SLAC-TN-78-4
Daryl Reagan
October 1978

A COMMENT ON NONUNIFORMITIES IN LONG-COILS USED IN DIPOLE STRENGTH MEASUREMENTS

Introduction

A rotating long-coil which extends longitudinally beyond the fringing fields of a bending dipole electromagnet will generate an e.m.f. whose amplitude is approximately proportional to the bending strength, $\int Bdz$, of the dipole. The constant of proportionality can be determined by measuring cross-section dimensions of the coil. Since making and measuring a long-coil to close tolerances is difficult and expensive, it is common practice, instead, to calibrate such a coil against a standard magnet which has been mapped by NMR and Hall probes. How will nonuniformities in the width of a long-coil affect the results of bending strength measurements made among a set of similar magnets? Think of an ideal long-coil having one turn of thin wire, with its two long legs separated by a variable width $A = A_0\left[1+a(z)\right]$. Let the coil rotate at a frequency ω in a mythical dipole magnet, with $B = 0$ for $|z| > \ell/2$, and with $B = B_0\left[1+b(z)\right]$ for $|z| \leq \ell/2$. Let A_0 and B_0 be constants representing averages, that is let $\int adz = \int bdz = 0$ over the interval $|z| \leq \ell/2$. The amplitude, V, of the signal produced in the coil by the magnet will then be

$$V = \omega\int ABdz = \omega A_0 B_0\left[\ell + \int abdz\right] \quad . \qquad (1)$$

This result indicates that a difference will exist between the signal produced by a uniform long-coil and that produced by a nonuniform one having the same average width if there are correlations between the deviations from average of the width of the coil and of the flux density of the magnet. Equation (1) is not useful in practical cases, since in fact b(z) must be large near the ends. Equation (1) does indicate, however, that unless one can guarantee that B(z) has the same shape for every dipole, one must expect that an imperfect long-coil will cause errors in comparisons of bending strengths among a set of similar magnets.

Comparing Two Dipoles

To arrive at an expression similar to Eq. (1) which will apply to practical problems, let us imagine that a "standard" dipole has been mapped by NMR and Hall probe, so that we know $B_s(z)$, where the subscript "s" will refer to the standard magnet. When a long-coil is calibrated in such a standard dipole it produces a signal whose amplitude is

$$V_s = \omega W \int [1 + w(z)]\, B_s(z)dz = \omega W \int B_s(z)dz \quad , \qquad (2)$$

where w(z) is the fractional deviation of the width of the long-coil from the constant value W. The first and last members of Eq. (2) represent a conventional interpretation of a long-coil calibration. It is important to recognize that this interpretation has the effect of choosing W to make $\int wB_s dz = 0$. In comparing a similar unit, which we will call a test dipole, with a standard dipole, a long-coil is used to determine k in the expression $\int B_t dz = k \int B_s dz$, which relates the relative strengths of the two dipoles for a particular value of excitation current. The subscript "t" will refer to the test dipole. For similar dipoles, the constant k will differ only slightly from unity. Let the z dependence of B in the test dipole be represented as $B_t(z) = kB_s(z) + kc(z)$. It follows from this definition that $\int cdz = 0$, so that the signal amplitude produced in the rotating long-coil by the test magnet will be

$$V_t = \omega Wk \int [1 + w(z)] [B_s(z) + c(z)] dz$$

$$= \omega Wk \left[\int B_s dz + \int wcdz \right] , \qquad (3)$$

remembering that $\int wB_s dz = 0$. When Eqs. (2) and (3) are combined, the result is

$$k = \frac{V_t}{V_s} \left[1 + \frac{\int wcdz}{\int B_s dz} \right]^{-1} . \qquad (4)$$

This shows that the measured value for $k = V_t/V_s$ will contain an error term of magnitude $\int wcdz \Big/ \int B_s dz$. Again the error term represents correlations between long-coil width deviations and flux density deviations, but now the flux density deviations are differences between the $B_t(z)$ and the $kB_s(z)$ functions belonging to the test and standard dipoles.

Estimating the Error

To estimate the magnitude of the error term in Eq. (4) in a practical case, it may be useful to divide the standard magnet conceptually into two "end" regions and a "middle" region. Let B_1 be the average value of B_s in the middle region and let c_1 be the maximum deviation of B_s from B_1 in this region. Then, assuming that each test magnet has flux density deviations from average which are the same size as those of the standard magnet, the contribution to the error term from the "middle" region cannot be larger than $2w_1c_1/B_1$, where w_1 is the maximum value of the deviation of the width of the long-coil from its average value.

The contributions of the "end" regions to the error may be largely due to misregistrations, that is to differences in the longitudinal positioning of the long-coil when it is rotated in the different magnets. A misregistration will be unavoidable at least at one end when magnets differ in length. The order of magnitude of the contribution to the error term from an end region due to misregistration is at most $w_1 e/\ell$, where e is the displacement and ℓ is the length of the magnet.

ACKNOWLEDGEMENT

Thanks are due to R. H. Helm for helpful suggestions.