ACCELERATING-FIELD QUADRUPOLE FOCUSING IN HIGH-CURRENT LINACS

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First proposed by Vladimirskii in 1956, the beam focusing method for a drift-tube proton linear accelerator, which uses a quadrupole configuration of an accelerating gap has been treated in /1-7/. However, originally it required rather a high injection energy /8/. In 1964, Teplyakov /9-11/ has found a way to expand the limits of the HF focusing by inserting two accelerating gaps with different focusing action into each accelerating period. Values of current considered in /1-11/ were low. It is the aim of this report to analyse the HF quadrupole focusing in a linac with suitably positioned lenses, for a high current beam of accelerated particles.

We shall assume that focusing period, L, holds an even number N of accelerating periods, D, each consisting of two accelerating gaps with equal transit time factors, T. The midgaps are spaced by 2ℓ, i.e. by a transit angle $2\varepsilon = 2\ell\cdot 2\pi/\beta\lambda$. An accelerated bunch is represented by a uniformly charged ellipsoid. A quadrupole lens is inserted only into the second gap to obtain optimum focusing action /11/. We shall suppose that the defocusing forces of both the accelerating field and the self charge are uniform over a period while the quadrupole forces are concentrated at the center of a lens.

With the above suppositions, the equation of particle motion as projected onto the transversal X-axis is /8/

$$\frac{d^2x}{dz^2} + \left[-A + \frac{\Lambda^2}{N}\sum_i f_i\,\delta(z-z_i)\right]x = 0, \qquad (1)$$

where the summation is made over all the lenses, f_i being ± 1 for focusing and defocusing half-periods, correspondingly. The equation in terms of Y is similar to this one but for the sign before Λ^2. In (1)

$$x = \sqrt{\frac{p}{p_{in}L_{in}L}}\,X, \qquad z = \int_0^Z \frac{dZ}{L} \qquad (2)$$

are dimensionless co-ordinates, $p = m_0 v/\sqrt{1-\beta^2}$ is the momentum of equilibrium particle, m_0 and v are the proton mass and velocity, $\beta = v/c$ and c is the velocity of light. The defocusing parameter, A, and the focusing parameter, Λ^2, are given by

$$\left.\begin{aligned} A &= A_c + PE_m\sin\varphi, \\ \Lambda^2 &= \Lambda_c^2 + QE_m(\cos\varphi + \mathrm{tg}\,\varepsilon\sin\varphi), \end{aligned}\right\} \qquad (3)$$

$$A_c = \mathcal{S}PE_mM_1, \qquad \Lambda_c^2 = k\mathcal{S}PE_mM_1, \qquad (4)$$

$$P = \frac{\pi c L^4(1-\beta^2)}{m_0c^2\lambda\beta^3}, \quad Q = \frac{e\varkappa\lambda L^2(1-\beta^2)^{3/2}}{m_0c^2a^2\beta}, \quad M_1 = \frac{1-M_z}{M_z}\sin\varphi_s \qquad (5)$$

In (3) the A_c and Λ_c^2 terms represent the defocusing and the quadrupole action due to the bunch self-charge (usually $k = \Lambda_c^2/A_c \leq 0,2$), and the P and Q terms represent the defocusing and the quadrupole action due to the accelerating field. Further, φ is the particle phase related to the accelerating wave, φ_s is the equilibrium phase, $E_m = \mathcal{U}_mT\cos\varepsilon/\mathcal{D}$ is the accelerating field amplitude, $\mathcal{U}_m$ is the voltage amplitude across the accelerating period, $2a$ is the distance between the opposing (of the same sign) lens electrodes. The quadrupole lens parameter is

$$\varkappa = \frac{a^2}{2T\mathcal{U}_m\beta\lambda}\int\left|\frac{\partial E_x}{\partial X} - \frac{\partial E_y}{\partial Y}\right|_{\omega t=0}\cdot\cos\frac{2\pi(Z-\ell)}{\beta\lambda}\,dZ, \qquad (6)$$

where E_x and E_y are the cross-sectional components of the gap field.

In (3) - (5)

$$\mathcal{S} = \frac{3\beta\lambda^2 I M_z}{2\pi c E_m\sin\varphi_s\, a_x a_y a_z\sqrt{1-\beta^2}} \qquad (7)$$

is a parameter proportional to the bunch charge density, I is the beam current, M_z is the ellipsoid form factor, and a_x, a_y, and a_z are the ellipsoid semiaxes.

Fig.1 illustrates the diagrams of radial oscillation stability in terms of A and Λ^2 plotted for two cases: one (that drown by dash-and-dot lines) for $N=2$ and the other (that drown by flat lines) for $N=4$. The focusing mode is represented in the (A, Λ^2)-plane by a point that moves during oscillations of the phase φ along an arc of ellipse described parametrically by (3). We shall limit an operating region of the diagram by a straight line at the left side and by a parabola at the right side (the dashed lines in Fig.1):

$$k_1\Lambda^2 - \ell_1 \leq A \leq k_2\Lambda^4 \qquad (8)$$

Since the arc of ellipse must fall entirely into the operating region (8) the value of E_m is limited by

$$\frac{P(\sin\varphi_M + \mathcal{S}M_1)}{k_2\left[Q\frac{\cos(\varphi_M-\varepsilon)}{\cos\varepsilon} + k\mathcal{S}PM_1\right]^2} \leq E_m \leq \frac{\ell_1}{\sqrt{\frac{k_1^2Q^2}{\cos^2\varepsilon} - 2k_1QP\,\mathrm{tg}\,\varepsilon + P^2} - \mathcal{S}PM_2} \qquad (9)$$

Here $M_2 = (1-kk_1)M_1$ and the phase $\varphi = \varphi_{max} = \varphi_M$ represents the intersection point of the arc of ellipse and the right-hand bound of the operating region. From (9) it is clear that a displacement of the lens off the center of acceleration period facilitates focusing by expanding the range of permitted values of E_m. It results from (9) that the injection energy can not be less than

$$\beta^2 \geq \frac{\pi a^2}{\lambda^2\varkappa}\left(\frac{Q}{P}\right)_{min} \qquad (10)$$

where

$$\left(\frac{Q}{P}\right)_{min} = 2G_1\cos\varepsilon\left[1 - G_2 + \sqrt{1 - 2G_2 - \frac{\sin\varepsilon + \mathcal{S}M_2}{k_1G_1}}\right], \qquad (11)$$

$$G_1 = \frac{k_1(\sin\varphi_M + \mathcal{S}M_1)}{4\ell_1k_2\cos^2(\varphi_M-\varepsilon)}, \quad G_2 = \frac{k\mathcal{S}M_1}{2G_1\cos(\varphi_M-\varepsilon)}$$

If the current increases from zero to its maximum value I_M then the upper limit of E_m increases due to the ellipse center (3) shifting to the right. An increase in current has a two-fold effect on the lower limit of E_m: ellipse shifting to the right in an increase of $\mathcal{S}M_1$ while the phase oscillation diminishing results in a smaller φ_M in the left-hand side of (9). Which effect dominates on the values of φ_s, M_1, and $\mathcal{S}$.

The table below gives the permitted injection energy W_{min} and the corresponding values of E_m for various shift angles ε determined from (9)-(11) with $N=4$, $K=1$, $\lambda=2$m, $a=1$ cm, $\varkappa=0.125$, $\varphi_s=30°$, $k=0.2$, $M_z=0.33$, $k_1=1.0$, $\ell_1=11.0$, $k_2=0.026$.

	$\varepsilon=0°$		$\varepsilon=20°$		$\varepsilon=30°$		$\varepsilon=40°$		$\varepsilon=50°$	
	$I=0$	$I=I_M$	$I=0$	$I=I_M$	$I=0$	$I=I_M$	$I=0$	$I=I_M$	$I=0$	$I=I_M$
W_{min}, MeV	3.57	3.57	1.33	1.33	0.88	0.93	0.58	0.69	0.28	0.51
E_m, kv/cm	7.4	7.4	12.3	12.3	15.1	14.8	18.7	16.3	36.2	17.5

As the velocity of particles grows the lens displacement decreases to become unnecessary at an energy of 10 or 15 MeV. This decrease results in a better field efficiency. The value of ε can be lessened smoothly down to 20° after which it is more desirable to turn to a one gap per accelerating period structure ($\mathcal{E}$ =0). To avoid a lowering of the upper limit of E_m it is better to use gradually weakening quadrupole lenses (with a decreasing æ).

Fig.2 illustrates an approximate variation of ε and æ as well as of their corresponding lower and upper limits of E_m for the input part of the accelerator beginning with W =0.75 MeV (β =0.04) up to W =12.25MeV (β =0.16). The curves are valid for any current between zero and with the above parameters.

It is concluded that the accelerating-field quadrupole focusing imposes directly no extra limitation on the beam current that is still limited by phase oscillation conditions alone. However, introduction of another gap lowers the efficiency of field utilization proportionally to cosε . In addition, gaps with a quadrupole geometry are more subjected to a breakdown so that the accelerating wave amplitude is limited by not so high a value. This results in a lower current limit as imposed by phase oscillation conditions.

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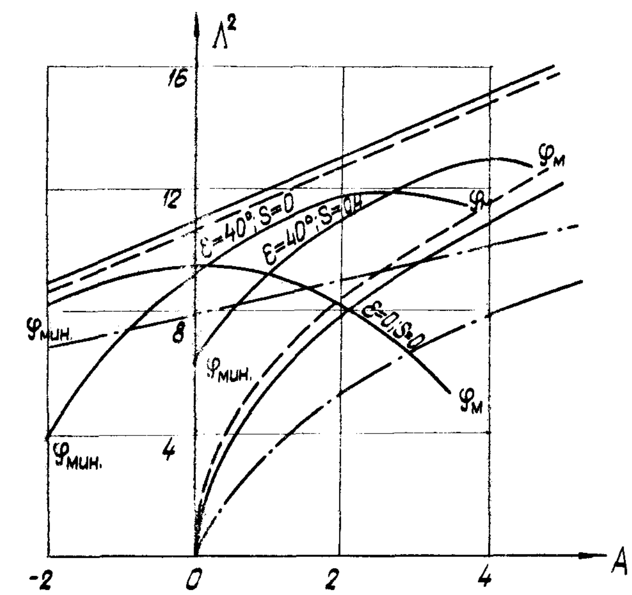

Fig.1.

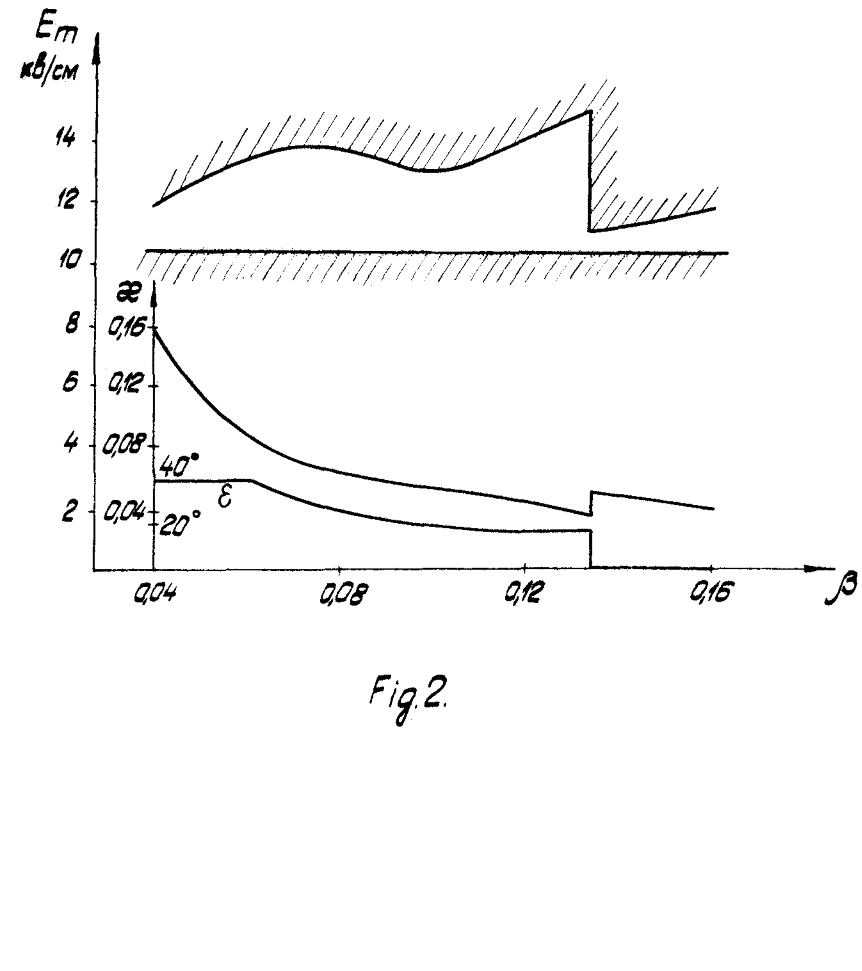

Fig.2.