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Correlations and Fluctuations in Relativistic Nuclear Collisions

Fluctuations in the statistical ensembles

Francesco Becattini*, **Antti Keränen, Lorenzo Ferroni, Tommaso Gabbriellini**

Università di Firenze and INFN Sezione di Firenze, Via G. Sansone 1, I-50019, Sesto F.no (Firenze), Italy

E-mail: `becattini@fi.infn.it`

Abstract. In this paper, we address multiplicity fluctuations of the ideal hadron-resonance gas in different ensembles: grand-canonical, canonical and microcanonical. Two different calculation methods are used: asymptotic expansions and full Monte Carlo simulations. The method based on asymptotic expansion allows a quick numerical calculation of dispersions in the hadron gas with three conserved charges at primary hadron level, while the Monte-Carlo simulation is suitable to study the effect of resonance decays. Even though mean multiplicities converge to the same values, major differences in fluctuations for these ensembles persist in the thermodynamic limit, as pointed out in recent studies. This difference is ultimately related to the non-additivity of the variances in the ensembles with exact conservation of extensive quantities.

1. Introduction

The statistical hadronization model has proven to be a very effective tool in describing average particle multiplicities in high energy heavy ion reactions as well as in elementary particle reactions. On the other hand, in this model it would be possible to calculate also fluctuations of particle multiplicities once the status of the hadronizing sources (*clusters* or *fireballs*) in terms of volume, mass, momentum and charges were known. Multiplicity and charge fluctuations have been indeed proposed to be a good discriminating tool between quark-gluon plasma and hadron gas [1, 2] provided that they survive the phase transition and the hadronic system freezes out in a non-equilibrium situation. However, in order to properly assess the discriminating power of such observables, one should firstly calculate fluctuations in a hadron gas by including all "trivial" effects, such as conservation laws, quantum statistics, resonance decays, kinematical cuts etc. The effects of conservation laws on fluctuations in thermal ensembles have been firstly addressed, in the perspective of heavy ion collisions, in ref. [3]. More recently, it has been pointed out [4, 5] that in the canonical ensemble (CE) with exact conservation of charges, scaled second moment (scaled variance) of the multiplicity distribution of any particle does not converge to the corresponding GC value even in the thermodynamic limit, unlike the mean [6, 7]. This was fairly understood among experts in statistical mechanics [8], but probably it has been shown explicitely for the canonical relativistic gas for the first time in refs. [4, 5]. Further deviations from the GC limit were found in the case of exact energy and energy-momentum conservation in the microcanonical ensemble (MCE) [9, 10]. Since in a heavy ion collision conservation of charges must be fulfilled, the difference between CE and GCE might have some impact on the estimated size of fluctuations in a statistical model.

The calculations performed in these recent studies [4, 5, 10, 11] were mainly concerned with simplified cases, such as pion gas and pion-nucleon gas. In this work, we address the fluctuations in the *general* multi-species hadron gas including all resonances up to 1.8 GeV mass and carrying three additive charges, that is baryon number B, strangeness S and electric charge Q, in the CE. A similar study has been performed for the MCE, which will not be reported here; this can be found, along with a more complete and detailed description of our work, in ref. [12]. Here, we also discuss the problem of the inequivalence between GCE and CE in the thermodynamic limit for scaled variance and we show that the ultimate reason thereof is the conceptual difference between additivity and extensivity [13]: while particle multiplicities are additive and extensive in both GCE and CE, variances are extensive (i.e. proportional to the volume) but they are non additive in the CE, so that the scaled variance turns out to be a *pseudo-intensive* quantity (according to the definition proposed in ref. [13]).

2. Asymptotic fluctuations in the canonical ensemble

Following refs. [4, 5], we describe fluctuations by means of the *scaled variance* of a multiplicity distribution:

$$\omega = \frac{\langle N^2 \rangle - \langle N \rangle^2}{\langle N \rangle}. \tag{1}$$

where N is meant to be the multiplicity of any hadron species, primary or final (i.e. after resonance decays) or the sum of an arbitrary number of hadron species (e.g. all negatively chraged). This is a finite quantity in the infinite volume limit because the difference between $\langle N^2 \rangle$ and $\langle N \rangle^2$ depends linearly on the volume for large volumes. It is worth reminding that, if quantum statistics is neglected, the multiplicity distribution of any *primary* hadron is a Poisson, thus $\omega = 1$.

For sake of simplicity, we will first keep our discussion at the level of the classical Maxwell-Boltzmann (MB) statistics. Indeed, none of our arguments is affected by this approximation, and at the end we will give the proper generalization to quantum statistics and discuss the corrections. In this framework, using the one-particle partition function:

$$z_{j(1)} = (2J_j + 1) \frac{V}{(2\pi)^3} \int \mathrm{d}^3\mathrm{p} \, \exp\left[-\sqrt{\mathrm{p}^2 + m_j^2}\right] \tag{2}$$

and the fugacity λ_j for each particle species j, the grand-canonical partition function can be written as:

$$Z_{\mathrm{GC}}(\{\lambda_j\}) = \prod_j \sum_{N_j=0}^{\infty} \frac{1}{N_j!} \left(z_{j(1)}\lambda_j\right)^{N_j}. \tag{3}$$

and, consequently, multiplicities of different species are uncorrelated and Poissonianly distributed:

$$P_{\mathrm{GC}}(N_j) = \frac{1}{N_j!} \langle N_j \rangle^{N_j} \mathrm{e}^{-\langle N_j \rangle}. \tag{4}$$

Since the sum of random Poisson variables is still Poisson, this also holds for any given subset of particles, e.g. negative hadrons or baryons.

In the canonical ensemble, the partition function does not factorize into one-species expressions because of the constraint of fixed charges. Let us consider a hadron gas with three abelian charges, i.e. baryon number B, strangeness S and electric charge Q. In the following, we will denote by $\vec{Q} = (Q_1, Q_2, Q_3) = (B, S, Q)$ a vector with components these charges and by $\vec{q}_j = (q_{1,j}, q_{2,j}, q_{3,j}) = (b_j, s_j, q_j)$ the vector of charges of the j^{th} hadron species [1]. The canonical

[1] For sake of clarity, it is worth stressing the difference between $q_{i,j}$, which is the i th charge of the hadron species j and q_j, which stands for its *electric charge*. Likewise, whilst Q_i stands for the net i th charge of the system, Q is its net electric charge throughout the paper

partition function with charges $\vec{Q}$ can be written as:

$$Z_{\vec{Q}} = \left[\prod_{i=1}^{3} \frac{1}{2\pi} \int_0^{2\pi} \mathrm{d}\phi_i \; \mathrm{e}^{-\mathrm{i}Q_i\phi_i}\right] Z_{GC}(\{\lambda_j\}), \tag{5}$$

where Wick-rotated fugacities $\lambda_j = \exp[\mathrm{i}\sum_i q_{i,j}\phi_i]$ are introduced in the grand-canonical partition function Z_{GC}. By setting $w_i = \exp[\mathrm{i}\phi_i]$, we may write Eq. (5) as a triple integral over the unitary circle in the complex w plane:

$$Z_{\vec{Q}} = \frac{1}{(2\pi \mathrm{i})^3} \oint \mathrm{d}w_B \oint \mathrm{d}w_S \oint \mathrm{d}w_Q \, w_B^{-B-1} w_S^{-S-1} w_Q^{-Q-1} \exp\left[\sum_j z_{j(1)} w_B^{b_j} w_S^{s_j} w_Q^{q_j}\right]. \tag{6}$$

The first and second moments of multiplicity distributions of a set h of hadron species can be calculated by inserting a suitable fictitious fugacity in the function Z_{GC}, i.e. replacing λ_j with $\lambda_h\lambda_j$ in Eq. (5) if $j \in h$ and taking the derivatives with respect to λ_h [14]:

$$\langle N_h \rangle = \frac{1}{Z_{\vec{Q}}} \frac{\partial Z_{\vec{Q}}}{\partial \lambda_h}\bigg|_{\lambda_h=1} = \sum_{j\in h} z_{j(1)} \frac{Z_{\vec{Q}-\vec{q}_j}}{Z_{\vec{Q}}} \tag{7}$$

$$\langle N_h^2 \rangle = \frac{1}{Z_{\vec{Q}}} \left[\frac{\partial}{\partial \lambda_h}\left(\lambda_h \frac{\partial Z_{\vec{Q}}}{\partial \lambda_h}\right)\right]_{\lambda_h=1} = \sum_{j\in h} z_{j(1)} \frac{Z_{\vec{Q}-\vec{q}_j}}{Z_{\vec{Q}}} + \sum_{j,k\in h} z_{j(1)} z_{k(1)} \frac{Z_{\vec{Q}-\vec{q}_j-\vec{q}_k}}{Z_{\vec{Q}}} \tag{8}$$

Using these, the scaled variance can be written as the sum of a Poissonian term, i.e. 1, and a canonical correction term:

$$\omega_h = 1 + \frac{\sum_{j\in h}\langle N_j\rangle \sum_{k\in h} z_{k(1)} \left(\dfrac{Z_{\vec{Q}-\vec{q}_k-\vec{q}_j}}{Z_{\vec{Q}-\vec{q}_j}} - \dfrac{Z_{\vec{Q}-\vec{q}_k}}{Z_{\vec{Q}}}\right)}{\sum_{j\in h}\langle N_j\rangle} \tag{9}$$

Therefore, in the canonical ensemble, the quantities appearing in the expressions of the moments of the multiplicity distributions are the canonical partition functions calculated for the difference between total charges and charges of hadrons, like $Z_{\vec{Q}-\vec{q}_j}$ and $Z_{\vec{Q}-\vec{q}_j-\vec{q}_k}$. The quantity within brackets in the equation above vanishes in the thermodynamic limit $V \to \infty$ [12]. However, the factor $\langle N_j\rangle z_{k(1)}$ is proportional to V^2 and, if the difference between brackets has terms proportional to $1/V$, they could give a finite contribution to ω_h in the thermodynamic limit. In fact, after a lenghty derivation based on asymptotic expansion of the canonical partition function (6), it can be proved that [12]:

$$\begin{aligned}
\frac{Z_{\vec{Q}-\vec{q}_j-\vec{q}_k}}{Z_{\vec{Q}-\vec{q}_j}} - \frac{Z_{\vec{Q}-\vec{q}_k}}{Z_{\vec{Q}}} &= -\frac{\lambda_B^{b_k}\lambda_S^{s_k}\lambda_Q^{q_k}}{V}\left[\frac{b_k b_j}{\lambda_B^2}M_{11} + \frac{s_k s_j}{\lambda_S^2}M_{22} + \frac{q_k q_j}{\lambda_Q^2}M_{33}\right.\\
&\quad \left. + \frac{b_k s_j + s_k b_j}{\lambda_B\lambda_S}M_{12} + \frac{b_k q_j + q_k b_j}{\lambda_B\lambda_Q}M_{13} + \frac{s_k q_j + q_k s_j}{\lambda_S\lambda_Q}M_{23}\right] + \mathcal{O}(V^{-2})\\
&\equiv \frac{\lambda_k}{V} C_{jk} + \mathcal{O}(V^{-2}).
\end{aligned} \tag{10}$$

where $\lambda_Q, \lambda_B, \lambda_S$ are fugacities related to the net charges of the system by:

$$\sum_j q_{i,j} z_{j(1)} \lambda_B^{q_j} \lambda_S^{s_j} \lambda_Q^{q_j} = Q_i \quad . \tag{11}$$

The matrix M in Eq. (10) is defined by:

$$M_{ln} = \sum_{i=1}^{3} \frac{A_{il} A_{in}}{h_i} \tag{12}$$

where h_i are the eigenvalues of the hessian matrix H of the function f:

$$f(\vec{w}) = -\rho_B \ln w_B - \rho_S \ln w_S - \rho_Q \ln w_Q + \sum_k \frac{z_{k(1)}}{V} w_B^{B_k} w_S^{S_k} w_Q^{Q_k} \tag{13}$$

($\rho_B = B/V$, $\rho_S = S/V$, $\rho_Q = Q/V$ being the baryon, strangeness and electric charge densities respectively), and A is the orthogonal matrix diagonalizing the hessian:

$$\mathsf{H}' = \mathrm{diag}(h_1, h_2, h_3) = \mathsf{A}\mathsf{H}\mathsf{A}^T \tag{14}$$

On the other hand, on the basis of the same asymptotic expansion, it can be shown that:

$$\lim_{V\to\infty} \frac{Z_{\vec{Q}-\vec{q}_j}}{Z_{\vec{Q}}} = \lambda_B^{b_j} \lambda_S^{s_j} \lambda_Q^{q_j} \equiv \lambda_j, \tag{15}$$

so that the usual grand-canonical expression is recovered for the average multiplicity in the large volume limit:

$$\lim_{V\to\infty} z_{j(1)} \frac{Z_{\vec{Q}-\vec{q}_j}}{Z_{\vec{Q}}} = z_{j(1)} \lambda_B^{b_j} \lambda_S^{s_j} \lambda_Q^{q_j} = \langle N_j \rangle_{\mathrm{GC}} \tag{16}$$

It is not difficult to realize, from their definition, that the matrix M and the C_{jk} factors do not depend on the volume, i.e. they stay finite and non-vanishing in the thermodynamic limit. Therefore, we can recast Eq. (9) as:

$$\omega_h = 1 + \frac{\sum_{j\in h} \langle N_j \rangle_{\mathrm{GC}} \sum_{k\in h} \langle N_k \rangle_{\mathrm{GC}} C_{jk}}{V \sum_{j\in h} \langle N_j \rangle_{\mathrm{GC}}} + \mathcal{O}(V^{-1}). \tag{17}$$

where the second term on the right hand side of the above equation is finite in the limit $V \to \infty$, thus giving rise to non trivial values of the scaled variance ω.

3. Numerical calculations in the canonical ensemble

We have calculated the scaled variances of several sets of hadrons in some cases of interest for ultra-relativistic heavy ion collisions. In our calculation, all light-flavoured hadron species up to a mass $\simeq 1.8$ GeV quoted in the 2002 issue of Particle Data Book [15] are included. The needed intensive input parameters for these calculations are the temperature T and the charge densities. The baryon density ρ_B is varied between 0 and 0.3 fm^{-3}, while the strangeness density ρ_S is set to zero and the electric charge density is set to $\rho_Q = 0.4\rho_B$, corresponding to the ratio Z/A of Pb-Pb and Au-Au collisions. The chemical potentials and fugacities are determined accordingly.

The scaled variances determined by means of analytical calculations have been compared with those obtained through Monte-Carlo simulations. The basic idea of this method is to extract randomly K-uples $\{N_j\}$ of multiplicities N_j for each hadron species j according to the multi-species multiplicity distribution of the canonical ensemble and averaging therafter. This method allows to determine numerically, with a finite statistical error, not only scaled variances but also higher order moments and, in general, to visualize the shape of the distributions. Furthermore, this method makes it possible to make calculations at final hadron level, taking into account resonance decays and thereby allowing a comparison of theoretical calculations with actual measurements. The multi-species multiplicity distribution in the canonical ensemble has been determined in the form of a cluster decomposition in ref. [9]:

$$P(\{N_j\}) = \frac{1}{Z_{\vec{Q}}}\left[\prod_j \sum_{\{h_{n_j}\}} \prod_{n_j=1}^{N_j} \frac{z_{j(n_j)}^{h_{n_j}}}{n_j^{h_{n_j}} h_{n_j}!}\right] \delta_{\vec{Q},\sum_j N_j \vec{q}_j} \tag{18}$$

where $\{h_{n_j}\}$ are partitions of the integers N_j in the multiplicity representations, i.e. such that $N_j = \sum_{n_j=1}^{N_j} n_j h_{n_j}$; $H_j = \sum_{n_j=1}^{N_j} h_{n_j}$; and $z_{j(n_j)}$ read:

$$z_{j(n_j)} = (\mp 1)^{n_j+1} \frac{(2J_j+1)V}{2\pi^2 n_j} T m_j^2 \mathrm{K}_2\left(\frac{n_j m_j}{T}\right) \tag{19}$$

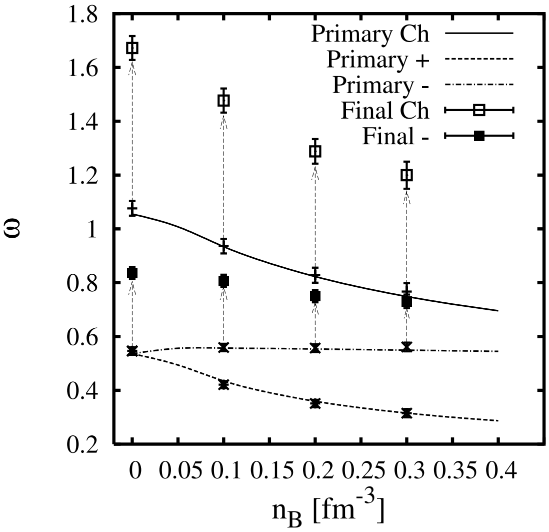
In the limit of the Boltzmann statistics, the distribution (18) reduces to a product of independent Poisson distributions, one for each species, with the constraint of charges conservation $\delta_{\vec{Q},\sum_j N_j \vec{q}_j}$. A direct sampling of the distribution (18) is very difficult though. The most effective method is the importance sampling technique, in which each event (namely a K-uple $\{N_j\}$) is weighted by the ratio w of the true distribution $P(\{N_j\})$ (18) and the actually sampled distribution $R(\{N_j\})$. The latter should be a distribution quickly and efficiently sampled and, moreover, as similar as possible to $P(\{N_j\})$ to minimize statistical errors. In our case, we have chosen $R(\{N_j\})$ as the product of unconstrained Poisson distributions, like in Eq. (4). Their mean multiplicities are chosen to be those of the GCE, that is $\langle N_j \rangle = z_{j(1)}\lambda_j$, where the fugacities λ_j are determined according to the Eq. (11); thereby, the mean values of the Poisson distributions in $R(\{N_j\})$ coincide with the actual CE average multiplicities in the thermodynamic limit.

Unfortunately, with this method, it is not possible to calculate observables straight in the thermodynamic limit because the simulation can be carried out only with a finite volume. Instead, one can study the variation of some observable of interest as a function of the volume, fixing total charges or charge densities, and estimate the thermodynamic limit by extrapolating. It should be pointed out that a too large volume cannot be used in order not to diminish too much the efficiency of the Monte-Carlo calculation. Fortunately, the statistical error on most averages decreases as volume increases because of the increase in multiplicity of single events, so that a lower efficiency at larger volumes does not spoil the accuracy.

In fig. (1) we have plotted ω for different sets of particles as a function of the baryon density ρ_B. Monte-Carlo results are shown as dots with errors bars and have been obtained by drawing 10^5 effective samples with $V = 200$ fm^3. This volume is large enough to ensure the effective reaching of the thermodynamic limit, yet the efficiency of these runs is very low, in the range $(1-6)\cdot 10^{-4}$. It can be clearly seen that Monte-Carlo and analytical calculations are in excellent agreement.

4. Discussion

We have found that the thermodynamic limit of the scaled variance is different in different ensembles. This effect has been understood for a long time in statistical mechanics. Though, from the previous derivations, the reader might have had the impression that this inequivalence between GCE, CE and MCE is a long-reaching consequence of a complicated analytical work not driven by a clear physical insight. Recently, it has been pointed out [11] that variances are qualitatively different from particle multiplicities in that their proportionality to the volume is not "primordial" but arises from the difference of two quantities whose leading term is $\mathcal{O}(V^2)$ (see Eq. (1)). As a consequence, the behaviour of variance in the thermodynamic limit is determined by sub-leading terms in both $\langle N^2 \rangle$ and $\langle N \rangle^2$ and different limits can be expected in different ensembles. We would like to point out here that the different behaviour in the thermodynamic



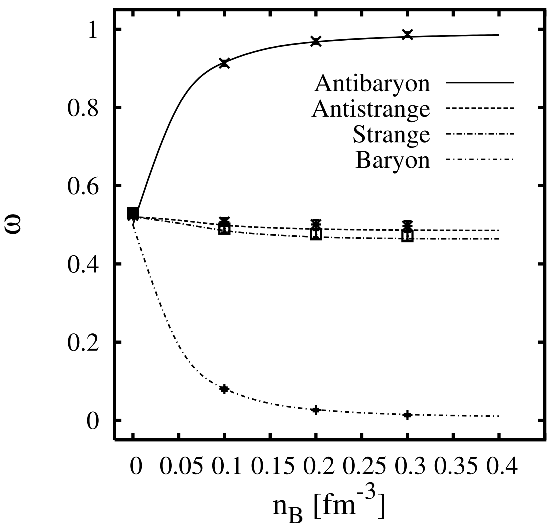


Figure 1. Scaled variances in the canonical ensemble of the full ideal hadron-resonance gas for different sets of hadrons at $T = 160$ MeV, $S = 0$ and $Q/B = 0.4$ as functions of baryon density ρ_B. Closed dots indicate the calculated values with Monte-Carlo simulations at primary level, open dots at final level with $V = 200$ fm^3. The lines depict the thermodynamic limits indendently calculated with the asymptotic expansions formulae. Left panel: charged, positive and negative hadrons; the arrows show the change in ω_- and $\omega_{\rm ch}$ from primary to final level. Right panel: baryons, antibaryons, strange and antistrange hadrons.

limit can indeed be understood more simply and with more physical insight, by observing that, unlike particle multiplicities, variances are non-additive quantities in both the CE and MCE.

The conceptual difference between extensivity and additivity has been recently discussed in ref. [13]. An additive quantity X is such that, if we split a general system in N subsystem:

$$X = \sum_{i=1}^{N} X_i \tag{20}$$

On the other hand, an extensive quantity is such that the limit:

$$\lim_{N\to\infty} \frac{X}{N} = x \tag{21}$$

has a non-vanishing and finite value. If a quantity is additive is also extensive except for some exceptional case [13]. Conversely, extensivity does *not* imply additivity. Quantities which are extensive and not additive are defined as *pseudo-extensive* and their corresponding limit x in Eq. (21) *pseudo-intensive* [13]. It can be shown quite easily that additive quantities have the same thermodynamic limit in all ensembles. In fact, if we split a CE or MCE with a very large volume into a large number of N parts with volume V/N, each part is, by definition, a GCE with the rest of the system acting as a reservoir; this is just the way the GCE is introduced in most statistical mechanics textbooks. Consequently, any X_i, where i labels a subsystem in the CE or MCE, has the same value as in the GCE with volume V/N in the limit $V, N \to \infty$. In other words, for any i:

$$\lim_{V,N\to\infty} X_i = \lim_{V,N\to\infty} \frac{X_{GCE}(V)}{N} \tag{22}$$

If X is additive, then:

$$\lim_{V,N\to\infty}\sum_{i=1}^{N} X_i = \lim_{V\to\infty} X_{CE,MCE}(V) \tag{23}$$

On the other hand, the left hand side of the previous expression also yields, according to (22)

$$\lim_{V,N\to\infty}\sum_{i=1}^{N} \frac{X_{GCE}(V)}{N} = \lim_{V\to\infty} X_{GCE}(V) \tag{24}$$

The comparison between (23) and (24) proves the equivalence between GCE and CE, MCE. Simplest examples of additive quantities are the energy and entropy for weakly interacting systems and particle mean multiplicities.

The previous argument does not apply to the variance σ^2 of particle multiplicity distribution. In fact, if we split a CE or MCE into N subsystems, the variance of any particle multiplicity distribution is not additive, as conservation laws involve non-vanishing correlations between different subsystems even for very large N. So:

$$\sigma^2 \neq \sum_{i=1}^{N} \sigma_i^2 \tag{25}$$

The equal sign would apply only if the subsystems were completely independent of each other, which is the case only in the GCE. Thus, if variances are not additive, their GCE and CE thermodynamic limits may and, in general, will differ. Yet, the variance itself is extensive because the limit (21) yields a finite value in both CE and MCE, as the leading behaviour is $\mathcal{O}(V)$.

In conclusion, being non additive, the variance is a pseudo-extensive quantity and the scaled variance is thus pseudo-intensive. For such quantities, the thermodynamic limit in the CE and MCE does not need to coincide with that in the GCE.

5. Fluctuations of charged particles

Fluctuations of charged particle ratios on an event-by-event basis in heavy ion collisions have been suggested as probes of the prehadronic phase [1, 2] and relevant measurements have been performed both at RHIC [16] and SPS [17]. To start with, it is very important to stress that the mere variance of a ratio of extensive quantities is an ill-defined observable in statistical mechanics because it is not a (pseudo-)intensive quantity and vanishes in the thermodynamic limit simply because it is proportional to $1/V$. Considering for instance the ratio N_j/N_k of particle multiplicities of two different species j and k we have:

$$\left\langle \delta\left(\frac{N_j}{N_k}\right)^2 \right\rangle \simeq \frac{1}{\langle N_k\rangle^2}\left(\langle \delta N_j^2\rangle + \frac{\langle N_j\rangle^2}{\langle N_k\rangle^2}\langle \delta N_k^2\rangle - 2\,\mathrm{cov}(N_j,N_k)\right) \tag{26}$$

Since $|\mathrm{cov}(N_j,N_k)| \leq \sqrt{\langle \delta N_j^2\rangle\langle \delta N_k^2\rangle}$ and $\langle \delta N_k^2\rangle \propto V$, one is left with an expression which decreases at least proportionally to $1/V$. Thus, in order to give a sensible fluctuation measure which does not vanish simply because the system gets larger, one should form some truly (pseudo-)intensive variable.

Many such variables have been proposed to measure charge fluctuations in heavy ion collisions (for a review see ref. [18]), e.g. D [1], Φ_Q [19], and $\nu_{\rm dyn}$ [18]. Their definitions read:

$$D = \langle N_{\rm ch}\rangle \left\langle \delta\left(\frac{N_+}{N_-}\right)^2 \right\rangle$$

$$\Phi_Q = \sqrt{\frac{\langle \Delta Q^2 \rangle}{\langle Q \rangle}} - \sqrt{\langle \delta q^2 \rangle} \tag{27}$$

$$\nu_{\rm dyn} = \left\langle \left(\frac{N_+}{\langle N_+ \rangle} - \frac{N_-}{\langle N_- \rangle} \right)^2 \right\rangle - \left(\frac{1}{\langle N_+ \rangle} - \frac{1}{\langle N_- \rangle} \right)$$

where N_+, N_- and $N_{\rm ch}$ is the number of positive, negative and charged particles respectively, $Q = \sum_j Q_j N_j$ is the net charge and q is the charge of a produced particle. Note that in Φ_Q definition, in the first term the random variables are the numbers N_j of particles with a given charge, whereas the random variable in the second term is the charge of each particle itself. These variables are indeed related to each other [18] and to the scaled variances of charged particles. It is not difficult to realize that the first two in Eq. (27) are pseudo-intensive whilst the latter is not and should be multiplied by an extensive variable, e.g. $\langle N_{\rm ch} \rangle$ to make it such. Being pseudo-intensive, they have different thermodynamic limits in the GCE, CE and MCE. Therefore, much care is needed in comparing the measurements to the predictions of statistical mechanics because the effect of conservation laws is crucial in determining their values even for very large systems. However, this comparison is in general difficult because of additional source of fluctuations which

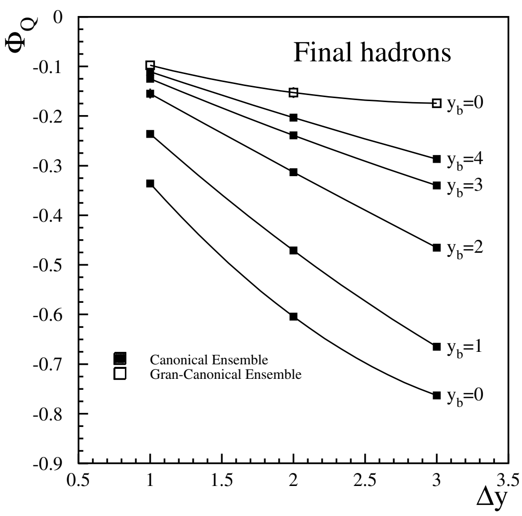


Figure 2. Calculated Φ_Q (see text for definition) in the canonical and grand-canonical ensembles of the full ideal hadron-resonance gas at $T = 160$ MeV and $\rho_B = 0.2$ fm^{-3}, in the thermodynamic limit, at final hadron level for different random boosts of primary particles y_b as a function of the acceptance rapidity window Δy.

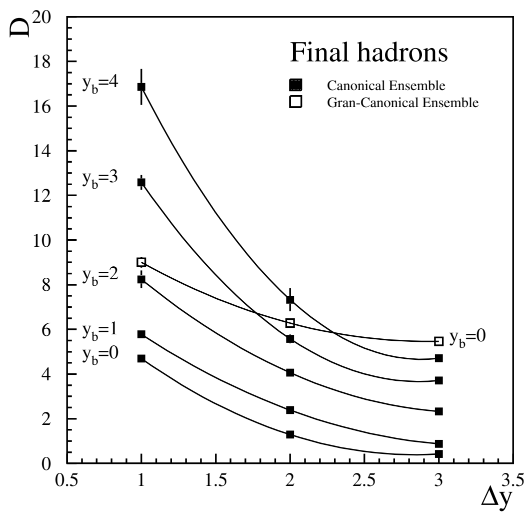


Figure 3. Calculated D (see text for definition) in the canonical and grand-canonical ensembles of the full ideal hadron-resonance gas at $T = 160$ MeV and $\rho_B = 0.2$ fm^{-3}, in the thermodynamic limit, at final hadron level for different random boosts of primary particles y_b as a function of the acceptance rapidity window Δy.

cannot be disregarded. Even if a statistical model framework would be essentially correct, there could be large fluctuations of thermodynamic parameters (volume, temperature, baryon density etc.) from event to event, which, being superimposed to the purely thermodynamic fluctuations, could swamp the thermodynamical fluctuations. Furthermore, experimental measurements are performed over a limited kinematical window and this introduces a further complication.

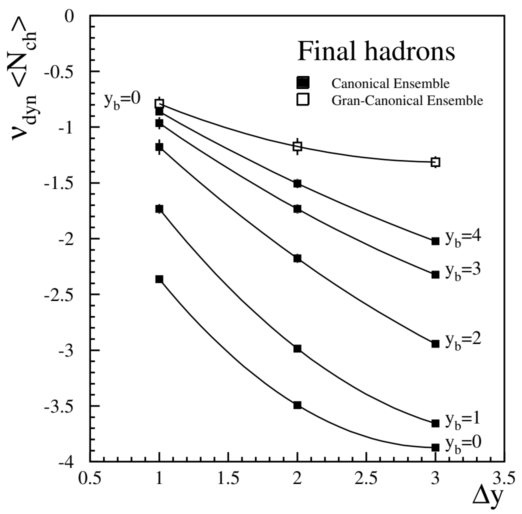


Figure 4. Calculated $\langle N_{\rm ch}\rangle\nu_{\rm dyn}$ (see text for definition) in the canonical and grand-canonical ensembles of the full ideal hadron-resonance gas at $T = 160$ MeV and $\rho_B = 0.2$ fm^{-3}, in the thermodynamic limit, at final hadron level for different random boosts of primary particles y_b as a function of the acceptance rapidity window Δy.

That said, it can be interesting to study the difference between the fluctuations of charged particles in the canonical and grand-canonical ensemble. In this respect, our Monte-Carlo method is especially suitable as it allows to predict the numerical values of the aforementioned variables taking into account all "trivial" effects including quantum statistics and resonance decays. Thus, we have calculated the values of these variables in the two ensembles at final hadron level for conditions relevant to heavy ion collisions at $\sqrt{s_{NN}} \approx 20$ GeV, i.e. $T = 160$ MeV, $S = 0$, $Q/B = 0.4$ and $\rho_B = 0.2$ fm^{-3} by using Monte-Carlo simulations. In the CE, the volume chosen was 200 fm^3, which is large enough to ensure in practice the reaching of the thermodynamic limit. In order to show the effect of a limited kinematic acceptance, we have also implemented a toy dynamical model, giving each primary generated particle (according to thermal distributions) a random longitudinal boost in rapidity uniformly between $-y_b$ and y_b. Though unrealistic, this model allows to understand the possible effect of measuring variables relevant to fluctuations over a finite rapidity window Δy. The calculated values of Φ_Q, D and $\langle N_{\rm ch}\rangle\nu_{\rm dyn}$ are shown in figs. 2, 3, 4 respectively. It can be seen that all of them are strongly affected by the dynamical boost y_b and the acceptance window Δy. Also, a relative strong difference is seen between CE and GCE. Yet, we note that, at least for Φ_Q and $\langle N_{\rm ch}\rangle\nu_{\rm dyn}$ their CE value converge to the GC one for small rapidity acceptance. This is not a trivial feature because such a behaviour is expected if we select a subsystem in *space* and not in *momentum space* as we have actually done. In fact, this behaviour is not seen in D (see fig. 3). Altogether, we can conclude that the spread of these variables is considerable and making a fairly accurate estimate of the theoretical expectation for a hadron gas in chemical equilibrium requires at least taking into account exact charges (i.e. B, S, Q) conservation.

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