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Research Article

Exact Solution of the Dirac Equation for the Yukawa Potential with Scalar and Vector Potentials and Tensor Interaction

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We present exact solutions of the Dirac equation with Yukawa potential in the presence of a Coulomb-like tensor potential. For this goal we expand the Yukawa form of the nuclear potential in its mesonic clouds by using Taylor extension to the power of seventh and bring out its simple form. In order to obtain the energy eigenvalue and the corresponding wave functions in closed forms for this potential (with great powers and inverse exponent), we use ansatz method. We also regard the effects of spin-spin, spin-isospin, and isospin-isospin interactions on the relativistic energy spectra of nucleon. By using the obtained results, we have calculated the deuteron mass. The results of our model show that the deuteron spectrum is very close to the ones obtained in experiments.

1. Introduction

It is well known that the exact energy eigenvalues of the bound state play an important role in quantum mechanics. In particular, the Dirac equation which describes the motion of a spin-1/2 particle has been used in solving many problems of nuclear and high-energy physics. Recently, there has been an increase in searching for analytic solution of the Dirac equation [1–11]. Recently, tensor couplings have been used widely in the studies of nuclear properties [12–22], and they were introduced into the Dirac equation by substitution $\vec{P} \rightarrow \vec{P} - im\omega\beta \cdot \hat{x}\, U(r)$ [16, 23], where m is one of the particles, mass and ω refers to harmonic oscillator. In this work, we are going to solve the relativistic Dirac equation for Yukawa potential in fresh approaches. We first discuss the Dirac equation with a tensor potential for Yukawa potential. Then, we solve the resulting equations for deuteron nuclei and obtain the parameters for it. We are also able to number the mass of Deuteron by using obtained parameters, and the equation is in conformity with experimental measure.

2. Yukawa Potential

The screened Coulomb potential, also known as the Yukawa potential in atomic physics and the Debye-Huckel potential in plasma physics, is of interest in many areas of physics. It was originally used to model strong nucleon-nucleon interactions due to meson exchange in nuclear physics by Yukawa [19, 20]. It is also used to represent a screened Coulomb potential due to the cloud of electronic charges around the nucleus in atomic physics or to account for the shielding by outer charges of the Coulomb field experienced by an atomic electron in hydrogen plasma. However, the Schrödinger equation for this potential cannot be solved exactly; hence, various numerical and pertubative methods have been devised to obtain the energy levels and related physical quantities [24]. The generic form of this potential is given by

$$U(x) = -\frac{g}{x} V(x) \quad g > 0, \tag{1}$$

that g is the potential strength, and

$$V(x) = e^{-kx}. \tag{2}$$

In this part, we expand the Yukawa potential in its meson clouds, $x = a$, by using Taylor extension to the power of seventh:

$$U(x) = -\frac{g}{x}\left[e^{-ka} - ke^{-ka}(x-a) + \frac{k^2}{2!}e^{-ka}(x-a)^2 - \frac{k^3}{3!}e^{-ka}(x-a)^3 + \frac{k^4}{4!}e^{-ka}(x-a)^4 - \frac{k^5}{5!}e^{-ka}(x-a)^5 + \frac{k^6}{6!}e^{-ka}(x-a)^6 - \frac{k^7}{7!}e^{-ka}(x-a)^7 + \cdots\right]. \tag{3}$$

We can reduce this equation to

$$U(x) = ax^6 - bx^5 + cx^4 - dx^3 + ex^2 - fx + h - \frac{L}{x}. \tag{4}$$

Now, we have the general Yukawa equation, that a, b, c, d, e, f, h, and L are

$$\begin{aligned}
a &= \frac{k^7}{7!},\\
b &= -\left(\frac{7}{7!}k^6 + \frac{7}{7!}k^6\right),\\
c &= \left(\frac{21}{7!}k^5 + \frac{6}{6!}k^5 + \frac{1}{5!}k^5\right),\\
d &= -\left(\frac{35}{7!}k^4 + \frac{15}{6!}k^4 + \frac{5}{5!}k^4 + \frac{1}{3!}k^4\right),\\
e &= \left(\frac{35}{7!}k^3 + \frac{20}{6!}k^3 + \frac{10}{5!}k^3 + \frac{6}{4!}k^3 + \frac{3}{3!}k^3 + \frac{1}{2!}k^3\right),\\
f &= -\left(\frac{21}{7!}k^2 + \frac{15}{6!}k^2 + \frac{10}{5!}k^2 + \frac{6}{4!}k^2 + \frac{3}{3!}k^2 + \frac{1}{2!}k^2\right),\\
h &= \left(\frac{7}{7!}k + \frac{6}{6!}k + \frac{5}{5!}k + \frac{4}{4!}k + \frac{3}{3!}k + \frac{1}{2!}k + k\right),\\
L &= -\left(\frac{7}{7!} + \frac{6}{6!} + \frac{5}{5!} + \frac{4}{4!} + \frac{3}{3!} + \frac{1}{2!} + 2\right).
\end{aligned} \tag{5}$$

3. Exact Analytical Solution of the Dirac Equation for Yukawa Potential

We are interested in the particular solution of the Dirac equation with the Yukawa interaction potential (4). In this section, we anticipate the answer and then obtain the parameters for highest nuclei, deuteron. A deuteron consists of a neutron and a proton. Let us assume one nucleon is fixed and another nucleon is moving around the center of mass. At last, by using single particle model, we investigate the effects of one pion exchange for a single nucleon. The Dirac Hamiltonian is

$$H = \vec{\alpha}\cdot\vec{P} + \beta(m+S) + V - i\beta\vec{\alpha}\cdot\hat{x}U, \tag{6}$$

where S is the scalar potential, V is the vector potential, and U is the tensor part. We have $\Delta = V - S$ and $\Sigma = V + S$. The Dirac equation can be solved exactly for the cases $\Delta = 0$ and $\Sigma = 0\cdot$ $\vec{\alpha} = \begin{pmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{pmatrix}$ and $\beta = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix}$ which are the usual Dirac matrices. We will indicate upper and lower components of Dirac spinor with Ψ_A and Ψ_B. Expressing the Dirac matrices in terms of the Pauli matrices: we obtain the following coupled equations, [25, 26]

$$\vec{\sigma}\cdot\vec{P}\Psi_A + (V - S - m)\Psi_B + i\vec{\sigma}\cdot xU(x)\Psi_A = E\Psi_B, \tag{7a}$$

$$\vec{\sigma}\cdot\vec{P}\Psi_B + (V + S + m)\Psi_A - i\vec{\sigma}\cdot xU(x)\Psi_B = E\Psi_A. \tag{7b}$$

We assume that V, S, and U are radial potentials. Using the identity

$$\vec{\sigma}\cdot\vec{P} = \frac{(\vec{\sigma}\cdot\vec{x})}{x^2}(\vec{\sigma}\cdot\vec{x})(\vec{\sigma}\cdot\vec{P}) = \frac{(\vec{\sigma}\cdot\vec{x})}{x^2}\left(-ix\frac{\partial}{\partial x} + i\vec{\sigma}\cdot\vec{L}\right). \tag{8}$$

With (7a) and (7b), we find the following second-order differential equations for Ψ_A and Ψ_B:

$$\begin{aligned}
&\vec{P}^2\Psi_A + \left(U^2 + \frac{du}{dx} + \frac{2u}{x} + \frac{(d\Delta/dx)}{(E+m-\Delta)}U\right)\Psi_A + \left(4U + 2\frac{(d\Delta/dx)}{(E+m-\Delta)}\right)\left(\frac{\vec{S}\cdot\vec{L}}{2}\right)\Psi_A - \frac{(d\Delta/dx)}{(E+m-\Delta)}\frac{\partial\Psi_A}{\partial x}\\
&= (E+m-\Delta)(E+m-\Sigma)\Psi_A,
\end{aligned} \tag{9a}$$

$$\begin{aligned}
&\vec{P}^2\Psi_B + \left(U^2 - \frac{du}{dx} - \frac{2u}{x} - \frac{(d\Sigma/dx)}{(E-m-\Sigma)}U\right)\Psi_B + \left(-4U + 2\frac{(d\Sigma/dx)}{(E-m-\Sigma)}\right)\left(\frac{\vec{S}\cdot\vec{L}}{2}\right)\Psi_B + \frac{(d\Sigma/dx)}{(E-m-\Delta)}\frac{\partial\Psi_B}{\partial x}\\
&= (E+m-\Delta)(E-m-\Sigma)\Psi_B.
\end{aligned} \tag{9b}$$

Here $\vec{S}$ stands for the $(1/2)\vec{\sigma}$ spin operator and $\vec{L}$ for orbital angular momentum operator. The Hamiltonian operator commutes with the total angular momentum operator which is given by $\vec{J} = \vec{L} + \vec{S}$ and with the parity operator. Therefore, the eigenfunctions defined by Hamiltonian can be expressed as

$$\Psi_{njmk} = \begin{pmatrix} i\dfrac{g_{nk}(x)}{x}\phi^{l}_{jmk}(\hat{x}) \\ \dfrac{f_{nk}(x)}{x}\phi^{\bar{l}}_{jm(-k)}(\hat{x}) \end{pmatrix}, \tag{10}$$

where $\phi^{l}_{jmk}(\hat{x})$ denotes the spin spherical harmonics. The quantum number k is related to ℓ and j as follows:

$$k = \begin{cases} -(\ell+1) = -\left(j+\dfrac{1}{2}\right) & j = \ell + \dfrac{1}{2}, \\ \ell = +\left(j+\dfrac{1}{2}\right) & j = \ell - \dfrac{1}{2}. \end{cases} \tag{11}$$

For spin spherical harmonics we have

$$\vec{\sigma}\cdot\hat{x}\phi^{l}_{jmk} = \phi^{\bar{l}}_{jm(-k)}. \tag{12}$$

Equations (9a) and (9b) reduced to a set of coupled equations for the radial wave functions g_k and f_κ:

$$\left(\frac{d^2}{dx^2} - \frac{k(k+1)}{x^2} + \frac{2k}{x}U - \frac{dU}{dx} - U^2\right) g_k(x) + \frac{d\Delta/dx}{(E+m-\Delta)}\left(\frac{d}{dx} + \frac{k}{x} - U\right) g_k(x) = -(E+m-\Delta)(E-m-\Sigma)\, g_k(x), \tag{13a}$$

$$\left(\frac{d^2}{dx^2} - \frac{k(k-1)}{x^2} + \frac{2k}{x}U + \frac{dU}{dx} - U^2\right) f_k(x) + \frac{d\Sigma/dx}{(E-m-\Delta)}\left(\frac{d}{dx} - \frac{k}{x} + U\right) f_k(x) = -(E+m-\Delta)(E-m-\Sigma)\, f_k(x). \tag{13b}$$

Using the following relations [13]:

$$\begin{aligned} 2\vec{S}\cdot\vec{L} &= \vec{J}^2 - \vec{L}^2 - \vec{S}^2, \\ \vec{\sigma}\cdot\vec{L}\phi^{l}_{jmk} &= -(k+1)\,\phi^{l}_{jmk}, \\ \vec{\sigma}\cdot\vec{L}\phi^{\bar{l}}_{jm(-k)} &= -(k+1)\,\phi^{\bar{l}}_{jm(-k)}, \end{aligned} \tag{14}$$

for particle of m mass and E energy, we have

$$\begin{aligned} S &= V = \frac{1}{2}\Sigma, \\ \Sigma(x) &= ax^6 - bx^5 + cx^4 - dx^3 + ex^2 - fx + h - \frac{L}{x}, \\ \Delta(x) &= 0, \\ U(x) &= -\frac{1}{x}. \end{aligned} \tag{15}$$

We assume the case $S(x) = V(x)$ [27–31] and consider $k(k+1) = \ell(\ell+1)$, then the upper component in (13a) is as follows:

$$\left(\frac{d^2}{dx^2} - \frac{\ell(\ell+1)+2+2k}{x^2} - \Sigma(r)(E+m)\right) g_k(x) = \left(m^2 - E^2\right) g_k(x). \tag{16}$$

As we know in deuteron, two nucleons have in D-state, so

$$\left[\frac{d^2}{dx^2} - \frac{\ell(\ell+1)+2-2(\ell+1)}{x^2}\right] g_k(x) = \left(\left(a_1x^6 - b_1x^5 + c_1x^4 - d_1x^3 + e_1x^2 - f_1x + h_1 - \frac{L_1}{x}\right) - \mathscr{E}\right) g_k(x). \tag{17}$$

Therefore, we have a Schrödinger-like equation [31]

$$g_1''(x) = \left[\left(a_1x^6 - b_1x^5 + c_1x^4 - d_1x^3 + e_1x^2 - f_1x + h_1 - \frac{L_1}{x}\right) - \mathscr{E} + \frac{\ell(\ell-1)}{x^2}\right]. \tag{18}$$

By setting

$$\begin{aligned} a(E_1+m) &= a_1, & b(E_1+m) &= b_1, \\ c(E_1+m) &= c_1, & d(E_1+m) &= d_1, \\ e(E_1+m) &= e_1, & f(E_1+m) &= f_1, \end{aligned} \tag{19}$$

$$h(E_1+m) = h_1, \qquad L(E_1+m) = L_1\left({E_1}^2 - m^2\right) = \mathscr{E}. \tag{20}$$

In order to solve (18), we suppose the following form for the wave function:

$$g(x) = M(x)\, e^{Z(x)}. \tag{21}$$

Now, for the functions $M(x)$ and $Z(x)$, respectively, we make use of the following ansatz [27, 32, 33]:

$$\begin{aligned} M(x) &= \begin{cases} 1 & \text{if ground state} \\ \prod_{i=1}^{\nu}\left(x - a_i^{\nu}\right) & \text{if } \nu > 0, \end{cases} \\ Z(x) &= -\frac{1}{4}\alpha x^4 - \frac{1}{3}\beta x^3 - \frac{1}{2}\eta x^2 - \tau x + \delta \ln x. \end{aligned} \tag{22}$$

For a particular grand angular quantum number γ, there are different solutions which are labeled by υ (υ determines the number of the nodes of the wave function). By substitution of $M(x)$ and $Z(x)$ into (21) and then taking the second-order derivative of the obtained equation, we can get

$$g''(x) = \left[Z'' + Z'^2 + \frac{M'' + 2M'Z'}{M}\right]. \tag{23}$$

We consider the ground state, which is called the 0th node solution of the above differential equation. By equating (18) and (23) for $\upsilon = 0$, it can be found that

$$\begin{aligned} \alpha &= \sqrt{a_1}, & \beta &= -\frac{b_1}{2}\sqrt{a_1}, \\ \eta &= \frac{c_1 - \beta_1}{2\alpha}, & \tau &= \frac{-d_1 - 2\beta\eta}{2\alpha}, \\ \delta &= \ell - 1, & \mathscr{E} &= \eta(1-2\delta) - \tau^2 - h_1. \end{aligned} \tag{24}$$

The upper component is as follows:

$$g_1(x) = N_0 x^{\ell-1} \exp\left(-\frac{1}{4}\alpha x^4 - \frac{1}{3}\beta x^3 - \frac{1}{2}\eta x^2 - \tau x\right) \tag{25}$$

and another component of wave function is

$$f_1(x) = \frac{\left(\vec{\sigma}\cdot\vec{P} + i\vec{\sigma}\cdot\hat{r}U\right)}{(E+m)} g_1(x). \tag{26}$$

By substituting (25) into (26), we obtained the following equation:

$$f_1(x) = \frac{-i\vec{\sigma}\cdot\widehat{x}}{(E+m)}\left[\frac{d}{dx} - i\vec{\sigma}\cdot\vec{L} - U\right] N_0 x^{\ell-1} \times \exp\left(-\frac{1}{4}\alpha x^4 - \frac{1}{3}\beta x^3 - \frac{1}{2}\eta x^2 - \tau x\right). \tag{27}$$

Therefore, the total wave function is

$$\Psi = N_0 \left(\begin{array}{c} 1 \\ \frac{-i\vec{\sigma}\cdot\widehat{x}}{(E+m)}\left[\alpha x^3 + \beta x + \eta x + \tau + \frac{1}{x}\right]\end{array}\right) \times \exp\left(-\frac{1}{4}\alpha x^4 - \frac{1}{3}\beta x^3 - \frac{1}{2}\eta - \tau x\right). \tag{28}$$

Now we have to obtain parameters a, b, c, d, e, f, h, and L, by using of (5):

$$b = \frac{14}{k}a, \quad c = \frac{105}{k^2}a, \quad d = \frac{440}{k^3}a, \quad e = \frac{2555}{k^4}a, \quad f = \frac{6846}{k^5}a, \quad h = \frac{8659}{k^6}a. \tag{29}$$

We know that $k = 1/x = 0.714\,(\mathrm{fm}^{-1})$ [34], by using of (24), we can get amount of parameters α, β, η, and τ:

$$\alpha = \sqrt{a_1}, \quad \beta = -9.8\sqrt{a_1}, \quad \eta = 83.08\sqrt{a_1}, \quad \tau = 83.08\sqrt{a_1}. \tag{30}$$

The bonding energy for deuteron is reported 2.224 MeV [35]. This is useful for calculating the numeric values of parameters in (30):

$$\varepsilon = E^2 - m^2 = \eta(1-2\delta) - \tau^2 - h_1. \tag{31}$$

By substituting the amounts of (31) and $\ell = 1$, we obtained the numeric values of parameters in (30). The parameters are reported in Table 1, while the parameters of Yukawa potential are reported in Table 2.

4. The Hyperfine Interaction

As the Baryons have spin and isospin, the deuteron neutron and proton can interact together. We introduce the hyperfine interaction potential. The complete potential is

$$\langle H_{\mathrm{int}}(x)\rangle = V(x) + \langle H_S(x)\rangle + \langle H_I(x)\rangle + \langle H_{SI}(x)\rangle. \tag{32}$$

In this work, we have added the hyperfine interaction potentials ($H_S(x)$, $H_I(x)$, and $H_{SI}(x)$) which yield properties very close to the experimental results. By regarding $V(x)$ as the nonpertubative potential and the other terms in (32) as pertubative ones according to this explanation. The spin and isospin potential contains a δ-like term. We have modified it by a Gaussian function of the nucleons pair relative distance [36]

$$H_S = A_S \frac{1}{\left(\sqrt{\pi}\sigma_S\right)^3} e^{-x^2/\sigma_S^2} \sum\left(\vec{S}_i\cdot\vec{S}_j\right), \tag{33}$$

where S_i is the spin operator of the ith nucleon and x is the relative nucleon pair coordinate. A_s and σ_S are constants: $\sigma_S = 2.87$ fm and $A_S = 67.4\,\mathrm{fm}^2$ [34, 37]. We know that the deuteron consists of two nucleons. Where 1 indicate the first nucleon, and 2 indicate the second nucleon. We have

$$H_S = A_S \frac{1}{\left(\sqrt{\pi}\sigma_S\right)^3} e^{-x^2/\sigma_S^2} \left(\frac{1}{2}\right)\left[S^2 - S_1^2 - S_2^2\right]. \tag{34}$$

Furthermore, we add two hyperfine interaction terms to the Hamiltonian nucleon pairs similar to (33) [38]

$$H_I = A_S \frac{1}{\left(\sqrt{\pi}\sigma_I\right)^3} e^{-x^2/\sigma_I^2} \sum\left(\vec{I}_i\cdot\vec{I}_j\right), \tag{35}$$

where I_i is the isospin operator of the ith nucleon, and the constants are described as $\sigma_I = 3.45$ fm and $A_I = 51.7\,\mathrm{fm}^2$ [37, 39]. The another one is a spin-isospin interaction, given by similar (33) [38, 40]

$$H_{SI} = A_{SI} \frac{1}{\left(\sqrt{\pi}\sigma_{SI}\right)^3} e^{-x^2/\sigma_{SI}^2} \sum\left(\vec{S}_i\cdot\vec{S}_j\right)\left(\vec{I}_i\cdot\vec{I}_j\right). \tag{36}$$

The fitted parameters are $\sigma_{SI} = 2.31$ fm and $A_{SI} = -106.2\,\mathrm{fm}^2$ [35, 41], where ψ_γ is the perturbed wave function, and we write it as

$$\psi_\gamma = \psi'_\gamma + \sum_{\gamma'=\gamma} \frac{\left\langle \psi^0_{\gamma'} \mid H_{\mathrm{int}} \mid \psi^0_{\gamma'}\right\rangle \psi'^0_{\gamma'}}{E^0_\gamma - E'^0_\gamma}. \tag{37}$$

The deuteron mass are given by two nucleons masses, and the eigenenergies of the Dirac equation E, (E is a function of η, δ, τ, h, m) with the first-order energy correction from potential H_{int} can be obtained by using the unperturbed wavefunction (33), (35), and (36). The total potential for the ground state as well as the other states can be written as [40, 42]

$$\langle H_{\mathrm{int}}\rangle = \frac{\int \psi^*_\gamma H_{\mathrm{in}} \psi_\gamma x^2 dx\, d\Omega}{\int \psi^*_\gamma \psi_\gamma x^2 dx\, d\Omega}. \tag{38}$$

We first assume $\gamma = 0$. The potentials can be extracted from (33), (35), and (36):

$$H_S = 0.10059\ (\mathrm{MeV}), \quad H_I = 0.04782\ (\mathrm{MeV}), \quad H_{SI} = -0.50053\ (\mathrm{MeV}). \tag{39}$$

Now, we can calculate the deuteron mass according to the following formula:

$$M_D = m_n + m_p + E_{\upsilon\gamma} + \langle H_{\mathrm{int}}\rangle. \tag{40}$$

Table 1: The best adaption with experimental value is $a_1 = 3.06136$, $\alpha = 1.74967$, $\beta = -17.146815$, $\eta = 145.362999$, $\tau = -141.02380$, and E (MeV) = 2.19020.

a_1	α	β	η	τ	E (MeV)
3.06131	1.74965	−17.146628	145.361420	−141.02227	2.89658
3.06134	1.74967	−17.146785	145.362749	−141.02356	2.49699
3.06135	1.74967	−17.146805	145.362915	−141.02372	2.34861
3.06136	1.74967	−17.146815	145.362999	−141.0238	2.19020
3.0638	1.74967	−17.147648	145.377006	−141.0234	1.83278

Table 2: The value of the Yukawa potential parameters.

a (fm^{-7})	b (fm^{-6})	c (fm^{-5})	d (fm^{-4})	e (fm^{-3})	f (fm^{-2})	h (fm^{-1})
0.21961	4.30589	45.24045	265.53556	2159.7519	8109.2235	14362.5603

By substituting $m_n = m_p = 938\,(\text{MeV}) = 4.65\,(\text{fm}^{-1})$, energy eigenvalue ($E_{\upsilon\gamma}$), and the expectation values of H_{int} in (40) we have

$$M_D = 1877.83808\ \left(\text{MeV}/\text{c}^2\right). \tag{41}$$

By comparing the experimental amount of deuteron mass ($M_D = 1875.612\,(\text{MeV}/\text{c}^2) = 9.29\,(\text{fm}^{-1})$) with our calculated mass for deuteron, we found that a good agreement has been obtained by our model [35].

5. Conclusion

In this work, we offer a new approach for the solving of Yukawa potential. We expand this potential around of its mesonic cloud that gets a new form with great powers and inverse exponent. We calculate wave function of the Dirac equation for a new form of Yukawa potential, including a coulomb-like tensor potential. By considering the effects of pertubative interaction potential, we see that the theoretical and experimental masses are in complete agreement. These improvements in reproduction of deuteron mass is obtained by using a suitable form for confinement potential and exact analytical solution of the Dirac equation for our proposed potential. Finally, one can use this model for another nuclei, by relativistic or nonrelativistic equation and get the properties of them and the amount of "g", the strength of nuclear force.

References

[1] H. Motavali, "Bound state solutions of the Dirac equation for the Scarf-type potential using Nikiforov-Uvarov method," *Modern Physics Letters A*, vol. 24, no. 15, pp. 1227–1236, 2009.

[2] M. R. Setare and Z. Nazari, "Solution of Dirac equations with five-parameter exponent-type potential," *Acta Physica Polonica B*, vol. 40, no. 10, pp. 2809–2824, 2009.

[3] L.-H. Zhang, X.-P. Li, and C.-S. Jia, "Analytical approximation to the solution of the Dirac equation with the Eckart potential including the spin-orbit coupling term," *Physics Letters A*, vol. 372, no. 13, pp. 2201–2207, 2008.

[4] C.-L. Ho, "Quasi-exact solvability of Dirac equation with Lorentz scalar potential," *Annals of Physics*, vol. 321, no. 9, pp. 2170–2182, 2006.

[5] H. Panahi and L. Jahangiry, "Shape invariant and Rodrigues solution of the Dirac-shifted oscillator and Dirac-Morse potentials," *International Journal of Theoretical Physics*, vol. 48, no. 11, pp. 3234–3240, 2009.

[6] A. D. Alhaidari, "L^2 series solution of the relativistic Dirac-Morse problem for all energies," *Physics Letters A*, vol. 326, no. 1-2, pp. 58–69, 2004.

[7] F. D. Adame and M. A. Gonzalez, "Solvable linear potentials in the Dirac equation," *Europhysics Letters*, vol. 13, no. 3, pp. 193–198, 1990.

[8] A. S. de Castro and M. Hott, "Exact closed-form solutions of the Dirac equation with a scalar exponential potential," *Physics Letters A*, vol. 342, no. 1-2, pp. 53–59, 2005.

[9] Y. Chargui, A. Trabelsi, and L. Chetouani, "Exact solution of the (1+1)-dimensional Dirac equation with vector and scalar linear potentials in the presence of a minimal length," *Physics Letters A*, vol. 374, no. 4, pp. 531–534, 2010.

[10] A. S. de Castro, "Bound states of the Dirac equation for a class of effective quadratic plus inversely quadratic potentials," *Annals of Physics*, vol. 311, no. 1, pp. 170–181, 2004.

[11] J.-Y. Guo, J. Meng, and F.-X. Xu, "Solution of the Dirac equation with special Hulthén potentials," *Chinese Physics Letters*, vol. 20, no. 5, pp. 602–604, 2003.

[12] H. Akcay and C. Tezcan, "Exact solutions of the dirac equation with harmonic oscillator potential including a coulomb-like tensor potential," *International Journal of Modern Physics C*, vol. 20, no. 6, pp. 931–940, 2009.

[13] H. Akcay, "Dirac equation with scalar vector quadratic potentials and coulomb-like tensor potential," *Physics Letters A*, vol. 373, pp. 616–620, 2009.

[14] O. Aydogdu and R. Sever, "Exact pseudospin symmetric solution of the Dirac equation for pseudoharmonic potential in the presence of tensor potential," *Few-Body Systems*, vol. 47, no. 3, pp. 193–200, 2010.

[15] M. Eshghi, "Dirac-Hua problem including a Coulomb-like tensor interaction," *Advanced Studies in Theoretical Physics*, vol. 5, no. 12, pp. 559–571, 2011.

[16] G. Mao, "Effect of tensor couplings in a relativistic Hartree approach for finite nuclei," *Physical Review C*, vol. 67, Article ID 044318, 12 pages, 2003.

[17] P. Alberto, R. Lisboa, M. Malheiro, and A. S. de Castro, "Tensor coupling and pseudospin symmetry in nuclei," *Physical Review C*, vol. 71, Article ID 034313, 2005.

[18] R. J. Furnstahl, J. J. Rusnak, and B. D. Serot, "The nuclear spin-orbit force in chiral effective field theories," *Nuclear Physics A*, vol. 632, no. 4, pp. 607–623, 1998.

[19] H. Yukawa, "On the interaction of elementary particles. I," *Proceedings of the Physico-Mathematical Society of Japan*, vol. 17, pp. 48–57, 1935.

[20] H. Yukawa, "On the interaction of elementary particles. II," *Proceedings of the Physico-Mathematical Society of Japan*, vol. 19, pp. 1084–1093, 1937.

[21] S. Zarrinkamar, A. A. Rajabi, and H. Hassanabadi, "Dirac equation for the harmonic scalar and vector potentials and linear plus Coulomb-like tensor potential; the SUSY approach," *Annals of Physics*, vol. 325, no. 11, pp. 2522–2528, 2010.

[22] H. Hassanabadi, E. Maghsoodi, S. Zarrinkamar, and H. Rahimov, "Dirac equation for generalized Pöschl-Teller scalar and vector potentials and a Coulomb tensor interaction by Nikiforov-Uvarov method," *Journal of Mathematical Physics*, vol. 53, no. 2, Article ID 022104, 2012.

[23] M. Moshinsky and A. Szczepaniak, "The Dirac oscillator," *Journal of Physics A*, vol. 22, no. 17, pp. L817–L819, 1989.

[24] H. Bahlouli, M. S. Abdelmonem, and I. M. Nasser, "Analytical treatment of the Yukawa potential," *Physica Scripta*, vol. 82, Article ID 065005, 2010.

[25] M. R. Shojaei and A. A. Rajabi, "Hypercentral constituent quark model and the hyperfine dependence potential," *Iranian Journal of Physics Research*, vol. 7, no. 2, 2007.

[26] A. A. Rajabi, "A three-body force model for the harmonic and anharmonic oscillator," *Iranian Journal of Physics Research*, vol. 5, no. 2, 2005.

[27] N. Salehi and A. A. Rajabi, "Proton static properties by using hypercentral constituent quark model and isospin," *Modern Physics Letters A*, vol. 24, no. 32, pp. 2631–2637, 2009.

[28] A. D. Alhaidari, H. Bahlouli, and A. Al-Hasan, "Dirac and Klein-Gordon equations with equal scalar and vector potentials," *Physics Letters A*, vol. 349, no. 1–4, pp. 87–97, 2006.

[29] A. A. Rajabi, "Hypercentral constituent quark model and isospin for the baryon static properties," *Journal of Sciences, Islamic Republic of Iran*, vol. 16, pp. 73–79, 2005.

[30] N. Salehi and A. A. Rajabi, "A new method for obtaining the spectrum of baryon resonances in the Killingbeck potential," *Physica Scripta*, vol. 85, Article ID 055101, 2012.

[31] C.-Y. Chen, "Exact solutions of the Dirac equation with scalar and vector Hartmann potentials," *Physics Letters A*, vol. 339, no. 3-5, pp. 283–287, 2005.

[32] A. A. Rajabi, "A method to solve the Schrödinger equation for any power hypercentral potentials," *Communications in Theoretical Physics*, vol. 48, no. 1, pp. 151–158, 2007.

[33] N. Salehi, A. A. Rajabi, and Z. Ghalenovi, "Spectrum of strange and nonstrange baryons by using generalized gürsey radicati mass formula and hypercentral potential," *Acta Physica Polonica B*, vol. 42, no. 6, pp. 1247–1259, 2011.

[34] S. S. M. Wong, "Introductory Nuclear Physics," *John Wiley & Sons*, 1998.

[35] N. Salehi, A. A. Rajabi, and Z. Ghalenovi, "Calculation of nonstrange baryon spectra based on symmetry group theory and the hypercentral potential," *Chinese Journal of Physics*, vol. 50, no. 1, p. 28, 2012.

[36] L. Y. Glozman, W. Plessas, K. Varga, and R. F. Wagenbrunn, "Unified description of light- and strange-baryon spectra," *Physical Review D*, vol. 58, no. 9, Article ID 094030, 1998.

[37] M. De Sanctis, M. M. Giannini, E. Santopinto, and A. Vassallo, "Electromagnetic form factors in the hypercentral CQM," *European Physical Journal A*, vol. 19, no. 1, pp. 81–85, 2004.

[38] L. Y. Glozman and D. O. Riska, "The spectrum of the nucleons and the strange hyperons and chiral dynamics," *Physics Report*, vol. 268, no. 4, pp. 263–303, 1996.

[39] M. F. de la Ripelle, "A confining potential for quarks," *Nuclear Physics A*, vol. 497, pp. 595–602, 1989.

[40] M. M. Giannini, E. Santopinto, and A. Vassallo, "An overview of the hypercentral Constituent Quark Model," *Progress in Particle and Nuclear Physics*, vol. 50, no. 2, pp. 263–272, 2003.

[41] H. Hassanabadi, "A simple physical approach to study spectrum of baryons," *Communications in Theoretical Physics*, vol. 55, no. 2, p. 303, 2011.

[42] H. Hassanabadi, A. A. Rajabi, and S. Zarrinkamar, "Spectrum of baryons and spin-isospin dependence," *Modern Physics Letters A*, vol. 23, no. 7, pp. 527–537, 2008.