A General Dark Energy Model

Changyu Huang[1] (a,b), Qing Zhang[2] (a) and Yong-Chang Huang[3] (a,c)

(a) *Institute of Theoretical Physics, Beijing University of Technology, Beijing, 100124, China*
(b) *Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing, 100190, China*
(c) *CCAST (World Lab.), P.O.Box 8730, Beijing, 100080, China*

Abstract

This Letter generally deduces a general dark energy model in Einstein's special relativity, which just shows a general dark energy may generally exist not being necessary in general relativity.

1 Introduction

Special Relativity is, built up by Einstein, a kind of special spacetime theory [1]. It overcomes the difficulties relative to Galilean transformation ((i) the equations of electromagnetic fields do not obey the Galilean transformation; (ii) the speed of light c is a constant —— the results of Michelson - Morley experiment; (iii) Particles move with high-speed), it generalized Lorentz invariant property of the equations of electromagnetic fields to the situation of mechanics. In addition, it expanded and modified Newtonian spacetime, thus Einstein gave the creation of a new era of Einstein's space-time theory. Beside, Poincare etc, had some contributions to the modern theory of special relativity [2]. Newtonian mechanics is the case of special relativity in the low-speed approximation.

2 A general dark energy model

For convenience of discussion, we first review some main relative results to be used in this Letter, then generally deduce a general dark energy model in Einstein's special relativity. Using proper time $d\tau = dt\sqrt{1-v^2/c^2}$ ($v = \frac{dl}{dt}$ ($l^2 = \Delta\mathbf{x}\cdot\Delta\mathbf{x}$) is the velocity of the object in the reference frame) and generalizing the speed to four dimensions, we may set up the Four-velocity

$$u_\alpha = \frac{dx_\alpha}{d\tau} = \frac{1}{\sqrt{1-v^2/c^2}}\frac{dx_\alpha}{dt} = \frac{v_\alpha}{\sqrt{1-v^2/c^2}},\ \alpha = 0,1,2,3. \tag{1}$$

Utilizing the invariance of the interval $\triangle x^0\triangle x_0+\triangle x^1\triangle x_1+\triangle x^2\triangle x_2+\triangle x^3\triangle x_3=c^2\triangle\tau\triangle\tau$, then we have $u^0u_0+u^1u_1+u^2u_2+u^3u_3=c^2=\eta^{\mu\nu}u_\mu u_\nu$. Therefore, we obtain

$$2\eta^{\mu\nu}u_\mu\frac{du_\nu}{d\tau}=2u_\mu\frac{du^\mu}{d\tau}=0. \tag{2}$$

Eq.(2) shows that the relation of 4-vector u_μ and $du^\mu/d\tau$ is vertical.

With Eq.(1), we define the four-momentum as

$$p_\alpha=m_0u_\alpha=m_0dx_\alpha/d\tau,\ \alpha\text{=0,1,2,3.} \tag{3}$$

Then the fourth component of the four-momentum is

$$p_4=m_0u_4=m_0ic/\sqrt{1-\beta^2}, \tag{4}$$

[1] Email address: cyhuang520@hotmail.com
[2] Email address: qingzhang@emails.bjut.edu.cn
[3] Email address: ychuang@bjut.edu.cn

the space component of the four-momentum is

$$p_i = \frac{m_0}{\sqrt{1-v^2/c^2}}\frac{dx_i}{dt} = mv_i,\ (m = m_0/\sqrt{1-\frac{v^2}{c^2}}). \tag{5}$$

In the equations of the special relativity, for an object acted by four-force K_μ, and considering Eq.(3), we have

$$K_\mu = \frac{dp_\mu}{d\tau} = m_0\frac{du_\mu}{d\tau}, \tag{6}$$

multipling Eq.(2) with m_0 , we achieve

$$u_\mu m_0 du^\mu/d\tau = u_\mu K^\mu = 0, \tag{7}$$

Eq. (7) shows that 4-vector u_μ is vertical to K^μ. With Eq.(6), we have

$$K_i = \frac{dp_i}{d\tau} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\frac{d(mv_i)}{dt} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}F_i. \tag{8}$$

The forth component of four-force is

$$K_4 = \frac{dp_4}{d\tau} = m_0\frac{d\mu_4}{d\tau}. \tag{9}$$

Substituting Eq.(8) and (9) into (7), we obtain

$$u_4K^4 + u_iK^i = u_4\frac{dp^4}{d\tau} + u_i\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}F^i = 0. \tag{10}$$

Then, we have

$$\frac{dp^4}{d\tau} = \frac{i}{c\sqrt{1-\frac{v^2}{c^2}}}v_iF^i, \tag{11}$$

$\mathbf{v}\cdot\mathbf{F}$ is the power of the force $\mathbf{F}$, and the power is equal to the rate of increase of energy with time, i.e., $\frac{dE}{dt}$, thus we have

$$\frac{dp^4}{d\tau} = \frac{i}{c\sqrt{1-\frac{v^2}{c^2}}}\frac{dE}{dt} = \frac{i}{c}\frac{dE}{d\tau}. \tag{12}$$

Therefore, we obtain

$$\frac{dp_4}{d\tau} = \frac{i}{c}\frac{dE}{d\tau}. \tag{13}$$

Then, we achieve

$$p_4 = \frac{i}{c}E + C_{01}, \tag{14}$$

where C_{01} is an integral constant, satisfies $\frac{C_{01}}{d\tau}=0$. It means that C_{01} is independent of the proper time or time. Therefore, C_{01} has a clear physical significance, which is relative to dark energy, cosmological constant and zero-point energy in quantum field theory. When neglecting C_{01}, it reduces to Einstein's theory of special relativity. Thus, using Eq.(4) we obtain

$$p_4 = \frac{i}{c}E + C_{01} = \frac{m_0ic}{\sqrt{1-\beta^2}}. \tag{15}$$

Thus, we achieve a general energy

$$E = m_0c^2/\sqrt{1-\beta^2} + E_{01} = mc^2 + E_{01}, \tag{16}$$

where $E_{01}=icC_{01}$ is a general invariant energy as a general dark energy, because its dimension is energy. In particular, it is just the dark energy that causes the accelerating expansion of the universe or relative to cosmological constant and so on, and which is determined by the relative cosmological experimental data.

3 Discussion and conclusion

When neglecting E_{01} , Eq.(16) reduces to the Einstein's Mass-Energy relation $E{=}m_0c^2/\sqrt{1{-}\beta^2}{=}mc^2$, which is another way to deduce the Einstein's Mass-Energy relation. It means that the achieved theory is consistent. $E{=}mc^2$ means that any matter has the great power, which predicts the existence of matter energy. Any rest object has a great rest energy $E_0{=}m_0c^2$. Combining Eqs. (8) and (12), we have $K_\mu{=}(\mathbf{K},i\mathbf{K}{\cdot}\mathbf{v}/c)$. If v changed, then $\dot{v}{\neq}0$, it is a non-inertial reference frame. In fact, the $\mathbf{F}{=}m\mathbf{a}$ is equivalent to $\mathbf{F}{=}d\mathbf{p}/dt$ in Newtonian physics. In the discussion above, we find that the former is not suitable for generalizing, becuase the property of m and $\mathbf{a}$ is not well defined. While the latter is a well defined vector, once we replace dt with $d\tau$, and generalize the 3-momentum to 4-momentum, it is just the relation between 4-force and 4-acceleration, which looks like the Newton's second law in four dimensional expression.

Therefore, we generally deduces a general dark energy model in Einstein's special relativity, which just shows that a general dark energy may generally exist, which is not necessary to exist in general relativity.

Acknowledgment: The work is partly supported by National Natural Science Foundation of China (Grant No. 10435080) and Beijing Natural Science Foundation (Grant No. 1072005).

References

[1] W. G. Dixon, W. Graham, Special relativity: the foundation of macroscopic physics, Cambridge University Press (1978).

[2] W. Rindler, Relativity: special, general, and cosmological, New York, Oxford University Press (2006).