Particle Accelerators, 1990, Vol. 31, pp. 7–19
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AN ADIABATIC FOCUSER*

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Abstract Theoretical analysis is made of an intense relativistic electron beam, such as would be available from a linear collider, moving through a plasma of increasing density, but density always less than that of the beam (underdense). In this situation the plasma electrons are expelled from the beam channel and the electrons are subject to an ever increasing focusing force provided by the channel ions. Analysis is made on the beam radiation energy loss in the classical, the transition, and the quantum regimes. It is shown that the focuser is insensitive to the beam energy spread due to radiation loss. Furthermore, because of the different scaling behaviors in the nonclassical regimes, the radiation limit on lenses (the Oide limit) can be exceeded. The sensitivity of the system to the optic mismatch and the nonlinearity is also analyzed. Examples are given with SLC-type and TLC-type parameters.

INTRODUCTION

To avoid increasing energy loss through synchrotron radiation in storage rings, it is generally agreed that future high energy e^+e^- colliders are necessarily linear.[1] To compensate for the much lower collision rates in linear colliders, one is forced to collide much tighter beams. For example, in the design of a TeV collider (TLC) by Palmer,[2] the beam sizes at the interaction point (IP) are as miniscule as $\sigma_x =$ 190 nm, $\sigma_y = 1$ nm. For multi-TeV colliders in the far future, the beam size is expected to be even smaller. This demanding requirement on the beam size imposes

⋆ Work supported by Department of Energy contract DE–AC03–76SF00515, DE–AC03–76SF00098, and W–7405–ENG–48.
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stringent constraints on the stability and tolerance in the final focusing beam optics system. Furthermore, it was recently demonstrated by one of us (K. O.)[3] that the chromatic effect due to the synchrotron radiation triggered at the final focusing lens imposes a strong limitation on the minimal possible beam size.

In this paper, we present a different concept of beam focusing, called *adiabatic focusing,* which promises to evade the synchrotron radiation limit set by Oide. This is achieved by implementing a beam optics system where the focusing gradient is continuosly and slowly increased along the direction of beam propagation, such that the β-function decreases linearly along the lens. In such a focusing system, beam particles with different energies would always oscillate within a definite envelope and eventually be focused down to within the designated size. The problem of chromatic aberration associated with conventional discrete focusing lenses can thus be alleviated.

The insensitivity of this focusing scheme to the particle energy does not imply that the system is entirely free from the constraint due to synchrotron radiation. For high energy physics purposes, the focused beams should not suffer from significant energy degradation. But as will be shown, the corresponding limitation on the attainable beam size is much milder so long as the focusing is strong enough that the synchrotron radiation enters into the nonclassical regime.

One possible way to realize the concept is to imploy an underdense plasma column with a graded density. When applied to the beam parameters similar to those of the SLAC End Station, where the beam energy is 15 GeV, and those of the Stanford Linear Collider (SLC), the necessary parameters for the plasma adiabatic focuser are shown to be very reasonable; and, in principle, to yield a significant increase in the luminosity for the SLC. To apply the scheme to TeV-range linear colliders—in particular, the TLC considered at SLAC—we find it necessary to invoke liquid or even solid-state materials. Although the necessary technology for the focuser is yet to be developed, such a focuser should in principle be more compact than the conventional focusing system. In particular, for a focuser relevant to the SLAC End Station-type parameters, the requirements for the system seems to be immediately realizable.

ADIABATIC FOCUSING

We start by introducing the basic concept of what we call *adiabatic focusing.* In this section, and throughout the paper, our discussions will be restricted to the one-dimensional analysis in the dimension transverse to the beam propagation. This treatment should be appropriate for *flat* beams, where the transverse beam size in one dimension is much smaller than that in the other dimension. In general, in a focusing (or defocusing) environment a particle with coordinate y satisfies the equation of motion

$$\frac{d^2y}{ds^2} + K(s)y = 0 \quad , \tag{1}$$

and the well-known solution is[4]

$$y(s) = \beta^{1/2}(s)\cos[\psi(s) + \phi] \quad , \tag{2}$$

where

$$\frac{d\beta}{ds} = -2\alpha(s) \quad ,$$

$$\psi(s) = \int^{s} \frac{ds'}{\beta(s')} \quad .$$

In an adiabatic focusing, we demand that the change in β, occuring in a length given by β, is small compared to β. For the sake of simplicity, we shall assume that

$$\frac{d\beta}{ds} = \text{constant} \quad . \tag{3}$$

Hence we take

$$\beta(s) = \beta_0 - 2\alpha_0 s \quad , \tag{4}$$

where α_0 is the initial condition and a constant of the system that characterizes the amount of adiabaticity.

Since $\alpha(s) = \alpha_0 =$ constant, we have $d\alpha/ds = 0$, and the focusing strength along the channel varies as

$$K(s) = \frac{1+\alpha_0^2}{\beta^2} = \frac{1+\alpha_0^2}{(\beta_0 - 2\alpha_0 s)^2} \quad . \tag{5}$$

Notice that the focusing strength scales inverse quadratically with $\beta(s)$. The phase advance, on the other hand, varies as

$$\psi(s) = \frac{1}{2\alpha_0}\ln\frac{\beta_0}{\beta_0 - 2\alpha_0 s} \quad . \tag{6}$$

For a particle with less energy than the design energy E_0 , i.e., $E = (1-\delta)E_0$, where $\delta \ll 1$, the focusing force K is larger by an amount $1/(1+\delta)$. According to Eq. (1), the matched β-function for the lower-energy particle becomes $\bar{\beta}(s) = \sqrt{1-\delta}\beta(s)$, and the α-function is also reduced to $\bar{\alpha}(s) = \sqrt{1-\delta}\alpha(s)$. The mismatched β-function can be shown to be

$$\tilde{\beta}(s) = \beta(s)[1 - \delta \sin^2 \bar{\psi}(s)] \leq \beta(s) \quad , \tag{7}$$

where $\bar{\psi}(s) \equiv \psi(s)/\sqrt{1-\delta}$.

Thus, the amplitude of the lower-energy particle never exceeds that of the reference particle. If one chooses the design energy of the focuser at the maximum energy of the incoming beam, the entire beam is expected to be focused. This *achromatic* nature of the focuser will hold true for a particle which emits radiation while traversing the focuser and is the very basis of the adiabatic focuser concept.

RADIATION LOSS

The rate of energy loss of a relativistic electron due to synchrotron radiation is well known.[5] In order to perform simple analytic calculations, it is convenient to approximate the exact formula by the following expressions in the *classical,* the *transition,* and the *quantum* regimes[6] (see Fig. 1):

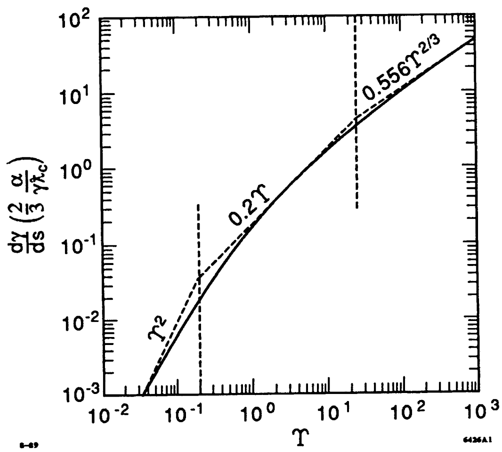

FIGURE 1 The rate of synchrotron radiation loss, in units of $2\alpha/3\gamma\lambda_c$, as a function of the dimensionless parameter Υ. The solid curve is from the exact expression, while the dashed lines are from our approximate formulae.

$$\frac{d\gamma}{ds} = -\frac{2}{3}\frac{\alpha}{\lambda_c}\begin{cases}\Upsilon^2 , & \Upsilon \lesssim 0.2 , \\ 0.2\Upsilon , & 0.2 \lesssim \Upsilon \lesssim 22 , \\ 0.556\Upsilon^{2/3} , & 22 \lesssim \Upsilon , \end{cases} \tag{8}$$

where γ is the Lorentz factor of the electron, α the fine structure constant, and $2\pi\lambda_c$ the Compton wavelength. We see that the energy loss is uniquely determined by the parameter Υ, which is Lorentz invariant and defined as

$$\Upsilon \equiv \gamma\frac{B}{B_c} . \tag{9}$$

Here $B_c = m^2c^3/e\hbar \simeq 4.4 \times 10^{13}$ Gauss is the Schwinger critical field.

Since the external magnetic field induces a bending of the electron trajectory, Υ can also be expressed in terms of the instantaneous radius of curvature ρ of the particle,

$$\Upsilon = \frac{\gamma^2\lambda_c}{\rho} = \frac{\gamma^2}{m\rho} . \tag{10}$$

In the above equation and for the rest of the paper we adopt the convention of natural units, i.e., $c = \hbar = 1$.

Since $1/\rho = K(s)y$, and from Eq. (6), we have

$$\Upsilon = \frac{\gamma^2}{m}\frac{1+\alpha_0^2}{(\beta_0 - 2\alpha_0 s)^2}y . \tag{11}$$

With the help of the relation $\sigma = \langle y^2\rangle = \beta\epsilon$, where ϵ is the emittance of the beam, and replacing $1/m$ by λ_c, we express the above equation as a function of s explicitly,

$$\Upsilon(s) = \lambda_c\sqrt{\epsilon}(1+\alpha_0^2)\frac{\gamma^2(s)}{[\beta_0 - 2\alpha_0 s]^{3/2}} . \tag{12}$$

Notice that one essential character of synchrotron radiation is that the actual emittance, not the normalized emittance ($\epsilon_n = \gamma\epsilon$), is conserved, to an accuracy of the order $\mathcal{O}(1/\gamma)$, by the radiation process. Thus, the energy loss as a function of the distance of travel becomes

$$\frac{d\gamma(s)}{ds} = -\frac{2}{3}\frac{\alpha}{\lambda_c}\begin{cases}{\lambda_c}^2\epsilon(1+\alpha_0^2)^2\frac{\gamma^4(s)}{[\beta_0 - 2\alpha_0 s]^3} , & \Upsilon(s) \lesssim 0.2 , \\ \frac{1}{5}\lambda_c\sqrt{\epsilon}(1+\alpha_0^2)\frac{\gamma^2(s)}{[\beta_0 - 2\alpha_0 s]^{3/2}} , & 0.2 \lesssim \Upsilon(s) \lesssim 22 , \\ 0.556\left[\lambda_c\sqrt{\epsilon}(1+\alpha_0^2)\right]^{2/3}\frac{\gamma^{4/3}(s)}{\beta_0 - 2\alpha_0 s} , & 22 \lesssim \Upsilon(s) . \end{cases} \tag{13}$$

First, we assume that conditions are such that upon injection into the adiabatic focuser one is in the classical regime of radiation. From Eq. (13):

$$\frac{1}{\gamma_0^3} - \frac{1}{\gamma^3(s)} = \frac{1}{2}\alpha\lambda_c\epsilon\frac{(1+\alpha_0^2)^2}{\alpha_0}\left[\frac{1}{\beta_0^2} - \frac{1}{\beta^2(s)}\right] \quad , \qquad \text{(classical)} \quad . \tag{14}$$

Assuming that the energy loss is small, i.e., $\gamma(s) = \gamma_0(1-\delta)$ and $\delta \ll 1$, we find the fractional energy loss to be

$$\delta_c(s) = \frac{1}{6}\alpha\lambda_c\gamma_0^3\epsilon\frac{(1+\alpha_0^2)^2}{\alpha_0}\left[\frac{1}{\beta^2(s)} - \frac{1}{\beta_0^2}\right] \quad . \tag{15}$$

If the conditions are such that upon injection into the adiabatic focuser one is in the transition regime, the scaling for energy loss follows the second expression in Eq. (13), and we find

$$\frac{1}{\gamma_0} - \frac{1}{\gamma(s)} = \frac{2}{15}\alpha\sqrt{\epsilon}\frac{(1+\alpha_0^2)}{\alpha_0}\left[\frac{1}{\sqrt{\beta_0}} - \frac{1}{\sqrt{\beta(s)}}\right] \quad , \qquad \text{(transition)} \quad . \tag{16}$$

Again, assuming small energy loss, we get

$$\delta_t(s) = \frac{2}{15}\alpha\gamma_0\sqrt{\epsilon}\frac{1+\alpha_0^2}{\alpha_0}\left[\frac{1}{\sqrt{\beta(s)}} - \frac{1}{\sqrt{\beta_0}}\right] \quad . \tag{17}$$

Finally, if the beam is injected directly into the quantum regime, then

$$\frac{1}{\gamma_0^{1/3}} - \frac{1}{\gamma^{1/3}(s)} = \frac{0.556}{9}\alpha\left(\frac{\epsilon}{\lambda_c}\right)^{1/3}\frac{(1+\alpha_0^2)^{2/3}}{\alpha_0}\ln\left(\frac{\beta(s)}{\beta_0}\right) \quad , \qquad \text{(quantum)} \quad . \tag{18}$$

and the energy loss formula in this regime is

$$\delta_q(s) = \frac{0.556}{3}\alpha\left[\frac{\gamma_0\epsilon(1+\alpha_0^2)^2}{\lambda_c\alpha_0^3}\right]^{1/3}\ln\left(\frac{\beta_0}{\beta(s)}\right) \quad . \tag{19}$$

In the situation where the focusing process continues across different regimes, matching of boundary conditions is necessary. The boundary between the classical and the transition regimes occurs at $\Upsilon = 1/5$. From Eq. (12), this corresponds to a β-function

$$\beta_1 = [5\lambda_c\gamma_1^2\sqrt{\epsilon}(1+\alpha_0^2)]^{2/3} \quad . \tag{20}$$

By definition, the purpose of the focuser is to effectively reduce the β-function, i.e., that $\beta_1 \ll \beta_0$. Thus, the total energy loss of the electron after traversing the entire classical regime is

$$\delta_c \simeq \frac{1}{5^{4/3}\cdot 6}\alpha\left[\frac{\gamma_0\epsilon(1+\alpha_0^2)^2}{\lambda_c\alpha_0^3}\right]^{1/3} \quad . \tag{21}$$

The boundary condition at the transition-quantum interface is $\Upsilon = 22$, which corresponds to

$$\beta_2 = \left[\frac{1}{22}\lambda_c\gamma_2^2\sqrt{\epsilon}(1+\alpha_0^2)\right]^{2/3} \quad . \tag{22}$$

The total energy loss within the transition regime is therefore

$$\delta_t \simeq \frac{2\cdot 22^{1/3}}{15}\alpha\left[\frac{\gamma_1\epsilon(1+\alpha_0^2)^2}{\lambda_c\alpha_0^3}\right]^{1/3} \quad . \tag{23}$$

For an adiabatic focuser where the beam is further focused into the quantum regime, the total energy loss throughout the focuser is then

$$\begin{aligned}\Delta &= \delta_c+\delta_t+\delta_q \\ &= \left[\frac{1}{5^{4/3}\cdot 6}\gamma_0^{1/3}+\frac{2\cdot 22^{1/3}}{15}\gamma_1^{1/3}+\frac{0.556}{3}\gamma_2^{1/3}\ln\left(\frac{\beta_2}{\beta^*}\right)\right]\frac{\alpha\epsilon^{1/3}(1+\alpha_0^2)^{2/3}}{\lambda_c{}^{1/3}\alpha_0} \quad .\end{aligned} \tag{24}$$

We now look for the optimal value of α_0 for attaining a desired β-function with minimum energy loss. From Eq. (15), Eq. (17), and Eq. (19), we see that the dependence of energy loss on α_0 is different in the three regimes. By imposing $d\delta/d\alpha_0 = 0$ on the three equations, we find the optimum α_0 to be

$$\alpha_0 = \begin{cases} \frac{1}{\sqrt{3}} \quad , & \text{(classical)} \quad , \\ 1 \quad , & \text{(transition)} \quad , \\ \sqrt{3} \quad , & \text{(quantum)} \quad . \end{cases} \tag{25}$$

It should, in principle, be possible to set up an adiabatic focuser where the increase of its focusing strength varies in accordance with the three different optimum values given above. But the focuser may be experimentally more convenient if α_0 is fixed throughout the system. If a focuser covers all three regimes of radiation, an obvious compromise would be $\alpha_0 = 1$. With this choice there will be about 15% additional radiation in the classical regime and about 30% more in the quantum regime. Alternatively, since the radiation loss occurs primarily near the end of an adiabatic focuser, a choice of α_0 according to the final regime is most advisable.

BEAM SIZE AND EMITTANCE LIMITS

In a conventional focusing of charged particle beams by discrete magnets, it has recently been shown by one of us (K. Oide)[3] that there exists a fundamental limit on the minimal attainable beam size due to unvoidable synchrotron radiation that the beam suffers during the passage through the final quadrupole. The fact that this occurs at the last focusing element, and that the radiation is stochastic in character, renders the induced aberration uncorrectable. This limit on beam size at the focus can be expressed as

$$\sigma \gtrsim 3.4 \times 10^{-4} \epsilon_n^{5/7} \quad , \tag{26}$$

in the vertical dimension for flat beams.

The situation is different in the case of a continous focusing environment such as the adiabatic focuser. Off-momentum particles in this case would still be focused down adiabatically within a certain beam envelope, as can be seen from the discussion at the end of Sec. 2. In so doing, the chromatic aberration is essentially eliminated by avoiding any drift space. However, the adiabatic focuser is not free from constraints.

Insensitive to the chromatic effect as it is, a beam would be useless if a substantial amount of energy is lost. The ultimate limitation is certainly that the fractional energy loss be less than unity. In the classical regime, from Eq. (15), this means that

$$\beta \gg \left(\frac{1}{6}\frac{(1+\alpha_0^2)^2}{\alpha_0}\alpha\lambda_c\gamma_0^3\epsilon\right)^{1/2} \quad , \tag{27}$$

where $1/\beta_0^2$ was neglected. Therefore, if the focuser is a purely classical one, then the beam size is limited as

$$\sigma_c = \sqrt{\beta\epsilon} \gg \left(\frac{1}{6}\frac{(1+\alpha_0^2)^2}{\alpha_0}r_e\epsilon_n^3\right)^{1/4} \quad , \qquad \text{(classical)} \quad . \tag{28}$$

where $r_e = \alpha\lambda_c$, and the normalized emittance $\epsilon_n = \gamma_0\epsilon$ has been restored. If we take $\alpha_0 = 1$, this gives $\sigma_y = 0.5$ nm for $\epsilon_n = 2.5 \times 10^{-8}$ m, which is numerically very close to the Oide limit with discrete focusing.

In the transition regime, the same constraint leads to a somewhat different scaling. From Eq. (17), we have

$$\sigma_t \gg \frac{2}{15}\frac{1+\alpha_0^2}{\alpha_0}\alpha\epsilon_n \quad , \qquad \text{(transition)} \quad . \tag{29}$$

With the same normalized emittance and α_0 as above, the limit on beam size is relaxed to 0.05 nm in the transition regime, which is about one order-of-magnitude smaller than the classical limit.

In the quantum regime, the same constraint on Eq. (19) leads to the condition

$$\sigma_q \gg \sigma_0 \exp\left\{-3\left[\frac{\alpha_0^3}{(1+\alpha_0^2)^2}\frac{\lambda_c}{\alpha^3\epsilon_n}\right]^{1/3}\right\} \quad , \qquad \text{(quantum)} \quad . \tag{30}$$

In order that the beam penetrates down to the quantum regime, there is, however, a requirement on the initial normalized emittance. Recall that the first boundary condition at the classical–transition interface is given by β_1 in Eq. (20), where the Lorentz factor γ_1 is related to δ_c by $\delta_c \equiv (\gamma_0 - \gamma_1)/\gamma_0$. Inserting these relations into Eq. (15), and demanding that $\delta_c \ll 1$, we find that

$$\epsilon_n \ll \frac{5^4 6 \lambda_c}{\alpha} \frac{\alpha_0^3}{(1+\alpha_0^2)^2} \quad , \qquad \text{(classical)} \quad . \tag{31}$$

This is the condition for the beam to penetrate through the classical regime. Taking $\alpha_0 = 1/\sqrt{3}$, we find $\epsilon_n \ll 3.76 \times 10^{10} \lambda_c = 0.014$ m.

One may go through a similar analysis on the condition for penetrate through the transition regime. With the help of the second boundary condition for β_2 in Eq. (22),

$$\epsilon_n \ll \frac{15^3}{2^3 22} \frac{\lambda_c}{\alpha^3} \frac{\alpha_0^3}{(1+\alpha_0^2)^2} \quad , \qquad \text{(transition)} \quad . \tag{32}$$

For $\alpha_0 = 1$, this condition requires that $\epsilon_n \ll 4.7 \times 10^{-6}$m, in order to enter the quantum regime.

When this condition on the emittance is satisfied, we replace the initial β-function in Eq. (19) by the second boundary condition and obtain

$$\sigma_q \gg \left[\frac{1}{22} \lambda_c \epsilon_n^2 (1+\alpha_0^2)\right]^{1/3} \exp\left\{-3\left[\frac{\alpha_0^3}{(1+\alpha_0^2)^2} \frac{\lambda_c}{\alpha^3 \epsilon_n}\right]^{1/3}\right\} \quad . \tag{33}$$

Notice that the limits on the emittance in Eq. (31) and Eq. (32) depend only on fundamental physical parameters and the adiabaticity of the system. In both equations, the dependence on α_0 has a maximum value at $\alpha_0 = \sqrt{3}$. At this value of α_0, the contraints on the emittance are least stringent. In fact, $\alpha_0 = \sqrt{3}$ is also the condition for least radiation in the quantum regime. We thus call [from Eq. (32)] the quantity

$$\epsilon_c \equiv \frac{3^{3/2} \cdot 15^3}{2^3 \cdot 4^2 \cdot 22} \frac{\lambda_c}{\alpha^3} = 6.17 \times 10^{-6} \text{ m} \tag{34}$$

the *critical emittance.*

The actual normalized emittance in the system can then be represented by the parameter

$$\xi = \left(\frac{\epsilon_n}{\epsilon_c}\right)^{1/3} \quad . \tag{35}$$

In terms of ξ, Eq. (33) can be rewritten as

$$\sigma_q \gg 1.39 \times 10^{-8} \, \xi^2 \exp\{-1.12 \, \xi^{-1}\} \text{ m} \quad , \tag{36}$$

where, again, $\alpha_0 = \sqrt{3}$ has been assumed. For an emittance $\epsilon_n = \epsilon_c/10$, we find that $\sigma_q \gg 2.68 \times 10^{-9}$ m.

SENSITIVITIES

Optical Mismatch

One essential issue for an optical element is to estimate the sensitivity of the element to the less than ideal initial condition caused by errors of other optical elements upstream. Since our consideration here on the adiabatic focuser is about its linear optics, one expects the sensitivity to be essentially the same as that from the linear analysis of the conventional optics.

In the conventional discrete optics, consider the final focusing quadrupole to have an error Δk in its focusing strength, and an error Δs in its position. Let the phase advance from the quadrupole to the interaction point (IP) be ψ, and the Twiss parameters at the quadrupole and the IP be (α_0, β_0) and (α^*, β^*), respectively. The induced degradation $\Delta\beta^*$ from the designed value, β^*, at the focus can be shown to be

$$\frac{\Delta\beta^*}{\beta^*} = \beta_0 \Delta k \sin 2\psi + (\beta_0 \Delta k)^2 \sin^2\psi \quad , \qquad (\Delta k \quad \text{error}) \quad , \tag{37}$$

and

$$\begin{aligned} \frac{\Delta\beta^*}{\beta^*} = & -2\frac{\Delta s}{\beta_0}(\cos\psi + \alpha_0 \sin\psi)(\sin\psi - \alpha_0\cos\psi) \\ & + \left(\frac{\Delta s}{\beta_0}\right)^2 (1+\alpha_0^2)\cos^2\psi \quad , \qquad (\Delta s \quad \text{error}) \quad , \end{aligned} \tag{38}$$

respectively. Since the phase advance, ψ, is generally determined by diverse elements upstream, it is not possible in practice to choose ψ to minimize the effect of Δk and Δs errors.

Consider now the situation for an adiabatic focuser. The degradation due to the errors in focusing strength along the focuser should have the same effect as that in Eq. (37), except that the error Δk should be acquired from a cumulation over the entire length of the focuser. On the other hand, the injection of an optically mismatched beam, with the actual Twiss parameters $(\alpha_0 + \Delta\alpha,\ \beta_0 + \Delta\beta)$, into a perfect focuser results in a degradation on the final β^*:

$$\frac{\Delta\beta^*}{\beta^*} = -\Delta\alpha \sin 2\psi + (\cos 2\psi + \alpha_0 \sin 2\psi)\frac{\Delta\beta}{\beta} \quad , \tag{39}$$

where the phase advance through the focuser is determined by

$$\psi = \frac{1}{2\alpha_0} \log \frac{\beta_0}{\beta^*} \quad . \tag{40}$$

We see from the above two equations that when the adiabaticity α_0 is large, the phase advance ψ becomes small, and the contribution to the degradation is dominated by the $\Delta\beta/\beta_0$ term. On the contrary, if α_0 is small then ψ gets large, and the contributions from the first and last terms in Eq. (39) dominate. Thus, the situation is not much different than that of the conventional case.

Nonlinearity

Next, we examine the effects due to the nonlinear force in the focuser. To elucidate the issue, we consider a sextupole-like nonlinear force which increases adiabatically as a fixed proportion of the linear force. The equation of motion is now

$$\frac{d^2y}{ds^2} + K(s)y = K(s)\frac{a}{\sigma}y^2 \quad , \tag{41}$$

where $\sigma = \sqrt{\beta\epsilon}$, $K(s)$ is given in Eq. (5), and the dimensionless parameter a characterizes the degree of nonlinearity of the force.

From particle tracking in the phase-space of such a Hamiltonian system, and from the direct particle-in-cell computer simulations, we find that a nonlinearity as large as $a = 0.12$ is still tolerable with no significant loss of beam particles.

FOCUSER EXAMPLES

We have generated, and checked with numerical simulations, three examples of the adiabatic focuser. The first is a proof-of-principle case using the beam in the SLAC End Station. The second involves the use of a focuser on the SLC, and the third is a focuser on a TLC being considered at SLAC. Parameters of the beam, the focuser, and the expected performance are displayed in Table I. In the first two cases, round beams, i.e., $\sigma_y = \sigma_x$, are assumed, whereas in the third case for the TLC, the beam is assumed to be flat ($\sigma_y \ll \sigma_x$).

In the End Station Focuser, we have a rather long device which employs differential pumping to form the variation in plasma density, ramping from the initial

TABLE I Three examples of the adiabatic focuser.

	SLAC End Station	SLC	TLC
Initial Beam Properties			
E_0 [GeV]	15	50	500
ϵ_n [m]	1×10^{-4}	3×10^{-5}	1×10^{-8}
σ_0 [μm]	20	3	5×10^{-3}
β_0 [cm]	12	3	0.25
Focuser Properties			
α_0	$5\ \times10^{-2}$	$1/\sqrt{3}$	$\sqrt{3}$
L [cm]	119	2.6	0.07
n_0 [cm^{-3}]	1.2×10^{14}	8.4×10^{15}	1.8×10^{19}
n^* [cm^{-3}]	1.2×10^{18}	8.4×10^{19}	1.8×10^{23}
Final Beam Properties			
δ	Negligible	3%	1%
σ^* [μm]	2	0.3	0.5×10^{-3}

value, n_0, to the final value, n^*, over a length L. Such a device appears to be possible to construct according to preliminary engineering estimates. The other two focusers require higher densities (ranging up to solid density), with variation over shorter distances.

We have not studied how to realize these focusers. Note, however, that they result in significantly involved luminosity in the colliders.

DISCUSSIONS

The concept of an adiabatic focuser has been proposed and analyzed. The device has a number of advantageous properties, but requires a plasma with very high density near the interaction point. This plasma will cause scattering of the beam and hence emittance blowup. The effect has been analyzed by Montague and Schnell.[7] It can be verified that the growth of emittance is negligibly small in all three examples which we discussed in the last section. In addition, the plasma will create background events. Two outstanding possible backgrounds are the inelastic scattering between the high energy electrons and the protons in the focuser,[8] and the e^+e^- pair creation by the radiated photons traversing the strong field in the focuser.[9] We have not analyzed the effect of these events on the design of a detector.

In this paper, we also have not analyzed the second synchrotron radiation limit given in Ref. 3, which has to do with the limitation on beam size due to the constraint on the rms energy spread of the beam. Presumably, since the focuser is insensitive to the deviations of beam particle energies, one expects that the situation be somewhat similar regarding the rms energy spread. In addition, our discussion has been concentrated on electron beams only. It is known that underdense plasmas generally respond rather differently to positron beams,[8] thus it awaits further efforts to see how our concept can be applied to positrons.

Most important, a focuser needs to be fabricated and, as has been seen, the required density demands for materials in the liquid, or even solid, states. We believe that for focusers in the gaseous state, a smooth increase of density (and therefore focusing strength) should be possible using differential pumping. In the extreme condition involving solids, multiple layers of different density solids, similar to those existing in microelectronics, may be invoked. Evidently, many more studies are necessary before one can realize the adiabatic focusing scheme.

Finally, experimental verification of the concept is required. A first test in the SLAC End Station would be most appropriate.

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