Gauge (In-)Dependence of the Gluon Propagator Poles and QCD Plasma Parameters

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Abstract

The recent controversy concerning the gauge dependence of the QCD plasma dispersion relations is discussed. A set of general Ward identities is used to argue that in a self-consistent perturbative calculation the QCD dispersion relations derived from the ordinary gluon propagator will be gauge fixing independent.

Hadronic matter at high temperatures and pressures is believed to undergo a phase transition to a deconfined plasma phase consisting of weakly interacting, deconfined quarks and gluons [1]. There has been a great deal of controversy in recent years, however, due to the gauge fixing dependence of the gluon plasma dispersion relations at the one loop level [2]. Much of the controversy has focussed on the correct definition of dispersion relations in hot QCD, and has led several groups [3] to propose dispersion relations derived from modified, manifestly gauge fixing independent gluon propagators. As well, calculations based on colour electric and magnetic correlation functions have been motivated in part by the apparent gauge fixing dependence of the damping constant [4]. Braaten and Pisarski [5], on the other hand, have argued that the gauge dependence results from the breakdown of the loop expansion, and have shown for the usual gluon propagator in covariant and Coulomb gauges that a resummation of higher loop graphs leads to gauge independent dispersion relations.

In the following, we will try to clarify the issues raised in the above controversy by showing that a set of generalized Ward identities govern the gauge dependence, in a wide class of gauges, of the QCD propagator and hence of the QCD plasma dispersion relations [6]. Given certain additional assumptions, the identities imply the gauge fixing independence of the **physical** poles in the gluon propagator. In the context of the gluon plasma, the identities explain the gauge fixing independence of the lowest order plasma frequency and imply that the gauge fixing dependence of the damping constant found in one loop calculations will be absent in a self consistent perturbative calculation.

Consider a Yang-Mills theory with classical action

$$S[A] = -\frac{1}{4}\int d^4x \left(F^a_{\mu\nu}\right)^2, \qquad (1)$$

where $F^a_{\mu\nu} \equiv \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\nu A^c_\mu$. The action is invariant under gauge transformations of the form $\delta A^a_\mu = D^{ab}_\mu \lambda^b$, where D^{ab}_μ is the covariant derivative. To define the quantum theory, a gauge condition must then be chosen; we shall consider such conditions defined by adding a term $\eta_{ab}F^a[A]F^b[A]$ to the action (1), where $F^a[A]$ is an arbitrary function of the quantum field. The Faddeev-Popov ghosts are then included as needed.

Changing the gauge fixing condition in the action will of course affect the calculation of the Green functions of the theory. Let us examine specifically how this change affects the gluon propagator. With global colour symmetry and translational invariance, we assume the propagator can be decomposed as

$$D_{\mu\nu}(k) = \frac{1}{\mathcal{T}(k)}A_{\mu\nu} + \\ +\frac{1}{\mathcal{L}(k)}\left[\frac{\tilde{n}_\mu \tilde{n}_\nu}{\tilde{n}^2} + \beta(\tilde{n}_\mu k_\nu + k_\mu \tilde{n}_\nu) + \delta\frac{k_\mu k_\nu}{k^2}\right], \quad (2)$$

with $\tilde{n}_\mu = P_{\mu\nu} n^\nu$, $P_{\mu\nu} = g_{\mu\nu} - k_\mu k_\nu / k^2$, and

$$A_{\mu\nu} = P_{\mu\nu} - \tilde{n}_\mu \tilde{n}_\nu / \tilde{n}^2 = \begin{pmatrix} 0 & 0 \\ 0 & -\delta_{ij} + k_j k_j / \vec{k}^2 \end{pmatrix} \tag{3}$$

projects out the spatially transverse mode. Note that the matrix $\tilde{n}_\mu \tilde{n}_\nu$ is transverse with respect to k_μ and is orthogonal to $A_{\mu\nu}$. At zero temperature the above decomposition restricts the gauge condition to depend at most on the momentum four-vector and on one arbitrary, fixed vector n_μ. At finite temperature the heat bath defines a preferred frame n_μ, so that the gauge conditions can depend at most on n_μ and derivatives.

At finite temperature, the structure function $1/\mathcal{L}$ represents the additional collective "plasmon" mode which propagates independently from the spatially transverse mode $1/\mathcal{T}$. The zeroes of $\mathcal{T}$ and $\mathcal{L}$ then determine the dispersion relations of the spatially transverse (plane-wave) and spatially longitudinal (plasmon) collective modes, respectively, of the quark-gluon plasma [7,8].

One can derive Ward identities that reflect how the propagator of Eq.(2) changes when the gauge fixing condition is changed : $F^a(A) \to F^a(A) + \Delta F^a(A)$. These identities are [6]:

$$\Delta D^{\mu\nu}(k) = -D^{\mu\lambda}(k) \Delta X^\nu_{,\lambda}(k) - \Delta X^\mu_{,\lambda}(k) D^{\lambda\nu}(k), \tag{4}$$

where the functional $\Delta X^\mu_{,\nu}$ involves the Faddeev-Popov operator and depends on how the gauge fixing condition is changed. Projecting Eq. (4) onto the orthogonal matrices $A_{\mu\nu}$ and $\tilde{n}_\mu \tilde{n}_\nu$, respectively, yields:

$$\Delta\mathcal{T}(k) = -\mathcal{T}(k) A^\lambda_\nu(k) \Delta X^\nu_{,\lambda}(k) \equiv \mathcal{T}(k) \Delta Y(k) \tag{5}$$

and

$$\Delta\mathcal{L}(k) = -2\mathcal{L}(k) \left[\frac{\tilde{n}_\nu \tilde{n}^\lambda}{\tilde{n}^2} \Delta X^\nu_{,\lambda} - \frac{B}{D} \tilde{n}_\nu k^\lambda \Delta X^\nu_{,\lambda} \right] \equiv \mathcal{L}(k) \Delta Z(k). \tag{6}$$

In the above, B and D are the components of the non-transverse parts of the bare propagator. Thus, as long as the coefficients of $\mathcal{T}$ and $\mathcal{L}$, respectively, on the right hand side of the above equations are well behaved in the neighbourhood of the solutions, the dispersion relations are gauge fixing independent: A solution of $\mathcal{T}(k) = 0$ is also a solution of $(\mathcal{T} + \Delta\mathcal{T})(k) = 0$, and similarly for solutions of $\mathcal{L}(k) = 0$.

However, not all zeroes in $\mathcal{T}$ and $\mathcal{L}$ are gauge fixing independent. One must exclude from this proof such zeroes which coincide with poles in ΔY and ΔZ, since in this case a cancellation in Eq. (5) or Eq. (6) may occur. Thus, it is difficult to make general statements concerning the gauge independence of the zeroes in $\mathcal{T}$ and $\mathcal{L}$ because the pole structure of ΔY and ΔZ may be complicated.

In perturbation theory, however, one does have a handle on the analytic structure of ΔY and ΔZ, and so definite statements can be made concerning the gauge fixing independence of the physical poles in the gluon propagator [6]. Consider, for example, the loop expansion of Eq. (5)

$$\Delta\mathcal{T}(k) = \mathcal{T}(k) \Delta Y(k) \tag{7}$$

with

$$\mathcal{T} = \mathcal{T}^{(0)} + \mathcal{T}^{(1)} + \mathcal{T}^{(2)} + \ldots, \tag{8}$$

$$\Delta Y = \Delta Y^{(0)} + \Delta Y^{(1)} + \Delta Y^{(2)} + \ldots \tag{9}$$

One finds to lowest order $\Delta\mathcal{T}^{(0)} = \mathcal{T}^{(0)} \Delta Y^{(0)}$. Since $\mathcal{T}^{(0)}(k)$ is gauge independent, we have $\Delta Y^{(0)} \equiv 0$. Thus, to one loop order

$$\Delta\mathcal{T}^{(1)} = \mathcal{T}^{(0)} \Delta Y^{(1)}. \tag{10}$$

When the loop expansion is consistent, the on-shell condition to this order is given by $\mathcal{T}^{(0)} = 0$, and therefore Eq. (10) guarantees gauge independence of one-loop corrections to the self energy of the transverse mode on-shell.

In cases when the loop expansion is not consistent, as in hot gauge theories where g is small but $gT \sim 1$, these identities can be used to explain the gauge dependence found in one-loop calculations. Consider the high temperature, long wavelength limit for pure QCD. In this case the transverse structure function to one loop order is of the form

$$\mathcal{T}^{(0)} = k_0^2 \tag{11}$$

$$\mathcal{T}^{(1)} = -\frac{1}{9} N (gT)^2 + C(\alpha) g (gT) k_0 + \mathrm{O}(1/T), \tag{12}$$

where $C(\alpha)$ is a complex, gauge parameter dependent numerical coefficient. The plasma parameters are obtained by finding solutions to

$\mathcal{T}^{(0)}+\mathcal{T}^{(1)}=0$ of the form $k_0=\omega_p-i\gamma$, where $\gamma << \omega_p$. This yields

$$\omega_p^2=\frac{1}{9}N(gT)^2+O(g), \tag{13}$$

$$\gamma=\frac{g}{2}ImC(\alpha)(gT). \tag{14}$$

Since $\mathcal{T}^{(0)} \sim (gT)^2 \sim 1$ on the plasmon mass shell, Eq. (10) implies that the gauge dependence of the dispersion relations will be of the same order as $\Delta Y^{(1)}$, and an explicit calculation shows that $\Delta Y^{(1)} \sim g^2T$. This then accounts for the gauge independence to leading order of the plasmon mass, as well as the gauge fixing dependence to order g^2T of the one loop damping constant.

For a self-consistent expansion one needs $\mathcal{T}^{(n+1)} \ll \mathcal{T}^{(n)}$. In the case of high temperature QCD this means that an adequate expansion is not just in powers g, but only in those powers of g in excess of powers of T. In such an expansion,

$$\mathcal{T}(k_0)=\overline{\mathcal{T}}^{(0)}(k_0;(gT))+g\overline{\mathcal{T}}^{(1)}(k_0;(gT))+\dots, \tag{15}$$

where it can be shown that $\overline{\mathcal{T}}^{(0)}=k_0^2-\frac{1}{9}N(gT)^2$. Thus, to this order there are no contributions in excess g from higher orders in the loop expansion. In such a self-consistent expansion, the plasmon mass shell is defined by $\overline{\mathcal{T}}^{(0)}=0$, and since the Ward identity (10) necessarily holds, it follows that the leading order damping constant, essentially determined by the imaginary part of $\overline{\mathcal{T}}^{(1)}$, will be gauge fixing independent, as long as the corresponding ΔY has no poles at the plasmon mass [6]. This gauge independence has in fact been recently verified by the calculation of Braaten and Pisarski [5] in covariant and Coulomb gauges.

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Q. **B. Ioffe** *(ITEP, Moscow)*: You know, of course, that in QCD the perturbation theory is violated inevitably in higher orders by infrared effects. Can you say something about the gauge dependence, taking into account these effects?

A. **R. Kobes**: One point is that we do not know how to handle the IR divergences—certainly they arise in a loop expansion, but there could be some other expansion in which they don't arise (eg., some generalization of 3rd order summation in the pressure). However, the proof we had of gauge independence of the pole assumes validity of (some) self-consistent expansion. It does not prove that such an expansion exists, but it does show that the question of one-loop gauge dependence and the validity of perturbation theory are separate questions.

G. Kunstatter: It is important to note that the main point of Dr. Kobes' talk was to prove, using generalized Ward identities, that the gauge dependence of the one loop damping constant is due to the fact that the pure one loop calculation is not a self-consistent approximation. Thus one must, to some extent, separate the question of gauge dependence of the damping constant at one loop, from the question of the validity of the perturbative QCD vacuum. While Dr. B. Bambah's comment presents an interesting approach to regulating infrared divergences, the results in no way contradict, or invalidate the discussion in Dr. Kobes' talk.