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IMPEDANCES OF STRIPLINE BEAM-POSITION MONITORS

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A systematic discussion of the coupling impedances of stripline beam-position monitors is given. Termination either at both ends or at the center is treated. Special emphasis has been given to the transverse impedance, which is derived in more than one way.

1. INTRODUCTION

Beam-position monitors are required to measure the horizontal and vertical positions of a beam so that it can be guided through the central region of a beam pipe and circulate around the storage ring many many times. Since beam signals are registered at the terminations, the monitors must exhibit impedances to the beam. For a storage ring of very high energy, the large number of beam position monitors required can become a significant portion of the total impedance of the whole ring (aside from the rf contribution of electron machines). Therefore, an accurate estimate of the monitor impedances is necessary.

The beam monitors that will be discussed here are cylindrical stripline pickups, used primarily because the computations are usually much simpler than for rectangular-geometry monitors. However, in many cases, as, for example, in the Fermilab main ring, rectangular geometry is preferred over cylindrical. Our cylindrical results described below should provide at least a rough estimation for the rectangular counterpart.

2. THE CYLINDRICAL STRIPLINE MONITORS

Consider a pair of cylindrical stripline pickups[1,2] exposed to a short beam bunch as shown in Fig. 1. Each stripline has a length l and subtends an angle ϕ_0 to the transverse axis of the beam pipe. The stripline together with the extruded beam pipe behind it can be considered as a section of transmission line with a characteristic impedance $Z_s = \sqrt{L/C}$, where L and C are the inductance and capacitance per unit length. Any signal propagating along this section of transmission line will have a velocity $\beta_s c = 1/\sqrt{LC}$. Each end of the stripline is

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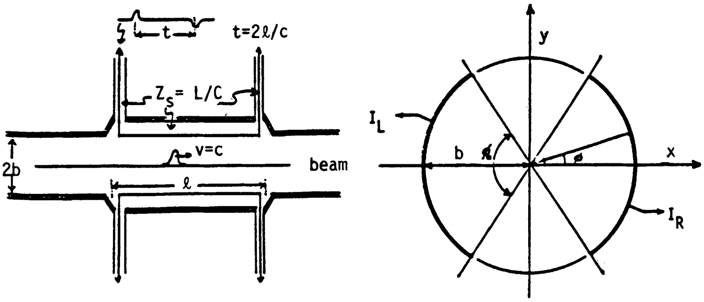

FIGURE 1 Geometry of the cylindrical stripline monitor.

attached via a port to a transmission line of the same characteristic impedance. Hence, any signal induced on the stripline will propagate through one of the ports into a transmission line without reflections. This is equivalent to terminating each end of the stripline by a resistance of Z_s as in Fig. 2.

When a beam bunch of time distribution $I(t)$ and velocity $\beta_p c$ traveling along the axis of the beam pipe crosses the first or upstream port, the image current on the walls of the pipe sees an impedance of $Z_s/2$, representing the parallel impedances of the upstream termination Z_s and the transmission line formed by the stripline, which, since terminated at the far end by Z_s, also has impedance Z_s. In other words, the image current splits into two equal parts, one traveling through the upstream termination where it is detected and the other half traveling along the stripling and ending up going through the downstream termination at a time $l/\beta_s c$ later.

When the bunched beam passes the downstream port, the same thing happens but the polarity reverses. One half of the signal travels through the downstream termination while the other half propagates up the stripline to be collected by the upstream termination. Thus, the voltage across the upstream termination is

$$V_u(t) = \frac{Z_s}{2}\left(\frac{\phi_0}{2\pi}\right)\left[I(t) - I\left(t - \frac{l}{\beta_p c} - \frac{l}{\beta_s c}\right)\right], \tag{1}$$

and that across the downstream termination is

$$V_d(t) = \frac{Z_s}{2}\left(\frac{\phi_0}{2\pi}\right)\left[I\left(t - \frac{l}{\beta_s c}\right) - I\left(t - \frac{l}{\beta_p c}\right)\right]. \tag{2}$$

the factor $\phi_0/2\pi$ represents the fraction of the image current that flows across the ports along the stripline.

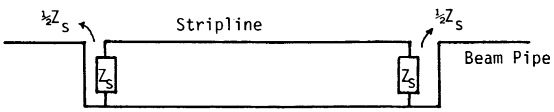

FIGURE 2 A stripline forming a transmission line with the beam pipe having characteristic impedance Z_s and terminated at both ends by Z_s.

The net signal seen at the upstream port is a bipolar doublet with each lobe having essentially the same time distribution as the beam bunch itself but speärated by a constant time of $(l/\beta_p c + l/\beta_s c)$. However, the signal at the donwnstream port will be completely cancelled if the beam velocity and the signal velocity are the same. This is in fact the situation, since the signal velocity in a transmission line with a free space medium is exactly c and the beam particle velocity is also very close to c. For simplicity, we shall set $\beta_p = \beta_s = 1$ below and forget the downstream port.

3. LONGITUDINAL COUPLING IMPEDANCE

In the frequency domain, the beam has at frequency $\omega/2\pi$ a current $I(t) = I_0 e^{j\omega t}$ at the upstream port. Note that I_0 is in general complex but is time-independent. The voltage across the upstream port becomes

$$V_u(\omega) = \frac{Z_s}{2}\left(\frac{\phi_0}{2\pi}\right) I_0 (1 - e^{-j2\omega l/c}). \tag{3}$$

This also happens to be the potential difference across the gap at the upstream end. The total image current in the walls of the beam pipe is of course $-I_0$. but only a fraction $\phi_0/2\pi$ will see this potential difference while the rest simply flows through without meeting any impedance. As a result, the *average* potential seen by the particle beam is

$$V_b(\omega) = \left(\frac{\phi_0}{2\pi}\right) V_u(\omega). \tag{4}$$

The longitudinal impedance for *one* strip plate is therefore $(Z_\parallel)_{\mathrm{BPM}} = V_b(\omega)/I_0$, or

$$(Z_\parallel)_{\mathrm{BPM}} = Z_s\left(\frac{\phi_0}{2\pi}\right)^2 \left(\sin^2\frac{\omega l}{c} + j \sin\frac{\omega l}{c}\cos\frac{\omega l}{c}\right). \tag{5}$$

The same impedance can be computed using Eq. (3) in another way. The average real power dissipated in the upstream termination is

$$P(\omega) = \frac{|V_u(\omega)|^2}{2Z_s}. \tag{6}$$

This is by definition equal to $\frac{1}{2}|I_0|^2 \operatorname{Re}(Z_\parallel)_{\mathrm{BPM}}$, which results in the same $\operatorname{Re}(Z_\parallel)_{\mathrm{BPM}}$ as Eq. (5). The imaginary part can be obtained using a Hilbert transformation and is left as an exercise for the reader. However, in this situation, $\operatorname{Re} Z_\parallel(\omega)$ does not vanish at infinity, and we have to work with $\operatorname{Re} Z_\parallel(\omega)/\omega$ instead. In using a Hilbert transformation, one has to keep in mind that we may not be getting a unique result. This is because a frequency-independent real impedance need not have an imaginary counterpart. Also the *ideal* inductance or capacitance that gives rise to $\operatorname{Im} Z_\parallel = \omega L$ or $-1/\omega C$ need not have a real counterpart. In other words, to the $\operatorname{Im} Z_\parallel$ obtained from $\operatorname{Re} Z_\parallel$ through a Hilbert transform, we can add any *ideal* inductive or capacitive terms.

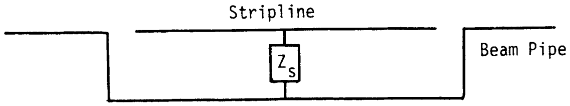

FIGURE 3 A stripline forming a transmission line with the beam pipe having characteristic impedance Z_s and terminated at the center by Z_s.

On the other hand, to the $\operatorname{Re} Z_{\|}$ obtained from $\operatorname{Im} Z_{\|}$ through a Hilbert transform, we can add any pure frequency-independent resistive term.

Note that the longitudinal impedance starts out as inductive at low frequencies and, after $\omega = \pi c/2l$, alternates between capacitive and inductive. However, there are no sharp resonances to the degree that the stripline is match-terminated at both ends.

There are some striplines, like those in the Fermilab main ring, that have only one termination at the center, which is chosen to be the same as the characteristic impedance formed by the stripline and the beam pipe (see Fig. 3). The impedance seen at each end is Z_s in parallel with an open transmission line of characteristic impedance of Z_s. At zero frequency, the impedance seen is therefore just Z_s, and the currents from each end of the stripline will be absorbed totally without any reflection. When the frequency is low, keeping the lowest reactive term, the impedance seen at either end of such a stripline is

$$Z_i = Z_s\left(1 - \frac{j\omega l}{2c}\right). \tag{7}$$

Here, we have set the particle velocity and the signal velocity along the stripline to be equal to c. The voltages V_u and V_d seen by the image current while crossing the upstream end and the downstream end are, respectively,

$$V_u = Z_i\left(\frac{\phi_0}{2\pi}\right)I(t) \tag{8}$$

and

$$V_d = -Z_i\left(\frac{\phi_0}{2\pi}\right)I(t - l/c). \tag{9}$$

The *average* voltage seen by the beam is therefore

$$V = Z_i\left(\frac{\phi_0}{2\pi}\right)^2 [I(t) - I(t - l/c)]. \tag{10}$$

The extra factor of $(\phi_0/2\pi)$ comes in because of the partial angular coverage process mentioned above. Putting in $I(t) = I_0 e^{j\omega t}$, we obtain the logitudinal impedance,

$$(Z_{\|})_{\mathrm{BPM}} = Z_s\left(\frac{\phi_0}{2\pi}\right)^2\left(1 - \frac{j\omega l}{c}\right)(1 - e^{-j\omega l/c}) \tag{11}$$

for one stripline terminated at the center at low frequencies. At high frequencies, the stripline can accept resonances with standing waves having a node at the middle, where the termination does not absorb any power. Such resonances will therefore not be dampled.[3] Resonances will occur whenever the length of the stripline l is a half-integral multiple of the wavelength.

4. TRANSVERSE COUPLING IMPEDANCE

Let us turn to the problem of transverse coupling impedance. Assume a dipole current source separated by Δ, i.e., I at $x = \Delta/2$ and $-I$ at $x = -\Delta/2$. Here, we assume that the pair of cylindrical striplines are positioned horizontally as in Fig. 1. Note that both currents have the same t and z dependence, such as $e^{j\omega(t-z/\beta c)}$, so that both are traveling in the same direction. Therefore only the upstream termination will see a signal (for striplines match-terminated at both ends).

When a current I_0 deviates from the pipe axis by an amount $x_0 = \xi b$, the image-surface current density at angle θ is

$$J(\theta; x_0) = -\frac{I_0}{2\pi b}\frac{1-\xi^2}{1+\xi^2-2\xi\cos\theta}. \tag{12}$$

This is obtained by the method of inversion by placing a current $-I_0$ at the point $x_1 = b^2/x_0$ as shown in Fig. 4. Then the beam-pipe cylinder is an equipotential surface (using the analogy of a line charge). The image-surface current density on the cylinder at angle θ can thus be computed directly from the two current sources at x_0 and x_1:

$$J(\theta; x_0) = \frac{I_0}{2\pi}\left(\frac{\cos\theta_0}{r_0} - \frac{\cos\theta_1}{r_1}\right), \tag{13}$$

where r_0 and r_1 are distances from the point of observation to the current and its image, respectively. Using the fact that $r_1 = br_0/x_0$, Eq. (13) leads to Eq. (12).

For our dipole current, the image-wall current density can be obtained from Eq. (12) by differentiation with respect to x_0. Since we are interested only in the dipole term, $J(\theta; x_0)$ can be expanded to give

$$J(\theta; x_0) = -\frac{I_0\Delta\cos\theta}{\pi b^2}. \tag{14}$$

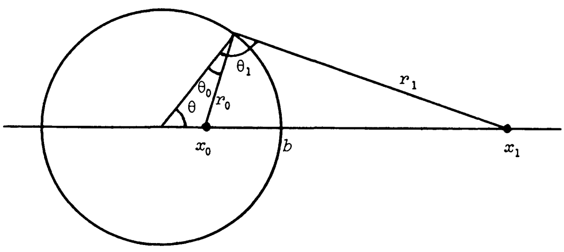

FIGURE 4 Computation of current density on cylinder by the method of inversion.

The current flowing into the right stripline system is, therefore,

$$I_R = \int_{-\phi_0/2}^{\phi_0/2} -\frac{I_0 \Delta \cos\theta}{\pi b^2} b\, d\theta, \tag{15}$$

and the current flowing into the left stripline system is

$$I_L = -I_R. \tag{16}$$

Using Eq. (3), the voltages at the right and left upstream gaps are, respectively,

$$V_R = Z_s \frac{I_0 \Delta}{\pi b} \sin\frac{\phi_0}{2}(1 - e^{-j2\omega l/c}),$$

$$V_L = -V_R. \tag{17}$$

The total average power dissipated is

$$P = \frac{1}{2Z_s}(|V_L|^2 + |V_R|^2), \tag{18}$$

or

$$P = 4Z_s\left(\frac{|I_0|\,\Delta}{\pi b}\right)^2 \sin^2\frac{\phi_0}{2}\sin^2\frac{\omega l}{c}. \tag{19}$$

There is a relation between the power dissipated and the real part of the transverse impedance derived by Nassibian and Sacherer,[4] which we are going to repeat here. For a length l of current loop, the interaction with the beam detector will give a magnetic field B_y through the loop so a back emf will be induced. Equivalently, the current in the loop will see an impedance Z given by

$$j\omega B_y l \Delta = Z I_0. \tag{20}$$

Substituting into the definition of transverse impedance, which is

$$Z_\perp(\omega) = \frac{1}{jI_0 \Delta \beta_p} \int_0^L dz[(\mathbf{E} + (\mathbf{v}_p \times \mathbf{B})]_\perp, \tag{21}$$

$v_p = \beta_p c$ being the particle velocity, gives a horizontal transverse impedance,

$$Z_\perp = \frac{cZ}{\omega \Delta^2}. \tag{22}$$

The power dissipated is $P = \frac{1}{2}|I_0|^2 \operatorname{Re} Z$, and we therefore get

$$P = \frac{1}{2}\frac{\omega}{c}(I_0\Delta)^2 \operatorname{Re} Z_\perp. \tag{23}$$

Thus, for a pair of striplines,

$$\operatorname{Re}(Z_\perp)_{\mathrm{BPM}} = \frac{8Z_s}{\pi^2 b^2}\frac{c}{\omega}\sin^2\frac{\phi_0}{2}\sin^2\frac{\omega l}{c}, \tag{24}$$

which has exactly the same frequency dependence as $\operatorname{Re}(Z_\parallel)_{\mathrm{BPM}}/\omega$. The imaginary part can be found by a Hilbert transform; it should have exactly the same frequency dependence as $\text{Im}\,(Z_{\|})_{\text{BPM}}/\omega$. Therefore, for a pair of striplines,

$$(Z_{\perp})_{\text{BPM}} = \frac{c}{b^2}\left(\frac{4}{\phi_0}\right)^2\left(\sin^2\frac{\phi_0}{2}\right)\left[\frac{(Z_{\|})_{\text{BPM}}}{\omega}\right] \qquad \perp \text{ to striplines}, \tag{25}$$

or the x-direction here. In Eq. (25), $(Z_{\|})_{\text{BPM}}$ is the longitudinal coupling impedance of a pair consisting of *two* striplines.

To compute the transverse impedance in the y-direction, we put the currents I at $y = \Delta/2$ and $-I$ at $u = -\Delta/2$ instead. Then the current flowing into the right stripline system is

$$I_{\text{R}} = \int_{-\phi_0/2}^{\phi_0/2} -\frac{I_0\Delta\cos(\pi/2+\theta)}{2\pi b^2}\, b\, d\theta, \tag{26}$$

which is identical to zero, and so is I_{L}. The voltages across each upstream gap are therefore also zero. Thus,

$$(Z_{\perp})_{\text{BPM}} = 0 \qquad \| \text{ to striplines}, \tag{27}$$

or the y-direction here. The reason is clear, because $(Z_{\perp})_{\text{BPM}}$ depends on the voltages across the stripline gaps, which, in turn, depend only on the *total* currents flowing across each gap but not on the actual distribution of the current density. However, a current dipole at $y = \pm\Delta/2$ will only produce an image current distribution that is antisymmetric with respect to the x-axis while the *total* currents crossing each gap are zero.

5. APPLICATION TO THE SSC

In the SSC, the most demanding requirements for the beam-position monitors are in the commissioning stage, when very low beam currents must be used in order not to quench the magnets. It appears that both coordinates must be measured at every quadrupole (one per half-cell); so the pickups must be a four-electrode design shown in Fig. 5. We take[5] $\phi_0 = 55°$, $l = 10$ cm, and $Z_s = 50$ ohms. The pipe radius is $b = 1.65$ cm. At low frequenceies, $\omega/2\pi \ll c/4l = 750$ MHz, the longitudinal impedance per harmonic per monitor (*four* striplines) is

$$\left(\frac{Z_{\|}}{n}\right)_{\text{BPM}} \simeq jZ_s\left(\frac{\phi_0}{\pi}\right)^2\frac{l}{R} = j3.53\times10^{-5}\,\Omega, \tag{28}$$

while the transverse impedance (only *two* striplines are contributing, either in the x-direction or the y-direction) is

$$(Z_{\perp})_{\text{BPM}} \simeq j\frac{8Z_s l}{\pi^2 b^2}\sin^2\frac{\phi_0}{2} = j3.17\text{ k}\Omega/\text{m}. \tag{29}$$

With almost 900 sets of monitors in each ring, the impedances are $(Z_{\|}n)_{\text{BPM}} \sim j0.0318\,\Omega$ and $(Z_{\perp})_{\text{BPM}} \sim j2.86\,\text{M}\Omega/\text{m}$ for either the horizontal or the vertical direction.

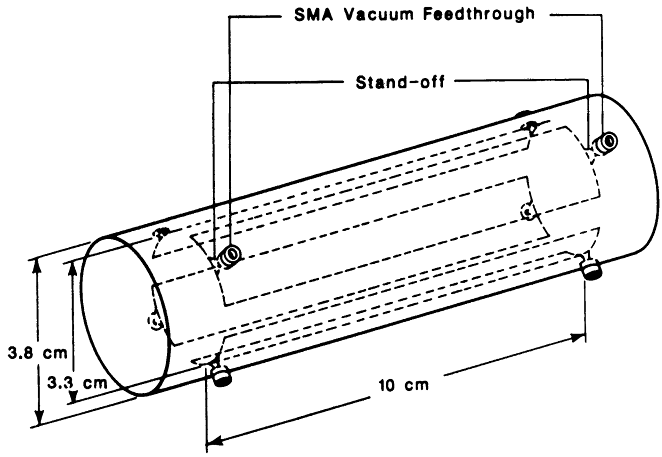

FIGURE 5 The four-electrode stripline monitor of the SSC.

6. OFF-CENTERED BEAM AND IMPEDANCES

We can also compute the longitudinal impedance of a beam passing through a pair of striplies as shown in Fig. 1 but deviating from the central axis by x_0 horizontally and y_0 vertically. Using Eq. (12), the total image currents flowing across the right and left upstream gaps are

$$I_{\mathrm{R,L}} = -I_0\left(\frac{\phi_0}{2\pi}\right)\left[1 \pm \frac{4x_0}{b}\frac{\sin(\phi_0/2)}{\phi_0} + \frac{2(x_0^2 - y_0^2)}{b^2}\sin\phi_0 + \cdots\right]. \tag{30}$$

Note that the second term is the dipole term, and that we have included a third term that is of a higher multipole. In the present situation, it is not so simple to compute the "average" voltage seen by the particle beam because the beam has been displaced and the distribution of the image current is no longer uniform. As a result, we shall compute the power dissipated at the upstream terminations and infer the real part of the longitudinal impedance. This power is proportional to the sum of $|I_{\mathrm{R}}|^2$ and $|I_{\mathrm{L}}|^2$ and is given by

$$P_{\mathrm{disp}} = 2G(x_0, y_0)P, \tag{31}$$

where P is the power dissipated through *one* stripline when the beam is at the central axis of the beam pipe as given by Eq. (6), and the function

$$G(x_0, y_0) = 1 + \left(\frac{x_0}{b}\right)^2\left(\frac{4}{\phi_0}\right)^2\sin^2\frac{\phi_0}{2} + \left(\frac{x_0^2 - y_0^2}{b^2}\right)\left(\frac{4}{\phi_0}\right)\sin\phi_0 \tag{32}$$

takes care of the fact that the beam is displaced. The longitudinal impedance of a pair of striplines is therefore equal to $G(x_0, y_0)$ times the impedance of an undisplaced beam. Note that the second term of $G(x_0, y_0)$ comes from the square of the dipole term while the third term is from the higher multipole. Here, the term linear in x_0 or the linear dipole term cancels out. This is in fact exactly what we expect. In order to measure the deviation of the beam from the central axis, one should measure the difference between the left and right terminations but not the sum; then the linear dipole term will emerge.

The longitudinal coupling impedance of a displaced beam provides a way to compute the transverse coupling impedances.[4] This method is very useful, so we shall derive it here. Let us concentrate on displacement in the x-direction only. We write $E_z(x, x_0)$ as the image electric field at x due to a current I_0 at x_0. For a length l, define

$$Z_{\parallel}(x, x_0) = \frac{lE_z(x, x_0)}{I_0}, \tag{33}$$

which is measurable quantity and reduces to the usual longitudinal impedance $Z_{\parallel}(x_0)$ at x_0 in the limit $x \to x_0$.

The image electric field at x due to a dipole current at x_0 separated by Δ is

$$E_z' = \frac{\partial E_z(x, x_0)}{\partial x_0}\Delta, \tag{34}$$

and the magnetic field at x perpendicular to the plane of the dipole is, by Faraday's law,

$$B_y' = \frac{\Delta}{j\omega}\frac{\partial^2 E_z(x, x_0)}{\partial x\,\partial x_0}. \tag{35}$$

Substituting into the definition of $Z_\perp$ in Eq. (21), the horizontal transverse impedance displaced by x_0 is

$$Z_\perp = \frac{c}{\omega}\frac{\partial^2 Z_{\parallel}(x, x_0)}{\partial x\,\partial x_0}\bigg|_{x=x_0}. \tag{36}$$

In many cases, the longitudinal impedance at (x_0, y_0) has the form

$$Z_{\parallel}(x_0, y_0) = [1 + F_x^2(x_0) + F_y^2(y_0)]Z_{\parallel}|_{x_0=0, y_0=0}, \tag{37}$$

so $Z_\perp$ can be obtained directly from the position dependence of $Z_{\parallel}$. In that case, the horizontal transverse impedance is

$$Z_\perp = \frac{c}{\omega}\left(\frac{dF_x}{dx_0}\right)^2 Z_{\parallel}|_{x_0=0, y_0=0}. \tag{38}$$

However, one should be careful that only dipole contributions should be included in F_x and F_y since the dipole impedances cannot receive contributions from other multipoles. Thus, in our situation, the third term of Eq. (32) must be omitted to obtain the square-bracketed term in Eq. (37). Then substitution into Eq. (37) will lead to exactly the same transverse impedances we obtained earlier in Eqs. (25) and (27). For striplines terminated at the center, the off-centered factor $G(x_0, y_0)$ is the same. Theorefore, the transverse impedances can also be obtained through Eq. (38).

In Ref. 1, the third or higher multipole term of $G(x_0, y_0)$ has been included when the differentiation with respect to x_0 or y_0 is carried out. As a result, both the horizontal and transverse impedances quoted there are not correct. In particular, for positive frequencies, the real part of the vertical transverse impedance quoted is negative, which is not possible. The transverse impedance is related to and has the same sign as the longitudinal impedance of the dipole mode when ω is positive.[6] The longitudinal impedance for each multipole must be positive because it is responsible for the power dissipation of a beam particle in that multipole mode.

7. HIGH-FREQUENCY LIMITS

The impedances derived above cannot be applied to the high-frequency region without reservation. The high-frequency limitations have been discussed by Shafer[2] and Cuperus.[7] Here, we just mention them briefly.

So far we have assumed that an image of a point charge in the wall of the beam pipe is also a charge of zero duration. This is in fact not true because the electric-field lines have a longitudinal spread. As a result, the wall current at frequency $\omega/2\pi$ is reduced. The 3-dB point occurs when

$$\omega = \frac{1.2\beta_p\gamma c}{b}, \tag{39}$$

where $\beta_p c$ is the particle veolicty and $\gamma = \sqrt{1-\beta_p^2}$.

The second limitation is the finite transit time across the upstream and downstream port gaps. For a centered beam, the signals from stripline pickups should include the additional factor

$$H(\omega) = \frac{\sin(\omega g/2\beta_p c)}{\omega g/2\beta_p c} \frac{1}{I_0(\omega b/\beta_p\gamma c)}, \tag{40}$$

where g is the port gap.

Also there is the problem of cutoff frequencies. When the thresholds for the lowest-order TE and TM modes are reached, the field structures between the striplines can be altered. At present, this problem has not yet been fully understood.

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