Study of barrier modification using Hill-Wheeler and WKB approximations in Wong model

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Introduction

The study of fusion involving nuclei at energies well below the s-wave barrier has attracted considerable theoretical and experimental interest. The fusion-fission process depends upon the barrier formed by the various contributing potentials like long-range Coulomb, short range nuclear and centrifugal potential. The incoming projectile must have sufficient energy to overcome the barrier, which in turn means that barrier characteristics are important in order to study the reaction dynamics. A little change in barrier characteristics can significantly modify the transmission probability. The barrier penetration approach is attractive because of its overall simplicity and elegance. At the same time it provides a reasonable description of fusion between two nuclei. The transmission can be calculated in different approaches, which are exact as well as approximated solutions. The approximated solution can provide easy way to solve the complex quantum mechanical problem. In literature the approximated ways, which are widely used, for calculating the transmission probability are Wentzel-Kramers-Brillouin (WKB) and Hill-Wheeler approach. Within the framework of Wong model, the penetrability function T(E) is usually calculated using the Hill-Wheeler formula, assuming a purely parabolic-barrier. The parabolic-barrier models have wide appeal because the calculations are relatively simpler and numerically inexpensive. The WKB approximations is also numerically less demanding and at the same time pertinent to predicting fusion cross-sections at very low energies. It would be very interesting to study the comparative behavior of these two approximations in terms of barrier lowering. In the present work, we will present our calculations of the study of barrier lowering required to fit the experimental fusion cross-section of ^{60}Ni+^{100}Mo reaction with in the framework of Wong model by using above mentioned approaximations. In this work the nuclear part is obtained from the Energy density formalism using SIII force. This work is accepted for publication in [4].

The Model

The transmission coefficient is obtained through a barrier, which is generated by a potential

$$V(r) = V_C(r) + V_N(r) + V_\ell(r), \qquad (1)$$

where $V_C(r)$ is the Coulomb interaction and $V_\ell(r)$ is the centrifugal potential. $V_N(r)$ represents the nuclear interaction between the projectile and the target and is obtained by using the semiclassical extended Thomas Fermi approach of energy density formalism with SIII force [1]. Using Hill-Wheeler [2] approximation, the transmission T_ℓ is given in terms of its barrier height $V_B^\ell(E_{c.m.}, \theta_i)$ and curvature $\hbar\omega_\ell(E_{c.m.}, \theta_i)$, as

$$T_\ell = \left[1 + \exp\left(\frac{2\pi(V_B^\ell(E_{c.m.}, \theta_i) - E_{c.m.})}{\hbar\omega_\ell(E_{c.m.}, \theta_i)}\right)\right]^{-1} \qquad (2)$$

with $\hbar\omega_\ell$ evaluated at the barrier position $R = R_B^\ell$ corresponding to barrier height V_B^ℓ. Using

TABLE I: Comparison of ℓ-summed Wong-formula calculated fusion cross-sections for SIII Skyrme force with experimental data for ^{60}Ni+^{100}Mo→^{160}Yb* reaction at sub-barrier energies in terms of the barrier lowering parameter ΔV_B, using Hill-Wheeler and WKB approximations. In WKB method the ΔV_B is given for the ℓ_{max} value.

	Hill-Wheeler		WKB		Expt.
$E_{c.m.}$ (MeV)	ΔV_B (MeV)	σ_{fus} (mb)	ΔV_B (MeV)	σ_{fus} (mb)	σ_{fus} (mb)
122.0	-4.0	0.002	-6.9	0.0019	0.0014
123.1	-3.5	0.005	-5.5	0.0047	0.005
123.7	-3.0	0.012	-4.2	0.012	0.012
124.2	-2.5	0.022	-3.0	0.022	0.021

the WKB integral transmission T_ℓ is given by

$$T_\ell = \exp\left[-\frac{2}{\hbar}\int_{R_a}^{R_b}\{2\mu[V(R)-Q_{eff}]\}^{1/2}dR\right], \tag{3}$$

with $V(R_a,T) = V(R_b,T) = TKE(T) = Q_{eff}$ for the entry and exit points of potential barrier. The barrier modification for each ℓ is defined as,

$$\Delta V_B = V - V_B^\ell, \tag{4}$$

where V=V_B^ℓ(modified) for the Hill-Wheeler case and V=$V(R_a,\ell)$ for WKB case.

Calculations and Discussion

In this section, we present our calculations and results of comparison of barrier modification obtained by fitting the fusion cross-section for ^{60}Ni+^{100}Mo→^{160}Yb* reaction [3] at four sub-barrier energies. The obtained results at below barrier energies are compared with the experimental data in Table 1. It is noted that the results obtained in two approaches (Hill-Wheeler and WKB) are nearly identical, though the ΔV_B values are different, as expected. It is clear from the table that both the transmission approaches need barrier modification to fit the experimental data at below barrier energies.

Here the two definitions of ΔV_B are different because in one case (Hill-Wheeler) the modification of barrier is arbitrarily fixed, whereas in the other case (WKB) it changes the area under the curve between R_a and R_b for each ℓ. We use this method of barrier lowering only to illustrate our results for one Skyrme force (see, Table 1), indicating the equal suitability of either of the two methods (Hill-Wheeler and WKB).

It would be of further interest to test the suitability of these approximations in different mass regions and energy scales.

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