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$\mathcal{N}$-extended Maxwell supergravities as Chern-Simons theories in three spacetime dimensions

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ABSTRACT

We present a new class of three-dimensional $\mathcal{N}$-extended supergravity theories based on the $\mathcal{N}$-extended Maxwell superalgebra with central charges and $\mathfrak{so}(\mathcal{N})$ internal symmetry generators. The presence of $\mathfrak{so}(\mathcal{N})$ generators is required in order to define a non-degenerate invariant inner product. Such symmetry allows us to construct an alternative supergravity action without cosmological constant term. Interestingly, the new theories can be obtained as a flat limit of a $\mathcal{N}$-extended AdS-Lorentz supergravity theories enlarged with $\mathfrak{so}(\mathcal{N})$ gauge fields.

1. Introduction

It is well assumed that a three-dimensional (super)gravity theory can be described by a Chern-Simons (CS) action as a gauge theory offering us an interesting toy model to approach higher-dimensional theories [1–11]. There has been a growing interest to go beyond Poincaré and AdS (super)groups to describe (super)gravity theories in order to study new models with different physical content [12–26]. In a previous work [27], we presented a new class of three-dimensional supergravity theories based on the Maxwell and AdS-Lorentz superalgebras.

At the bosonic level, the Maxwell symmetry has been initially introduced in [28–30] in order to describe a Minkowski space in presence of a constant classical electromagnetic field background. Its generalization has been useful to recover General Relativity from CS and Born-Infeld (BI) gravity actions [31–33]. Recently, there has been a particular interest in exploring CS gravity model invariant under the Maxwell algebra [34–37]. In particular, the presence of the additional gauge field influences the vacuum energy and the vacuum angular momentum of the stationary configuration [37].

On the other hand, the AdS-Lorentz symmetry was first introduced in [13,38] and can be seen as a semi-simple enlargement of the Poincaré symmetry. Recently, the AdS-Lorentz algebra and its generalizations have been used to recover diverse (pure)Lovelock Lagrangian from CS and BI ones [39–41]. More recently, it has been showed that the asymptotic symmetry of a three-dimensional CS gravity action invariant under the AdS-Lorentz group is given by a semi-simple enlargement of the $\mathfrak{bms}_3$ symmetry which is isomorphic to three copies of the Virasoro algebra [25].

At the supersymmetric level, the Maxwell superalgebra has been introduced to describe the presence of a constant abelian supersymmetric gauge field background in a four-dimensional superspace [42]. Interestingly, a pure supergravity action in four dimensions can be constructed based on the Maxwell superalgebra using a geometric procedure [43,44]. Recently it has been shown in [45] that the bosonic extra field and the additional Majorana gauge field, appearing in the Maxwell superalgebra, are crucial to recover supersymmetry invariance of flat supergravity on a manifold with non-trivial boundary. Extensions and generalizations of the supersymmetric version of the Maxwell symmetries have been extensively studied by diverse authors [46–56].

In this paper we extend the results of [27] and present a new class of $\mathcal{N}$-extended CS supergravity theories in three spacetime dimensions based on the central $\mathcal{N}$-extended Maxwell superalgebra. We show that the Maxwell superalgebra has to be enlarged with $\mathfrak{so}(\mathcal{N})$ generators in order to have non-degenerate invariant inner product. Interestingly, the Maxwell superalgebra allows us to define an alternative supergravity model in absence of cosmological constant term. The introduction of the cosmological constant term is done by considering an enlarged superalgebra which corresponds to a $\mathcal{N}$-extended AdS-Lorentz superalgebra. In order to establish a well-defined Maxwell limit we extend the superalgebra with $\mathfrak{so}(\mathcal{N})$ internal symmetry algebra.

This work is organized as follows: In Section 2, we give a brief review of the three-dimensional minimal Maxwell CS supergravity theory. The section 3 is devoted to the construction of the CS supergravity action invariant under the $\mathcal{N}=2$ Maxwell superalgebra. In section 4, we present a new class of $\mathcal{N}$-extended supergrav-

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ity models expressed as CS action for the $\mathcal{N}$-extended Maxwell superalgebra. The obtention of the $\mathcal{N}$-extended Maxwell supergravity action through a Maxwell limit is explored in Section 5. In Section 6, we give a brief discussion about future possible developments.

2. Minimal Maxwell Chern-Simons supergravity

In this section, following [27], we give a brief review of the minimal Maxwell supergravity theory based on the CS formalism. The minimal supersymmetric extension of the Maxwell algebra in three spacetime dimensions is spanned by the set $\{J_a, P_a, Z_a, Q_a, \Sigma_\alpha\}$ whose generators satisfy the following non-vanishing (anti-)commutation relations:

$$\begin{aligned}
&[J_a, J_b] = \epsilon_{abc} J^c, \qquad [J_a, P_b] = \epsilon_{abc} P^c, \\
&[J_a, Z_b] = \epsilon_{abc} Z^c, \qquad [P_a, P_b] = \epsilon_{abc} Z^c, \\
&[J_a, Q_\alpha] = \frac{1}{2} (\Gamma_a)_\alpha^{\ \beta} Q_\beta, \\
&[J_a, \Sigma_\alpha] = \frac{1}{2} (\Gamma_a)_\alpha^{\ \beta} \Sigma_\beta, \\
&[P_a, Q_\alpha] = \frac{1}{2} (\Gamma_a)_\alpha^{\ \beta} \Sigma_\beta, \\
&\{Q_\alpha, Q_\beta\} = -\frac{1}{2} (C\Gamma^a)_{\alpha\beta} P_a, \\
&\{Q_\alpha, \Sigma_\beta\} = -\frac{1}{2} (C\Gamma^a)_{\alpha\beta} Z_a,
\end{aligned} \tag{2.1}$$

where the Lorentz indices $a, b, \dots = 0, 1, 2$ are lowered and raised with the Minkowski metric η_{ab}. Here, C is the charge conjugation matrix,

$$C_{\alpha\beta} = C^{\alpha\beta} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \tag{2.2}$$

and satisfies $C^T = -C$ and $C\Gamma^a = (C\Gamma^a)^T$ where Γ^a are the Dirac matrices in three spacetime dimensions.

Such Maxwell superalgebra has been first introduced in [42] in four spacetime dimensions and subsequently in three dimensions [47]. Although the supersymmetrization of the Maxwell symmetry is not unique [12,48,51,52], this is the minimal supersymmetric extension providing us a consistent three-dimensional CS supergravity action [27]. Their (anti-)commutators differ from those of the super Poincaré ones in the presence of the abelian generators Z_a and the additional Majorana spinor generators Σ_α. On the other hand, the introduction of a second abelian spinors charges has been first considered in the context of $D = 11$ supergravity [57] and superstring theory [58]. Here, the second spinorial generator assure that the Jacobi identities hold.

A CS action,

$$I_{CS} = \frac{k}{4\pi} \int_M \left\langle AdA + \frac{2}{3} A^3 \right\rangle, \tag{2.3}$$

invariant under the Maxwell superalgebra has been explicitly constructed in [27] using the gauge connection one-form $A = A_\mu dx^\mu$ and the corresponding invariant tensor. In particular, the connection one-form is given by

$$A = \omega^a J_a + e^a P_a + \sigma^a Z_a + \bar{\psi} Q + \bar{\xi} \Sigma, \tag{2.4}$$

where ω^a corresponds to the spin connection, e^a is the vielbein, σ^a is the so-called gravitational Maxwell gauge field [37] while ψ and ξ are the respective fermionic gauge fields.

In order to construct the CS supergravity action we shall require the following non-vanishing components of the invariant tensor [27],

$$\begin{aligned}
&\langle J_a J_b \rangle = \alpha_0 \eta_{ab}, \qquad \langle P_a P_b \rangle = \alpha_2 \eta_{ab}, \\
&\langle J_a P_b \rangle = \alpha_1 \eta_{ab}, \qquad \langle Q_\alpha Q_\beta \rangle = \alpha_1 C_{\alpha\beta}, \\
&\langle J_a Z_b \rangle = \alpha_2 \eta_{ab}, \qquad \langle Q_\alpha \Sigma_\beta \rangle = \alpha_2 C_{\alpha\beta},
\end{aligned} \tag{2.5}$$

where α_0, α_1 and α_2 are arbitrary constants. Then, using the connection one-form (2.4) and the non-vanishing components of the invariant tensor (2.5), the explicit form of the CS supergravity action reads,

$$\begin{aligned}
I = \frac{k}{4\pi} \int \alpha_0 \left(\omega^a d\omega_a + \frac{1}{3} \epsilon_{abc} \omega^a \omega^b \omega^c \right) \\
+ \alpha_1 \left(2e^a R_a - \bar{\psi} \nabla \psi \right) + \alpha_2 \left(2R^a \sigma_a + e^a T_a - \bar{\psi} \nabla \xi - \bar{\xi} \nabla \psi \right),
\end{aligned} \tag{2.6}$$

where the Lorentz curvature R^a, the torsion T^a and the fermionic curvatures are respectively given by

$$\begin{aligned}
R^a &= d\omega^a + \frac{1}{2} \epsilon^{abc} \omega_b \omega_c, \\
T^a &= de^a + \epsilon^{abc} \omega_b e_c, \\
\nabla \psi &= d\psi + \frac{1}{2} \omega^a \Gamma_a \psi, \\
\nabla \xi &= d\xi + \frac{1}{2} \omega^a \Gamma_a \xi + \frac{1}{2} e^a \Gamma_a \psi.
\end{aligned} \tag{2.7}$$

The Maxwell gravitational field σ^a and the additional Majorana spinor field ξ have only contribution on the exotic part (α_2) of the CS action. This is due to the presence of the non-vanishing component $\langle J_a Z_b \rangle$, $\langle P_a P_b \rangle$ and $\langle Q_\alpha \Sigma_\beta \rangle$ of the invariant tensor which do not appear for the Poincaré superalgebra. Interestingly, as was noticed at the bosonic level in [37], the vacuum energy and the vacuum angular momentum of the stationary configuration are influenced by the presence of the gravitational Maxwell field σ^a. Here one can see that, for $\alpha_2 \neq 0$, the field equations reduce to the vanishing of the curvature two-forms,

$$\begin{aligned}
&R^a = 0, \qquad \mathcal{T}^a = 0, \qquad \mathcal{F}^a = 0, \\
&\nabla \psi = 0, \qquad \nabla \xi = 0,
\end{aligned} \tag{2.8}$$

where

$$\mathcal{T}^a = T^a + \frac{1}{4} \bar{\psi} \Gamma^a \psi, \tag{2.9}$$

$$\mathcal{F}^a = d\sigma^a + \epsilon^{abc} \omega_b \sigma_c + \frac{1}{2} \epsilon^{abc} e_b e_c + \frac{1}{2} \bar{\psi} \Gamma^a \xi. \tag{2.10}$$

The $\mathcal{N}$-extension of generalized Maxwell superalgebras have been already presented in [59,60]. Such $\mathcal{N}$-extended superalgebras contain not only internal symmetry generators but also require the presence of additional bosonic generators different to the Maxwell one. However, as we shall see in the next sections, the construction of a proper CS supergravity action based on a $\mathcal{N}$-extended Maxwell superalgebra will only require to introduce $\mathfrak{so}(\mathcal{N})$ generators.

3. Chern-Simons formulation of $\mathcal{N} = 2$ Maxwell supergravity

The $\mathcal{N} = 2$ supersymmetric extension of the Maxwell algebra is not unique and can be subdivided into two inequivalent cases: the $(1, 1)$ and the $(2, 0)$ cases. Here we shall refer to the $\mathcal{N} = 2$ Maxwell theory as $(2, 0)$ Maxwell supergravity theory.

The extension to $\mathcal{N} \geq 2$ of the Maxwell superalgebra allows us to include a central charge Z to the usual Maxwell generators $\left\{J_a, P_a, Z_a, Q^i\right\}$ with $i = 1, \ldots, \mathcal{N}$. In particular the (anti)-commutation relations of the $\mathcal{N} = 2$ centrally extended Maxwell superalgebra are given by

$$\begin{aligned}
&[J_a, J_b] = \epsilon_{abc} J^c , \qquad [J_a, P_b] = \epsilon_{abc} P^c , \\
&[J_a, Z_b] = \epsilon_{abc} Z^c , \qquad [P_a, P_b] = \epsilon_{abc} Z^c , \\
&\left[J_a, Q^i_\alpha\right] = \frac{1}{2} (\Gamma_a)^\beta_{\ \alpha} Q^i_\beta , \quad \left[J_a, \Sigma^i_\alpha\right] = \frac{1}{2} (\Gamma_a)^\beta_{\ \alpha} \Sigma^i_\beta , \\
&\left[P_a, Q^i_\alpha\right] = \frac{1}{2} (\Gamma_a)^\beta_{\ \alpha} \Sigma^i_\beta , \\
&\left\{Q^i_\alpha, Q^j_\beta\right\} = -\frac{1}{2} \delta^{ij} \left(C\Gamma^a\right)_{\alpha\beta} P_a , \\
&\left\{Q^i_\alpha, \Sigma^j_\beta\right\} = -\frac{1}{2} \delta^{ij} \left(C\Gamma^a\right)_{\alpha\beta} Z_a + C_{\alpha\beta} \epsilon^{ij} Z .
\end{aligned} \tag{3.1}$$

Nevertheless, such central extension of the $\mathcal{N} = 2$ Maxwell superalgebra cannot reproduce a $\mathcal{N} = 2$ CS supergravity action in three spacetime dimensions. The $(2, 0)$ Maxwell superalgebra (3.1) does not have an invariant non-degenerate inner product. Indeed the central charge Z is orthogonal to the super Maxwell generators and to itself. In order to have a non-degenerate invariant inner product, it is necessary to enlarge the Maxwell superalgebra by introducing $\mathfrak{so}(2)$ internal symmetry generators. In particular, we consider two internal symmetry generators T and B such that they satisfy the following non-trivial commutation relations:

$$\begin{aligned}
&\left[Q^i_\alpha, T\right] = \epsilon^{ij} Q^j_\alpha , \\
&\left[Q^i_\alpha, B\right] = \epsilon^{ij} \Sigma^j_\alpha , \\
&\left[\Sigma^i_\alpha, T\right] = \epsilon^{ij} \Sigma^j_\alpha .
\end{aligned} \tag{3.2}$$

Furthermore, one can see that the Jacobi identity requires that the anticommutator of the Majorana spinor generators Q^i has the following form,

$$\left\{Q^i_\alpha, Q^j_\beta\right\} = -\frac{1}{2} \delta^{ij} \left(C\Gamma^a\right)_{\alpha\beta} P_a + C_{\alpha\beta} \epsilon^{ij} B . \tag{3.3}$$

Remarkably, the $\mathcal{N} = 2$ Maxwell superalgebra endowed with a central charge Z and two additional bosonic $\mathfrak{so}(2)$ generators admits the following non-vanishing components of the invariant tensor,

$$\begin{aligned}
&\langle J_a J_b \rangle = \alpha_0 \eta_{ab} , \qquad \langle P_a P_b \rangle = \alpha_2 \eta_{ab} , \\
&\langle J_a P_b \rangle = \alpha_1 \eta_{ab} , \qquad \left\langle Q^i_\alpha Q^j_\beta \right\rangle = \alpha_1 C_{\alpha\beta} \delta^{ij} , \\
&\langle J_a Z_b \rangle = \alpha_2 \eta_{ab} , \qquad \left\langle Q^i_\alpha \Sigma^j_\beta \right\rangle = \alpha_2 C_{\alpha\beta} \delta^{ij} , \\
&\langle T B \rangle = -\alpha_1 , \qquad \langle T Z \rangle = -\alpha_2 ,
\end{aligned} \tag{3.4}$$

where α_0, α_1 and α_2 are real constants. Well-defined invariant tensor for a $\mathcal{N} = 2$ Maxwell superalgebra has also been presented in [52]. Nevertheless, the bosonic field content is larger than our case since the $\mathcal{N} = 2$ super Maxwell considered in [52] corresponds to a supersymmetric extension of a generalized Maxwell algebra. Indeed, such generalization contains an extra bosonic gauge field $\tilde{Z}_{ab}$ in addition to the usual Maxwell gauge fields.

The gauge connection one-form for the $\mathcal{N} = 2$ super Maxwell reads

$$A = \omega^a J_a + e^a P_a + \sigma^a Z_a + \bar{\psi}^i Q^i + \bar{\xi}^i \Sigma^i + aT + bB + cZ , \tag{3.5}$$

which, in addition to the usual Maxwell fields and $\mathcal{N} = 2$ gravitini, contains three additional gauge fields given by a, b and c, respectively.

The curvature two form $F = dA + A^2$ is given by

$$\begin{aligned}
F = {} & R^a J_a + \mathcal{T}^a P_a + \mathcal{F}^a Z_a + \nabla\bar{\psi}^i Q^i + \nabla\bar{\xi}^i \Sigma^i + F(a) T \\
& + F(b) B + F(c) Z ,
\end{aligned} \tag{3.6}$$

with

$$\begin{aligned}
R^a &= d\omega^a + \frac{1}{2} \epsilon^{abc} \omega_b \omega_c , \\
\mathcal{T}^a &= de^a + \epsilon^{abc} \omega_b e_c + \frac{1}{4} \bar{\psi}^i \Gamma^a \psi^i , \\
\mathcal{F}^a &= d\sigma^a + \epsilon^{abc} \omega_b \sigma_c + \frac{1}{2} \epsilon^{abc} e_b e_c + \frac{1}{2} \bar{\psi}^i \Gamma^a \xi^i , \\
\nabla\psi^i &= d\psi^i + \frac{1}{2} \omega^a \Gamma_a \psi^i + a\epsilon^{ij} \psi^j , \\
\nabla\xi^i &= d\xi^i + \frac{1}{2} \omega^a \Gamma_a \xi^i + \frac{1}{2} e^a \Gamma_a \psi^i + a\epsilon^{ij} \xi^j + b\epsilon^{ij} \psi^j ,
\end{aligned} \tag{3.7}$$

and

$$\begin{aligned}
F(a) &= da, \\
F(b) &= db - \epsilon^{ij} \psi_i \psi_j , \\
F(c) &= dc - 2\epsilon^{ij} \psi_i \xi_j .
\end{aligned} \tag{3.8}$$

The CS supergravity form for the connection (3.5) constructed with the invariant tensor (3.4) defines a gauge-invariant supergravity action for the $\mathcal{N} = 2$ Maxwell superalgebra which is given by

$$\begin{aligned}
I = \frac{k}{4\pi} \int & \alpha_0 \left(\omega^a d\omega_a + \frac{1}{3} \epsilon_{abc} \omega^a \omega^b \omega^c \right) \\
& + \alpha_1 \left(2e^a R_a - \bar{\psi}^i \nabla\psi^i - 2adb \right) \\
& + \alpha_2 \left(2R^a \sigma_a + e^a T_a - \bar{\psi}^i \nabla\xi^i - \bar{\xi}^i \nabla\psi^i - 2adc \right) ,
\end{aligned} \tag{3.9}$$

up to a boundary term. Here, $T^a = de^a + \epsilon^{abc} \omega_b e_c$ is the usual torsion two-form. Let us note that the term proportional to α_1 coincides with the $\mathcal{N} = 2$ Poincaré supergravity Lagrangian of [9]. The Maxwell gravitational field σ^a and the additional Majorana spinor field ξ appear only in the exotic term proportional to α_2. Thus, the $\mathcal{N} = 2$ supergravity action (3.9) can be seen as a Maxwell extension of the $\mathcal{N} = 2$ Poincaré supergravity action considered in [9].

4. $\mathcal{N}$-extended Maxwell Chern-Simons supergravity

In this section we present the three-dimensional $\mathcal{N}$-extended Maxwell supergravity theory based on the CS formulation. In particular, we shall focus on the generic $\mathcal{N} = (\mathcal{N}, 0)$ case. Nevertheless, our approach can be extended to the $\mathcal{N} = (p, q)$ case.

A centrally $\mathcal{N}$-extended Maxwell superalgebra can be obtained by considering $\mathcal{N}$ spinor generators Q^i_α and Σ^i_α, with $i = 1, \ldots, \mathcal{N}$, in addition to the usual Maxwell bosonic generators $\{J_a, P_a, Z_a\}$ and the $\mathcal{N}(\mathcal{N} - 1)/2$ central charges $Z^{ij} = -Z^{ji}$. Then, an $\mathcal{N}$-extended supersymmetric central extension of the Maxwell algebra is given by the following non-vanishing (anti-)commutation relations

$$\begin{aligned}
&[J_a, J_b] = \epsilon_{abc} J^c , \qquad [J_a, P_b] = \epsilon_{abc} P^c , \\
&[J_a, Z_b] = \epsilon_{abc} Z^c , [P_a, P_b] = \epsilon_{abc} Z^c ,
\end{aligned}$$

$$\begin{aligned}
\left[J_a, Q^i_\alpha\right] &= \frac{1}{2}\left(\Gamma_a\right)^\beta_\alpha Q^i_\beta ,\\
\left[J_a, \Sigma^i_\alpha\right] &= \frac{1}{2}\left(\Gamma_a\right)^\beta_\alpha \Sigma^i_\beta ,\\
\left[P_a, Q^i_\alpha\right] &= \frac{1}{2}\left(\Gamma_a\right)^\beta_\alpha \Sigma^i_\beta ,\\
\left\{Q^i_\alpha, Q^j_\beta\right\} &= -\frac{1}{2}\delta^{ij}\left(C\Gamma^a\right)_{\alpha\beta} P_a ,\\
\left\{Q^i_\alpha, \Sigma^j_\beta\right\} &= -\frac{1}{2}\delta^{ij}\left(C\Gamma^a\right)_{\alpha\beta} Z_a + C_{\alpha\beta} Z^{ij} .
\end{aligned} \tag{4.1}$$

Although such supersymmetrization is well-defined, it does not allow to construct a CS supergravity action. Indeed, the superalgebra (4.1) does not have a non-degenerate invariant inner product. This can be seen by noting that the Z^{ij} generator is orthogonal to all generators of the $\mathcal{N}$-extended Maxwell superalgebra and to itself. As in the $\mathcal{N}=2$ super Maxwell case, we have to enlarge the $\mathcal{N}$-extended Maxwell superalgebra by adding $\mathfrak{so}(N)$ generators. In particular, we have to include the new generators $T^{ij}=-T^{ji}$ and $B^{ij}=-B^{ji}$ which satisfy the following non-trivial commutation relations:

$$\begin{aligned}
\left[T^{ij}, T^{kl}\right] &= \delta^{jk}T^{il} - \delta^{ik}T^{jl} - \delta^{jl}T^{ik} + \delta^{il}T^{jk} ,\\
\left[T^{ij}, B^{kl}\right] &= \delta^{jk}B^{il} - \delta^{ik}B^{jl} - \delta^{jl}B^{ik} + \delta^{il}B^{jk} ,\\
\left[B^{ij}, B^{kl}\right] &= \delta^{jk}Z^{il} - \delta^{ik}Z^{jl} - \delta^{jl}Z^{ik} + \delta^{il}Z^{jk} ,\\
\left[T^{ij}, Z^{kl}\right] &= \delta^{jk}Z^{il} - \delta^{ik}Z^{jl} - \delta^{jl}Z^{ik} + \delta^{il}Z^{jk} ,\\
\left[T^{ij}, Q^k_\alpha\right] &= \left(\delta^{jk}Q^i_\alpha - \delta^{ik}Q^j_\alpha\right) ,\\
\left[B^{ij}, Q^k_\alpha\right] &= \left(\delta^{jk}\Sigma^i_\alpha - \delta^{ik}\Sigma^j_\alpha\right) ,\\
\left[T^{ij}, \Sigma^k_\alpha\right] &= \left(\delta^{jk}\Sigma^i_\alpha - \delta^{ik}\Sigma^j_\alpha\right) .
\end{aligned} \tag{4.2}$$

Additionally, the anticommutator of the Majorana spinors Q^i_α is modified as

$$\left\{Q^i_\alpha, Q^j_\beta\right\} = -\frac{1}{2}\delta^{ij}\left(C\Gamma^a\right)_{\alpha\beta} P_a + C_{\alpha\beta}B^{ij} . \tag{4.3}$$

Let us note that such central $\mathcal{N}$-extension of the Maxwell superalgebra endowed with $\mathfrak{so}(\mathcal{N})$ internal symmetry algebra is the three-dimensional version of the $D=4$ $\mathcal{N}$-extended Maxwell superalgebras obtained in [59,60], without additional bosonic charges $\{\tilde{Z}_{\mu\nu}, Z_\mu\}$. In particular, the present superalgebra can be seen as a deformation and enlargement of the $\mathcal{N}$-extended Poincaré superalgebra discussed in [9]. Interestingly, the $\mathcal{N}$-extended Maxwell superalgebra with central charge and $\mathfrak{so}(\mathcal{N})$ internal symmetry algebra can be alternatively obtained as a semigroup expansion [61] of a $\mathcal{N}$-extended Lorentz superalgebra following the procedure used in [27].

The central $\mathcal{N}$-extension of the Maxwell superalgebra endowed with bosonic $\mathfrak{so}(\mathcal{N})$ generators admits the following non-vanishing components of the invariant tensor,

$$\begin{aligned}
&\langle J_a J_b\rangle = \alpha_0\eta_{ab} , \qquad \langle P_a P_b\rangle = \alpha_2\eta_{ab} ,\\
&\langle J_a P_b\rangle = \alpha_1\eta_{ab} , \qquad \left\langle Q^i_\alpha Q^j_\beta\right\rangle = \alpha_1 C_{\alpha\beta}\delta^{ij} ,\\
&\langle J_a Z_b\rangle = \alpha_2\eta_{ab} , \qquad \left\langle Q^i_\alpha \Sigma^j_\beta\right\rangle = \alpha_2 C_{\alpha\beta}\delta^{ij} ,\\
&\left\langle T^{ij}T^{kl}\right\rangle = \alpha_0\left(\delta^{ik}\delta^{lj} - \delta^{il}\delta^{kj}\right) ,\\
&\left\langle B^{ij}T^{kl}\right\rangle = \alpha_1\left(\delta^{ik}\delta^{lj} - \delta^{il}\delta^{kj}\right) ,\\
&\left\langle Z^{ij}T^{kl}\right\rangle = \alpha_2\left(\delta^{ik}\delta^{lj} - \delta^{il}\delta^{kj}\right) ,\\
&\left\langle B^{ij}B^{kl}\right\rangle = \alpha_2\left(\delta^{ik}\delta^{lj} - \delta^{il}\delta^{kj}\right) ,
\end{aligned} \tag{4.4}$$

where α_0, α_1 and α_2 are real constants. Let us note that the central charges Z^{ij} are orthogonal to the generators of the $\mathcal{N}$-extended Maxwell superalgebra. The presence of internal symmetries generators allows to achieve appropriately a non-degenerate invariant inner product.

The other crucial ingredient for the construction of a CS action is the connection one-form which, for our case, reads

$$A = \omega^a J_a + e^a P_a + \sigma^a Z_a + \bar{\psi}^i Q^i + \bar{\xi}^i\Sigma^i + \frac{1}{2}a^{ij}T_{ij} + \frac{1}{2}b^{ij}B_{ij} + \frac{1}{2}c^{ij}Z_{ij} , \tag{4.5}$$

where the coefficients in front of the generators are the gauge potential one-forms. In particular, the theory contains $\mathcal{N}$ gravitini.

The corresponding curvature two-form is given by

$$F = R^a J_a + \mathcal{T}^a P_a + \mathcal{F}^a Z_a + \nabla\bar{\psi}^i Q^i + \nabla\bar{\xi}^i\Sigma^i + \frac{1}{2}F^{ij}(a)\,T_{ij} + \frac{1}{2}F^{ij}(b)\,B_{ij} + \frac{1}{2}F^{ij}(c)\,Z_{ij} , \tag{4.6}$$

where

$$\begin{aligned}
R^a &= d\omega^a + \frac{1}{2}\epsilon^{abc}\omega_b\omega_c ,\\
\mathcal{T}^a &= de^a + \epsilon^{abc}\omega_b e_c + \frac{1}{4}\bar{\psi}^i\Gamma^a\psi^i ,
\end{aligned} \tag{4.7}$$

are the Lorentz and supertorsion curvature, respectively. On the other hand, we have

$$\begin{aligned}
\mathcal{F}^a &= d\sigma^a + \epsilon^{abc}\omega_b\sigma_c + \frac{1}{2}\epsilon^{abc}e_b e_c + \frac{1}{2}\bar{\psi}^i\Gamma^a\xi^i ,\\
\nabla\psi^i &= d\psi^i + \frac{1}{2}\omega^a\Gamma_a\psi^i + a^{ij}\psi^j ,\\
\nabla\xi^i &= d\xi^i + \frac{1}{2}\omega^a\Gamma_a\xi^i + \frac{1}{2}e^a\Gamma_a\psi^i + a^{ij}\xi^j + b^{ij}\psi^j ,\\
F^{ij}(a) &= da^{ij} + a^{ik}a^{kj} ,\\
F^{ij}(b) &= db^{ij} + a^{ik}b^{kj} + b^{ik}a^{kj} - \bar{\psi}^i\psi^j ,\\
F^{ij}(c) &= dc^{ij} + a^{ik}c^{kj} + c^{ik}a^{kj} + b^{ik}b^{kj} - 2\bar{\psi}^i\xi^j .
\end{aligned} \tag{4.8}$$

Then, using the non-vanishing components of the invariant tensor (4.4) and the connection one-form (4.5) in the explicit form of the CS action (2.3) we find,

$$\begin{aligned}
I = \frac{k}{4\pi}\int \alpha_0&\left[\omega^a d\omega_a + \frac{1}{3}\epsilon_{abc}\omega^a\omega^b\omega^c - \frac{1}{2}\left(a^{ij}da^{ji} + \frac{2}{3}a^{ik}a^{kj}a^{ji}\right)\right]\\
&+\alpha_1\left[2e^a R_a - \bar{\psi}^i\nabla\psi^i + b^{ji}F^{ij}(a)\right]\\
&+\alpha_2\left[2R^a\sigma_a + e^a T_a - \bar{\psi}^i\nabla\xi^i - \bar{\xi}^i\nabla\psi^i + c^{ji}F^{ij}(a) + b^{ji}f^{ij}(b)\right] ,
\end{aligned} \tag{4.9}$$

where

$$f^{ij}(b) = db^{ij} + a^{ik}b^{kj} + b^{ik}a^{kj} . \tag{4.10}$$

It is interesting to note that the CS terms proportional to α_0 and α_1 is analogous to the centrally $\mathcal{N}$-extended Poincaré supergravity Lagrangian. A subtle difference appears in the nature of the b^{ij} field which in the Poincaré case is related to the central charge meanwhile here b^{ij} is a $\mathfrak{so}(\mathcal{N})$ gauge field. The gravitational Maxwell gauge field σ_a and the additional Majorana spinor charge ξ^i appear on the exotic sector α_2. This is quite different to the minimal four-dimensional Maxwell supergravity theory in which the extra fields σ_a and ξ appear only on the boundary term without modifying the dynamics of flat supergravity [45]. As was discussed in [45], such extra field were crucial to restore supersymmetry of the theory on a manifold with boundary. Here, the supergravity action (4.9) is invariant under the following supersymmetry transformation laws

$$\begin{aligned}
\delta\omega^a &= 0\,,\\
\delta e^a &= \frac{1}{2}\bar{\zeta}^i\Gamma^a\psi^i\,,\\
\delta\sigma^a &= \frac{1}{2}\bar{\zeta}^i\Gamma^a\xi^i + \frac{1}{2}\bar{\varrho}^i\Gamma^a\psi^i\,,\\
\delta\psi^i &= d\zeta^i + \frac{1}{2}\omega^a\Gamma_a\zeta^i + a^{ij}\zeta^j,\\
\delta\xi^i &= d\varrho^i + \frac{1}{2}\omega^a\Gamma_a\varrho^i + a^{ij}\varrho^j + \frac{1}{2}e^a\Gamma_a\zeta^i + b^{ij}\zeta^j\\
\delta a^{ij} &= 0\,,\\
\delta b^{ij} &= -2\bar{\psi}^{[i}\zeta^{j]}\,,\\
\delta c^{ij} &= -2\left(\bar{\psi}^{[i}\varrho^{j]} + \bar{\xi}^{[i}\zeta^{j]}\right)\,,
\end{aligned} \tag{4.11}$$

where ζ^i and ϱ^i are the fermionic gauge parameters related to Q^i and Σ^i, respectively.

The equations of motion derived from the supergravity action (4.9) reduce to the vanishing of the curvatures, for $\alpha_2 \neq 0$,

$$\begin{aligned}
&R^a = 0\,, && \mathcal{T}^a = 0\,, && \mathcal{F}^a = 0\,,\\
&\nabla\psi^i = 0\,, && \nabla\xi^i = 0\,, && F^{ij}(a) = 0\,,\\
&F^{ij}(b) = 0\,, && F^{ij}(c) = 0\,.
\end{aligned} \tag{4.12}$$

It is interesting to note that the vanishing of $\mathcal{F}^a$ is analogue to the constancy of an abelian SUSY field strength background. Although σ^a appears only on the exotic part, one could expect that its presence would influence the boundary dynamics. In particular, one could argue that the asymptotic symmetries of the $\mathcal{N}=2$ Maxwell supergravity are spanned by a deformation and enlargement of the $\mathcal{N}=2$ super $\mathfrak{bms}_3$ algebra presented in [62,63] as occurs in the bosonic sector [37,64].

The centrally $\mathcal{N}$-extended Maxwell supergravity enlarged with $\mathfrak{so}(\mathcal{N})$ gauge fields allows us to define an alternative supergravity model. It is interesting to note that we do not introduce a cosmological constant term although we have enlarged the field content. Similarly to the minimal case [27], one could recover our result as a flat limit of a particular supergravity theory. Indeed, as we shall see, one can add a length scale by modifying the superalgebra and the supergravity action from which a Maxwell limit can be properly applied.

5. $\mathcal{N}$-extended Maxwell supergravity theory as a flat limit

The incorporation of a cosmological constant term in a $\mathcal{N}>2$ supergravity theory can be done by considering an extension of the (p,q) AdS supergravity by an $\mathfrak{so}(p)\oplus\mathfrak{so}(q)$ automorphism algebra allowing a well-defined flat limit [9]. Here we propose an alternative approach to incorporate a cosmological constant to a $\mathcal{N}$-extended supergravity by considering an enlarged superalgebra such that the flat limit leads us to the $\mathcal{N}$-extended Maxwell theory.

A length parameter ℓ can be introduced to the set of generators $\left\{J_a, P_a, Z_a, Q^i, \Sigma^i, T^{ij}, B^{ij}, Z^{ij}\right\}$ by considering a supersymmetric $\mathcal{N}$-extension of the AdS-Lorentz symmetry. Such generators satisfy not only the non-vanishing (anti-)commutators of the $\mathcal{N}$-extended Maxwell superalgebra (4.1)-(4.3) but also

$$\begin{aligned}
\left[Z_a, Z_b\right] &= \frac{1}{\ell^2}\epsilon_{abc}Z^c\,,\\
\left[P_a, Z_b\right] &= \frac{1}{\ell^2}\epsilon_{abc}P^c\,,\\
\left[Z_a, Q^i_\alpha\right] &= \frac{1}{2\ell^2}\left(\Gamma_a\right)^\beta_{\ \alpha}Q^i_\beta\,,\\
\left[P_a, \Sigma^i_\alpha\right] &= \frac{1}{2\ell^2}\left(\Gamma_a\right)^\beta_{\ \alpha}Q^i_\beta\,,\\
\left[Z_a, \Sigma^i_\alpha\right] &= \frac{1}{2\ell^2}\left(\Gamma_a\right)^\beta_{\ \alpha}\Sigma^i_\beta\,,\\
\left\{\Sigma^i_\alpha, \Sigma^j_\beta\right\} &= -\frac{1}{2\ell^2}\delta^{ij}\left(C\Gamma^a\right)_{\alpha\beta}P_a + \frac{1}{\ell^2}C_{\alpha\beta}B^{ij}\,.
\end{aligned} \tag{5.1}$$

$$\begin{aligned}
\left[B^{ij}, Z^{kl}\right] &= \frac{1}{\ell^2}\left(\delta^{jk}B^{il} - \delta^{ik}B^{jl} - \delta^{jl}B^{ik} + \delta^{il}ZB^{jk}\right)\,,\\
\left[Z^{ij}, Z^{kl}\right] &= \frac{1}{\ell^2}\left(\delta^{jk}Z^{il} - \delta^{ik}Z^{jl} - \delta^{jl}Z^{ik} + \delta^{il}Z^{jk}\right)\,,\\
\left[B^{ij}, \Sigma^k_\alpha\right] &= \frac{1}{\ell^2}\left(\delta^{jk}Q^i_\alpha - \delta^{ik}Q^j_\alpha\right)\,,\\
\left[Z^{ij}, Q^k_\alpha\right] &= \frac{1}{\ell^2}\left(\delta^{jk}Q^i_\alpha - \delta^{ik}Q^j_\alpha\right)\,,\\
\left[Z^{ij}, \Sigma^k_\alpha\right] &= \frac{1}{\ell^2}\left(\delta^{jk}\Sigma^i_\alpha - \delta^{ik}\Sigma^j_\alpha\right)\,.
\end{aligned} \tag{5.2}$$

The superalgebra given by (4.1)-(4.3) and (5.1)-(5.2) corresponds to a supersymmetric extension of the so called AdS-Lorentz algebra enlarged with $\mathfrak{so}(\mathcal{N})$ generators. Although its supersymmetrization is not unique and have been explored with different purposes [24,65–67], this is the smallest one with $2\mathcal{N}$ spinor charges. Let us note that the $\mathfrak{so}(\mathcal{N})$ generators, which satisfy (5.2), are required in order to relate the $\mathcal{N}$-extended AdS-Lorentz superalgebra with the centrally $\mathcal{N}$-extended Maxwell algebra endowed with $\mathfrak{so}(\mathcal{N})$ internal symmetry. Naurally, the Maxwell limit $\ell\to\infty$ can be applied properly leading to the $\mathcal{N}$-extended Maxwell superalgebra with central charge and $\mathfrak{so}(\mathcal{N})$ algebra. It is interesting to point out that the Z^{ij} generator becomes a central charge after the flat limit. As we shall see, such enlargement allows us to relate the non-degenerate invariant inner product to the Maxwell ones through the Maxwell limit.

The $\mathcal{N}$-extended AdS-Lorentz superalgebra with $\mathfrak{so}(\mathcal{N})$ generators admits the following non-vanishing components of the invariant tensor,

$$\begin{aligned}
&\langle J_aJ_b\rangle = \alpha_0\eta_{ab}\,, && \langle P_aP_b\rangle = \alpha_2\eta_{ab}\,, && \left\langle Q^i_\alpha Q^j_\beta\right\rangle = \alpha_1C_{\alpha\beta}\delta^{ij}\,,\\
&\langle J_aP_b\rangle = \alpha_1\eta_{ab}\,, && \langle P_aZ_b\rangle = \frac{\alpha_1}{\ell^2}\eta_{ab}\,, && \left\langle Q^i_\alpha \Sigma^j_\beta\right\rangle = \alpha_2C_{\alpha\beta}\delta^{ij}\,,\\
&\langle J_aZ_b\rangle = \alpha_2\eta_{ab}\,, && \langle Z_aZ_b\rangle = \frac{\alpha_2}{\ell^2}\eta_{ab}\,, && \left\langle \Sigma^i_\alpha \Sigma^j_\beta\right\rangle = \frac{\alpha_1}{\ell^2}C_{\alpha\beta}\delta^{ij}\,,
\end{aligned} \tag{5.3}$$

$$\begin{aligned}
&\left\langle T^{ij}T^{kl}\right\rangle = \alpha_0\left(\delta^{ik}\delta^{lj} - \delta^{il}\delta^{kj}\right)\,, && \left\langle B^{ij}T^{kl}\right\rangle = \alpha_1\left(\delta^{ik}\delta^{lj} - \delta^{il}\delta^{kj}\right)\,,\\
&\left\langle Z^{ij}T^{kl}\right\rangle = \alpha_2\left(\delta^{ik}\delta^{lj} - \delta^{il}\delta^{kj}\right)\,, && \left\langle B^{ij}B^{kl}\right\rangle = \alpha_2\left(\delta^{ik}\delta^{lj} - \delta^{il}\delta^{kj}\right)\,,\\
&\left\langle B^{ij}Z^{kl}\right\rangle = \frac{\alpha_1}{\ell^2}\left(\delta^{ik}\delta^{lj} - \delta^{il}\delta^{kj}\right)\,, && \left\langle Z^{ij}Z^{kl}\right\rangle = \frac{\alpha_2}{\ell^2}\left(\delta^{ik}\delta^{lj} - \delta^{il}\delta^{kj}\right)\,.
\end{aligned} \tag{5.4}$$

One can see that the limit $\ell \to \infty$ reproduces the invariant tensor of the $\mathcal{N}$-extended Maxwell superalgebra with central charges and $\mathfrak{so}(\mathcal{N})$ generators.

Although the connection one-form A is analogous to the Maxwell one,

$$A = \omega^a J_a + e^a P_a + \sigma^a Z_a + \bar{\psi}^i Q^i + \bar{\xi}^i \Sigma^i + \frac{1}{2} a^{ij} T_{ij} + \frac{1}{2} b^{ij} B_{ij} + \frac{1}{2} c^{ij} Z_{ij} , \tag{5.5}$$

the corresponding curvature two-form is subtle different due to the new commutators. Indeed, we have

$$F = R^a J_a + \mathcal{T}^a P_a + \mathcal{F}^a Z_a + \nabla \bar{\psi}^i Q^i + \nabla \bar{\xi}^i \Sigma^i + \frac{1}{2} F^{ij}(a) T_{ij} + \frac{1}{2} F^{ij}(b) B_{ij} + \frac{1}{2} F^{ij}(c) Z_{ij} , \tag{5.6}$$

where

$$\begin{aligned}
R^a &= d\omega^a + \frac{1}{2}\epsilon^{abc}\omega_b\omega_c , \\
\mathcal{T}^a &= de^a + \epsilon^{abc}\omega_b e_c + \frac{1}{\ell^2}\epsilon^{abc}\sigma_b e_c + \frac{1}{4}\bar{\psi}^i \Gamma^a \psi^i , \\
\mathcal{F}^a &= d\sigma^a + \epsilon^{abc}\omega_b\sigma_c + \frac{1}{2\ell^2}\epsilon^{abc}\sigma_b\sigma_c + \frac{1}{2}\epsilon^{abc}e_b e_c + \frac{1}{2}\bar{\psi}^i\Gamma^a\xi^i , \\
F^{ij}(a) &= da^{ij} + a^{ik}a^{kj} , \\
F^{ij}(b) &= db^{ij} + a^{ik}b^{kj} + b^{ik}a^{kj} + \frac{1}{\ell^2}\left(b^{ik}c^{kj} + c^{ik}b^{kj}\right) - \bar{\psi}^i\psi^j - \frac{1}{\ell^2}\bar{\xi}^i\xi^j , \\
F^{ij}(c) &= dc^{ij} + a^{ik}c^{kj} + c^{ik}a^{kj} + b^{ik}b^{kj} + \frac{1}{\ell^2}c^{ik}c^{kj} - 2\bar{\psi}^i\xi^j .
\end{aligned} \tag{5.7}$$

Here the covariant derivative acting on spinors read

$$\begin{aligned}
\nabla\psi^i &= d\psi^i + \frac{1}{2}\omega^a\Gamma_a\psi^i + a^{ij}\psi^j + \frac{1}{\ell^2}\left(b^{ij}\xi^j + c^{ij}\psi^j\right) , \\
\nabla\xi^i &= d\xi^i + \frac{1}{2}\omega^a\Gamma_a\xi^i + \frac{1}{2}e^a\Gamma_a\psi^i + a^{ij}\xi^j + b^{ij}\psi^j + \frac{1}{\ell^2}c^{ij}\xi^j .
\end{aligned} \tag{5.8}$$

The CS supergravity action based on the $\mathcal{N}$-extended AdS-Lorentz superalgebra with $\mathfrak{so}(\mathcal{N})$ generators and the invariant tensor (5.3)-(5.4) reads, up to boundary terms

$$\begin{aligned}
I = \frac{k}{4\pi}\int & \alpha_0\left[\omega^a d\omega_a + \frac{1}{3}\epsilon_{abc}\omega^a\omega^b\omega^c - \frac{1}{2}\left(a^{ij}da^{ji} + \frac{2}{3}a^{ik}a^{kj}a^{ji}\right)\right] \\
&+\alpha_1\left[2e^aR_a + \frac{1}{3\ell^2}e^ae^be^c + \frac{2}{\ell^2}e^aF_a - \bar{\psi}^i\nabla\psi^i - \frac{1}{\ell^2}\bar{\xi}^i\nabla\xi^i + b^{ji}F^{ij}(a) + \frac{1}{\ell^2}b^{ji}f^{ij}(c)\right] \\
&+\alpha_2\left[2R^a\sigma_a + e^aT_a + \frac{2}{\ell^2}F^a\sigma_a + \frac{1}{\ell^2}\epsilon_{abc}e^a\sigma^be^c - \bar{\psi}^i\nabla\xi^i - \bar{\xi}^i\nabla\psi^i + c^{ji}F^{ij}(a) + b^{ji}f^{ij}(b) + \frac{1}{\ell^2}c^{ji}f^{ij}(c)\right] ,
\end{aligned} \tag{5.9}$$

where we have defined

$$\begin{aligned}
F^a &= d\sigma^a + \epsilon^{abc}\omega_b\sigma_c + \frac{1}{2\ell^2}\epsilon^{abc}\sigma_b\sigma_c , \\
f^{ij}(b) &= db^{ij} + a^{ik}b^{kj} + b^{ik}a^{kj} + \frac{1}{\ell^2}\left(b^{ik}c^{kj} + c^{ik}b^{kj}\right) , \\
f^{ij}(c) &= dc^{ij} + a^{ik}c^{kj} + c^{ik}a^{kj} + b^{ik}b^{kj} + \frac{1}{\ell^2}c^{ik}c^{kj} .
\end{aligned} \tag{5.10}$$

The $\mathcal{N}$-extended AdS-Lorentz symmetry offer us an alternative procedure to introduce a cosmological constant term to a three-dimensional CS supergravity action. In particular, the inclusion of $\mathfrak{so}(\mathcal{N})$ gauge fields allows to establish a well-defined flat limit $\ell \to \infty$ leading to the central $\mathcal{N}$-extension Maxwell supergravity action enlarged with $\mathfrak{so}(\mathcal{N})$ CS terms. A particular difference with the Maxwell theory is the explicit presence of the σ^a and ξ^i field on the term proportional to α_1. Such presence allows us to recover the vanishing of the AdS-Lorentz curvature two forms (5.7) as field equations when $\alpha_2 \neq 0$. Naturally, the Maxwell limit applied to the equations of motions reproduces the field equations (4.12) of the $\mathcal{N}$-extended Maxwell supergravity theory with central charges and $\mathfrak{so}(\mathcal{N})$ gauge fields.

6. Discussion

We have presented a new class of three-dimensional $\mathcal{N}$-extended CS supergravity theories based on the centrally $\mathcal{N}$-extended Maxwell superalgebra enlarged with $\mathfrak{so}(\mathcal{N})$ internal symmetry algebra. The construction of the supergravity action requires to introduce $\mathfrak{so}(\mathcal{N})$ generators which are essential to the obtention of non-degenerate invariant inner product. The new theories correspond to a $\mathcal{N}$-extension of the minimal Maxwell supergravity theory presented in [27] and can be seen as alternative $\mathcal{N}$-extended supergravity theories without cosmological constant. Let us note that the CS supergravity action is characterized by three coupling constants α_0, α_1 and α_2. Interestingly, the term proportional to α_0 and α_1 is the usual $\mathcal{N}$-extended Poincaré action with $\mathfrak{so}(\mathcal{N})$ gauge fields. On the other hand, the gravitational Maxwell field σ_a and the additional Majorana spinor field ξ appear only on the α_2 sector.

The introduction of a cosmological constant term to our model is achieved by considering the $\mathcal{N}$-extended AdS-Lorentz superalgebra enlarged with $\mathfrak{so}(\mathcal{N})$ generators. The presence of the $\mathfrak{so}(\mathcal{N})$ generators allows us to have a well-defined Maxwell limit $\ell \to \infty$ in which the $\mathcal{N}$-extended AdS-Lorentz theory reduces properly to the $\mathcal{N}$-extended Maxwell one.

A proper characteristic of the Maxwell symmetries is the presence of the gravitational Maxwell gauge field σ_a which is related to the abelian generator Z_a. Such generator modifies the asymptotic $\mathfrak{bms}_3$ symmetry of standard Einstein gravity to a deformed and enlarged $\mathfrak{bms}_3$ symmetry [37]. It would then be interesting to extend the results of [37] to the minimal Maxwell supergravity presented in [27] and to the $\mathcal{N}$-extended Maxwell supergravity theories presented here [work in progress].

On the other hand, it was recently shown that the deformed and enlarged $\mathfrak{bms}_3$ algebra can be obtained as a flat limit of three copies of the Virasoro algebra [64]. Such asymptotic symmetry results to be the asymptotic structure of the AdS-Lorentz CS gravity [25]. It would be interesting to study these relations at the supersymmetric level.

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