Synchrotron oscillation damping by beam-beam collisions in DAΦNE

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In DAΦNE, the Frascati e^+/e^- collider, the crab waist collision scheme has been successfully implemented and tested during the years 2008 and 2009. During operations for the Siddharta experiment an unusual synchrotron damping effect induced by beam-beam collisions has been observed. Indeed, the positron beam becomes unstable above currents in the order of 200–300 mA when the longitudinal feedback is off. The longitudinal instability is damped by colliding the positron beam with a high current electron beam (~ 2 A) and a shift of ≈ -600 Hz in the residual synchrotron sidebands is observed. Precise measurements have been performed by using both a commercial spectrum analyzer and the diagnostic capabilities of the DAΦNE longitudinal bunch-by-bunch feedback. This damping effect has been observed in DAΦNE for the first time during collisions with the crab waist scheme. Our explanation is that beam collisions with a large crossing angle produce longitudinal tune shift and spread, providing Landau damping of synchrotron oscillations.

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I. INTRODUCTION

DAΦNE, the Φ-factory built at Frascati in the years 1991–1996 [1,2] and running since 1997, consists of a linac, which accelerates electrons up to $\sim$700 MeV or positrons up to 510 MeV, an accumulator-damping ring, transfer lines, and two main rings (MR) with one or two interaction points for collisions at 1.02 GeV in the center of mass. The linac, accumulator ring, and transfer line can be set to inject a positron or an electron bunch every $\frac{1}{2}$ second. In the typical injection scheme, the electron bunches are stored in the MR before the positron ones because the electron injection rate is higher and the beam lifetime rather short (less than half hour). Electrons are therefore injected up to ≈ 2 A, then the injection system is switched to positron mode in ≈ 1 minute, and then positrons are injected. The electron beam current decays rather rapidly, due to the low beam energy (510 MeV) and small transverse emittance, and finally the two beams collide at approximately the same currents, starting usually in the range between 1 and 1.5 A.

After electron injection and during the transfer line switch, there are beam collisions with very high electron currents (between 2 and 1.5 A) and relatively low positron ones (between 500 and 200 mA). In this particular situation, a longitudinal damping of the positron beam has been observed even with the longitudinal bunch-by-bunch positron feedback turned off. This damping effect has been observed in DAΦNE for the first time during collisions with the crab waist scheme [3,4], implemented in 2008–2009. After the first observations of this behavior, three dedicated machine studies have been carried out with the goal of precisely measuring the features of this effect [5]. Here we describe their results, we obtain an analytical formula for the longitudinal beam-beam tune shift evaluation, and compare the measured tune shift with analytical estimates and numerical calculations.

II. MEASUREMENTS DESCRIPTION

In order to perform the measurements in the positron MR, we have used and compared two different diagnostic tools: a commercial real-time spectrum analyzer RSA3303A by Tektronix, working from DC to 3 GHz, connected to a high bandwidth beam pickup and the longitudinal bunch-by-bunch feedback with its beam diagnostic capability both in real time and off-line which can be used both in closed and open loop.

Figure 1 shows the screen of the spectrum analyzer with the 118th revolution harmonic (highest peak at 362.484 MHz) of the positron beam together with the synchrotron sidebands at 35 ± 1 kHz distance. The e^+ longitudinal feedback is off (i.e. in open loop) and the total beam current is $I^+ \cong 130$ mA in 103 bunches. The upper part of the screen is the frequency spectrum at a time corresponding to the baseline of the lower part, which shows the time evolution of the strongest frequency lines (time flows from up to down).

The following plot (Fig. 2) shows again the positron beam with the longitudinal feedback off in collision with

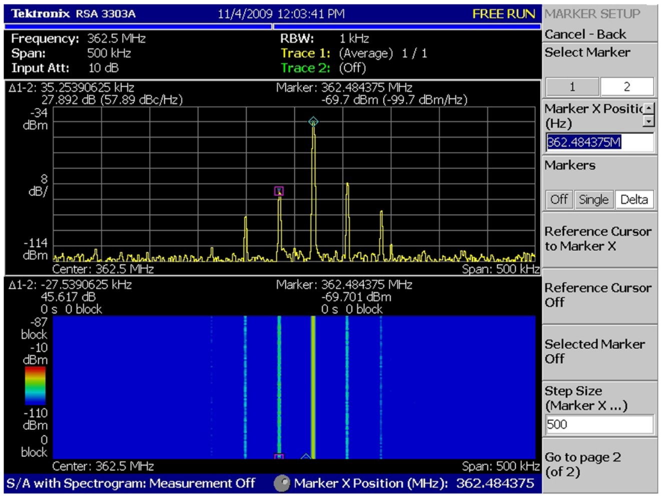

FIG. 1. Positron 118th rf harmonic with synchrotron sidebands.

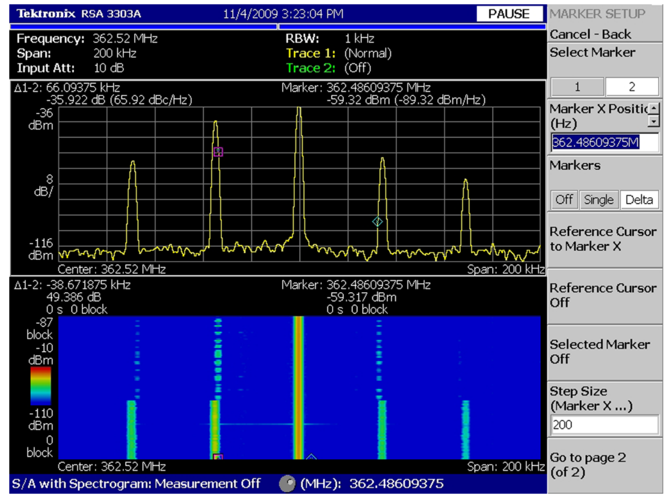

FIG. 3. Positron spectrum in collision (before) and off collision (after).

the electron beam at a current of ~1700 mA and all its feedbacks on (i.e. in closed loop): the e^+ synchrotron sidebands are almost completely damped.

Figure 3 shows the difference between the tunes with and without collisions, obtained by rapidly separating the beams at the crossing point, with the same setup as the previous ones, namely, with the e^+ longitudinal feedback off and all the others on. A frequency shift of the order of -1 kHz in the sidebands is clearly visible but the resolution of the instrument is not accurate enough to exactly measure this shift. It is evident that the beam-beam collisions induce a damping effect and decrease the synchrotron frequency on both sidebands. In the case of Fig. 3 the total currents are 1550 mA for the electrons and 390 mA for the positrons.

Downloading new e^+ power spectrum traces from RSA 3303A, transferring them to a PC/MATLAB environment

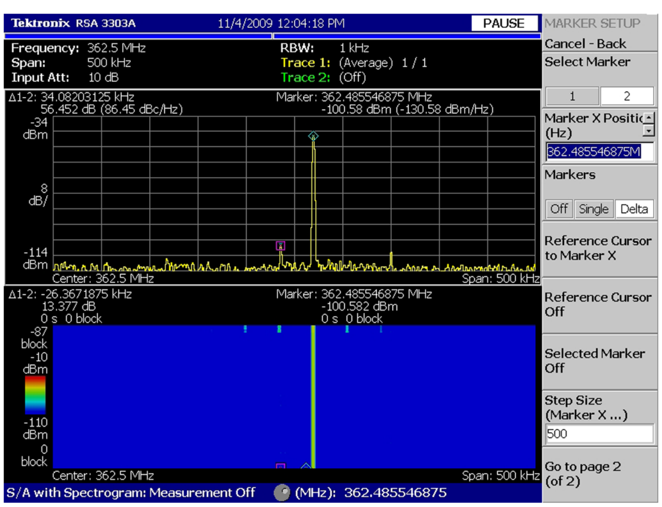

FIG. 2. Synchrotron sidebands damped by beam-beam collisions.

and zooming the plots, the following Fig. 4 has been created.

In this figure the highest peak is the e^+ 118th harmonic; the red trace shows the positron spectrum without collision, while the blue one corresponds to the case of colliding beams. The vertical scale is in dBm, the horizontal axis is the number of bins (proportional to the frequency, with 1 bin $\approx$ 86 Hz). This case is interesting because it shows a situation where the electron beam damps longitudinally and shifts in frequency the synchrotron oscillation of the positron one. Moreover, it is possible to see that the beam collisions produce also a horizontal betatron tune shift.

With the goal to confirm the measurements done with the spectrum analyzer and to evaluate more precisely the effect, the beam diagnostic tools of the DAΦNE longitudinal feedback, developed in collaboration with SLAC and

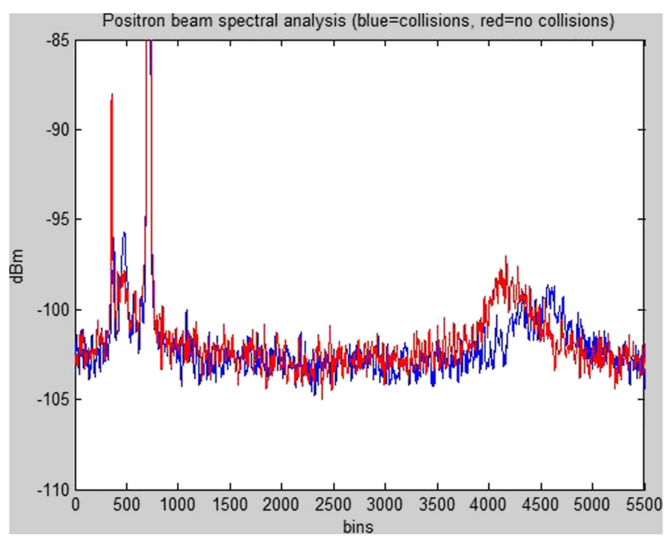

FIG. 4. Positron spectrum in collision (blue) and off collision (red), showing the longitudinal and horizontal tunes.

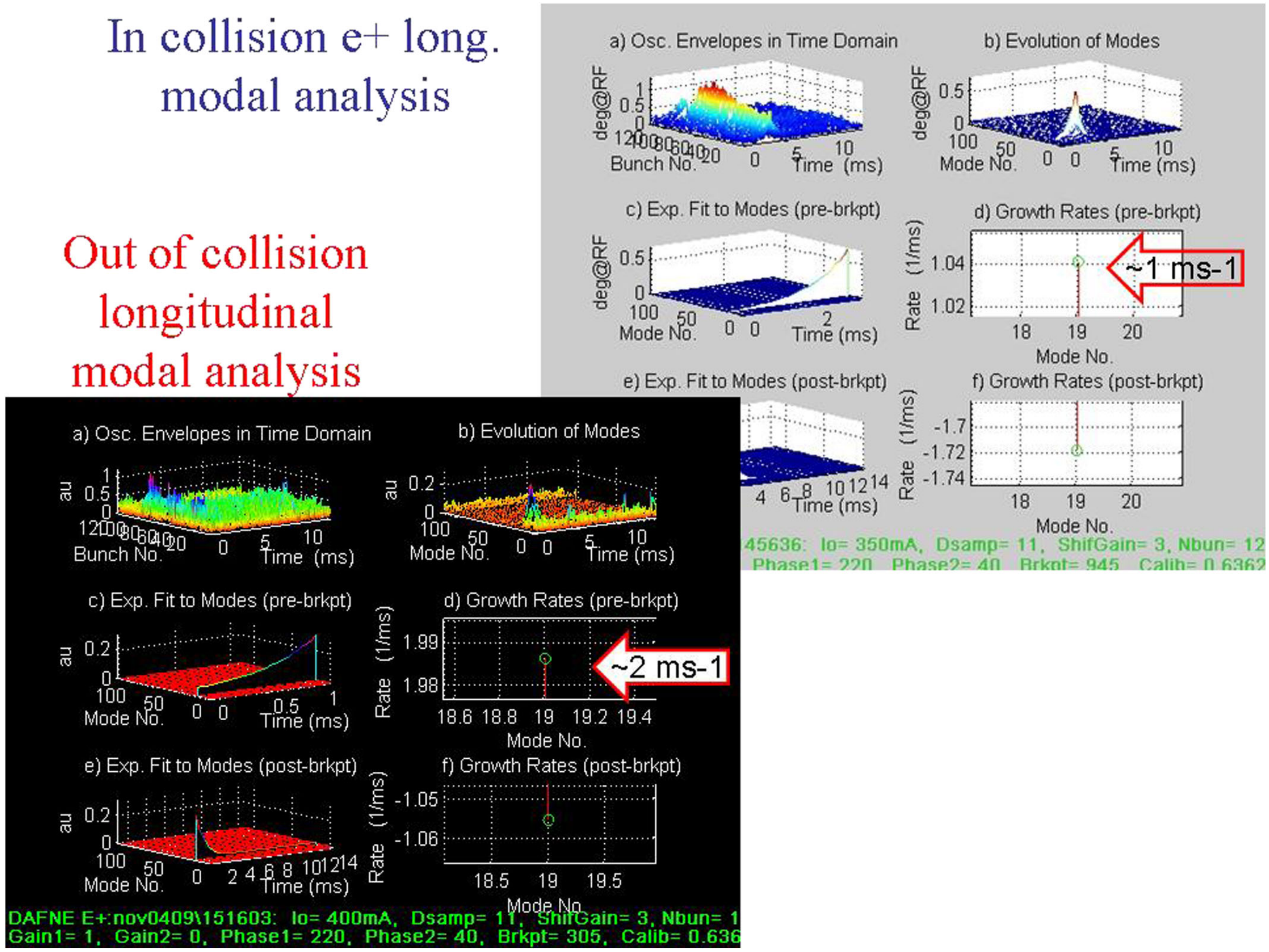

FIG. 5. The growth rate of mode 19 out of collision is 1.99 ms^{-1}, in collision 1.04 ms^{-1}.

LBL, have been used [6–8]. With this system it is possible to record longitudinal data separately for each bunch. Data can be recorded both in closed and in open loops.

Figure 5 shows the modal growth rate analysis with and without collisions, turning off for a short time the bunch-by-bunch feedback.

In both cases the 19th mode is the strongest unstable longitudinal mode; without collisions it has a growth rate, in inverse units, of 1.99 ms^{-1}, (corresponding to 502 microseconds), while in collision the growth rate is almost halved (1.04 ms^{-1}, corresponding to 961 microseconds). This, once again, confirms the damping effect of beam-beam interaction.

Analyzing these data in detail, it is possible to measure the synchrotron frequency shift induced on the positron beam by the beam-beam collisions with the e^+ longitudinal feedback off. As shown in Figs. 6 and 7, the synchrotron frequency without collisions is 34.86 kHz, dropping to 34.23 kHz in collision. The frequency shift induced by the beam-beam collisions is therefore -630 Hz at a beam

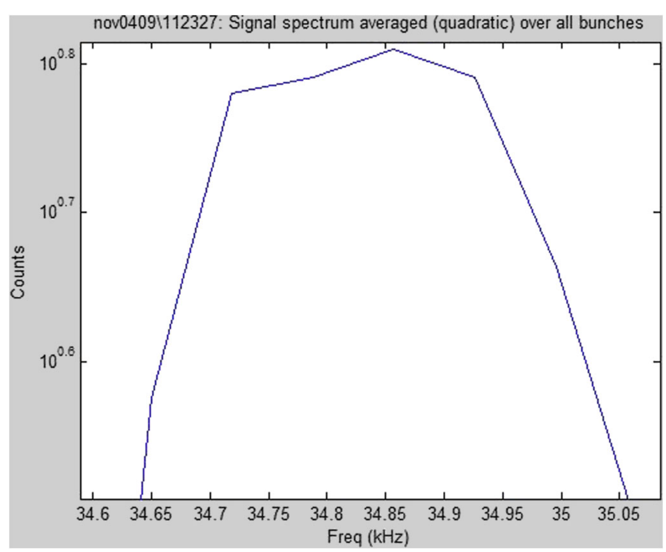

FIG. 6. The average synchrotron frequency (off collision) is 34.86 kHz.

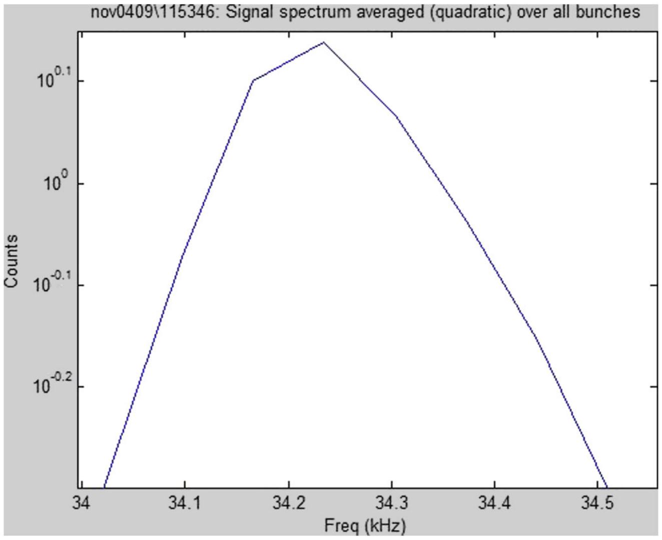

FIG. 7. The average synchrotron frequency (in collision) is 34.23 kHz.

current of ~285 mA (in and out) for the positrons, while the e^- beam currents were ~500 mA (in and out). In both rings 100 bunches were stored contiguously, so the bunch currents were respectively ~2.85 mA for e^+ (corresponding to 5.8×10^9 particles) and ~5 mA for e^- (corresponding to 1.0×10^{10} particles).

III. ANALYTICAL FORMULA AND COMPARISON WITH NUMERICAL SIMULATIONS

Summarizing, experimental observations and measurements at DAΦNE have shown that beam-beam collisions can damp the longitudinal coupled bunch instability. Colliding with a high current electron beam, the synchrotron oscillations of an unstable positron one were stabilized, even with the longitudinal feedback system off. Moreover, a negative frequency shift of positron beam synchrotron sidebands has been observed.

The authors explain these two effects with a nonlinear longitudinal kick arising from beam-beam interaction under a large crossing angle, namely, when the length of the overlap region of the two beams is shorter than the bunch length. It is worthwhile remarking that we observed this effect clearly only after the implementation of the crab waist scheme of beam-beam collisions at DAΦNE, which exploits a twice larger horizontal crossing angle with respect to the previous standard collision scheme [9].

In the following, an analytical expression for the synchrotron tune shift is obtained [10], giving also an evaluation of the synchrotron tune spread. The formula results are also compared with numerical simulations.

A. Tune shift analytical formula

In collisions with a crossing angle, the longitudinal kick of a test particle is created by a projection of the transverse electromagnetic fields of the opposite beam onto the longitudinal axis of the particle motion. The kicks that the test particle receives while passing through the strong opposite bunch with rms sizes σ_x, σ_y, σ_z under a horizontal crossing angle θ are [11]

$$\begin{aligned}
x' &= \frac{2r_eN}{\gamma}[x - ztg(\theta/2)] \\
&\quad\times\int_0^\infty dw \frac{\exp\{-\frac{[x-ztg(\theta/2)]^2}{2[\sigma_x^2+\sigma_z^2tg^2(\theta/2)]+w} - \frac{y^2}{(2\sigma_y^2+w)}\}}{\{2[\sigma_x^2+\sigma_z^2tg^2(\theta/2)]+w\}^{3/2}(2\sigma_y^2+w)^{1/2}} \\
y' &= \frac{2r_eN}{\gamma}y\int_0^\infty dw \frac{\exp\{-\frac{[x-ztg(\theta/2)]^2}{2[\sigma_x^2+\sigma_z^2tg^2(\theta/2)]+w} - \frac{y^2}{(2\sigma_y^2+w)}\}}{\{2[\sigma_x^2+\sigma_z^2tg^2(\theta/2)]+w\}^{1/2}(2\sigma_y^2+w)^{3/2}} \\
z' &= x'tg(\theta/2), \qquad (1)
\end{aligned}$$

where x, y, and z are, respectively, the horizontal, vertical, and longitudinal deviations from the synchronous particle traveling on axis. N is the number of particles in the strong bunch, γ is the relativistic factor of the weak beam. Then, for the on-axis test particle ($x = y = 0$) the longitudinal kick is given by

$$z' = -\frac{2r_eN}{\gamma}ztg^2(\theta/2) \times\int_0^\infty dw \frac{\exp\left\{-\frac{[ztg(\theta/2)]^2}{\{2[\sigma_x^2+\sigma_z^2tg^2(\theta/2)]+w\}}\right\}}{\{2[\sigma_x^2+\sigma_z^2tg^2(\theta/2)]+w\}^{3/2}(2\sigma_y^2+w)^{1/2}}. \qquad (2)$$

For small synchrotron oscillations $z \ll \sigma_z$, the exponential factor in the integral can be approximated by 1 and, taking into account that

$$\int_0^\infty dt \frac{1}{(a+t)^{3/2}(b+t)^{1/2}} = \frac{2}{a+\sqrt{a}\sqrt{b}}, \qquad (3)$$

we obtain an expression for the linearized longitudinal kick:

$$z' = -\frac{2r_eN}{\gamma}ztg^2(\theta/2) \times\frac{1}{[\sigma_x^2+\sigma_z^2tg^2(\theta/2)]+\sqrt{[\sigma_x^2+\sigma_z^2tg^2(\theta/2)]\sigma_y^2}}. \qquad (4)$$

Then, similarly to the transverse cases, we can write the expression for the synchrotron tune shift:

$$\xi_z = -\frac{r_eN}{2\pi\gamma}\beta_z \times\frac{tg^2(\theta/2)}{\{[\sigma_x^2+\sigma_z^2tg^2(\theta/2)]+\sigma_y\sqrt{[\sigma_x^2+\sigma_z^2tg^2(\theta/2)]}\}}. \qquad (5)$$

Remembering that the longitudinal beta function can be written as

$$\beta_z = \frac{c|\eta|}{\nu_{z0}\omega_0} = \frac{\sigma_{z0}}{(\sigma_E/E)_0}, \tag{6}$$

c being the velocity of light, η the slippage factor, ν_{z0} the unperturbed synchrotron frequency, and ω_0 the angular revolution frequency, we obtain the final expression for the linear tune shift:

$$\xi_z = -\frac{r_e N^{\text{strong}}}{2\pi\gamma^{\text{weak}}} \times \frac{(\frac{\sigma_{z0}}{\sigma_E/E})^{\text{weak}} tg^2(\theta/2)}{\left\{[\sigma_x^2 + \sigma_z^2 tg^2(\theta/2)] + \sigma_y\sqrt{[\sigma_x^2 + \sigma_z^2 tg^2(\theta/2)]}\right\}^{\text{strong}}}. \tag{7}$$

Here we have added notations "weak" and "strong" just to remark which beam parameters should be used in tune shift calculations. The minus sign in (5)–(7) reflects the fact that the longitudinal beam-beam fields acts against the rf voltage thus reducing the longitudinal focusing provided by the rf system.

For the case of flat beams with $[\sigma_y \ll \sqrt{\sigma_x^2 + \sigma_z^2 tg^2(\theta/2)}]$, the tune shift expression can be further simplified to

$$\xi_z = -\frac{r_e N^{\text{strong}}}{2\pi\gamma^{\text{weak}}} \frac{(\frac{\sigma_{z0}}{\sigma_E/E})^{\text{weak}}}{[(\frac{\sigma_x}{tg(\theta/2)})^2 + \sigma_z^2]^{\text{strong}}}. \tag{8}$$

As we see from (8), for flat bunches the synchrotron tune shift practically does not depend on the vertical beam parameters and so one should not expect any large variation due to crabbing and/or hourglass effects.

Since particles with very large synchrotron amplitudes practically do not "see" the opposite beam (except for a small fraction of the synchrotron period), their synchrotron frequencies remain very close to the unperturbed value ν_{z0}. For this reason, like in the transverse cases, the linear tune shift can be used as an estimate of the nonlinear tune spread. For the sake of completeness, we should mention that the longitudinal tune shift and spread can be also evaluated by exploiting another approach reported in [12]. However, in our opinion, that approach is much more complicated and less transparent in obtaining simple analytical formulas like our expression (7).

B. Numerical simulations

In order to check the validity of (7), we performed numerical simulations with the beam-beam code LIFETRAC [13]. The synchrotron and betatron tunes in the presence of beam-beam effects are calculated by tracking in the following way. First of all, a test particle is tracked for one turn with initial conditions:

$$X_i = \delta(i,j)\sigma_i q, \qquad i = 1, 2, \ldots, 6, \qquad q \ll 1,$$
$$\delta(i,j) = \begin{cases} 0, & \text{if } i \neq j \\ 1, & \text{if } i = j, \end{cases} \tag{9}$$

where X_i are the coordinates in the 6D phase space and σ_i are the corresponding rms sizes.

Repeating this 6 times for $j = 1, 2, \ldots, 6$ we obtain the 6×6 revolution matrix. Then the matrix eigenvalues are calculated, giving the tunes. For these simulations we use a simple model of a collider with linear transformations from IP to IP. In order to reproduce correctly the Gaussian longitudinal distribution, we divide a strong bunch in much more longitudinal slices than in ordinary beam-beam simulations. In these conditions the following equation is valid:

$$\cos(2\pi\nu_z) = \cos(2\pi\nu_{z0}) - 2\pi\xi_z \sin(2\pi\nu_{z0}), \tag{10}$$

where ν_{z0} is the initial synchrotron tune without beam-beam interaction and ν_z is the tune calculated by tracking. Thus, we can find the synchrotron tune shift ξ_z.

For sake of comparison we use typical parameters of SuperB [14] and DAΦNE listed in Table I. The last three rows show ξ_z calculated analytically from (7), the nominal synchrotron tune and the tune in beam-beam collisions obtained from (10), respectively.

First, our numerical simulations have confirmed that, in agreement with (8), the synchrotron tune shift does not depend on the parameters of the vertical motion, such as β_y and ν_y. Second, the agreement between the analytical and numerical estimates is quite reasonable for horizontal tunes far from integers, see Fig. 8. Quite naturally, in a scheme with a horizontal crossing angle, synchrotron oscillations are coupled with the horizontal betatron oscillations. One of the side effects of this coupling is the ν_z dependence on ν_x, which becomes stronger near the main coupling resonances $\nu_x \pm \nu_z = k$. Obviously this effect is not accounted for in (7) and (8). Therefore, in order to make comparisons with the analytical formula, we need to choose the

TABLE I. DAΦNE and SuperB parameters and synchrotron tune shifts

Parameters	SuperB	DAΦNE
N, strong beam	5.74×10^{10}	3.3×10^{10}
γ, weak beam	8180	998
σ_z, mm, weak beam	5	12.8
σ_z, mm, strong beam	5	19
σ_x, μm, strong beam	5.65	255
$\sigma_{E/E}$, weak beam	6.57×10^{-4}	5.0×10^{-4}
θ, mrad	60	50
ф, weak beam	16.58	1.255
ξ_z, analytical	−0.001 02	−0.000 811
ν_{z0}	0.0100	0.011 50
ν_z	0.008 93	0.010 66

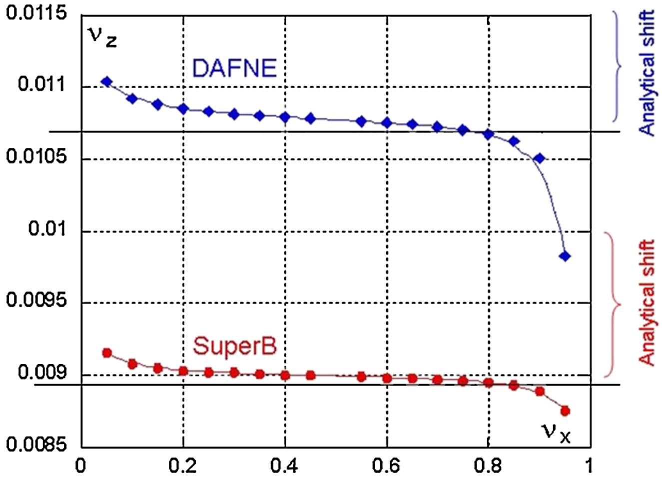

FIG. 8. Synchrotron tune dependence on the horizontal tune. The solid straight lines correspond to the analytically predicted synchrotron tunes (last row in Table I).

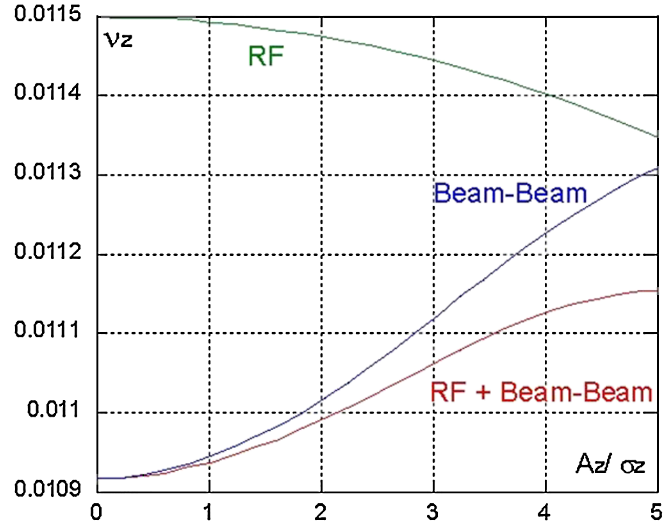

FIG. 9. Synchrotron tune dependence on normalized amplitude of synchrotron oscillations (blue curve—tune dependence created by beam-beam collisions alone; green—rf nonlinearity alone; red—both contributions).

horizontal betatron tune ν_x closer to half integer, where its influence on ν_z is weaker. The coupling vanishes for very large Piwinski angles, and that is why the ν_z dependence on ν_x is stronger for DAΦNE than for SuperB.

Since ν_x for DAΦNE is rather close to the coupling resonance, we will use numerical simulations in order to compare the calculated synchrotron tune shift with the measured one. In particular, when the weak positron beam collides with an electron beam of ≈ 500 mA, the measured synchrotron frequency shift is about -630 Hz (peak to peak). In our simulations we use the DAΦNE parameters listed in Table I with, respectively, lower electron bunch current ($N = 0.9 \times 10^{10}$) and shorter bunch length ($\sigma_z = 1.6$ cm). This results in a synchrotron tune shift of $-0.000\,232$ corresponding to a frequency shift of -720 Hz. The agreement is reasonable taking into account the experimental measurement errors and the non-negligible width of the synchrotron sidebands (see Figs. 6 and 7).

We have also calculated numerically the synchrotron tune dependence on synchrotron oscillation amplitudes since this amplitude dependent spread of synchrotron frequencies can give Landau damping of the longitudinal coupled bunch instability. For this purpose we track on-axis particles with different initial longitudinal coordinates over 2048 turns and perform the Fourier transform in order to extract the corresponding synchrotron frequencies.

In Fig. 9 the blue curve shows the calculated synchrotron tune dependence on the normalized synchrotron amplitude for the DAΦNE weak positron beam interacting with the strong 1.7 A electron beam. For comparison, the green curve shows the tune dependence on amplitude arising from the nonlinearity of the rf voltage. As we can see, the synchrotron tune spread due to the beam-beam interaction is significantly larger than that due to the rf voltage alone, at least within $5\sigma_z$. In the past it was shown that the rf voltage nonlinearity is strong enough to damp quadrupole longitudinal coupled bunch instability [15]. Therefore, we can expect a strong Landau damping of longitudinal coupled bunch oscillations by the beam-beam collision. This conclusion is in agreement with the performed measurements.

VI. CONCLUSION

Experimental data on synchrotron oscillation damping due to beam-beam collisions in DAΦNE have been collected by a commercial spectrum analyzer and by the bunch-by-bunch longitudinal feedback diagnostic tools [16].

In order to explain the experimental observations, a simple analytical formula for the longitudinal tune shift and tune spread due to beam-beam collisions was obtained and numerical simulations of beam-beam interactions were carried out.

It was shown that the formula agrees well with the simulations when the horizontal tune is far from the synchrobetatron resonances $\nu_x \pm \nu_z = k$. The agreement improves with large Piwinski angles.

Measured and simulated synchrotron frequency shifts are in a reasonable agreement.

Calculations have also shown that at high beam currents the synchrotron tune spread induced by the beam-beam interaction at DAΦNE can be larger than the tune spread due to the nonlinearity of the rf voltage. This may result in additional Landau damping of the longitudinal coupled bunch oscillations.

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