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LARGE MASS TIME-LIKE MUON PAIRS IN HADRONIC INTERACTIONS[†]

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ABSTRACT

Utilizing a simple field-theoretic model we consider the production of time-like muon pairs via a massive virtual photon in high energy hadronic collisions. We predict the differential cross section in the mass-squared of the muon pair as well as the total cross section. We find that our form factors do not possess the analogue of the Bjorken scaling associated with deep inelastic electron scattering, but are scale functions multiplied by the usual Bremsstrahlung factor $< t_{average} >/s$.

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We consider the interaction initiated by a hadron (proton or pion)

$$p(\pi) + \text{nucleon} \rightarrow (\mu^+\mu^-) + (\text{hadronic states}) \qquad (1)$$

where the muons are produced via a very virtual photon. We investigate the asymptotic region analogous to that of deep inelastic electron scattering. In this kinematic region the invariant mass-squared of the muon pair q^2 and the center of mass energy squared, s, of the initial hadrons are both very large. We take P^μ and P_1^μ to be the four momenta of the two initial hadrons and q^μ to be four momentum of the muon pair (see Figure 1). We work in the kinematic region where the variables q^2, $q\cdot P$, $q\cdot P_1$, $P\cdot P_1 \rightarrow \infty$ but the various ratios of these variables are held finite; i.e., the ratios $(q^2/P\cdot P_1)$, $(q\cdot P_1/q^2)$, etc. are all finite. It is our hope that these limits are relevant in the experimentally accessible kinematical regions.

If process (1) is summed over all hadronic states and averaged over initial spins then the differential cross section $d\sigma_{\mu^+\mu^-}$ is proportional to the following tensor:

$$W_{\mu\nu} = \left(\frac{E_1}{M}\right)\left(\frac{E_2}{M}\right)\int d^4x\, e^{-iqx} < P.P_1 \mid J_\mu(x)\, J_\nu(0) \mid P.P_1 >$$

$$= \frac{W_{2A}}{M^2}\left(P - \frac{q\cdot P}{q^2}q\right)^\mu \left(P - \frac{q\cdot P}{q^2}q\right)^\nu + \frac{W_{2B}}{M^2}\left(P \rightleftarrows P_1\right)^{\mu\nu} \qquad (2)$$

$$+ \frac{W_{2C}}{2M^2}\left\{\left(P - \frac{q\cdot P}{q^2}q\right)^\mu \left(P_1 - \frac{q\cdot P_1}{q^2}q\right)^\nu + P \rightleftarrows P_1\right\} - \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) W_1$$

where M is the nucleon mass and $J_\mu(x)$ is the full electromagnetic current. The matrix element involved in Eq. (2) is similar to the case considered by Bjorken for inelastic electron scattering except that the initial state contains two hadrons rather than one photon, and the mass of the virtual photon is time-like rather than space-like[1]. It will turn out that the form factors anologous to νW_2 and W_1 of deep inelastic electron scattering, namely $(q\cdot P)W_{2A}$, $(q\cdot P_1)W_{2B}$, $\sqrt{(q\cdot P)(q\cdot P_1)}\,W_{2C}$, and W_1 do not scale in the Bjorken sense but are functions of the form

$$\frac{<t_{average}>}{s}\, F\left(\frac{q^2}{s}, \frac{q\cdot P_1}{s}, \ldots, \frac{q\cdot P}{q^2}\right),$$

where $<t_{average}> \approx (300\ \mathrm{MeV})^2$ is the average momentum transfer in purely hadronic interactions.

In an attempt to predict the total muon pair cross section as well as the differential cross section in the photon mass q^2, we have abstracted certain "insights" gained by studying inelastic electron scattering. These may be summarized as follows.

The Bjorken scaling of the inelastic form factors, such that they are functions of the ratio $q^2/q\cdot P$ (P_μ = initial proton's momentum) rather than independent functions of q^2 and $q\cdot P$ separately, is most simply understood in terms of point-like scattering of the virtual photon on elementary constituent particles. These constituents may be parton-like (see Bjorken and Paschos[2]) or the more ordinary field theoretic virtual nucleons and pions which make up hadronic structures (see Drell, Levy, and Yan[3]). Considering for the moment that this kind of model is capable of explaining processes involving both time- and space-like photons, then we note that there should be a significant qualitative difference between the time- and space-like cases. An elementary constituent may scatter elastically with a very massive space-like photon (i. e. remain on its mass shell before and after the scattering). However in the time-like case, the initial constituent must be very massive (far off-shell) to decay into a very massive photon plus itself. This property reflects itself in the propagator of the internal nucleon (constituent) being evaluated far off-shell for the time-like photon (see Figure 1). (In this kinematical region use of the Ward-Takahashi[4] identity for the proper proton-photon vertex plus the D.G.S.[5] representation suggests that the proper vertex function $\Gamma(q^2, M_1^2, M_2^2)$ when $q^2 \approx M_1^2 \rightarrow \infty$ is indeed of order 1 or point-like.)

Here we perform a model calculation for the process

$$p(\pi) + \text{nucleon} \rightarrow p(\pi) + \text{nucleon} + (\mu^+\mu^-) \qquad (3)$$

We consider only the elastic part of the hadronic interactions, but since in pn interactions this is about one fourth the total cross section, we hope that our calculation gives a reasonable first approximation to the total process (1). This calculation will have two advantageous properties: i) it will be explicitly gauge invariant, and ii) it will involve no artificial cut-offs or extra parameters. We mock up the purely strong high energy interactions with the exchange of a single neutral vector meson with its propagator chosen such that it correctly reproduces the high energy elastic differential cross-sections[6]. (The exchange of more vector mesons essentially leaves our final result unchanged and will be discussed in a longer forthcoming paper.) We introduce electromagnetic interactions through minimal point-like couplings.

We find after doing all integrations except over the virtual photon the differential cross-section for $P + N \rightarrow P + N + (\mu^+\mu^-)$,

$$d\sigma_{\mu^+\mu^-} = \sigma_{pn}\left(\frac{\alpha}{\pi}\right)^2\left(\frac{4}{3}\right)\frac{<t_{average}>}{s}\frac{1}{q^2}\frac{d^4q}{2\pi}\frac{(2P\cdot P_1 - 2(q\cdot P_1 + q\cdot P) + q^2)}{(q^2-2q\cdot P)^2} \quad (4)$$

$$\times\ e^{\dfrac{-M^2(q^2-2q\cdot P)^2}{<t_{average}>(2P\cdot P_1 - 2q\cdot P_1)(q^2-2q\cdot(P+P_1)+2P\cdot P_1)}}$$

where the constant total elastic pn cross section, σ_{pn}, is taken as 10 millibarns and $< t_{average} > \approx M_p^2/12.$ [7] The factor $< t_{average} >/s$ is analogous to the classical acceleration factor in bremsstrahlung emission[8]. To calculate the process $\pi + p \rightarrow \pi + p + (\mu^+\mu^-)$, we replace σ_{pn} by $\sigma_{\pi p}$ and $< t_{average} >$ by the appropriate average momentum transfer for πp elastic scattering. After integration over the photon three-momentum $d\sigma_{\mu^+\mu^-}$ becomes a distribution in q^2

$$d\sigma_{\mu^+\mu^-} \approx \sigma_{pn}\left(\frac{\alpha}{\pi}\right)^2\left(\frac{4}{3}\right)\frac{<t_{average}>}{s}\frac{dq^2}{q^2}\frac{1}{q^2}\left\{\frac{(s-q^2)(s+q^2)}{8s} - \frac{q^2}{4}\ln\frac{s}{q^2}\right\} \quad (5)$$

When we perform this integration in the overall c.m. system the range of integration of $|\vec{q}|$ is $|\vec{q}| = 0$ to $|\vec{q}| = (s-q^2)/2\sqrt{s}.$ We stress that this expression is approximate and only appropriate in the region $q^2 \rightarrow \infty$ and $s \rightarrow \infty$ with the ratio q^2/s finite. This formula is exact only if the masses of the hadrons are zero. Upon doing the integration over q^2 we find for the total cross-section $\sigma_{\mu^+\mu^-} \approx 2.0 \times 10^{-34}\,cm^2$, when $s = 60\ (GeV)^2$. Here we let the lower bound on q^2 be 1.5 $(GeV)^2$ as suggested by experiment[9]. We restate here that our structure functions $(q\cdot P\, W_{2A}, \ldots, W_1)$ do not exhibit the analogue of Bjorken scaling as suggested by other authors[10]. This is a consequence of using vector particles to mediate the strong forces and not allowing large momentum transfers in the purely hadronic interactions.

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References

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2. J. Bjorken and E. Paschos, Phys. Rev. (1969) to be published.
3. S. Drell, J. Levy, T. M. Yan, Phys. Rev. Letters 22, 744 (1969).
4. Y. Takahashi, Prog. Theoret. Phys. 11, 251 (1954).
5. S. Deser, W. Gilbert, E. Sudarshan, Phys. Rev. 115, 731, (1959).
6. The momentum transfer dependence is taken from the measured elastic pn data as a simple exponential $e^{t/< t_{average} >}$.
7. To simplify the kinematics we used all scalar hadronic particles in obtaining our numerical results, although explicit calculations show that our results are essentially unchanged when we use spin 1/2 kinematics. In Figure 1 the contact term necessary for the scalar theory is not shown.
8. We thank J. D. Jackson for this comment.
9. L. Lederman, invited talk at Daresbury.
10. V. A. Matveyev, R. M. Muradyan, and A. N. Tavkhelidze, Dubna Preprint, JINR-P2-4543, (1969).

Figure Caption

Figure 1 — Model for $P + N \rightarrow P + N + (\mu^+\mu^-)$.

Figure 2 — Plots of q^2 distribution when masses of hadrons are taken equal to zero (expression 5) and when masses are non-zero (expression 4).

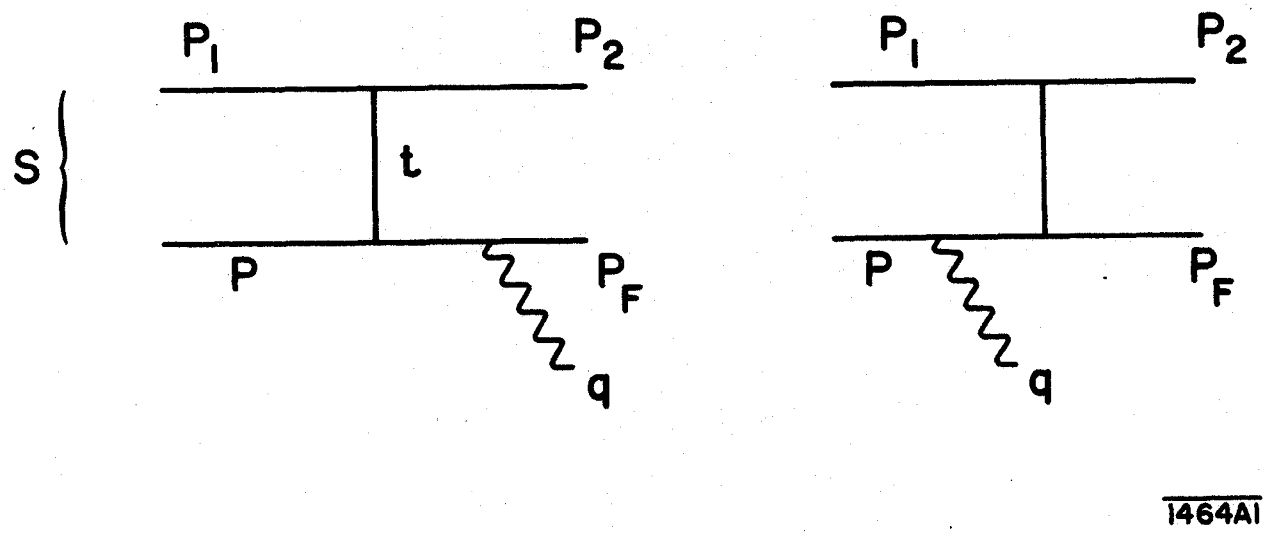
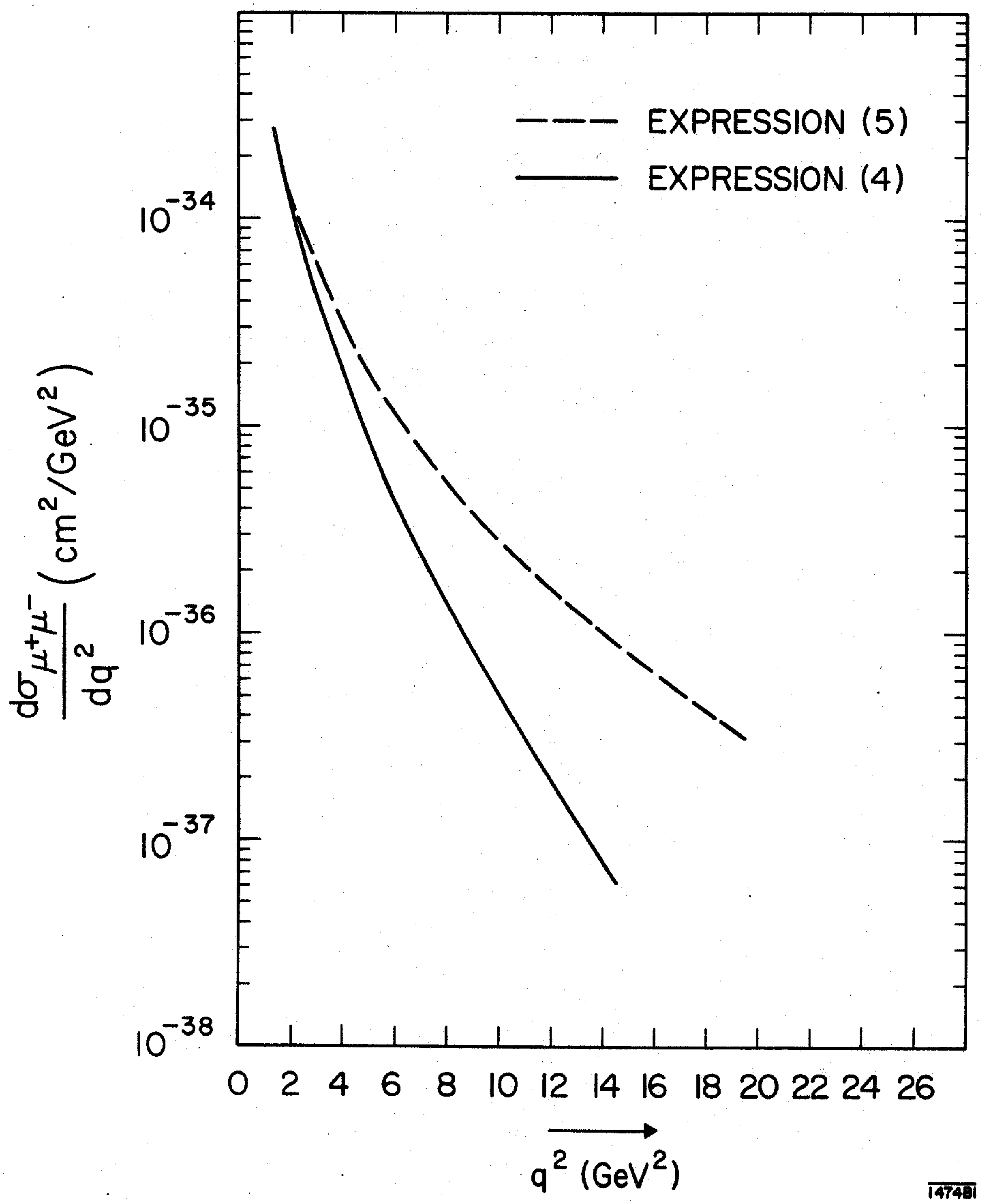


Fig. 1



EXPRESSION (5)
EXPRESSION (4)
10^{-34}
10^{-35}
10^{-36}
10^{-37}
10^{-38}
$\frac{d\sigma_{\mu^+\mu^-}}{dq^2}$ (cm^2/GeV^2)
0 2 4 6 8 10 12 14 16 18 20 22 24 26
q^2 (GeV^2)
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Fig. 2