Fermi National Accelerator Laboratory

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Momentum Compaction and Slippage Factor

K.Y. Ng

Fermi National Accelerator Laboratory
P.O. Box 500, Batavia, Illinois 60510

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Fermi National Accelerator Laboratory, PO Box 500, Batavia, IL 60540

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0.0.1 Momentum Compaction and Slippage Factor

K.Y. Ng, FNAL

The slippage factor η is the relative slip in revolution period T for a particle with fractional off-momentum $\delta = \Delta p/p_0$, or

$$\frac{\Delta T}{T_0} = \eta\delta .$$

where the subscript zero stands for on-momentum. The various orders of momentum-compaction factor α_i give the relative increase in closed-orbit length C for an off-momentum particle, or $C = C_0[1+\alpha_0\delta(1+\alpha_1\delta+\alpha_2\delta^2+\cdots)]$. With the slippage factor expanded as $\eta = \eta_0+\eta_1\delta+\eta_2\delta^2+\cdots$, we have [1]

$$\eta_0 = \alpha_0 - \frac{1}{\gamma_0^2}, \quad \eta_1 = \alpha_0\alpha_1 - \frac{\eta_0}{\gamma_0^2} + \frac{3\beta_0^2}{2\gamma_0^2},$$
$$\eta_2 = \alpha_0\alpha_2 - \frac{\eta_1}{\gamma_0^2} + \frac{3\beta_0^2\eta_0}{2\gamma_0^2} + \frac{(1-5\beta_0^2)\beta_0^2}{2\gamma_0^2},$$
$$\eta_3 = \alpha_0\alpha_3 - \frac{\eta_2}{\gamma_0^2} + \frac{3\beta_0^2\eta_1}{2\gamma_0^2} + \frac{(1-5\beta_0^2)\beta_0^2\eta_0}{2\gamma_0^2} - \frac{5(3-7\beta^2)\beta_0^4}{8\gamma_0^2},$$

where β_0 and γ_0 are the Lorentz factors of the on-momentum particle. The transition gamma is defined as $\gamma_t = \sqrt{1/\alpha_0}$. To lowest order, all off-momentum particles have the same transition gamma when $\alpha_1 \approx -\frac{1}{2}$, and cross transition at the same time when $\alpha_1 \approx -\frac{3}{2}$.

For a FODO lattice with *thin* quadruples of integrated strength $B'\ell/(B\rho) = S/L$, where L is the half cell length with dipole bending angle θ, we have approximately [2, 3]

$$\alpha_0 \approx 1 - \frac{S(\hat{D}_0-\check{D}_0)}{L\theta}, \quad \alpha_0\alpha_1 \approx -\frac{S(\hat{D}_1-\check{D}_1)}{L\theta},$$
$$\alpha_0\alpha_2 \approx -\frac{S(\hat{D}_2-\check{D}_2)}{L\theta} - \frac{S^3(\hat{D}_0^3-\check{D}_0^3)}{6L^3\theta},$$

where the dispersions at the F- and D-quadrupoles have been expanded, respectively, as $\hat{D} = \hat{D}_0 + \hat{D}_1\delta + \hat{D}_2\delta^2 + \cdots$, and $\check{D} = \check{D}_0 + \check{D}_1\delta + \check{D}_2\delta^2 + \cdots$, When $S \ll 12$, $\alpha_1 \to +\frac{3}{2}$ and reduces to $+\frac{1}{2}$ when chromaticities are corrected by sextupoles.

For an isochronous or quasi-isochronous ring, we must require the spread in η for off-momentum particles to be small also. Therefore, α_1 and α_2 need to be controlled in addition to α_0. In fact, first-order effect of sextupoles can alter α_1, that of octupoles can alter α_2, etc. For example, when a *thin* quadrupole of integrated strength $S_1 = B'\ell/(B\rho)$, a *thin* sextupole of integrated strength $S_2 = B''\ell/(B\rho)$, or an octupole of integrated strength $S_3 = B'''\ell/(B\rho)$ is placed at a location where the horizontal and vertical dispersions are, respectively D_x and D_y, their first-order effects are [4]

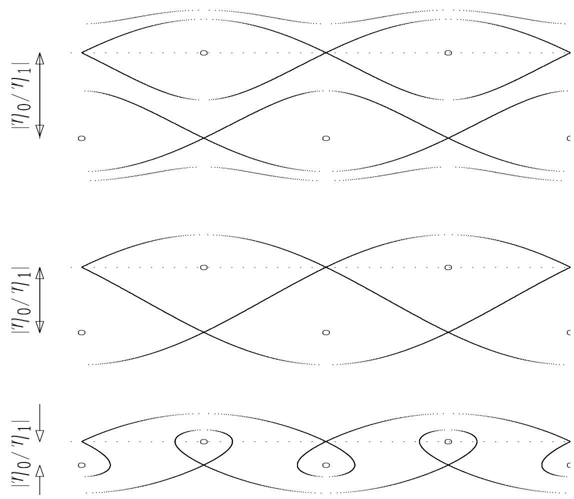
$$\Delta\alpha_0 = -\frac{S_1}{C_0}(D_x^2 - D_y^2),$$
$$\Delta\alpha_0\alpha_1 = -\frac{S_2}{C_0}(D_x^3 - 3D_xD_y^2),$$
$$\Delta\alpha_0\alpha_2 = -\frac{S_3}{C_0}(D_x^4 - 6D_x^2D_y^2 + D_y^4).$$

The Hamiltonian describing the longitudinal phase space can be written as [5]

$$H = h\left(\frac{\eta_0\delta^2}{2} + \frac{\eta_1\delta^3}{3} + \frac{\eta_2\delta^4}{4} + \cdots\right) + \frac{eV}{2\pi\beta^2E}\left[\cos(\phi_s+\Delta\phi) + \Delta\phi\sin\phi_s\right],$$

where V is the rf voltage with synchronous phase ϕ_s and harmonic h while E the particle energy. If only the η_0 and η_1 terms are considered, the two series of distorted pendulum-like buckets in the top figure (ϕ_s is set to zero) begin to merge to the middle figure when $|\eta_0/\eta_1|$ is lowered to

$$\left|\frac{\eta_0}{\eta_1}\right| = \left\{\frac{6eV}{\pi\beta^2h\eta_0E}\left[\left(\frac{\pi}{2}-\phi_s\right)\sin\phi_s - \cos\phi_s\right]\right\}^{1/2}.$$



With further reduction of $|\eta_0/\eta_1|$, the buckets become α-like (lower figure), which shrink to zero when $|\eta_0/\eta_1| = 0$. The α-like bucket of total height $|3\eta_0/(2\eta_1)|$ is small. It is asymmetric with momentum spread and is susceptible to longitudinal head-tail instability. If the η_1 term is eliminated, the Hamiltonian will be dominated by η_0 and η_2 and the bucket becomes pendulum-like again [3]. If the Hamiltonian is dominated by the η_2 term alone, the kinetic term is similar to a quartic potential providing maximal amount of synchrotron-frequency spread and therefore Landau damping.

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