Generalised Dark Matter:
Imprints on the CMB and mapping to non-perturbative models

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Dark Matter (DM) is a crucial component of the universe and is successfully modelled as a pressureless perfect fluid for calculations of the cosmic microwave background (CMB). With data from Planck it becomes possible to test generalisations of this model, searching for DM properties beyond the pressureless perfect fluid and thereby testing the CDM paradigm itself. Although there is no unique way to generalise the pressureless perfect fluid, the Generalised Dark Matter (GDM) model has proven useful in CMB applications. In this model, DM is an imperfect fluid with pressure and shear viscosity. We will present the GDM closure equations for pressure and shear that are parameterised by 3 new model parameters w, c_s^2 and $c_{\rm vis}^2$ and elucidate their physical meaning and main effects on the CMB.[1] This will shed light on our parameter constraints we obtain using Planck data, see also the contribution by D. Thomas. Assuming constant values we constrained those parameters $|w| < \mathcal{O}(10^{-3})$ and $c_s^2, c_{\rm vis}^2 < \mathcal{O}(10^{-6})$, both at the 99.7% CL using the CMB, finding no evidence for properties beyond the pressureless perfect fluid.[2] We will also discuss how several non-perturbative models can be related to GDM,[1] which will prove useful for extending the parameterisation to the non-linear regime of structure formation. These models include the non-equilibrium thermodynamics of Landau and Lifshitz, the so-called effective field theory of large scale structure and the effective field theory of fluids.

1 Generalized Dark matter

The evidence for Cold Dark Matter (CDM) has been mounting up for over 80 years [3] culminating in a precise measurement of its abundance to be $26 \pm 1\%$ of the total energy budget,[4] and spectacular demonstrations of its gravitational footprint and collisionless nature through observations of colliding galaxy clusters.[5] Many more independent astrophysical and cosmological observations created this concordant picture. In addition many extensions of the standard model of particle physics (SM), whose purpose was to solve problems within the SM, predict as byproducts the existence of particles, like axions, wimps or a sterile neutrino, which are perfect dark matter candidates. They are non-baryonic, electrically neutral and for most practical purposes cold and collisionless and therefore approximately described as pressureless perfect fluid,

$$T_c^{\mu\nu} = \rho_c u_c^\mu u_c^\nu \,. \tag{1}$$

On the other hand despite extensive searches [6,7] there is still no non-gravitational evidence for DM. We therefore should exploit all the available cosmological data to test the CDM paradigm by searching for properties beyond the perfect pressureless fluid. This requires modelling the DM component with a more general stress-energy-momentum-tensor

$$T_g^{\mu\nu} = \rho_g u_g^\mu u_g^\nu + P_g(g^{\mu\nu} + u_g^\mu u_g^\nu) + \Sigma_g^{\mu\nu} \,. \tag{2}$$

Although axions, wimps and sterile neutrinos can be reasonably well described as pressureless perfect fluid in the regime where linear perturbation theory applies, there are subtle differences and these manifest themselves as pressure P_g and shear $\Sigma_g^{\mu\nu}$.[8,9,10,11,12] Since there exist many more (and also more complicated) DM candidates,[13,14,15,16,17,18] and since there is no strong theoretical prior for any particular class of models, a phenomenological approach is best suited.

1.1 GDM definition

One such phenomenological model is the Generalised Dark Matter (GDM) model,[19] a specific ansatz for P_g and $\Sigma_g^{\mu\nu}$, with three parametric functions: *equation of state* w, *sound speed* c_s^2, and *viscosity* $c_{\rm vis}^2$. In more detail, this ansatz is for the GDM pressure and shear $P_g = \bar{P}_g + \bar{\rho}_g\,\Pi_g$, $\Sigma^i_{gj} = (\bar{\rho}_g + \bar{P}_g)\big(\vec{\nabla}^i\vec{\nabla}_j - \frac{1}{3}\vec{\nabla}^2\delta^i_j\big)\,\Sigma_g$, where Π_g and Σ_g are linear scalar perturbations of GDM pressure and shear and $\bar{P}_g$ the Friedmann-Robertson-Walker (FRW) background value of the GDM pressure. Hu postulated the following closure equations [19]

$$\bar{P}_g = w(a)\,\bar{\rho}_g \tag{3a}$$

$$\Pi_g = c_a^2(w)\,\delta_g + \big(c_s^2(a,k) - c_a^2(w)\big)\,\hat{\Delta}_g \tag{3b}$$

$$\dot{\Sigma}_g + 3\mathcal{H}\Sigma_g = \frac{4}{(1+w(a))}c_{\rm vis}^2(a,k)\,\hat{\Theta}_g\,, \tag{3c}$$

where $\hat{\Delta}_g = \delta_g|_{\text{rest frame}}$ and $\hat{\Theta}_g = \theta_g|_{\text{Newtonian}}$ are gauge invariant combinations, $\mathcal{H} = \dot{a}/a$, where a dot refers to a derivative w.r.t. conformal time τ, and $c_a^2 = w - \frac{1}{3}d\ln(1+w)/d\ln a$ is the *adiabatic sound speed*. The conservation equations $\nabla_\mu T_g^{\mu\nu} = 0$ give the remaining equations for the GDM background density $\bar{\rho}_g$ and density perturbation δ_g and as well as the GDM velocity perturbation θ_g. Together with the Einstein equation $G_{\mu\nu} = 8\pi G T_{\mu\nu}$, where $T^\mu{}_\nu = T_g{}^\mu{}_\nu + T_\Lambda{}^\mu{}_\nu + T_{\rm SM}{}^\mu{}_\nu + \dots$, is the total stress-energy-momentum tensor, the system of equations is closed. It should be noted that although the parameters c_s^2 and $c_{\rm vis}^2$ are allowed to depend on scale k, they are not allowed to depend on the particular solution. It is worth emphasising that the *non-adiabatic pressure*

$$\Pi_{\rm nad} \equiv \Pi_g - c_a^2\delta_g = \big(c_s^2(a,k) - c_a^2(w)\big)\,\hat{\Delta}_g \tag{4}$$

is algebraically related to the rest frame density perturbation $\hat{\Delta}_g$, while the Σ_g is dynamical and sourced by the Newtonian velocity perturbation $\hat{\Theta}_g$. We have investigated those choices implicit in (3), but refer the reader to our paper for more details.[1]

1.2 DM models encompassed by GDM

To get a feeling for the size and expected time dependence of those parameters, we compare in Fig. 1 two important cases. The dotted line shows the prediction for CDM by the effective field theory of large scale structure (EFTofLSS) [11] and the dashed line shows the upper bound for freely streaming warm dark matter. Note that while the former takes into account that even initially cold DM warms up through the backreaction of unresolved small scale non-linearities, the latter describes initially Maxwell-Boltzmann distributed warm dark matter without taking into account this backreaction. In these two cases $w \simeq c_s^2 \simeq c_{\rm vis}^2$, and it is interesting that the expectation for CDM comes close to the value of our upper bound (the constant line).

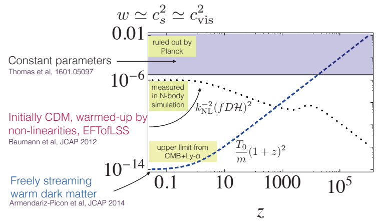


Figure 1 – Redshift evolution of GDM parameters for two physical DM models and the constant case.

1.3 *GDM imprints on the CMB*

The basic imprints of GDM on the CMB can be understood by expanding analytic solutions for $\bar{\rho}_g$ and the Newtonian potential $\hat{\Phi}$ (sourced only by GDM) in small $w, c_s^2, c_{\rm vis}^2 \ll 1$ giving [1]

$$a^3\bar{\rho}_g \propto \omega_g(1 + 3w\ln(1+z))\,, \qquad k_{\rm d}^{-1}(\tau) \simeq \tau\sqrt{c_s^2 + \tfrac{8}{15}c_{\rm vis}^2}\,, \tag{5}$$

where $k_{\rm d}^{-1}$ is the scale below which the potential $\hat{\Phi}$ starts to decay. This explains most features in

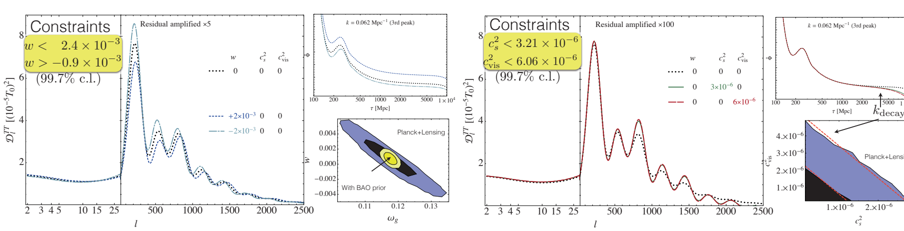
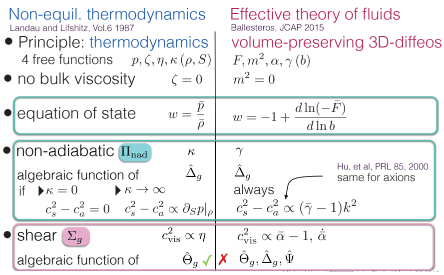


Figure 2 – Comparison of the effects of constant GDM parameter on the temperature power spectrum of the CMB, the potential evolution. Also shown are the constraints and important degeneracies.

the temperature power spectrum shown in Fig. 2, where we take the Planck best fit value for ω_g and the other standard parameters and switch on one GDM parameter with a size corresponding to our upper limits. The smaller upper plots show the effect on the evolution of a single k-mode of the potential $\hat{\Phi}$ and the contours show the results of our MCMC analysis confirming (5), which states that w, like ω_g, affect the freeze-out value of $\hat{\Phi}$ via matter-radiation equality and $c_s^2, c_{\rm vis}^2$ cause $\hat{\Phi}$ to decay below $k_{\rm d}^{-1}$, manifesting in a reduction of the lensing potential, amplifying slightly acoustic peaks and throughs. The yellow boxes in the left corners show our constraints.[2] Earlier studies only constrained one or two parameters.[20]

2 Non-pertubative Extensions

In order extend the GDM model into the non-linear regime of structure formation, necessary if we want to take into account the wealth of data that cannot be described by linear perturbation theory, we considered several models for imperfect fluids that are defined non-perturbatively. Here we focus on the non-equilibrium thermodynamics of Landau and Lifshitz (LL) [21] and the effective field theory of fluids (EFT) by Ballesteros.[22] Both theories use physical or mathematical principles to restrict the possible form of $T_g^{\mu\nu}$, Eq. (2), see the table below. In both causes the full $T_g^{\mu\nu}$ contains only a few free functions. To make the connection to GDM we to set to zero the bulk viscosity in both cases. The remaining free function κ and η for LL, as well as γ and α for EFT describe non-adiabatic pressure $\Pi_{\rm nad}$ and shear Σ_g.



While $\Pi_{\rm nad}$ in EFT is automatically of GDM form, although with a particular scale dependence, reminiscent of axions,[8] in LL $\Pi_{\rm nad}$ is in general dynamical. Nevertheless it is sourced by the particular density perturbation $\hat{\Delta}_g$. It can be made algebraic in two limits, either very large or very small heat conduction κ. In LL theory, the Σ_g is algebraically related to $\hat{\Theta}_g$, which gives a behaviour similar to Eq. 3c and connects the viscosity parameters η and $c_{\rm vis}^2$.

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