signals in the bispectrum, with the “clock signal” being the oscillatory component of this bispectrum. Additionally, non-zero spins generate an additional dependence on the angle between the two modes, creating an incredibly rich phenomenology.^{196,214}

We define the shape $S(k_1, k_2, k_3)$ of the three-point function as

$$\langle \zeta^3 \rangle \equiv (2\pi)^3 \delta_D(\mathbf{k}_{123}) \frac{A^2}{(k_1 k_2 k_3)^2} S(k_1, k_2, k_3). \quad (4.99)$$

These behaviors have been shown to lead to a characteristic scaling of the squeezed limit of the bispectrum given by

$$S_{\text{squeezed}} \propto \left(\frac{k_{\text{long}}}{k_{\text{short}}} \right)^{1/2 \pm i\mu}. \quad (4.100)$$

This form reproduces the power-law of the squeezed limit of the local shape when $m/H \approx 0$ and interpolates between the local and equilateral shapes up to $m/H = 3/2$. For higher masses, it becomes an oscillatory function of $k_{\text{long}}/k_{\text{short}}$. For compactness we denote $\nu \equiv i\mu$ for imaginary μ . Motivated by this behavior, we therefore propose a template of the form

$$S^{\text{clock}}(k_1, k_2, k_3) = f_{\text{NL}} \frac{3^{7/2}}{2} A(\alpha_{123}) (\alpha_{123})^{-1/2} \times \sin \left(\mu \ln \left(\frac{\alpha_{123}}{2} \right) + \delta_\mu \right) + 2 \text{perm}, \quad (4.101)$$

where $\alpha_{123} = \frac{k_1 + k_2}{k_3}$ and where δ_μ is a calculable but model-dependent phase. Here $A(\alpha)$ is a window function meant to remove equilateral contributions. We will consider two different

⁹In models that temporarily break scale-invariance, it is also possible that much heavier fields can be excited non-adiabatically and leave different signatures in the density perturbations.^{207, 208, 213}

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window functions, a smooth generalized Gaussian of the form

$$A_G(\alpha_{123}) = 1 - \exp^{-\left(\frac{\alpha_{123}-1}{a}\right)^b}, \quad (4.102)$$

and a sharp cutoff

$$A_H(\alpha_{123}) = \mathcal{H}(\alpha_{123} - \alpha_0), \quad (4.103)$$

where $\mathcal{H}$ is the Heaviside step function. For details about the choice of the window function and its parameters a, b or α_0 the reader is encouraged to visit Ref.²¹⁵

Instead here we will focus on the effect of the secondary nongaussianities mentioned above. We define the signal-to-noise-ratio (SNR) degradation as the amount of signal lost with respect to the case without secondaries, i.e. $\sqrt{F_{00}^{-1}F_{00}} - 1$, assuming the subindex 0 corresponds to f_{NL} and the Fisher matrix F_{ij} is computed using $B_{\delta T}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$ and $P_{\delta T}(\mathbf{k})$ at $z = 50$. We show a histogram of the degradations in signal to noise when adding each of the 21 independent shapes in Figure 4.19 for the μ case, and in Fig. 4.20 for the ν case. We find that most secondary shapes are orthogonal to the μ primordial shapes, especially for higher μ values, due to more-rapid oscillations. In the ν case, however, there is significant overlap between several secondaries and the primordial signal, which is to be expected since the ν shapes interpolate between the local and equilateral templates, already found to be highly correlated with secondaries in Ref.²¹⁶ We show the total degradation in signal to noise in Tab. 4.5, where we have marginalized over the 21 secondary shapes simultaneously. All the degradation factors are $\mathcal{O}(1)$, and are particularly small for the oscillatory (μ) case. We note that this degradation is driven by a few highly-correlated shapes, so restricting the analysis to 4 linear combinations of the 21 shapes, as done originally in Ref.,²¹⁶ would not change results significantly. We thus conclude that secondaries would not strongly affect cosmic-collider forecasts.

4.4 Cosmic Anisotropies

One of the key principles of cosmology is the notion of isotropy and homogeneity—there is no preferred location nor preferred direction in the Universe. This, though, is violated by small primordial perturbations. Still, the prevailing single-field slow-roll model for the

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Model	SNR Deg.
$\mu = 0.3$	0.99
$\mu = 0.7$	0.75
$\mu = 1$	0.47
$\mu = 2$	0.56
$\nu = 0.3$	1.05
$\nu = 0.7$	1.92
$\nu = 1$	1.44

Table 4.5: Degradation in SNR computed as $\sqrt{F_{00}^{-1}F_{00}} - 1$, when considering the 21 secondary shapes of non-Gaussianities simultaneously.

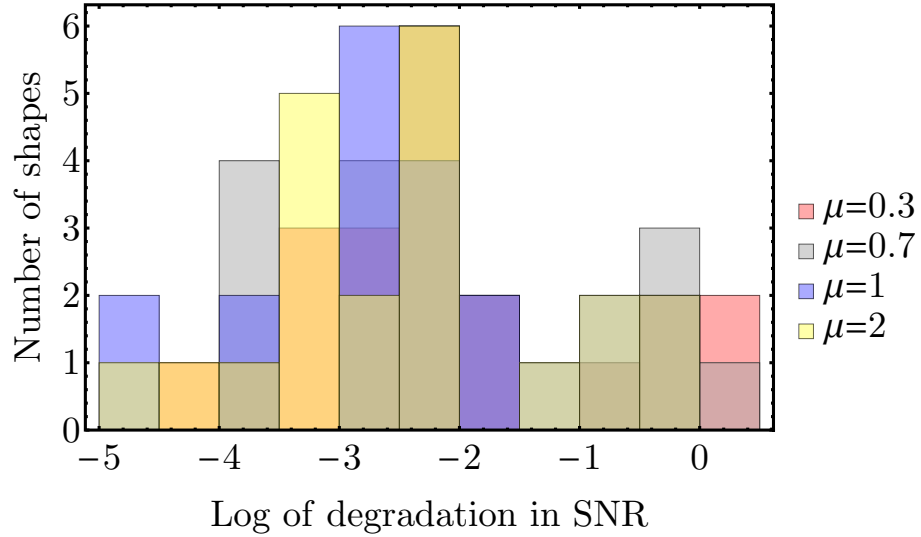


Figure 4.19: Histogram of the (natural logarithm of the) degradation in the SNR, defined as $\sqrt{F_{00}^{-1}F_{00}} - 1$, with each of the 21 secondary shapes. In red we show the $\mu = 0.3$ case, in gray $\mu = 0.7$, in blue $\mu = 1$, and in yellow $\mu = 2$.

origin of these perturbations predicts that isotropy and homogeneity should be preserved in a *statistical* sense. A significant detection of a deviation from statistical isotropy or homogeneity would falsify some of the simplest models of inflation, making it necessary to postulate new physics, such as non-scalar degrees of freedom. Moreover, it would open a window into the physics of the early Universe, shedding light upon the primordial degrees of freedom responsible for inflation. Departures from statistical isotropy and homogeneity can take different forms. We will study the two main cases, a dipolar power asymmetry and a quadrupolar asymmetry.

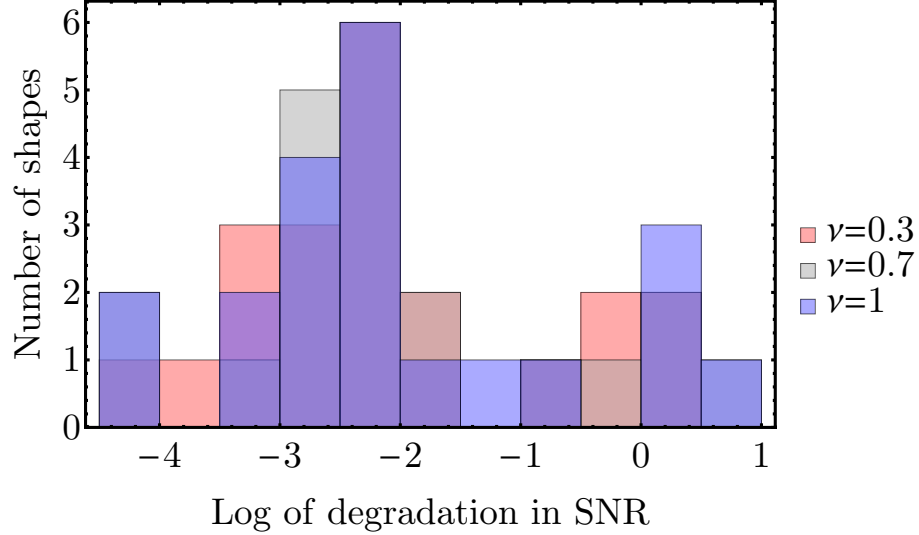


Figure 4.20: Degradation in the SNR, defined as $\sqrt{F_{00}^{-1}F_{00}} - 1$, with each of the 21 secondary shapes independently. In red we plot the $\nu = 0.3$ case, in gray $\nu = 0.7$, in blue $\nu = 1$.

4.4.1 Primordial dipolar asymmetry

The CMB measurements indicate that the microwave sky is almost perfectly gaussian and rotationally-symmetric, as predicted by single-field inflation. There are, however, a few observed anomalies that require explaining, most important of which is the dipolar asymmetry. First detected by WMAP,^{217–220} and later on confirmed by *Planck*,^{118,221–223} this anomaly can be phrased as a dipolar modulation of the CMB sky at large angular scales. When including smaller CMB angular scales, however, this anomaly seems to disappear completely. Moreover, study of the density of quasars at lower redshifts showed no evidence for a dipolar modulation at a corresponding angular scale of $k \sim 1 \text{ Mpc}^{-1}$.²²⁴ We will explore models for a dipole modulation in which the signal vanishes at $k = 1 \text{ Mpc}^{-1}$, being non-zero at smaller and bigger angular scales, as predicted by theoretical models.²²⁵

In general statistical anisotropy can be expressed by a curvature perturbation with a position-dependent dipolar modulation, written as

$$\zeta_{\mathbf{k}}(\mathbf{x}) = \zeta_{\mathbf{k}}^{\text{iso}} \left[1 + \sum_M A_{1M} f(k) Y_{1M}(\hat{n}) \right], \quad (4.104)$$

where $\mathbf{x} \equiv x\hat{n}$ with $x \equiv |\mathbf{x}|$. A constant $f(k) = 1$ was originally introduced to explain a $\sim 3\sigma$

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evidence of dipolar asymmetry in the CMB sky at very large scales ($\ell \lesssim 60$).^{118,221–223} This asymmetry can be expressed as an amplitude $A \sim 0.07$ in $T(\hat{n}) = T^{\text{iso}}(\hat{n})(1 + A\hat{n} \cdot \hat{p})$, with $\hat{p}$ denoting the preferred direction of the modulation, and T^{iso} being the usual temperature fluctuation. On the other hand, several detailed analyses of the CMB maps have also shown that such dipolar modulation is highly damped for $\ell \gtrsim 60$.^{118,221–223} This scaling seems to be consistent with a different constraint obtained by quasar abundances, leading to a vanishing dipolar asymmetry at $k \sim 1 \text{ Mpc}^{-1}$.²²⁴ We implement a heuristic model for such observationally-motivated scale dependence as a function, $f(k) = (1 - k/k_A)^n$ with $k_A \equiv 1 \text{ Mpc}^{-1}$, for $n = 1$ and 2 , in our parametrization of Eq. (4.104).

While the precise scale dependence is different for different models,^{225–229} we choose the following 2 shapes as proxies

$$f(k) = 1 - \frac{k}{k_A}, \quad \left(1 - \frac{k}{k_A}\right)^2. \quad (4.105)$$

These reconstruct the observed decaying shapes for $k < k_A (\equiv 1 \text{ Mpc}^{-1})$. On the other hand, on unobserved small scales as $k > k_A$, these grow larger, and with opposite signs.

We can define the mode-coupling matrix as

$$G_{\ell_1 \ell_2} = \frac{2}{\pi} \int_0^\infty k^2 dk P(k) f(k) \mathcal{T}_{\ell_1}(k, \nu) \mathcal{T}_{\ell_2}(k, \nu), \quad (4.106)$$

where the additional factor $f(k)$ is 1 in the usual case, and will have different values for alternative inflationary models. This is the standard general way to express the angular correlations; in the following section we will compute the angular power spectra for different models describing rotational asymmetries.

Figure 4.21 plots the correlations $G_{\ell, \ell} + G_{\ell+1, \ell+1}$ for the 2 different models for the function $f(k)$, taking $z = 30$ and $\Delta\nu = 1.0 \text{ MHz}$. We can confirm there that a dip is located at $\ell \sim \ell_A \equiv k_A x(z = 30) \simeq 1.3 \times 10^4$, with $x(z = 30) \simeq 13 \text{ Gpc}$ denoting the conformal distance to $z = 30$, as expected. The difference between these 2 models show up for $\ell \gtrsim \ell_A$ because of the difference of the signs and the different power-law exponent. We also notice that $G_{\ell, \ell} + G_{\ell+1, \ell+1}$ is substantially enhanced compared with the isotropic power spectrum C_ℓ , which we also show in Figure 4.21 for reference, for $\ell \gtrsim \ell_A$, as expected from Eq. (4.105). This results in a sharp rise in the signal-to-noise ratio for $\ell \gtrsim \ell_A$.

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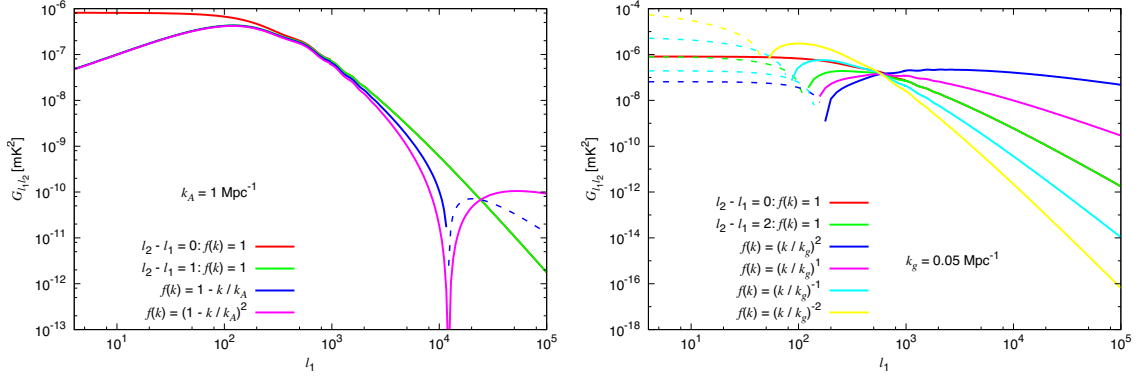


Figure 4.21: $G_{\ell,\ell} + G_{\ell+1,\ell+1}$ for A_{1M} (left panel) and $G_{\ell,\ell+2}$ for g_{2M} (right panel). Solid (dashed) lines describe positive (negative) values. We here adopt $z = 30$ and $\Delta\nu = 1.0 \text{ MHz}$. For the pivot scales, we choose $k_A = 1 \text{ Mpc}^{-1}$ and $k_g = 0.05 \text{ Mpc}^{-1}$, corresponding to $\ell_A \equiv k_A x(z = 30) \simeq 1.3 \times 10^4$ and $\ell_g \equiv k_g x(z = 30) \simeq 650$ in ℓ space. For comparison, we also show the isotropic power spectrum $C_\ell = G_{\ell\ell}^{f(k)=1}$ (purple), which acts as a cosmic variance in the Fisher matrix computation.

4.4.2 Primordial quadrupolar asymmetry

We now turn to study quadrupolar asymmetries, which can be generated by a wide range of inflation models involving anisotropic sources, such as vector fields. The signatures of these anomalies may be discovered in observables other than CMB. In this section we investigate the 21-cm temperature power spectrum generated by such anomalies.

When including a quadrupolar asymmetry, we can write the curvature power spectrum as

$$\langle \zeta_{\mathbf{k}_1} \zeta_{\mathbf{k}_2} \rangle = (2\pi)^3 P(k_1) \left[1 + \sum_M g_{2M} f(k_1) Y_{2M}(\hat{k}_1) \right] \delta^{(3)}(\mathbf{k}_1 + \mathbf{k}_2). \quad (4.107)$$

A non-vanishing g_{2M} arises in inflation models where the inflaton couples to a vector field with a non-zero vacuum expectation value, via $\mathcal{L} = -\frac{1}{4} I^2(\phi) F^2$.^{230–237} In these cases, the time dependence of the coupling function $I(\phi)$ determines the scale dependence of $f(k)$. The nearly scale-invariant spectrum, i.e., $f(k) = 1$, is realized by choosing $I(\phi) \propto a^{-2}$, with a denoting the scale factor.^{233,234,236} In other words, except for this case, $f(k)$ has

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a scale dependence. A nearly scale-invariant $f(k)$ is also generated in the solid inflation models.^{238,239} The magnitude of g_{2M} relies strongly on the model parameters. For vector-field models, g_{2M} is proportional to the ratio of the energy density of the vector field to that of the scalar field, ρ_A/ρ_ϕ .^{233,234} If the inflaton is identified to the pseudoscalar field, the coupling strength between the pseudoscalar and the vector field also affects g_{2M} .^{235–237} The data analysis with the *Planck* map gives upper bounds for the scale-invariant case $f(k) = 1$ of $|g_{2M}| \lesssim 10^{-2}$,^{118,240} leading to several constraints on the model parameters e.g., $\rho_A/\rho_\phi \lesssim 10^{-9}$.²³⁶

In what follows, we discuss the detectability of g_{2M} in a general model-independent way, and do not translate the results into any specific model parameters. To do so, we assume 5 different power-law shapes for $f(k)$,

$$f(k) = 1, \quad \left(\frac{k}{k_g}\right)^{\pm 1}, \quad \left(\frac{k}{k_g}\right)^{\pm 2}, \quad (4.108)$$

with $k_g = 0.05 \text{ Mpc}^{-1}$ being the pivot scale adopted in the *Planck* collaboration.¹¹⁸

The right panel of Fig. 4.21 plots the off-diagonal correlations $G_{\ell, \ell+2}$ generated from the 5 different models in Eq. (4.108), for $z = 30$ and $\Delta\nu = 1.0 \text{ MHz}$. It is obvious in this Figure that all the lines intersect each other at the multipole corresponding to the pivot scale, $\ell_g \equiv k_g x(z = 30) \simeq 650$, and they are tilted depending on their spectral indices, respectively. We can also observe sign changes happen at $\ell \sim 100$.

4.4.3 Results and Conclusions

Here we forecast measurements of rotational asymmetries with the 21-cm power spectrum. We compute $G_{\ell_1 \ell_2}$ for the 7 different models of the function $f(k)$ (2 for the dipolar and 5 for the quadrupolar asymmetry), and then calculate the forecasted detectability of the coefficients A_{1M} and g_{2M} , via a Fisher matrix analysis, including three different instrumental noise configurations.

First, we will use specifications consistent with those of the Square Kilometre Array, currently under construction in South Africa. We will then assume a Futuristic Radio Array (SKA-fut), as an example for a futuristic, but still Earth-based, experiment. Finally, we

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will show results for the cosmic-variance-limited (CVL) case, where $C_\ell^N = 0$.

For the SKA case we take a baseline of 6 km, a coverage fraction of $f_{\text{cover}} = 0.02$, and a time of data collection of 5 whole years. In the SKA-fut case we consider an increased baseline of $D = 100$ km, a coverage fraction of 0.2, and 10 whole years of observations. The baseline of this SKA-fut case would be large enough to reach $\ell \gtrsim 10^4$ at redshift $z \lesssim 50$, enabling us to prove the very small scales anisotropy predicted by some models.²²⁵

We show, in Fig. 4.22, the ℓ_{max} dependence of $\sigma(A_{1M})$ and $\sigma(g_{2M})$ for $z = 30$ and $\Delta\nu = 0.1$ MHz. As seen in this figure, the sensitivities to A_{1M} and g_{2M} in SKA-fut get better as ℓ_{max} increases up to $\ell_{\text{max}} \sim 10^4$, after where the sensitivity plateaus. The sensitivities for the SKA-fut and CVL cases are comparable for $\ell_{\text{max}} \lesssim 10^3$, departing after that due to the noise becoming dominant. The SKA-fut sensitivities actually exceed the SKA ones by ~ 4 orders of magnitude; thus, highly accurate measurements are possible with SKA-fut, being the constraints comparable to, or better than, *Planck* 2015.^{118,240} In the 2 top panels for $\sigma(A_{1M})$, we notice that the sensitivities in the CVL cases drastically increase beyond the pivot scale $\ell_A \simeq 1.3 \times 10^4$ as expected from Fig. 4.13. Likewise, in the blue-tilted cases for $\sigma(g_{2M})$ ($f(k) \propto k^{1,2}$) the error bars decline drastically at around $\ell_g \simeq 650$, corresponding with the crossing point in Figure 4.13, as seen in the two bottom panels. Moreover, as in the previous sections we can employ tomography, i.e., to coadd information from different redshift slices. This increases sensitivity by a factor of ~ 5 with respect to a fixed redshift slice, as shown in Ref.²⁶

4.5 Scale-Invariance of the Power Spectrum

The CMB has allowed us to very precisely measure a narrow band of the universe at redshift $z = 1100$, as described in Chapter 3, whereas galaxy surveys map the local universe up to $z \sim O(1)$. The region between these two probes has very valuable cosmological information, which can be probed with the 21-cm hydrogen line.^{136,241}

In order to reach scales beyond $k \sim 1 \text{ Mpc}^{-1}$ one can observe at the dark ages, where the matter distribution of the universe remains linear down to much smaller scales,¹³⁶ enabling access to a tremendous wealth of cosmological information. This era, however,

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will be extremely difficult to probe, due to the increase in Galactic synchrotron emission at low frequencies, as well as the atmospheric absorption of radio frequencies. LOFAR¹⁰ will access the edge of the frequency range required for this task, although it is not likely to reach enough sensitivity to observe primordial fluctuations. Instead, this will require building an interferometric array on the far side of the moon.¹³⁵ The proposed DARE¹¹ satellite will serve as a stepping stone to explore the end of the dark ages, although to constrain the runnings to any significant level one will need a large moon-based interferometer, which we will model as different FFTT long-baseline arrays.

4.5.1 Formalism

As opposed to the angular power spectrum noise we employed above we use a k -space noise here. Given an antenna array with a baseline D_{base} uniformly covered up to a fraction $f_{\text{cover}} \leq 1$, observing for a time t_o , we can write the instrumental-noise power spectrum in k -space as^{23, 242}

$$P_{21}^N(z) = \frac{\pi T_{\text{sys}}^2}{t_o f_{\text{cover}}^2} \chi^2(z) y_\nu(z) \frac{\lambda^2(z)}{D_{\text{base}}^2}, \quad (4.109)$$

where $\lambda(z)$ is the 21-cm transition wavelength at redshift z , $y_\nu(z) = 18.5 \sqrt{(1+z)}/10$ Mpc/MHz is a conversion function from frequency ν to $k_{\parallel}$.

We take a FFTT-like experiment,²⁴² with $f_{\text{cover}} = 1$ and a variable baseline D_{base} , observing 2π steradians of the sky (so $f_{\text{sky}} = 0.5$). For these experiments we will employ a pivot scale of $k_* = 0.1 \text{ Mpc}^{-1}$, to help break the degeneracy between α_s and β_s arising from the augmented observable k range with respect to CMB experiments.

The baseline of each array will determine the maximum perpendicular wavenumber it can observe, calculated as

$$k_{\perp}^{\text{max}} = \frac{2\pi D_{\text{base}}}{\chi(z)\lambda(z)} \approx \frac{2 \text{ Mpc}^{-1}}{(1+z) + 1.1 \sqrt{1+z}} \times \frac{D_{\text{base}}}{\text{km}}. \quad (4.110)$$

For simplicity we choose a matching line-of-sight resolution for the FFTT experiments to have a single maximum k_{max} that will vary with D_{base} , although in practice line-of-sight

¹⁰<http://www.lofar.org/>

¹¹<http://lunar.colorado.edu/dare/>

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resolution might be easier to achieve through finer frequency binning. We also assume that astrophysical foregrounds will cut off line-of-sight wavenumbers smaller than²⁴³

$$k_{\parallel}^{\min} \approx \frac{2\pi}{y_{\nu} \Delta\nu}, \quad (4.111)$$

where $\Delta\nu$ is the total bandwidth probed by our experiment, and is $\Delta\nu \approx 50$ MHz for the dark ages ($z = 20 - 100$), which translates into a minimum wavenumber $k_{\parallel}^{\min} \approx 10^{-2}$ Mpc⁻¹. Our results do not depend sensitively on this cutoff, so we set both $k_{\parallel}^{\min}$ and $k_{\perp}^{\min}$ to $k_{\min} = 10^{-2}$.

Similarly to the CMB case, the 21-cm power spectrum $P_{21}(k)$ depends on the primordial parameters n_s , A_s , α_s , and β_s only through the matter power spectrum $P_{\delta}(k)$. Unfortunately, the redshift functions $\mathcal{A}(z)$ and $T_{21}(z)$ depend on the rest of Λ CDM parameters. For simplicity we will perform an order-of-magnitude forecast of the errors only on the primordial parameters, ignoring ω_b , ω_c , H_0 , and the reionization parameters. This is a vast oversimplification, adopted because the primordial parameters should be moderately decoupled from the rest of Λ CDM.

Fisher Formalism in k -space

To forecast we separate the available comoving volume in redshift bins, chosen to be small enough that all redshift-dependent parameters are constant within each bin. We then compute the Fisher matrix for one of these slices, centered at redshift z_i , as²⁴⁴

$$F_{ab}^{(i)} = \frac{f_{\text{sky}}}{2} \frac{\text{Vol}_i}{(2\pi)^3} \int_{k_{\min}}^{k_{\max}} dk (2\pi k^2) \int_{-1}^1 d\mu \frac{\frac{\partial P_{21}(\mathbf{k}, z)}{\partial p_a} \frac{\partial P_{21}(\mathbf{k}, z)}{\partial p_b}}{[P_{21}(\mathbf{k}, z) + P_{21}^N(z)]^2}, \quad (4.112)$$

where Vol_i is the comoving volume of the slice, and $p_a = \{A_s, n_s, \alpha_s, \beta_s\}$. We will then incorporate the information from all the redshift bins by just adding the Fisher matrices, i.e.,

$$F_{ab} = \sum_i F_{ab}^{(i)}. \quad (4.113)$$

4.5.2 Results

During the dark ages the spin temperature is coupled to the gas temperature through collisions.¹³⁶ This makes $\bar{T}_{21}$ negative, causing absorption of CMB photons at radio frequencies. The redshift range in which $\bar{T}_{21}, \mathcal{A} < 0$ is $z \sim 20 - 200$. During this range $\bar{T}_{21}$, as well as $\mathcal{A}(z)$, including perturbations to gas temperature, can be found in Ref.²⁵ We note that the $z \sim 20$ range might be contaminated by astrophysical effects. In any case our results are not altered dramatically by changes in the starting redshift.

We can now compute the uncertainty in α_s , when marginalizing over A_s and n_s , for a FFTT-like experiment. Likewise for β_s , marginalizing in this case over α_s as well. We show these errors in Fig. 4.23, where it can be seen that an experiment with a baseline of $D_{\text{base}} \sim 5$ km could confirm the slow-roll prediction for α_s , whereas to detect $\beta_s \sim (1 - n_s)^3$ one would need $D_{\text{base}} = O(100)$ km. With the goal of detecting a non-vanishing β_s we propose a 300-km perfectly covered array, which we just label FFTT₃₀₀. We show the results for this very-futuristic array in Table 4.6. More details can be found in Ref.²²

Array	$\sigma(A_s)$	$\sigma(n_s)$	$\sigma(\alpha_s)$	$\sigma(\beta_s)$
FFTT ₃₀₀	1.4×10^{-15}	4.1×10^{-6}	7.0×10^{-6}	1.2×10^{-5}

Table 4.6: $1 - \sigma$ uncertainties in the scalar amplitude A_s , tilt n_s , running α_s , and second running β_s for different 21-cm arrays. For A_s , n_s , and α_s we only marginalize over A_s and n_s , whereas for β_s we marginalize over α_s as well, and we do not marginalize over nuisance parameters. Here we have increased the pivot scale to $k_* = 0.1 \text{ Mpc}^{-1}$.

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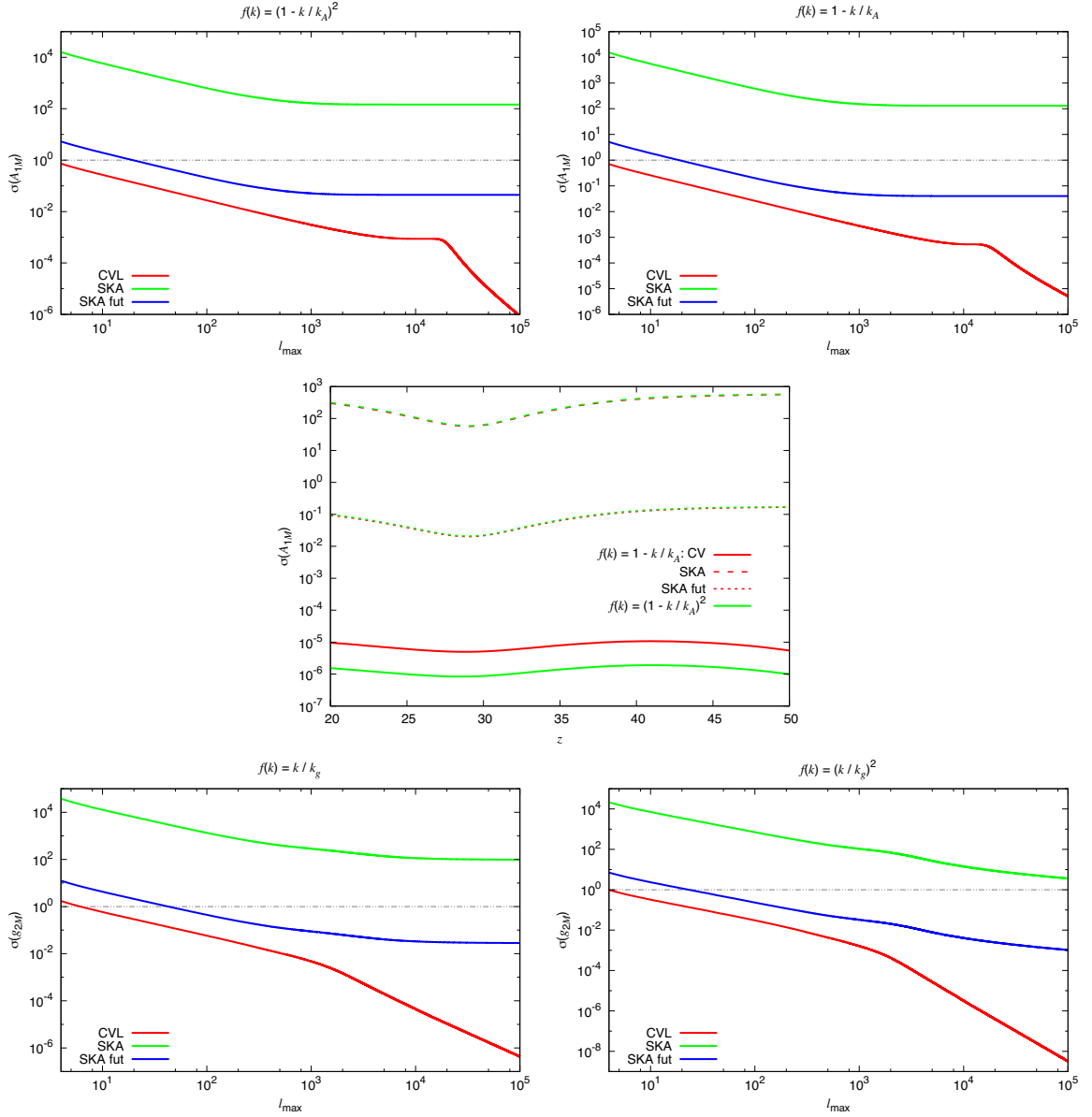


Figure 4.22: Expected 1σ errors on A_{1M} and g_{2M} for a single redshift slice at $z = 30$ and $\Delta\nu = 0.1$ MHz with $\ell_{\min} = 2$. We show the results for $f(k) = (1 - k/k_A)^{1,2}$ for the dipolar case, and for $f(k) = (k/k_g)^{0,1,2}$ for the quadrupolar case. For comparison, we draw the 1σ errors obtained from the *Planck* 2015 bounds, i.e., $\sigma(A_{1M}) \sim \sigma(g_{2M}) \sim 0.01$.^{118,240}

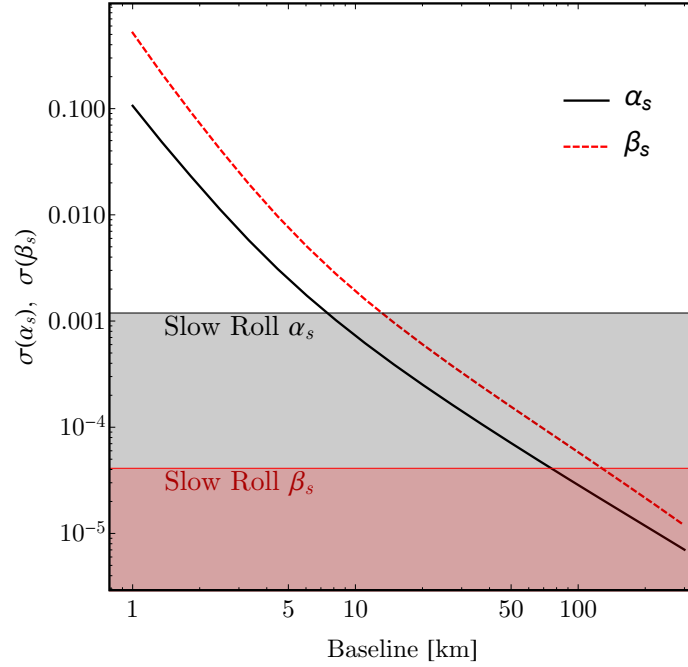


Figure 4.23: $1\text{-}\sigma$ uncertainties in α_s and β_s for an experiment measuring the dark ages with a dense core ($f_{\text{cover}} = 1$) extending for a baseline D_{base} in km. We show the slow-roll-inflation prediction for α_s in gray and for β_s in red, and in dashed green we show the specifications for the proposed FFTT. In this figure we have marginalized over A_s and n_s to calculate $\sigma(\alpha_s)$, and also over α_s to compute $\sigma(\beta_s)$, and not marginalized over nuisance parameters. Here we have chosen a pivot scale of $k_* = 0.1 \text{ Mpc}^{-1}$.

“Is such a thing even possible? Yes it is.”

— Giorgio Tsoukalos

Chapter 5

Gravitational Waves

Einstein’s theory of general relativity predicts that two massive objects orbiting each other will emit radiation, therefore inspiraling and eventually merging. This radiation would be in the form of gravitational waves, with a characteristic spectrum given by the orbital frequency of the binary.

Ever since Einstein himself predicted gravitational waves there has been a strong effort by the community to detect them. However, only the waves emitted by extremely massive objects, orbiting very close to Earth, are within our reach. It can be argued that the first “indirect” detection of gravitational waves was made by Hulse, Taylor, and Weisberg, who studied the binary pulsar system PSR B1913+16.²⁴⁵ Over thirty years they carefully studied the pulses from this system, and found that the Doppler shift induced by the orbit are accelerating, due to the inspiral of the system. The gravitational waves emitted, however, are orders of magnitude fainter than what could be detected from Earth.

It has taken nearly a century to directly detect the first gravitational waves (GWs). The LIGO collaboration recently detected the merger of two pairs of black holes (BHs) of tens of solar masses.^{246–248} These kinds of events have the best chances of being detected by an interferometer like LIGO, since most of the energy in GWs is emitted in the last few orbits before the merger, where the BHs orbit with a frequency of ~ 100 Hz, within the LIGO band.

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In this chapter I will review the GW signatures of the merger of BHs, as well as its applications for the search of dark matter in the form of primordial black holes.

5.1 General Formalism

Here we will discuss the coalescence of two black holes. In general, we can split this event in three phases: *(i)* the inspiral, where the two objects are far away and can be described through a post-newtonian approximation, and the emission is mostly caused by their quadrupole; *(ii)* the merger, starting when the objects cross their innermost stable circular orbit (ISCO), and a burst of radiation is emitted; and *(iii)* the ringdown of the resulting black hole into a stable Kerr solution, where the first few normal modes are excited and emit gravitational waves as they relax.

Let us now review this process.

5.1.1 Energetics

We can define the chirp mass as

$$M_c = \frac{(m_1 \cdot m_2)^{3/5}}{m_{\text{tot}}^{1/5}}, \quad (5.1)$$

where m_1 and m_2 are the masses of the two objects and m_{tot} is their sum. This will prove to be an useful quantity.

Inspiral

Following Ref.,²⁴⁹ the spectral energy density at the source of the emitted GWs during the inspiral of a circularized orbit is

$$\left. \frac{dE}{df_s} \right|_{\text{inspiral}} = \frac{1}{3} \left(\frac{\pi^2}{f_s} \right)^{1/3} M_c^{5/3}, \quad (5.2)$$

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where f_s is the GW frequency at the source (related to the observed frequency f_{obs} as $f_{\text{obs}} = f_s/(1+z)$).

Merger

The frequency at the end of the inspiral and the beginning of the merger phase is (at the source),

$$f_{\text{merger}}(m_1, m_2) = 0.02/m_{\text{tot}}, \quad (5.3)$$

and the merger phase lasts

$$\tau_{\text{merger}}(m_1, m_2) = 14.7\text{s} \times \frac{m_{\text{tot}}}{10^5 M_{\odot}}. \quad (5.4)$$

During the merger phase, the spectral energy density is given by

$$\left. \frac{dE}{df_s} \right|_{\text{merger}} = \frac{16\mu^2\epsilon}{m_{\text{tot}}(f_{\text{ringdown}} - f_{\text{merger}})}, \quad (5.5)$$

where μ is the reduced mass and ϵ is the fraction of the energy in the initial BH binary that is emitted in GWs during the merger phase, and f_{ringdown} is defined below. Throughout we take $\epsilon = 0.04$, in agreement with the uncertainties of the GW150914 event.³³

Ringdown

The perturbed Kerr black hole resulting from the merger oscillates with a frequency of quasi-normal ringdown of

$$f_{\text{ringdown}}(m_1, m_2) = \frac{(1 - 0.63(1 - \alpha)^{3/10})}{2\pi m_{\text{tot}}}, \quad (5.6)$$

where the dimensionless spin α of the final BH is simply $\alpha = \frac{cS}{Gm_f^2}$, assuming $m_f \simeq m_{\text{tot}}$.

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5.1.2 Amplitude and Noise

Given an energy flux dE/df_s per unit frequency we can compute the observed strain amplitude as

$$h_c(f_{\text{obs}}) = \sqrt{2} \frac{1+z}{\pi d_L(z)} \sqrt{\frac{dE}{df_s}}, \quad (5.7)$$

where for all practical purposes we include the inspiral and merger phases, but ignore the contribution from the ringdown, which is short and characterized by a very fast reduction of the h_c with time (even in the high signal-to-noise GW150914 event).

As an example, in Fig. 5.1, we show the evolution of the strain amplitude over frequency and time during the last second of the coalescence of two $30 M_\odot$ BHs following a circularized orbit.

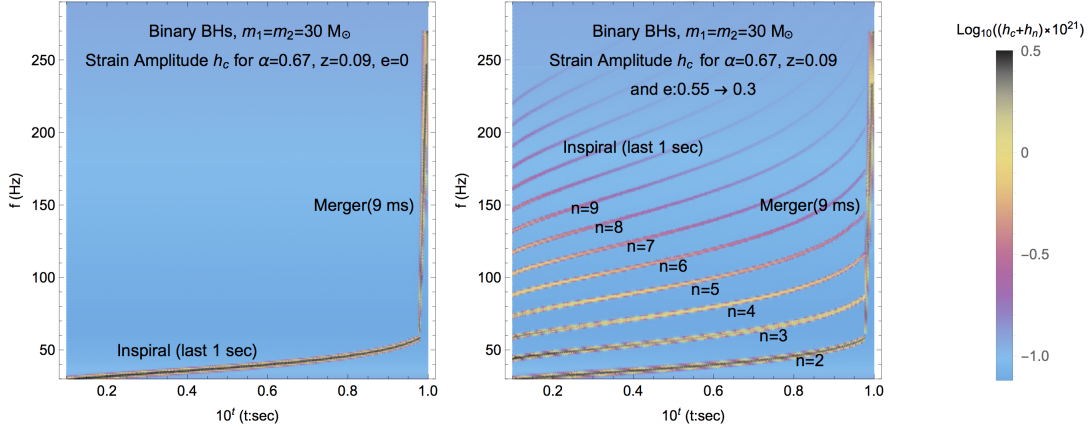


Figure 5.1: Evolution over the last second of the amplitude of the signal strain, superimposed on the noise as expected with the LIGO design sensitivity. Time t here is between -1 to 0 seconds. *Left*: assuming no significant remaining eccentricity over that last one second. *Right*: assuming that during the last one second there is a remaining eccentricity that evolves from 0.55 to 0.3, resulting in the presence of higher GW modes than just the quadrupole ($n=2$) on. The different mode amplitudes also evolve with time.

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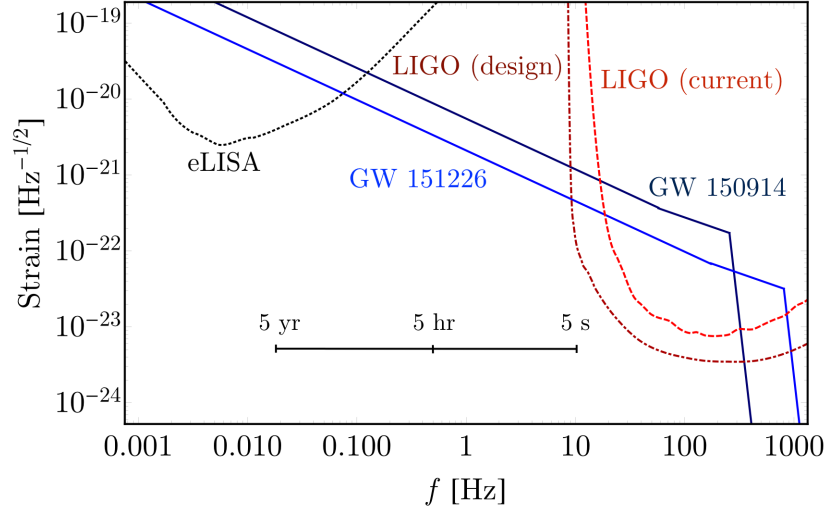


Figure 5.2: Noise and strain for gravitational-wave observations as a function of frequency f . In blue solid I show the characteristic strain h_c for both GW events detected by LIGO so far. In red-dashed I show the current LIGO sensitivity, computed as $\sqrt{S_h}$, whereas the red-dash-dotted line is the design sensitivity of LIGO. The black-dotted line shows the futuristic eLISA satellite sensitivity.

Notice that this characteristic strain h_c can be related to the Fourier amplitude via

$$h_c(f) = 2f\tilde{h}(f). \quad (5.8)$$

Given a stochastic signal (such as noise) we can compute its power spectrum as

$$\langle \tilde{h}(f)\tilde{h}(f') \rangle = \frac{1}{2}S_h(f)\delta_D(f - f'). \quad (5.9)$$

To find whether some given GW experiment can detect a signal we define the signal-to-noise ratio ρ as

$$\rho^2 \equiv \frac{4}{5} \int_0^\infty df \frac{|\tilde{h}(f)|^2}{S_h(f)}, \quad (5.10)$$

where the $1/5$ factor accounts from accounting for non-optimal orientation of the detector, and S_h is the noise power spectrum of the interferometer.

For reference we show the noise (computed as $\sqrt{S_h}$) from LIGO (both current and the design sensitivity), and from the futuristic space-based eLISA, in Fig. 5.2. We also show the strain (h_c) from the two BH-BH mergers observed by LIGO so far. It is interesting that

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LIGO can detect the last ~ 5 seconds before the merger, whereas eLISA can observe the system for $\sim$ years prior to their merger.

5.2 Search for Primordial Black Holes

The nature of the dark matter (DM) is one of the most longstanding and puzzling questions in physics. Cosmological measurements have now determined with exquisite precision the abundance of DM,^{1,104} and from both observations and numerical simulations we know quite a bit about its distribution in Galactic halos. Still, the nature of the DM remains a mystery. Given the efficacy with which weakly-interacting massive particles—for many years the favored particle-theory explanation—have eluded detection, it may be warranted to consider other possibilities for DM. Primordial black holes (PBHs) are one such possibility.^{30,31,250,251}

In this section we will use the knowledge from gravitational waves from above, as well as the early results from the LIGO experiment, to find out whether primordial black holes (PBHs) can be the dark matter.

5.2.1 Formation

If they are to be the dark matter, PBHs could have formed in the primordial universe from very-dense pockets of plasma that collapsed under their own gravitational pull. The scales in which stellar-mass PBHs were formed are orders of magnitude beyond the reach of any cosmological observable. However, if the inflationary dynamics were fully determined by a single field, one could extract information about the potential $V(\phi)$ at the smallest scales from $V(\phi_*)$ at the pivot scale (and its derivatives) by extrapolation. With this idea in mind we attempt to find what are the values of the running α_s and second running β_s , as defined in Chapter 3, which allow PBH formation.

The formation process of the PBHs is poorly understood,²⁵² so we will not attempt to model it. For several reviews see for example Refs.^{30,253,254} Here we will assume that PBHs form at the scale at which the fluctuations become of order unity. It is clear that any positive running, if not compensated by a negative running of higher order, will create

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enough power in some small-enough scale to have $\Delta_s^2(k) = 1$. Nonetheless, we will require that the mass of the formed PBHs is larger than $\sim 10^{15}$ gr, to prevent PBH evaporation before $z = 0$, which sets a limit on the smallest scale where PBHs can form of $k_{\text{pbh}} = 10^{15} \text{ Mpc}^{-1}$. We compute the $\alpha_s - \beta_s$ range in parameter space that produces enough power in scales $k < k_{\text{pbh}}$ to generate PBHs and show it in Fig. 3.11, along with the constraints from future experiments. In order to produce PBHs of $\sim 30 M_\odot$, as suggested in Ref.³² to be the dark matter, the relevant scale is $k \sim 10^5 \text{ Mpc}^{-1}$, forcing the second running to be as large as $\beta_s \approx 0.03$, which will be tested at high significance by the S4 CMB experiment.

5.2.2 Rate of GW events

Here we consider whether part of the BH-BH events detected by LIGO²⁴⁸ could be PBHs. There may be a window for PBHs to be DM if the BH mass is in the range $20 M_\odot \lesssim M \lesssim 100 M_\odot$.^{254,255} Lower masses are excluded by microlensing surveys,^{36,256,257} and pulsar timing arrays.²⁵⁸ Higher masses would disrupt wide binaries.^{255,259,260}

We will study different constraints to PBHs from different sources in the next Section. Here we instead focus on what can GWs teach us about this possibility.

In any galactic halo, there is a chance two BHs will undergo a hard scatter, lose energy to a soft gravitational wave (GW) burst and become gravitationally bound. This BH binary will merge via emission of GWs in less than a Hubble time. Note that this analysis differs from Refs.,^{251,261} where the binaries are formed very early on and take a Hubble time to merge. Below we first estimate roughly the rate of such mergers and then present the results of more detailed calculations. We discuss uncertainties in the calculation and some possible ways to distinguish PBHs from BH binaries from more traditional astrophysical sources.

Consider two PBHs approaching each other on a hyperbolic orbit with some impact parameter and relative velocity v_{pbh} . As the PBHs near each other, they produce a time-varying quadrupole moment and thus GW emission. The PBH pair becomes gravitationally bound if the GW emission exceeds the initial kinetic energy. The cross section for this

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process is,²⁶²

$$\begin{aligned}\sigma &= 2^{3/7} \pi \left(\frac{85\pi}{6\sqrt{2}} \right)^{2/7} R_s^2 \left(\frac{v_{\text{pbh}}}{c} \right)^{-18/7} \\ &= 1.37 \times 10^{-14} M_{30}^2 v_{\text{pbh}-200}^{-18/7} \text{pc}^2,\end{aligned}\tag{5.11}$$

where $R_s = 2GM_{\text{pbh}}/c^2$ is the Schwarzschild radius, M_{30} the PBH mass in units of $30 M_\odot$, and $v_{\text{pbh}-200}$ the relative velocity in units of 200 km sec^{-1} .

We begin with a rough but simple and illustrative estimate of the rate per unit volume of such mergers. Suppose that all DM in the Universe resided in Milky-Way like halos of mass $M = M_{12} 10^{12} M_\odot$ and uniform mass density $\rho = 0.002 \rho_{0.002} M_\odot \text{pc}^{-3}$ with $\rho_{0.002} \sim 1$. Assuming a uniform-density halo of volume $V = M/\rho$, the rate of mergers per halo would be

$$\begin{aligned}N &\simeq (1/2)V(\rho/M_{\text{pbh}})^2\sigma v \\ &\simeq 3.10 \times 10^{-12} M_{12} \rho_{0.002} v_{\text{pbh}-200}^{-11/7} \text{yr}^{-1}.\end{aligned}\tag{5.12}$$

The relative velocity $v_{\text{pbh}-200}$ is specified by a characteristic halo velocity. The mean cosmic DM mass density is $\rho_{\text{dm}} \simeq 3.6 \times 10^{10} M_\odot \text{Mpc}^{-3}$, and so the spatial density of halos is $n \simeq 0.036 M_{12}^{-1} \text{Mpc}^{-3}$. The rate per unit comoving volume in the Universe is thus

$$\Gamma \simeq 1.1 \times 10^{-4} \rho_{0.002} v_{\text{pbh}-200}^{-11/7} \text{Gpc}^{-3} \text{yr}^{-1}.\tag{5.13}$$

The factor M_{12} drops out, as it should. The merger rate per unit volume also does not depend on the PBH mass, as the capture cross section scales like M_{bh}^2 .

This rate is small compared with the $2 - 53 \text{ Gpc}^{-3} \text{yr}^{-1}$ estimated by LIGO for a population of $\sim 30 M_\odot - 30 M_\odot$ mergers,²⁶³ but it is a very conservative estimate. As Eq. (5.13) indicates, the merger rate is higher in higher-density regions and in regions of lower DM velocity dispersion. The DM in Milky-Way like halos is known from simulations²⁶⁴ and analytic models²⁶⁵ to have substructure, regions of higher density and lower velocity dispersion. DM halos also have a broad mass spectrum, extending to very low masses where the densities can become far higher, and velocity dispersion far lower, than in the Milky Way. To get a very rough estimate of the conceivable increase in the PBH merger rate

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due to these smaller-scale structures, we can replace ρ and v in Eq. (5.13) by the values they would have had in the earliest generation of collapsed objects, where the DM densities were largest and velocity dispersions smallest. If the primordial power spectrum is nearly scale invariant, then gravitationally bound halos of mass $M_c \sim 500 M_\odot$, for example, will form at redshift $z_c \simeq 28 - \log_{10}(M_c/500 M_\odot)$. These objects will have virial velocities $v \simeq 0.2 \text{ km sec}^{-1}$ and densities $\rho \simeq 0.24 M_\odot \text{ pc}^{-3}$.²⁶⁶ Using these values in Eq. (5.13) increases the merger rate per unit volume to

$$\Gamma \simeq 1400 \text{ Gpc}^{-3} \text{ yr}^{-1}. \quad (5.14)$$

This would be the merger rate if *all* the DM resided in the smallest haloes. Clearly, this is not true by the present day; substructures are at least partially stripped as they merge to form larger objects, and so Eq. (5.14) should be viewed as a conservative upper limit.

Having demonstrated that rough estimates contain the merger-rate range $2\text{--}53 \text{ Gpc}^{-3} \text{ yr}^{-1}$ suggested by LIGO, we now turn to more careful estimates of the PBH merger rate. As Eq. (5.13) suggests, the merger rate will depend on a density-weighted average, over the entire cosmic DM distribution, of $\rho_{0.002} v_{\text{pbh}}^{-11/7}$. To perform this average, we will (a) assume that DM is distributed within galactic halos with a Navarro-Frenk-White (NFW) profile³ with concentration parameters inferred from simulations; and (b) try several halo mass functions taken from the literature for the distribution of halos.

The PBH merger rate $\mathcal{R}$ within each halo can be computed using

$$\mathcal{R} = 4\pi \int_0^{R_{\text{vir}}} r^2 \frac{1}{2} \left(\frac{\rho_{\text{nfw}}(r)}{M_{\text{pbh}}} \right)^2 \langle \sigma v_{\text{pbh}} \rangle dr \quad (5.15)$$

where $\rho_{\text{nfw}}(r) = \rho_s [(r/R_s)(1 + r/R_s)^2]^{-1}$ is the NFW density profile with characteristic radius r_s and characteristic density ρ_s . R_{vir} is the virial radius at which the NFW profile reaches a value 200 times the comoving mean cosmic density and is cutoff. Here, M_{pbh} is the PBH mass and v_{pbh} is the relative velocity of two PBHs. The angle brackets denote an average over the PBH relative velocity distribution in the halo. The merger cross section σ is given by Eq. (5.11). We define the concentration parameter $C = R_{\text{vir}}/R_s$. To determine the profile of each halo, we require C as a function of halo mass M . We will use the concentration-mass relations fit to DM N-body simulations by both Ref.²⁶⁷ and Ref.⁴

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We now turn to the average of the cross section times relative velocity. The one-dimensional velocity dispersion of a halo is defined in terms of the escape velocity at radius $R_{\max} = 2.1626 R_s$, the radius of the maximum circular velocity of the halo. i.e.,

$$v_{\text{dm}} = \sqrt{\frac{GM(r < r_{\max})}{r_{\max}}} = \frac{v_{\text{vir}}}{\sqrt{2}} \sqrt{\frac{C}{C_m} \frac{g(C_m)}{g(C)}}, \quad (5.16)$$

where $g(C) = \ln(1 + C) - C/(1 + C)$, and $C_m = 2.1626 = R_{\max}/R_s$. We approximate the relative velocity distribution of PBHs within a halo as a Maxwell-Boltzmann (MB) distribution with a cutoff at the virial velocity. i.e.,

$$P(v_{\text{pbh}}) = F_0 \left[\exp\left(-\frac{v_{\text{pbh}}^2}{v_{\text{dm}}^2}\right) - \exp\left(-\frac{v_{\text{vir}}^2}{v_{\text{dm}}^2}\right) \right], \quad (5.17)$$

where F_0 is chosen so that $4\pi \int_0^{v_{\text{vir}}} P(v) v^2 dv = 1$. This model provides a reasonable match to N-body simulations, at least for the velocities substantially less than the virial velocity which dominate the merger rate (e.g., Ref.²⁶⁸). Since the cross-section is independent of radius, we can integrate the NFW profile to find the merger rate in any halo:

$$\mathcal{R} = \left(\frac{85\pi}{12\sqrt{2}}\right)^{2/7} \frac{9G^2 M_{\text{vir}}^2}{cR_s^3} \left(1 - \frac{1}{(1+C)^3}\right) \frac{D(v_{\text{dm}})}{g(C)^2}, \quad (5.18)$$

where

$$D(v_{\text{dm}}) = \int_0^{v_{\text{vir}}} P(v, v_{\text{dm}}) \left(\frac{2v}{c}\right)^{3/7} dv, \quad (5.19)$$

comes from Eq. (5.17).

Eq. (5.11) gives the cross section for two PBHs to form a binary. However, if the binary is to produce an observable GW signal, these two PBHs must orbit and inspiral; a direct collision, lacking an inspiral phase, is unlikely to be detectable by LIGO. This requirement imposes a minimum impact parameter of roughly the Schwarzschild radius. The fraction of BHs direct mergers is $\sim v^{2/7}$ and reaches a maximum of $\sim 3\%$ for $v_{\text{pbh}} = 2000 \text{ km s}^{-1}$. Thus, direct mergers are negligible. We also require that once the binary is formed, the time until it merges (which can be obtained from Ref.²⁶⁹) is less than a Hubble time. The characteristic time it takes for a binary BH to merge varies as a function of halo velocity dispersion. It can be hours for $M_{\text{vir}} \simeq 10^{12} M_{\odot}$ or kyrs for $M_{\text{vir}} \simeq 10^6 M_{\odot}$, and is thus

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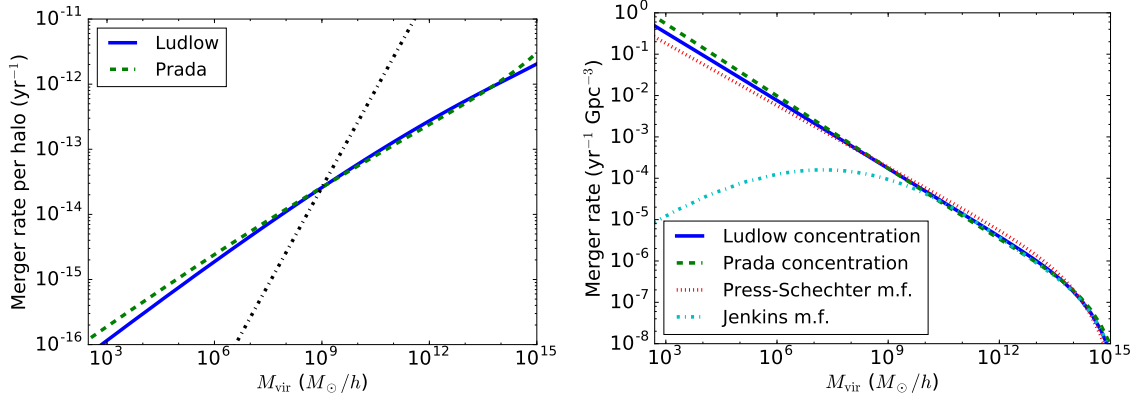


Figure 5.3: (*Left*) The PBH merger rate per halo as a function of halo mass. The solid line shows the trend assuming the concentration-mass relation from Ref.⁴ and the dashed line that from Ref.²⁶⁷ To guide the eye, the dot-dashed line shows a constant BH merger rate per unit halo mass. (*Right*) The total PBH merger rate as a function of halo mass. Dashed and dotted lines show different prescriptions for the concentration-mass relation and halo mass function.

instantaneous on cosmological timescales. Given the small size of the binary, and rapid time to merger, we can neglect disruption of the binary by a third PBH once formed. BH binaries can also form through non-dissipative three-body encounters. The rate of these binary captures is non-negligible in small halos, but they generically lead to the formation of wide binaries that will not be able to harden and merge within a Hubble time. This formation mechanism should not affect our LIGO rates. The merger rate is therefore equal to the rate of binary BH formation, Eq. (5.18).

Fig. 5.3 shows the contribution to the merger rate, Eq. (5.18), for two concentration-mass relations. As can be seen, both concentration-mass relations give similar results. An increase in halo mass produces an increased PBH merger rate. However, less massive halos have a higher concentration (since they are more likely to have virialized earlier), so that the merger rate per unit mass increases significantly as the halo mass is decreased.

To compute the expected LIGO event rate, we convolve the merger rate $\mathcal{R}$ per halo with the mass function dn/dM . Since the redshifts ($z \lesssim 0.3$) detectable by LIGO are relatively low we will neglect redshift evolution in the halo mass function. The total merger rate per unit volume is then,

$$\mathcal{V} = \int (dn/dM)(M) \mathcal{R}(M) dM. \quad (5.20)$$

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Given the exponential falloff of dn/dM at high masses, despite the increased merger rate per halo suggested in Fig. 5.3, the precise value of the upper limit of the integrand does not affect the final result.

At the lower limit, discreteness in the DM particles becomes important, and the NFW profile is no longer a good description of the halo profile. Furthermore, the smallest halos will evaporate due to periodic ejection of objects by dynamical relaxation processes. The evaporation timescale is

$$t_{\text{evap}} \approx (14\mathcal{N}/\ln\mathcal{N}) [R_{\text{vir}}/(C v_{\text{dm}})], \quad (5.21)$$

where $\mathcal{N}$ is the number of individual BHs in the halo, and we assumed that the PBH mass is $30 M_{\odot}$. For a halo of mass $400 M_{\odot}$, the velocity dispersion is 0.15 km sec^{-1} , and the evaporation timescale is $\sim 3 \text{ Gyr}$. In practice, during matter domination, halos which have already formed will grow continuously through mergers or accretion. Evaporation will thus be compensated by the addition of new material, and as halos grow new halos will form from mergers of smaller objects. However, during dark-energy domination at $z \lesssim 0.3$, 3 Gyr ago, this process slows down. Thus, we will neglect the signal from halos with an evaporation timescale less than 3 Gyr, corresponding to $M < 400 M_{\odot}$. This is in any case 13 PBHs, and close to the point where the NFW profile is no longer valid.

The halo mass function dn/dM is computed using both semi-analytic fits to N-body simulations and with analytic approximations. Computing the merger rate in the small halos discussed above requires us to extrapolate both the halo mass function and the concentration-mass relation around six orders of magnitude in mass beyond the smallest halos present in the calibration simulations. High-resolution simulations of $10^{-4} M_{\odot}$ cold dark matter micro-halos^{270,271} suggest that our assumed concentration-mass relations underestimate the internal density of these halos, making our rates conservative.

The mass functions depend on the halo mass through the perturbation amplitude $\sigma(R_{\text{vir}})$ at the virial radius R_{vir} of a given halo. Due to the scale invariance of the window functions on small scales, $\sigma(R_{\text{vir}})$ varies only by a factor of two between $M_{\text{vir}} = 10^9 M_{\odot}$ and $10^3 M_{\odot}$. Thus the extrapolation in the mass function is less severe than it looks. We also note that the scale-invariant nature of the initial conditions suggests that the shape of the

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halo mass function should not evolve unduly until it reaches the scale of the PBH mass, or evaporation cutoff.

To quantify the uncertainty induced by the dn/dM extrapolation, we obtained results with two different mass functions: the classic analytic Press-Schechter calculation²⁷² and one calibrated to numerical simulations from Tinker et al.²⁷³ The agreement between the two small-scale behaviors suggests that extrapolating the mass functions is not as blind as it might otherwise seem. We also include a third mass function, due to Jenkins,²⁷⁴ that includes an artificial small-scale mass cutoff at a halo mass $M_{\text{vir}} \sim 10^6 M_{\odot}$. This cutoff is inserted to roughly model the mass function arising if there is no power on scales smaller than those currently probed observationally. We include it to provide a *very* conservative lower limit to the merger rate if, for some reason, small-scale power were suppressed. We do not, however, consider it likely that this mass function accurately represents the distribution of halo masses in our Universe.

Fig. 5.3 shows the merger rate per logarithmic interval in halo mass. In all cases, halos with $M_{\text{vir}} \lesssim 10^9 M_{\odot}$ dominate the signal, due to the increase in concentration and decrease in velocity dispersion with smaller halo masses. The Tinker mass function, which asymptotes to a constant number density for small masses, produces the most mergers. Press-Schechter has $\sim 50\%$ fewer events in small halos, while the Jenkins mass function results in merger rates nearly four orders of magnitude smaller (and in rough agreement with Eq. (5.13)).

We integrate the curves in Fig. 5.3 to compute the total merger rate $\mathcal{V}$. All mass functions give a similar result, $\sim (3 \pm 1) \times 10^{-4} \text{ Gpc}^{-3} \text{ yr}^{-1}$, from halos of masses $\gtrsim 10^9 M_{\odot}$, representing for the Tinker and Press-Schechter mass function a small fraction of the events. When we include all halos with $M_{\text{vir}} > 400 M_{\odot}$, the number of events increases dramatically, and depends strongly on the lower cutoff mass M_c for the halo mass. Both the Press-Schechter and Tinker mass functions are for small halos linear in the integrated perturbation amplitude $\propto 1/\sigma(R_{\text{vir}})$ at the virial radius R_{vir} of the collapsing halo. In small halos, $1/\sigma(R_{\text{vir}})$ is roughly constant. Thus for a mass function $\text{MF}(\sigma)$, we have

$$(dn/dM) \sim (C \log \sigma / dM) [\text{MF}(\sigma) / M_{\text{vir}}] \sim M_{\text{vir}}^{-2}. \quad (5.22)$$

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The concentration is also a function of $1/\sigma(R_{\text{vir}})$ and it too becomes roughly constant for small masses. Assuming a constant concentration, the merger rate per halo scales as $\mathcal{R} \sim M^{10/21}$. Thus, Eq. (5.20) suggests that $\mathcal{V} \sim M_c^{-11/21}$. This compares well to numerical differentiation of Fig. 5.3, which yields $\mathcal{V} \sim M_c^{-0.51}$.

The integrated merger rate is thus

$$\mathcal{V} = 2 f(M_c/400 M_\odot)^{-11/21} \text{Gpc}^{-3} \text{yr}^{-1}, \quad (5.23)$$

with $f \simeq 1$ for the Tinker mass function, and $f \simeq 0.6$ for the Press-Schechter mass function (the Jenkins mass function results in an event rate $\mathcal{V} \simeq 0.02 \text{Gpc}^{-3} \text{yr}^{-1}$, independent of $M_c \lesssim 10^6 M_\odot$).

A variety of astrophysical processes may alter the mass function in some halos, especially within the dwarf galaxy range, $10^9 - 10^{10} M_\odot$. However, halos with $M_{\text{vir}} \lesssim 10^9 M_\odot$ are too small to form stars against the thermal pressure of the ionized intergalactic medium²⁷⁵ and are thus unlikely to be affected by these astrophysical processes. Inclusion of galactic substructure, which our calculation neglects, should boost the results. However, since the event rate is dominated by the smallest halos, which should have little substructure, we expect this to make negligible difference to our final result.

In the above, we have assumed an NFW profile, which has a singular density as $r \rightarrow 0$. The Einasto profile²⁷⁶ avoids this feature, and is

$$\rho(R) = \rho_0 \exp \left(-\frac{2}{\alpha} \left[\left(\frac{R}{R_s} \right)^\alpha - 1 \right] \right) \quad (5.24)$$

where $\alpha = 0.18$. This profile shape reduces the amplitude of the central density peak, spreading the mass to wider radii. As most of the mergers occur outside the central peak, this increases the implied merger rate by 50% to $\sim 3 \text{Gpc}^{-3} \text{yr}^{-1}$.

Our assumption of an isotropic MB-like velocity distribution in the halo may also underestimate the correct answer, as any other velocity distribution would have lower entropy and thus larger averaged $v^{-11/7}$. Finally, the discreteness of PBH DM will provide some Poisson enhancement of power on $\sim 400 M_\odot$ scales. More small-scale power would probably lead to an enhancement of the event rate beyond Eq. (5.23).

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The recent LIGO detection of two merging $\sim 30 M_\odot$ black holes suggests a 90% C.L. event rate²⁶³ of $2 - 53 \text{ Gpc}^{-3} \text{ yr}^{-1}$ if all mergers have the masses and emitted energy of GW150914. It is interesting that—although there are theoretical uncertainties—our best estimates of the merger rate for $30 M_\odot$ PBHs, obtained with canonical models for the DM distribution, fall in the LIGO window.

We have assumed a population of PBHs with the same mass. The basic results obtained here should, however, remain unaltered if there is some small spread of PBH masses, as expected from PBH-formation scenarios, around the nominal value of $30 M_\odot$.

PBH mergers may also be interesting for LIGO/VIRGO even if PBHs make up only a fraction f_{pbh} of the DM. In this case, the number density of PBHs will be reduced by f_{pbh} . The cutoff mass will increase as $M_c \sim f_{\text{pbh}}^{-1}$ if we continue to require > 13 PBHs in each halo to avoid halo evaporation. The overall event rate will be $\mathcal{V} \sim 2f_{\text{pbh}}^{53/21} \text{ Gpc}^{-3} \text{ yr}^{-1}$. Advanced LIGO will reach design sensitivity in 2019,^{277,278} and will probe $z < 0.75$, an increase in volume to $\approx 500 \text{ Gpc}^3$. Thus over the six planned years of aLIGO operation, while we should expect to detect ~ 6000 events with $f_{\text{pbh}} = 1$, we will expect at least one event if $f_{\text{pbh}} > 0.03$.

Distinguishing whether any individual GW event, or even some population of events, are from PBH DM or more traditional astrophysical sources will be extremely complicated. PBH mergers are expected to have no electromagnetic/neutrino counterparts whatsoever. A DM component could conceivably show up in the BH mass spectrum as an excess of events with BH masses near $30 M_\odot$ over a more broadly distributed mass spectrum from astrophysical sources.^{279,280} Still, there are some prospects, which we now study.

5.2.3 Distribution

Measurements of the cross-correlation of the GW events with overlapping galaxy catalogs may provide an useful tool to determine if BH mergers trace the stellar mass of the Universe, as would be expected from mergers of the endpoints of stellar evolution. If on the other hand the BHs are of primordial origin, their merging would be preferentially distributed more like small-scale DM halos instead of luminous galaxies, and thus have a lower cross-correlation with galaxy surveys.³⁴ Here we forecast the expected precision

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of the cross-correlation measurement for current and future GW detectors such as LIGO and the Einstein Telescope. We then predict how well these instruments can distinguish the model that identifies high-mass BH-BH mergers as the merger of primordial black holes that constitute the dark matter in the Universe from more traditional astrophysical sources.

In order to measure the correlation between the host halos of BH-binaries and galaxies, we use measurements of their number counts. We consider angular projections C_ℓ , that can be calculated from the underlying 3D matter power spectrum by using

$$C_\ell^{XY} = r \int \frac{4\pi dk}{k} \Delta^2(k) W_\ell^X(k) W_\ell^Y(k), \quad (5.25)$$

where $W_\ell^{\{X,Y\}}$ are the source distribution window functions for the different observables (here X and Y stand for galaxies and GWs), $\Delta^2(k)$ is the dimensionless matter power spectrum today, and r is a cross-correlation coefficient ($r \equiv 1$ for the auto-correlation case, $X = Y$).

The window function for the number count distributions can be written as:

$$W_\ell^X(k) = \int \frac{d\bar{N}_X(z)}{dz} b_X(z) j_\ell[k\chi(z)] dz. \quad (5.26)$$

$d\bar{N}_X(z)/dz$ is the source redshift distribution, normalized to unity within the same redshift range as the window function; $b_X(z)$ is the bias that relates the observed correlation function to the underlying matter distribution, that we assume to be scale-independent on large scales; $j_\ell(x)$ is the spherical Bessel function of order ℓ , and $\chi(z)$ is the comoving distance. The integral in Equation (5.26) is performed over the redshift range corresponding to the selection function of the galaxy survey.

For the galaxy catalog we assume a constant redshift distribution of galaxies. As for GW events, their number can be estimated by

$$\frac{dN_{GW}(z)}{dz} \approx \mathcal{R}(z) T_{\text{obs}} \frac{4\pi\chi^2(z)}{(1+z)H(z)}, \quad (5.27)$$

where $\mathcal{R}(z)$ is the redshift-dependent merger rate, T_{obs} is the observation time and $H(z)$ is

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the Hubble parameter. The errors in the auto- and cross-correlations are given by

$$\sigma_{C_\ell^{\text{g GW}}} = \sqrt{\frac{\left(C_\ell^{\text{g GW}}\right)^2 + \left[\left(C_\ell^{gg} + \frac{1}{\bar{n}_g}\right)\left(C_\ell^{\text{GW GW}} + \frac{1}{\bar{n}_{\text{GW}}}\right)\right]}{(2\ell + 1)f_{\text{sky}}}}, \quad (5.28)$$

and

$$\sigma_{C_\ell^{\text{gg}}} = \sqrt{\frac{2\left[C_\ell^{gg} + \frac{1}{\bar{n}_g}\right]^2}{(2\ell + 1)f_{\text{sky}}}}, \quad (5.29)$$

where f_{sky} is the fraction of the sky observed and $\bar{n}_{\{\text{g}, \text{GW}\}}$ is the average number of sources per steradian, i.e. the integral of dN/dz (Eq. (5.27) in the GW case).

We now define the *effective correlation amplitude* $A_c \equiv r \times b_{\text{GW}}$, where r is the cross-correlation coefficient of Equation (5.25). This cross-correlation coefficient parametrizes the extent to which two biased tracers of the matter field are correlated. How well this amplitude can be measured is shown in Fig. 5.4. I show results as a function of the minimum scale probed ℓ_{max} and the number of BH-BH mergers detected, defined as $N_{\text{GW}} = T_{\text{obs}} \mathcal{R} V_{\text{obs}}$, where $\mathcal{R}$ is the integrated average merger rate in units of $\text{Gpc}^{-3} \text{ yr}^{-1}$, and T_{obs} and V_{obs} are the relevant observation time and volume. I keep the maximum redshift as $z_{\text{max}} = 1.5$. An error of $\sigma(A_c) = 0.5$ can be achieved with those parameters, which translates into a prospective $2\text{--}\sigma$ “measurement” of a less-biased GW population, and hence a PBH origin. We refer the reader to Ref.³⁴ for more details.

5.2.4 Eccentricities

The binaries in the local Universe are formed on very elongated orbits, so the GW waveforms will initially have high ellipticity, shown as higher frequency harmonics in the GW signal.²⁶⁹ For example, for an event occurring at a redshift $z = 0.09$ and with a final-BH spin of $\alpha = 0.67$ (roughly corresponding to the best-fit values for the GW150914 event), we see that such an event would be easily traced over the expected final design noise of LIGO,²⁷⁷ during a full second before the merger. If instead, those black holes were on an elliptical orbit with eccentricity e evolving from 0.55 to 0.3 during that last second of the coalescence (as in Fig. 5.1), then the GW power would be emitted also in other modes at frequencies $f_{n_{\text{source}}} = n \cdot f_{\text{orb}}$, where f_{orb} is the Keplerian orbital frequency of the binary. As

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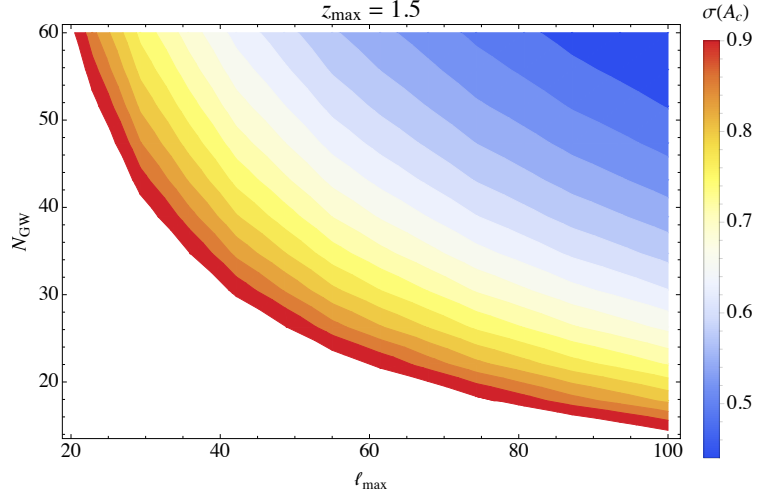


Figure 5.4: Forecast errors on the cross-correlation amplitude A_c (colorbar) as a function of $\ell_{\max}$, the maximum multipole accessible, and the number N_{GW} of GW events observed, assuming they are distributed up to a maximum redshift $z_{\max} = 1.5$.

can be clearly seen, in addition to observing the $n = 2$ (quadrupole) mode, LIGO with its expected final-design sensitivity should clearly be able to identify higher modes at least up to $n = 8$, since for frequencies $> 50\text{Hz}$ all these additional modes have a strain amplitude that is at least a factor of 3 higher than that of the noise. We note that the extent to which the higher modes can be identified relies on the waveforms used by the LIGO collaboration. We also note the eccentricity gets reduced within the last second of the inspiral, which changes the relative power of GWs between modes.

Observing higher modes can also allow the identification of events with eccentricity at the last stages of the inspiral with a higher signal-to-noise ratio (S/N) than that of the equivalent circularized objects. In fact, properly accounting for the presence of higher modes is relevant to understanding the physical properties (mainly the masses and the distance) of the binary. In Table 5.1, we present the expected S/N at LIGO and ET. For LIGO we assume the final design sensitivity while for the Einstein Telescope we used the design option "ET-B" of Ref.,²⁸¹ which is the more pessimistic at low frequencies (relevant for high-mass BH coalescence events). As can be seen, the exact contribution to the S/N from the various GW modes at the inspiral depends on the eccentricity of the binary once its GWs enter the frequency band of the observatories, denoted here as e_{in} (not to be confused with the eccentricity at formation of the binary e_0), as well as the final e_{LSO} . Yet, for events

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z	$e_{\text{in}} \rightarrow e_{\text{LSO}}$	S/N Merger & Ringdown	$\Delta(\text{S/N})$ Ins. ($n=2$)	$\Delta(\text{S/N})$ Ins. ($n=3$)	$\Delta(\text{S/N})$ Ins. ($n=4$)	$\Delta(\text{S/N})$ Ins. ($n=5$)
0.09	$0 \rightarrow 0$	44	+25	-	-	-
0.09	$0.6 \rightarrow 0$	44	+22	+33	+35	+31
0.09	$0.6 \rightarrow 0.3$	44	+8.0	+30	+37	+33
0.75	$0 \rightarrow 0$	6.8	+1.3	-	-	-
0.75	$0.6 \rightarrow 0.3$	6.8	+1.2	+3.3	+4.2	+4.1

Table 5.1: The contribution of higher GW modes at elliptical orbits of $30 M_{\odot}$ PBHs detected by the LIGO experiment. We assume that when entering the frequency band of observation the binary has an initial eccentricity e_{in} that evolves down to e_{LSO} . The total S/N is the linear sum of columns 2-6 (we have accounted for the quadratic sum from the various phases). We take $\alpha = 0.67$ in Eq. (5.6) and $\epsilon = 0.04$ in Eq. (5.5).

with significant eccentricities during observation, higher modes can contribute significantly to the total S/N. Furthermore, these modes would reduce the overall contribution of the quadrupole mode to the S/N.

The results for two cases are shown in Table 5.1, where it is clear that very-eccentric events have significant power in $n > 2$ modes. In this Table we do not include the $n = 1$ mode since its contribution to the S/N is small, and depends on the exact assumptions of the instrument sensitivity at the lowest frequencies. For LIGO, we assume the frequency band starts as 20 Hz. Any further advances allowing that conservative value to be reduced would increase the S/N values quoted and the capacity of these observatories to identify elliptical orbits at coalescence.

With the numbers here mentioned it is expected that LIGO will detect $\mathcal{O}(1)$ eccentric events during its 10-year life, assuming that the entirety of the dark matter is made of PBHs. Only futuristic telescopes will be able to observe these events in a large-enough quantity to confirm whether the dark matter is made of PBHs.³⁵

“They already put my ideas into practice.”

— Tommy Wiseau

Chapter 6

Fast Radio Bursts

Although observations indicate that dark matter accounts for a significant share of the energy density of our Universe,¹ we do not know its composition. Longtime candidates to make up the dark matter are massive compact halo objects (MACHOs).³⁶ They were originally proposed to be as light as $10^{-7}M_{\odot}$ and as heavy as the first stars ($\sim 10^3M_{\odot}$).²⁸² Over the years, different experiments have progressively constrained the fraction f_{DM} of dark matter that can reside in MACHOs with a given mass, placing tight upper bounds over most of the vast range above. High-mass ($\gtrsim 100M_{\odot}$) MACHOs, for example, are constrained by the fact that they would perturb wide stellar binaries in our Galaxy.²⁶⁰ Meanwhile, lower-mass ($\lesssim 20M_{\odot}$) MACHOs are effectively ruled out as the sole component of Galactic dark matter, as they would create artificial variability in stars, due to gravitational microlensing.²⁵⁶

However, there remains a window of masses between 20 and $100M_{\odot}$, where the constraints are weaker, and in which arguably all the cosmological dark matter could be in the form of MACHOs.^{283,284} This is a particularly interesting window, as it has been recently argued in Ref.³² that if primordial black holes (PBHs)^{30,31} in the $\sim 30M_{\odot}$ mass range are the constituents of dark matter, they form binaries in halos, coalesce, and emit observable gravitational waves, with an event rate consistent with the published LIGO detection.²⁴⁷

6.1 Gravitational lensing

In this chapter we propose to use the strong lensing of fast radio bursts (FRBs) to probe MACHOs of masses $\gtrsim 20 M_\odot$, including PBHs, and either confirm that they make up the dark matter or close this window.

6.1.1 The FRB Properties

FRBs are strong radio bursts with a very short duration, which makes them ideal as microlensing targets. Their temporal width is increased by the dispersion measure (DM), which measures the time delay of photons with different radio frequencies due to scattering by free electrons on their way to Earth. All detected FRBs to date possess high DMs, which yield burst widths of $\sim 1 - 10$ ms.^{38,285–288} These values of the DM are several times larger than the expected contribution from free electrons within the Milky Way,²⁸⁹ suggesting their origin is extragalactic (some authors, however, prefer a Galactic origin²⁹⁰). Proposed sources of extragalactic FRBs include merging neutron stars²⁹¹ or white dwarfs,²⁹² as well as bursts from pulsars.²⁹³

Strong lensing of a FRB by a MACHO will generate two images of the burst. While their angular separation may be too small to be resolved, the time delay between them, on the order of milliseconds for a MACHO lens with mass $M_L \sim 20 - 100 M_\odot$, might be large enough to enable a detection of two separate peaks, rather than one, if the time delay is bigger than the pulse width. Fortunately, the lensing of FRBs by compact objects is not necessarily an unlikely occurrence. In fact, if all the dark matter is in MACHOs, roughly one in 50 FRBs originating at $z = 0.5$ should be lensed. If there are $\sim 10^4$ FRBs on the full sky each day,²⁹⁴ then as many as ~ 20 microlensed FRBs may be reaching Earth daily. Upcoming surveys, like APERTIF,²⁹⁵ UTMOST,²⁹⁶ HIRAX,²⁹⁷ or CHIME,²⁹⁸ which will map a considerable fraction of the sky, may thus see a significant number of lensed FRBs.

Below, we calculate the effects of microlensing on a given FRB and compute the optical depth for strong lensing by compact objects. We then combine those results with different redshift distributions of FRBs and estimate how many lensed bursts are expected if MACHOs make up all the dark matter. We also estimate the smallest fraction f_{DM} that will

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give rise to a detectable rate of microlensed events.

6.1.2 Lensing Formalism

A MACHO of mass M_L can be treated as a point lens with an (angular) Einstein radius:

$$\theta_E = 2\sqrt{\frac{GM_L}{c^2} \frac{D_{LS}}{D_S D_L}}, \quad (6.1)$$

where D_S , D_L , and D_{LS} are the (angular-diameter) distances to the source, to the lens, and between the source and the lens, respectively.²⁹⁹ A point lens produces two images, at positions $\theta_{\pm} = (\beta \pm \sqrt{\beta^2 + 4\theta_E^2})/2$, where β is the (angular) impact parameter. The time delay between these two images is

$$\Delta t = \frac{4GM_L}{c^3} (1 + z_L) \left[\frac{y}{2} \sqrt{y^2 + 4} + \log \left(\frac{\sqrt{y^2 + 4} + y}{\sqrt{y^2 + 4} - y} \right) \right], \quad (6.2)$$

where $y \equiv \beta/\theta_E$ is the normalized impact parameter and z_L is the redshift of the lens. We also define the flux ratio R_f as the absolute value of the ratio of the magnifications μ_+ and μ_- of both images; i.e.,

$$R_f \equiv \left| \frac{\mu_+}{\mu_-} \right| = \frac{y^2 + 2 + y\sqrt{y^2 + 4}}{y^2 + 2 - y\sqrt{y^2 + 4}} > 1. \quad (6.3)$$

To claim that a FRB is strongly lensed we will require three conditions. First is that the brighter image has a signal-to-noise ratio of 10. Second is that the observed time delay is larger than some reference time $\overline{\Delta t}$, which will place a lower bound on the impact parameter $y > y_{\min}(M_L, z_L)$, calculated via Eq. (6.2). Finally, we demand that the flux ratio R_f is smaller than some critical $\overline{R}_f$ (which we take to be redshift independent), to ensure that both events are observed (note that for the fainter image the look-elsewhere effect is no longer relevant). This forces the impact parameter to be smaller than $y_{\max} = \left[(1 + \overline{R}_f)/\sqrt{\overline{R}_f} - 2 \right]^{1/2}$.

6.2 Lensing Probability

We now calculate the probability for a FRB to be lensed.

6.2.1 Lensing Optical Depth

The lensing optical depth of a source at redshift z_S is given by

$$\tau(M_L, z_S) = \int_0^{z_S} d\chi(z_L) (1 + z_L)^2 n_L \sigma(M_L, z_L), \quad (6.4)$$

where $\chi(z)$ is the comoving distance at redshift z , n_L is the comoving number density of lenses, and σ is the lensing cross section of a point lens of mass M_L , given by an annulus between the maximum and minimum impact parameters by

$$\sigma(M_L, z_L) = \frac{4\pi G M_L}{c^2} \frac{D_L D_{LS}}{D_S} [y_{\max}^2 - y_{\min}^2(M_L, z_L)]. \quad (6.5)$$

Equation (6.4) can be recast by using the Hubble parameter both at the redshift of the lens, $H(z_L)$, and today, H_0 , as

$$\begin{aligned} \tau(M_L, z_S) &= \frac{3}{2} f_{\text{DM}} \Omega_c \int_0^{z_S} dz_L \frac{H_0^2}{c H(z_L)} \frac{D_L D_{LS}}{D_S} \\ &\quad \times (1 + z_L)^2 [y_{\max}^2 - y_{\min}^2(M_L, z_L)], \end{aligned} \quad (6.6)$$

where $\Omega_c = 0.24$ is the cold-dark-matter density today, and the only remaining dependence on the lens mass M_L is through $y_{\min}$. Lower MACHO masses result in a lower optical depth, especially at lower source redshifts, due to our minimum time-delay requirement.

To calculate the integrated lensing probability, the optical depth for lensing of a single burst has to be convolved with the redshift distribution of incoming FRBs. We will consider two possible redshift distributions. First, we assume FRBs have a constant comoving number density, in which case the number of FRBs in a shell of width dz at redshift z is proportional to the shell's comoving volume $dV(z) = [4\pi\chi^2(z)/H(z)] dz$,³⁰⁰ divided by $(1 + z)$ to account for the effect of cosmological time dilation in the rate of bursts. To represent an instrumental signal-to-noise threshold we introduce a Gaussian cutoff at some

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redshift z_{cut} , so the constant-density redshift distribution function would be

$$N_{\text{const}}(z) = \mathcal{N}_{\text{const}} \frac{\chi^2(z)}{H(z)(1+z)} e^{-d_L^2(z)/[2d_L^2(z_{\text{cut}})]}, \quad (6.7)$$

where d_L is the luminosity distance, and $\mathcal{N}_{\text{const}}$ is a normalization factor to ensure that $N_{\text{const}}(z)$ integrates to unity. Second, we consider a scenario in which FRBs follow the star-formation history (SFH),³⁰¹ whose density is parametrized as

$$\dot{\rho}_*(z) = h \frac{a + bz}{1 + \left(\frac{z}{c}\right)^d}, \quad (6.8)$$

with $a = 0.0170$, $b = 0.13$, $c = 3.3$, $d = 5.3$, and $h = 0.7$.^{302,303} In this case, the SFH-based redshift distribution function $N_{\text{SFH}}(z)$ is,

$$N_{\text{SFH}}(z) = \mathcal{N}_{\text{SFH}} \frac{\dot{\rho}_*(z) \chi^2(z)}{H(z)(1+z)} e^{-d_L^2(z)/[2d_L^2(z_{\text{cut}})]}, \quad (6.9)$$

and the normalization factor $\mathcal{N}_{\text{SFH}}$ is chosen to have N_{SFH} integrate to unity.

In Figure 6.1 we plot a histogram of the estimated redshifts for the current FRB catalog,³⁸ which is well fit by the two FRB distribution functions above, if a cutoff of $z_{\text{cut}} = 0.5$ is chosen.

6.2.2 Detection Prospects

To estimate the total number of FRBs observable in the near future, we consider an experiment like CHIME.²⁹⁸ In Ref.²⁹⁴ it was estimated that CHIME will detect $\sim 730 - 15000$ FRBs per year, and so we will take a fiducial, albeit optimistic, value of $N_{\text{FRB}} = 10^4$ bursts per year.

Interchannel dispersion broadens the FRB pulse arrival time to

$$\delta t_{\text{DM}} = 0.3 \text{ ms} \times \frac{\text{DM}}{800 \text{ pc cm}^{-3}} \frac{\Delta\nu}{24 \text{ kHz}} \left(\frac{800 \text{ MHz}}{\nu} \right)^3, \quad (6.10)$$

where ν is the frequency, $\Delta\nu$ is the bandwidth, which will be 24 MHz, or smaller, for transient studies with CHIME,²⁹⁸ and DM is the dispersion measure, given by the integrated

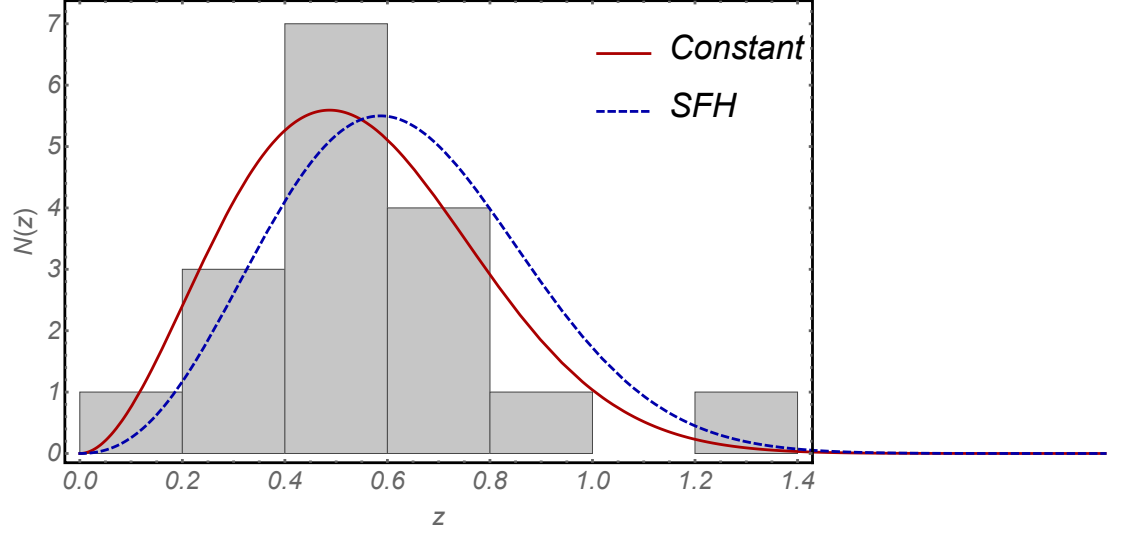


Figure 6.1: A histogram of the 17 FRBs observed to date, with inferred redshifts.³⁸ FRB redshift distributions are plotted assuming a constant comoving density (solid-red), and following the star-formation history (dashed-blue), both with a cutoff at $z_{\text{cut}} = 0.5$, and normalized to match the total number of detected events.

column density of electrons.³⁰⁴ The total pulse width of a FRB will have a contribution from its (unknown) intrinsic pulse profile, as well as scattering with the intergalactic medium,³⁰⁵ and the lensing time delay has to be bigger than its total width to be easily detectable. To account for this, we will require a lensing time delay longer than $\overline{\Delta t} = 1$ ms as our baseline case. FRBs might have a distribution of intrinsic widths; wider bursts would give rise to more pessimistic results, whereas narrower FRBs might produce more optimistic ones. We will therefore show results for $\overline{\Delta t} = 0.3$ ms and $\overline{\Delta t} = 3$ ms, as well.

Given how little is known about the luminosity function of FRBs,^{294,300} we will not attempt to model the lensing magnifications μ_+ and μ_- observable at each source redshift. Instead, we will simply require a constant flux ratio $\overline{R_f} = 5$ as a threshold, since this will make the echoed image detectable.

Now, given a distribution function $N(z)$ for FRBs, we can calculate their integrated optical depth $\bar{\tau}(M_L)$, due to MACHOs of mass M_L , as

$$\bar{\tau}(M_L) = \int dz \tau(z, M_L) N(z). \quad (6.11)$$

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We show this quantity in Figure 6.2 for the same two distribution functions discussed above. It is clear that the distribution mimicking the SFH produces a higher optical depth, due to the higher redshift of most sources.

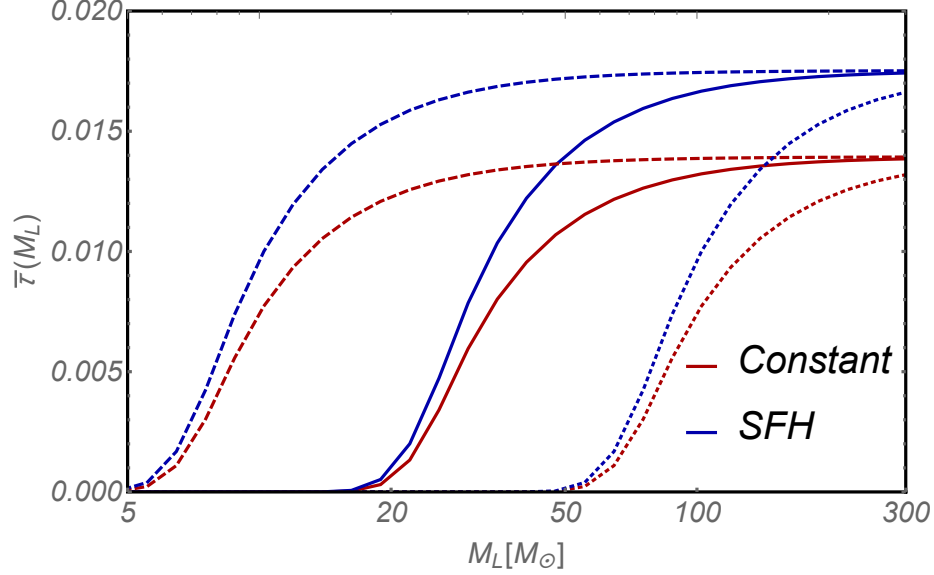


Figure 6.2: Integrated optical depth, with weightings corresponding to a population of FRBs with constant comoving density (red curves) and following the SFH (blue curves), both with a cutoff at $z_{\text{cut}} = 0.5$. In dashed, solid, and dotted lines we require a time delay $\Delta t > 0.3, 1$, and 3 ms, respectively. In all cases, $f_{\text{DM}} = 1$.

We can finally forecast the number N_{lensed} of lensed FRBs that a fraction f_{DM} of dark matter, in the form of point lenses of different masses, will yield. In all cases we are in the optically thin regime, where the probability to be lensed is just $P_{\text{lens}} = 1 - e^{-\bar{\tau}} \approx \bar{\tau}$. Thus, if we observe a number N_{FRB} of FRBs, $\bar{\tau}N_{\text{FRB}}$ of them should be lensed. Notice that, even if all the dark matter was composed of compact objects of a single mass M_L , the lensing time delays induced on FRBs would not have a unique value, due to the different impact parameters and redshifts of the lenses.

In Fig. 6.3 we show the joint probability distribution function (PDF) for a time delay Δt and a flux ratio R_f , assuming a $30 M_\odot$ lens. This PDF has been calculated by convolving Eqs. (6.2) and (6.3), assuming a flat distribution in impact parameters squared up to y_{max}^2 , with a population of FRBs following $N_{\text{const}}(z)$, and shows a clear correlation between the lensing time delays and the flux ratios. We also show the probability $P(\Delta t)$ to find a time delay Δt between the two events, calculated by marginalizing the PDF over $R_f < 5$. This

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time-delay distribution would be broadened further if the MACHOs had some range of masses instead of a single M_L .

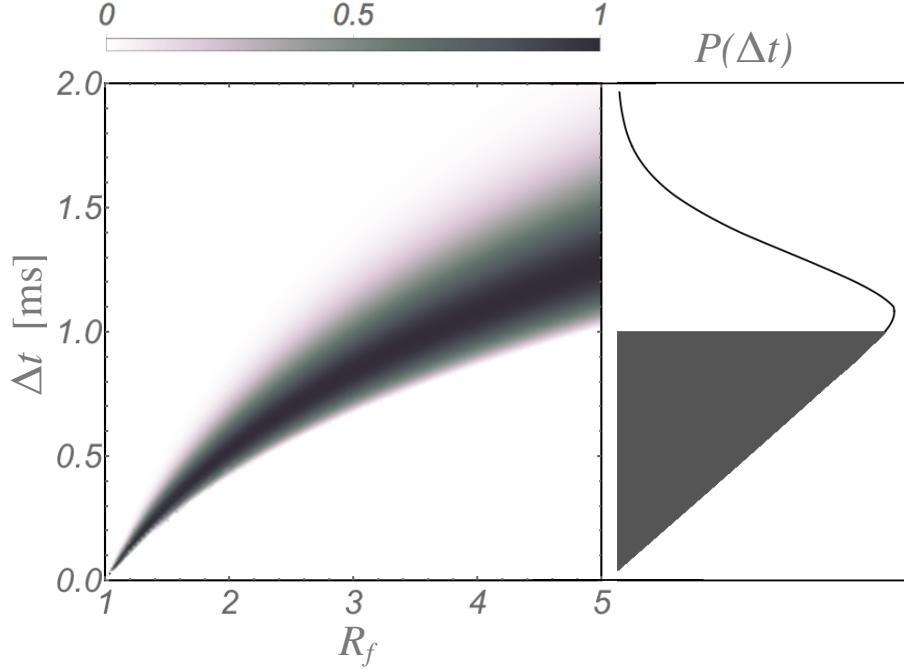


Figure 6.3: Joint probability distribution for the flux ratio R_f and time delay Δt between the two peaks of a FRB lensed by a $30 M_\odot$ MACHO. On the right, we marginalize over R_f , and show the probability to find a time delay Δt . The shaded region corresponds to time delays smaller than 1 ms, too short to be detectable.

Considering the most conservative case of a constant-density distribution of FRBs, with a cutoff at redshift $z_{\text{cut}} = 0.5$ as discussed above, and 10^4 total detected FRBs, corresponding to one year of observation with CHIME, we will see a number $N_{\text{lensed}} = 13$ of lensed bursts with a time delay longer than 1 ms, if all the dark matter is in the form of $20 M_\odot$ MACHOs. If, however, the dark matter is made of $30 M_\odot$ PBHs, as suggested in Ref.,³² the number of lensed events that will be detected is $N_{\text{lensed}} = 60$. For all MACHO masses larger than $M_L = 100 M_\odot$ the number of lensed events is simply $N_{\text{lensed}} = 130$. Here we have required a flux ratio smaller than $\overline{R_f} = 5$ to observe both bursts, although high MACHO masses produce time delays much in excess of the threshold values of $\overline{\Delta t}$, so the cross section annulus in Eq. (6.6) becomes a circle, and N_{lensed} scales roughly linearly with $\overline{R_f}$.

6.3 Constraints to Compact Dark Matter

We can also determine the smallest fraction f_{DM} that will produce at least one lensed event in a survey with 10^4 FRBs. Fig. 6.4 shows the regions of the $f_{\text{DM}}-M_L$ parameter space that give rise to at least one such event for lensing time delays longer than 0.3, 1, and 3 ms. We also show the current constraints to f_{DM} from the EROS Collaboration,²⁵⁶ the MACHO Collaboration,³⁶ and wide-binary disruption.²⁶⁰

From Figure 6.4 we see that, if none of the 10^4 upcoming FRBs is lensed, the amount of dark matter in MACHOs will be constrained to $f_{\text{DM}} < 0.8\%$ above a cutoff mass of $\sim 100 M_\odot$, under the assumption that the smallest time delay detectable is 1 ms. This will thus place more stringent constraints over this mass range than those coming from wide-binary disruption,²⁶⁰ by more than an order of magnitude.

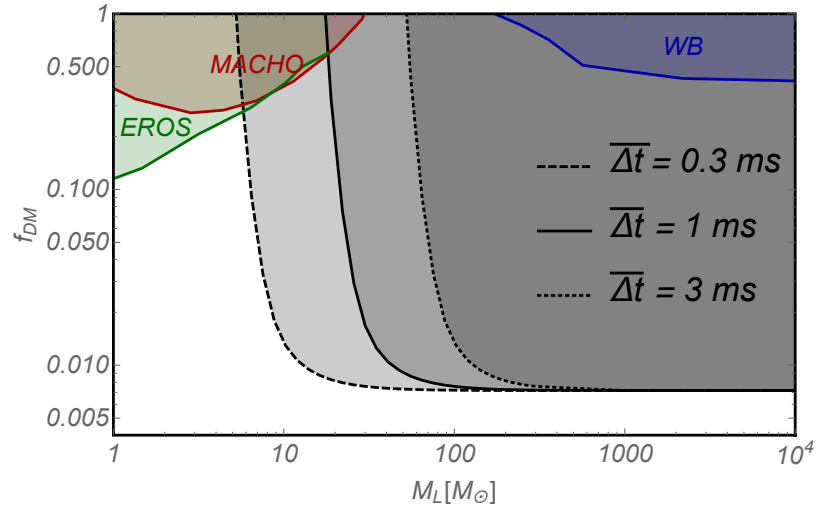


Figure 6.4: Fraction f_{DM} of dark matter allowed in the form of point lenses of mass M_L , if no events out of $N_{\text{FRB}} = 10^4$ are lensed, where the FRBs have a constant comoving density with a cutoff at $z_{\text{cut}} = 0.5$. In dashed, solid, and dotted black we show our constraints when we require a time delay $\Delta t > 0.3$, 1, and 3 ms, respectively. In red we show the current constraints from the MACHO Collaboration, in green the ones from the EROS Collaboration, and in blue the constraints from galactic wide binaries.

For masses in the $20 - 100 M_\odot$ window, outside the reach of the Galactic-lensing surveys,³⁶ a dark-matter fraction of $f_{\text{DM}} \sim 8\%$, at the lower-mass end of this range, and 0.8% at the higher-mass end, would suffice to detect one lensed FRB, if 10^4 FRBs are ob-

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served with a time resolution of 1 ms. As the number of lensed events scales trivially with $N_{\text{FRB}}/10^4$, even a smaller number of $\sim 10^3$ FRBs per year should suffice to detect $\sim 1 - 10$ lensed FRBs in the first year of operation of CHIME, if MACHOs in this window made up the dark matter. This conservative number will still allow us to place constraints on f_{DM} , comparable in magnitude to all current surveys ($f_{\text{DM}} \leq 10\%$), but over the whole mass range $M_L > 20 M_\odot$, if no lensed events are observed. Interestingly, even with a time resolution of 3 ms one would detect at least one lensed event, if MACHOs of mass $M_L \gtrsim 50 M_\odot$ were the main component of dark matter.

6.3.1 Disentangling the Lensing Signal

FRBs might suffer intrinsic repetition. For example, the event FRB 121102 has been observed repeating as quickly as over minutes.²⁸⁸ Lensing by a MACHO of mass $M_L \sim 10^5 M_\odot$ creates a time delay also on the scale of minutes, which then sets a natural ceiling to the MACHO mass that can be unequivocally probed with lensing of FRBs. In general, the correlation between time delays and flux ratios of the bursts, as shown in Fig. 6.3, will be of invaluable help to statistically determine whether repetition of FRBs is caused by microlensing.

Throughout this work we have assumed that an upcoming CHIME-like experiment will detect events up to a cutoff redshift of $z_{\text{cut}} = 0.5$, as this fits the current FRB data. We can also calculate constraints for an increased cutoff redshift, e.g., $z_{\text{cut}} = 0.7$, representing a more optimistic redshift distribution. In that case, for $\overline{\Delta t} = 1$ ms and $\overline{R_f} = 5$, we expect $N_{\text{lensed}} = 35$ lensed events out of 10^4 if dark matter is made of MACHOs of mass $M_L = 20 M_\odot$, $N_{\text{lensed}} = 110$ if this mass is $M_L = 30 M_\odot$, and $N_{\text{lensed}} \gtrsim 200$ for masses higher than $75 M_\odot$. Were none of these 10^4 FRBs to show lensing, however, we could constrain f_{DM} at $30 M_\odot$ to be smaller than 0.9% (or 0.5% for $\overline{\Delta t} = 0.3$ ms, where this last number would apply to larger masses, and smaller $\overline{\Delta t}$). The increase in the lensing optical depth, due to the higher redshift of the events, leads to either more FRBs being lensed, or better constraints on f_{DM} .

6.4 Conclusions

Let us add a few comments before concluding.

It has been argued that we could be preferentially observing strongly-lensed FRBs.^{306,307} If this is the case, most observed FRBs will be lensed by intervening objects, such as galactic halos, on their way to Earth. This would create a double image with a time delay on the order of weeks.³⁰⁸ More importantly, when crossing those galactic halos the probability to be microlensed by a MACHO is close to unity, which would help detect more microlensed FRBs, or improve our constraints on f_{DM} .

Note that, due to our requirement that they behave as point lenses, MACHOs need to be smaller than their Einstein radii. This constrains the size of a MACHO of mass M_L to be more compact than $\sim 0.1 \text{ pc} \times \sqrt{M_L/30 M_\odot}$.

Other Lensing Applications

An effect similar to femto- or nanolensing of gamma-ray bursts could be observed in FRBs,^{309,310} albeit, given the relatively low frequency ($\nu \sim \text{GHz}$) of FRBs, one could probe lenses only with masses higher than $M_L \sim 10^{-5} M_\odot$, since lower masses would create a time delay smaller than $1/\nu$ and not cause interference. An experiment with bandwidth $\Delta\nu \sim 20 \text{ kHz}$ could probe a maximum mass $M_L \sim 0.1 M_\odot$ with nanolensing (higher masses would cause time delays longer than $1/\Delta\nu$ and interfere within each bandwidth). The unknown FRB frequency spectrum poses a challenge to modeling this effect, so it is left for future work.

A Lensed FRB?

Among the FRBs found to date, there is one particular event, FRB 121002, which has been observed with a double peak delayed by 5.1 ms.³¹¹ This delay could have been caused by a MACHO lens of mass $M_L \gtrsim 200 M_\odot$. The second image of FRB 121002 appears brighter, however, which contradicts the usual lensing prediction. In future work will assess

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how likely it is that this delay is due to lensing and study further cosmological applications of lensing of FRBs.

Closure

In conclusion, upcoming interferometers will open up the radio sky, which will allow us to detect FRBs at a staggering pace. By studying whether these FRBs are doubly peaked we can conclude if they have been microlensed or not. Given the existing constraints, compact objects (MACHOs) are allowed to make up a large fraction of dark matter in our Universe (and even all of it in the mass window between 20 and 100 $M_{\odot}$). We will be able to detect from tens to hundreds of lensed FRBs if the dark matter is indeed composed of these MACHOs. Alternatively, if no FRBs are microlensed, we will place the strongest constraints yet on the fraction of dark matter in the form of compact objects.

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Vita



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