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A STUDY OF ELASTIC NEUTRINO SCATTERING USING A DEUTERIUM BUBBLE CHAMBER

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We propose to study the "elastic" scattering $\nu + n \rightarrow p + \mu^-$ in deuterium, using the physical reaction $\nu + d \rightarrow p_1 + p_2 + \mu^-$, in order to measure the axial form factor. In the Appendix, the effects encountered using deuterium as a target are given in some detail. The principal conclusions are

(1) For noninteracting final state protons, the deuterium cross section goes over into the free neutron cross section at high Q^2 (squared four-momentum transfer).

(2) The ratio of the differential scattering cross section for deuterium to that for the free neutron is, to a high degree of accuracy, independent of both the neutrino energy and the choice of the form factor, for $E_\nu \gtrsim 1$ BeV. It is a universal curve of the squared momentum transfer variable, Q^2, and is illustrated in Figure 1. We note that for $Q^2 \gtrsim 0.10$ (BeV/c)^2, the neutron and deuterium cross sections for elastic scattering are essentially the same. We have evaluated the cross sections for neutrino scattering using two sets of form factors, with Set I being a modified version of the Nambu[1] analysis of electron pion production and Set II being the conventional $m_A = m_V$. The Nambu Set I was

(1) Y. Nambu and M. Yoshimura, Phys. Rev. Letters 24, 25 (1970).

chosen such that it was forced, for $Q^2 = 0$, to go to the value 1.

In particular, we have assumed for Set I

$$f_A = \left(1 + \frac{Q^2}{(2.67)^2}\right)^{-2}, \quad \text{for } Q^2 < 0.5 \ (\text{BeV/c})^2$$

$$f_A = 1.43 \left(1 + \frac{Q^2}{(1.34)^2}\right)^{-2}, \quad \text{for } Q^2 \geq 0.5 \ (\text{BeV/c})^2,$$

$$f_V = f_M = \left(1 + \frac{Q^2}{(0.84)^2}\right)^{-2}, \text{ for all } Q^2$$

and $f_P = 0$

For Set II, we assume

$$f_V = f_M = f_A = \left(1 + \frac{Q^2}{(0.84)^2}\right)^{-2}, \text{ for all } Q^2$$

and $f_P = 0$.

In the above, f_V is the vector, f_M is the weak magnetism, f_A is the axial and f_P is the induced pseudoscalar form factor, all normalized to 1 at $Q^2 = 0$.

The cross sections for high energy are essentially energy independent. The integrated elastic scattering cross sections, for Sets I and II, are given in Table I.

We propose that the NAL large chamber, filled with deuterium, be exposed to a pure beam of neutrinos (i.e., using a horn) to study the elastic scattering process. The following design parameters are employed:

(1) The fiducial volume of the chamber is assumed to be 20 m^3.

(2) The latest flux estimates of Nezrick, for both 200 and 500 BeV beams, are used, appropriately scaled for the action of the horn (see figure 7).

(3a) For 200 BeV, it is assumed that the number of interacting protons is 3×10^{13} per pulse.

(3b) For 500 BeV, the number of interacting protons is taken to be 1×10^{13} per pulse.

(4) The elastic scattering cross sections for deuterium are taken from Table I.

(5) The inelastic cross section per nucleon is taken to be σ/nucleon = $0.8\ E_\nu \times 10^{-38}\ cm^2$, with E_ν in BeV.

(6) The number of photos is 250,000.

The design differences for the 200 and 500 BeV exposure reflect the fact that we wish to limit the number of interactions per picture to $\lesssim 3$ per photo.

The design estimates, in terms of numbers of events per energy bin, are summarized in Table II.

Depending on which form factor is chosen, the total number of elastic scattering events ranges from 22,500 to 52,000 at 200 BeV and from 18,000 to 41,000 at 500 BeV, making either beam choice a high statistics experiment. For example, we estimate that the numerical value of the axial form factor at $Q^2 \approx 1\ (BeV/c)^2$ can be measured to about 2%.

We propose to measure the absolute neutrino flux, as a function of energy, in the same exposure, by selecting those events with $Q^2 \leq 0.05\ (BeV/c)^2$. The average ratio of $(d\sigma/dQ^2)_d/(d\sigma/dQ^2)_n$ in this interval is 0.672 and is energy independent. The neutron cross section, at $Q^2 = 0$, is known, independently of any knowledge of the axial form factor (the argument is simply seen by noting that $f_A = 1$ at $Q^2 = 0$, by definition) and the cross section for the free neutron is

2.03×10^{-38} $cm^2/(BeV/c)^2$. Thus, the deuterium cross section, for small Q^2, is given by 1.26×10^{-38} $cm^2/(BeV/c)^2$, and is independent of the neutrino energy and the form factors. We propose to use this fact to normalize the spectrum. For both Set I and II, we expect ~ 2500 events with $Q^2 < 0.05$ at 200 BeV, and about 2000 for the 500 BeV exposure. Thus, the total incident number of neutrinos should be able to be determined with an accuracy of $\sim 2\%$. Once the form factors have been determined from the lower energy events, we will have been able to then compute (from measurements) the asymptotic total cross section for elastic scattering. In turn, we can then use <u>all</u> of the highest energy events to calculate the spectrum in the largely unknown region in which K-meson decays dominate, i.e., the energy region above 50 BeV.

This experiment could also be performed with plates on the downstream side of the chamber used to both identify the muon as well as to materialize possible background π^0 events. The rates would be slightly lower, but the analysis would be easier. It is clear that the latitude in possible beams and configurations makes this exposure completely compatible with other searches and experiments that could simultaneously be performed in the same film.

TABLE I

Set I		Set II	
σ_d	σ_n	σ_d	σ_n
$1.68 \times 10^{-38}\,cm^2$	$1.72 \times 10^{-38}\,cm^2$	$0.73 \times 10^{-38}\,cm^2$	$0.76 \times 10^{-38}\,cm^2$

TABLE II

Table of Expected Events in Deuterium for 250,000 Pictures

	200 BeV*			500 BeV**		
	Elastic			Elastic		
ΔE(BeV)	Set I	Set II	Inelastic	Set I	Set II	Inelastic
0-5	6,820	2,980	16,250	2,270	990	5,800
5-10	18,640	8,150	133,000	10,350	4,540	74,400
10-15	13,830	6,040	165,000	10,350	4,540	123,500
15-20	6,540	2,850	108,800	6,930	3,030	115,300
20-30	3,750	1,640	89,200	5,530	2,420	132,000
30-40	948	414	32,000	2,070	910	68,000
40-50	412	181	17,700	1,060	460	45,300
50-60	248	109	13,100	690	300	36,200
60-75	206	90	13,250	760	330	48,700
75-100	153	67	12,000	790	345	63,000
100-125	34	15	3,650	335	145	35,500
125-150				175	75	22,800
TOTAL	51,581	22,546	603,950 = 2.45/picture	41,310	17,985	770,500 = 3.08/picture

* Number of interacting protons is 3×10^{13}/pulse.

** Number of interacting protons is 1×10^{13}/pulse.

APPENDIX

I have made a recent analysis, presented at the last Washington APS meeting, of the effects of the deuterium on elastic scattering. A full manuscript of these results is now being prepared for publication. In this Appendix, I will outline the principal conclusions, on the following hypothesis.

(1) An impulse model of the collision is made, using a nonrelativistic expansion of the weak current including terms of p^2/m^2.

(2) A Hulthén wave function for the deuteron is employed.

(3) The two protons from the reaction $\nu + d \rightarrow p_1 + p_2 + \mu^-$ are assumed to be in a relative plane wave.

Under these circumstances, the deuterium elastic cross section is given by Figure 1, where the exclusion integrals are given by

$$I(Q) = \int e^{i\vec{Q}\cdot\vec{\rho}}\, \phi_d^2(\rho)\, d\tau_\rho$$

$$H(Q) = -\int e^{i\vec{Q}\cdot\vec{\rho}}\, \nabla^2\, \phi_d(\rho)\, d\tau_\rho$$

and $\mathcal{K} = H(0)$ and $I(0) = 1$.

In the above, ϕ_d is the deuteron wave function, $\lambda = -G_A/G_V = 1.2$, $\mu = 3.71$, M is the nucleon mass, and G is the Fermi constant.

The results of this formula were numerically computed, and it was found that, to $\sim \pm 1\%$, the ratio of $(d\sigma/dQ^2)d$ to $(d\sigma/dQ^2)n$ (i.e., the deuterium to the free neutron cross section) is independent of both the neutrino energy (for $E_\nu \gtrsim 1$ BeV) and the assumed choice of form factors, and is essentially a universal curve of momentum transfer. It is illustrated in Figure 2. In Figures 3 and 4, we plot the elementary cross section from the free nucleon for both neutrino and antineutrino beams, for the two choices of form factors. Figure 3 is for the Nambu Set I and Figure 4 is for Set II ($m_A = m_V$). In Figure 5, we plot the exclusion integral I(Q) and in Figure 6, the function $H(Q)/m^2$, for an assumed Hulthén wave function.

CLOSURE, WITH PLANE WAVE

FINAL STATE PROTONS, $\nu + d \rightarrow p_1 + p_2 + \mu^-$

$$\frac{d\sigma}{dQ^2} = \frac{G^2}{2\pi}\Big\{f_V^2\left[1 - \frac{Q^2}{2ME_\nu} - \frac{Q^2}{4E_\nu^2} - \frac{K}{mE_\nu} + \frac{1}{3}\frac{K}{m^2}\right]$$

$$+ \lambda^2 f_A^2\left[1 - \frac{Q^2}{2mE_\nu} + \frac{Q^2}{4E_\nu^2} - \frac{K}{mE_\nu} + \frac{1}{3}\frac{K}{m^2}\right] + (\mu f_M)^2 \frac{Q^2}{4m^2}$$

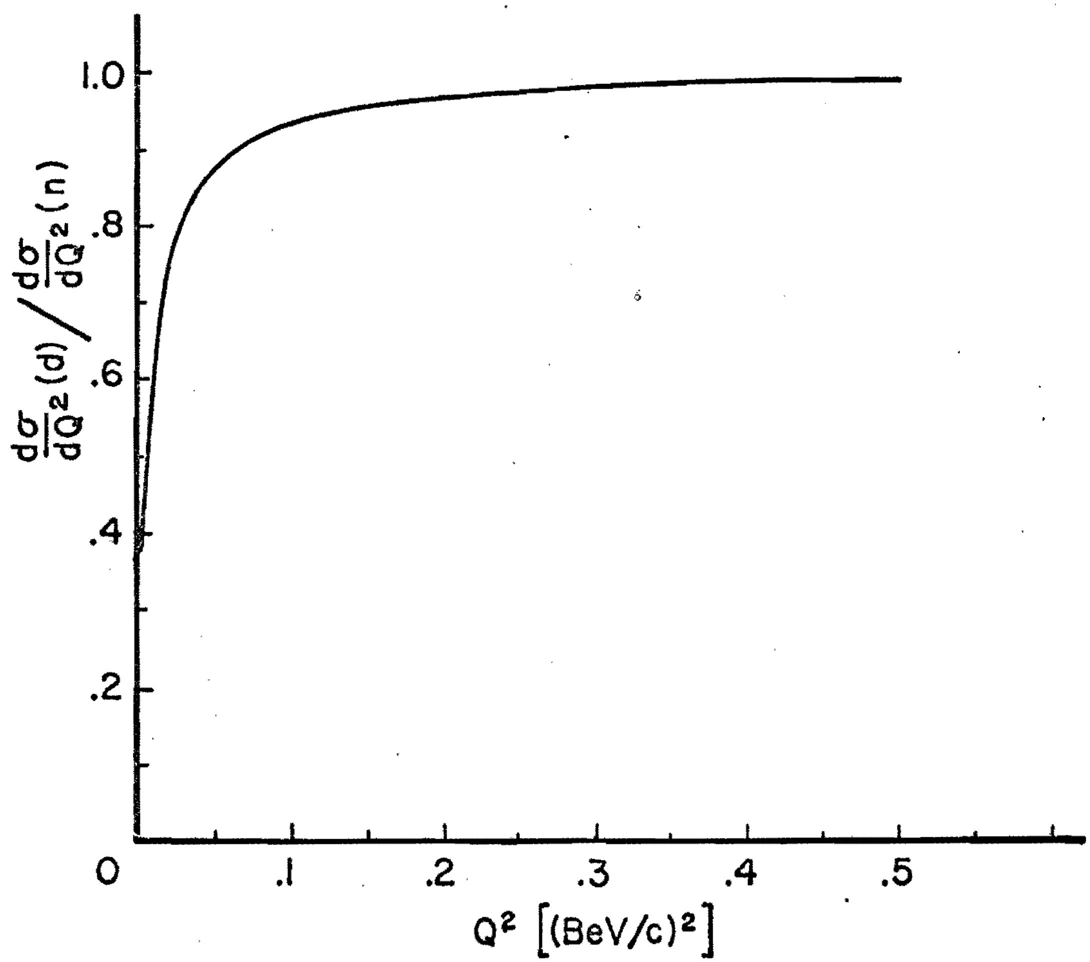
$$- (\mu f_M) f_V \frac{K}{2m^2} + \lambda f_A (f_V + \mu f_M)\frac{Q^2}{mE_\nu} - \lambda f_A (bf_p)\left(\frac{m_\mu}{m}\right)^2 \frac{Q^2}{8E_\nu^2}$$

$$- \Big(f_V^2\left[I(Q)\left(1 - \frac{Q^2}{4E_\nu^2} - \frac{Q^2}{6m^2}\right) + \frac{1}{6}\frac{H(Q)}{m^2} - \frac{H(Q)}{mE_\nu}\right]$$

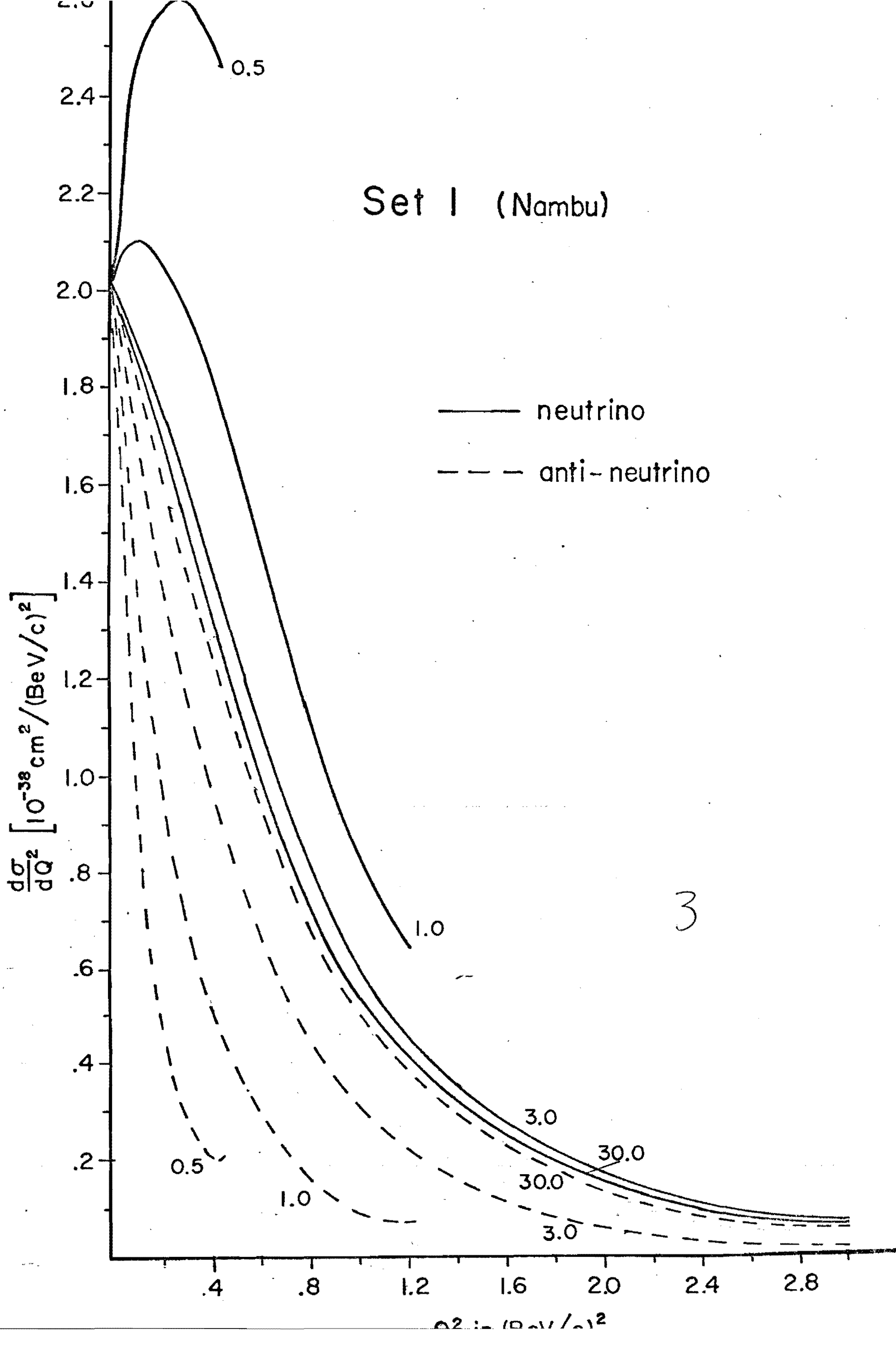
$$+ \frac{\lambda^2 f_A^2}{3}\left[I(Q)\left(1 + \frac{Q^2}{4E_\nu^2} + \frac{Q^2}{2m^2}\right) - \frac{7}{6}\frac{H(Q)}{m^2} - \frac{H(Q)}{mE_\nu}\right]$$

$$+ (\mu f_M)^2 \frac{Q^2}{12m^2} I(Q) - (\mu f_M) f_V\left[\frac{Q^2}{12m^2} I(Q) - \frac{H(Q)}{2m^2}\right]$$

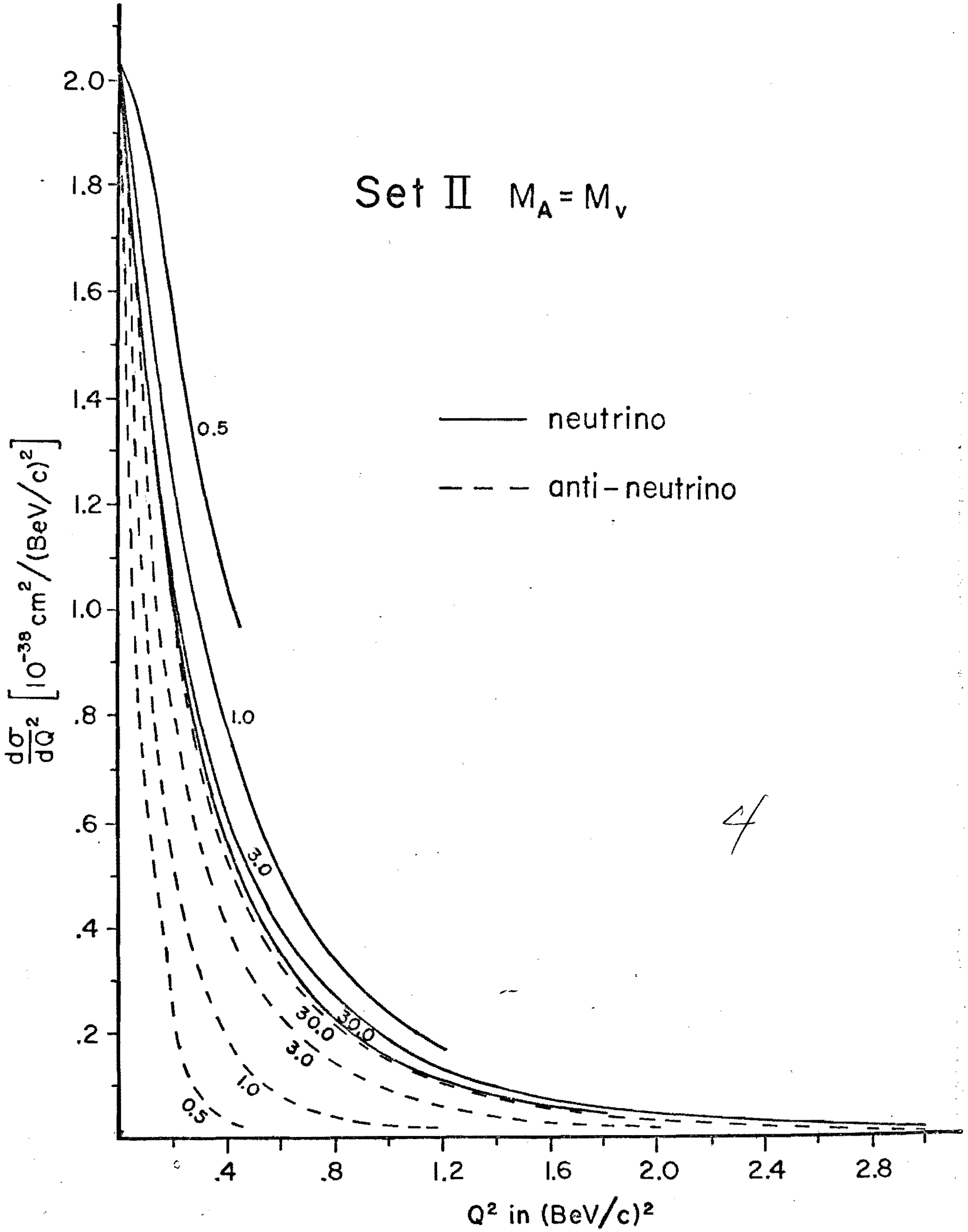
$$+ \lambda f_A (f_V + \mu f_M)\frac{Q^2}{3mE_\nu} I(Q) - \lambda f_A (bf_p)\left(\frac{m_\mu}{m}\right)^2 \frac{Q^2}{24E_\nu^2} I(Q)\Big)\Big\}$$



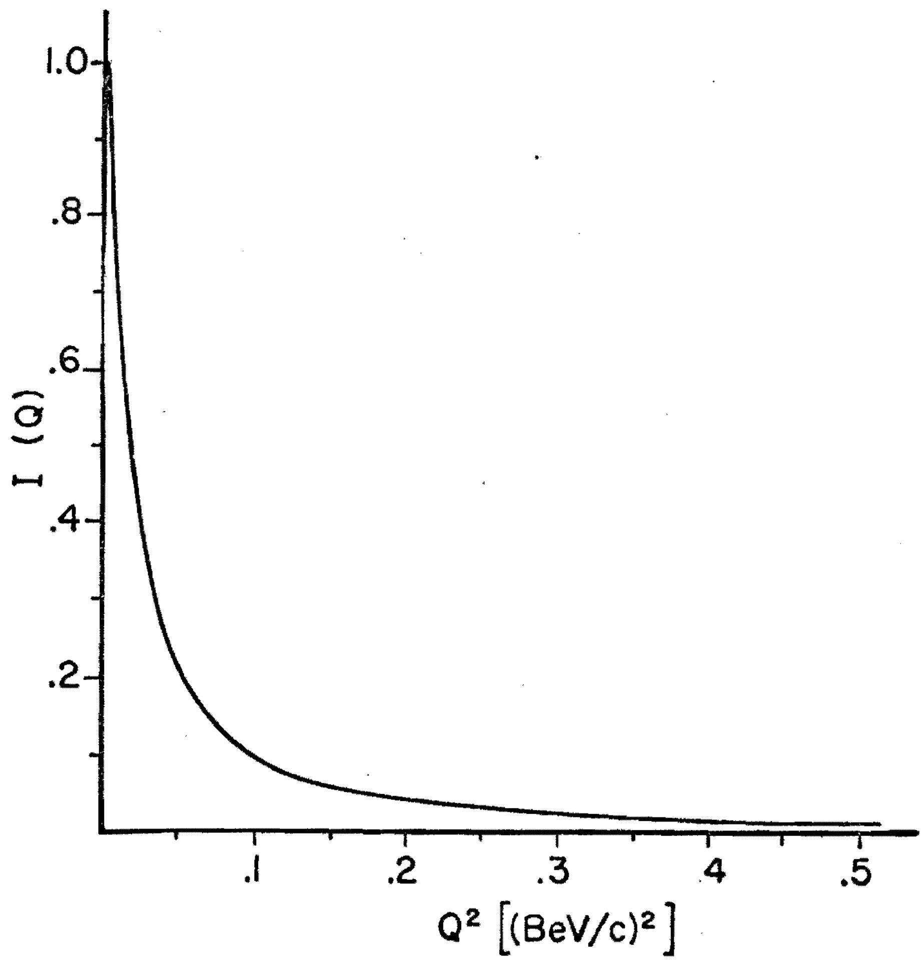
2



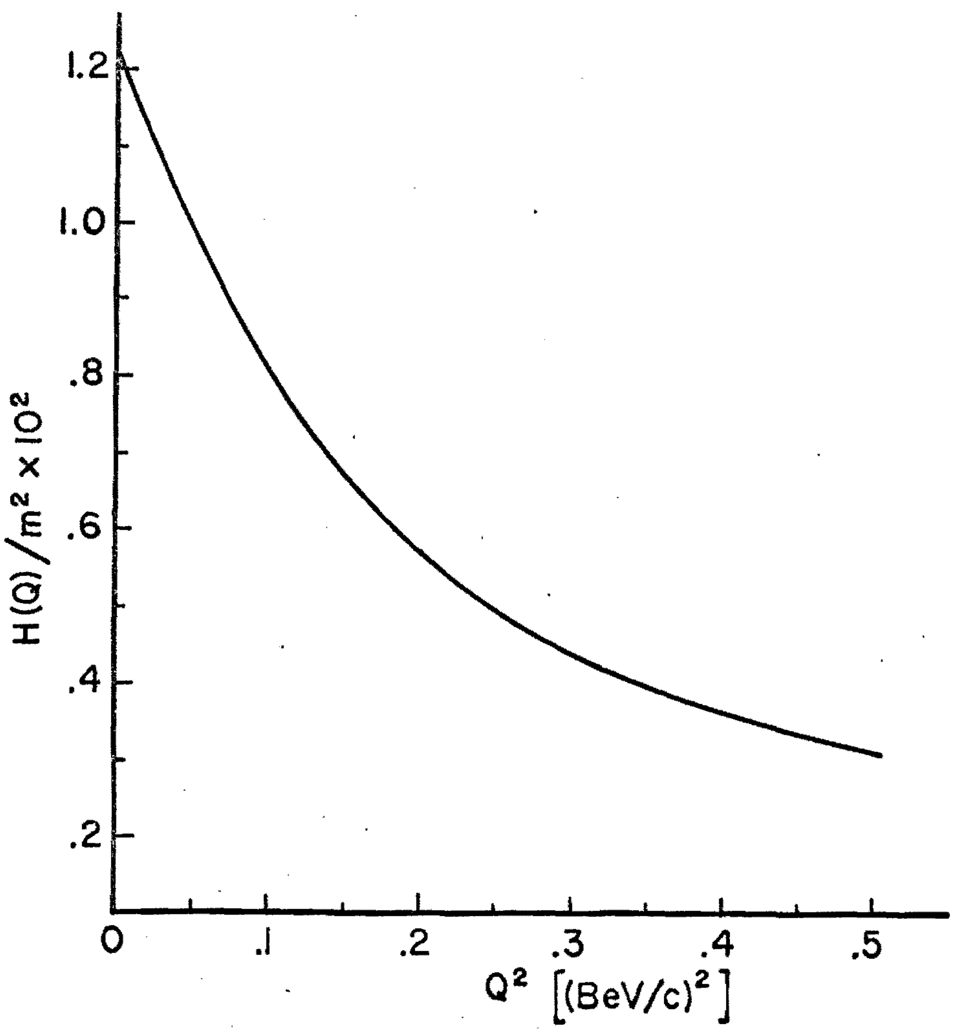
Set I (Nambu)
neutrino
anti-neutrino
2.4
2.2
2.0
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
.2
$\frac{d\sigma}{dQ^2}$ $[10^{-38}cm^2/(BeV/c)^2]$
.4
.8
1.2
1.6
2.0
2.4
2.8
0.5
1.0
3.0
30.0
30.0
3.0
1.0
0.5
3

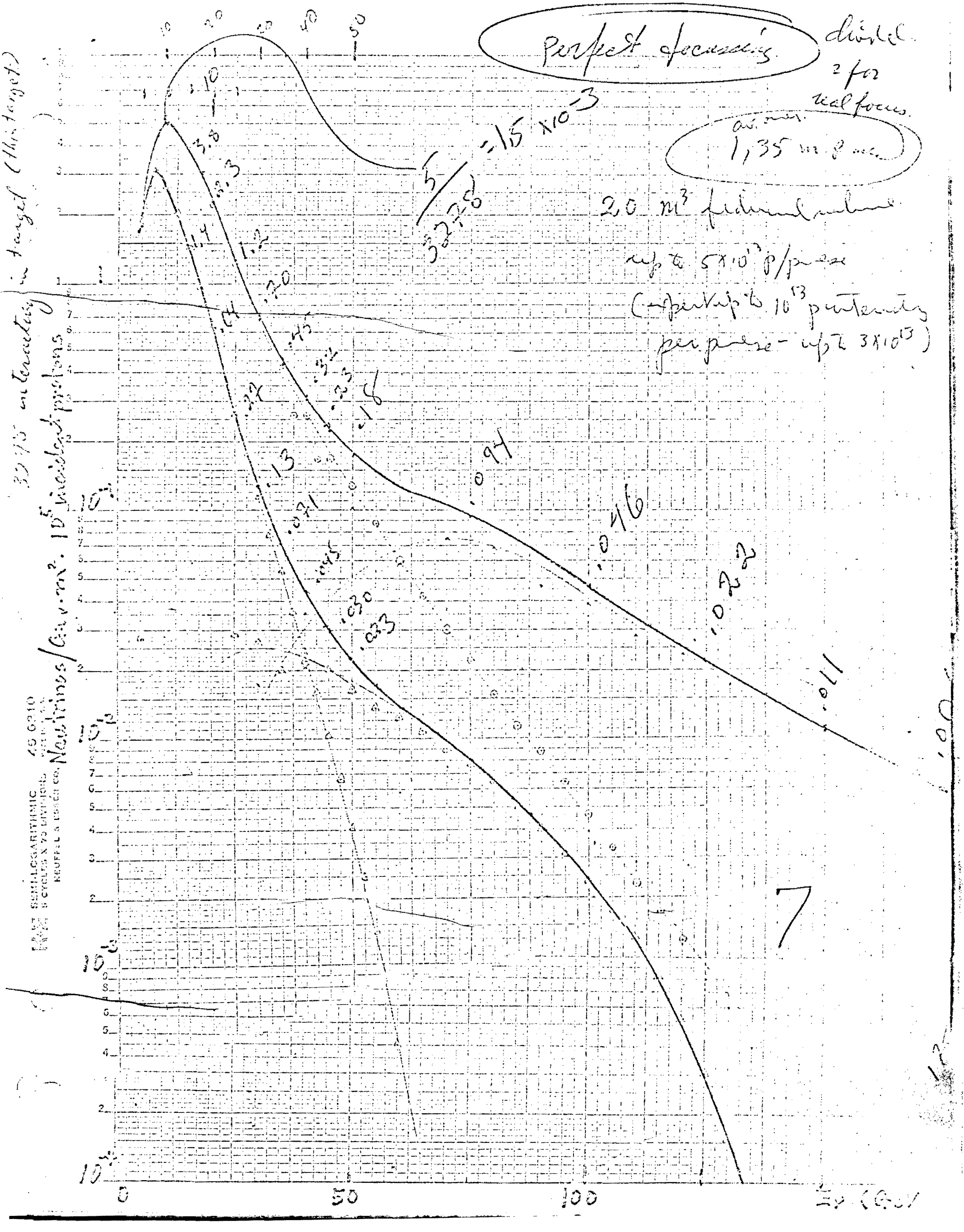


Set II $M_A = M_v$
neutrino
anti-neutrino
$\frac{d\sigma}{dQ^2}$ $[10^{-38} cm^2/(BeV/c)^2]$
Q^2 in $(BeV/c)^2$
2.0
1.8
1.6
1.4
1.2
1.0
.8
.6
.4
.2
.4
.8
1.2
1.6
2.0
2.4
2.8
0.5
1.0
3.0
30.0
30.0
3.0
1.0
0.5
4



5


1.2
1.0
.8
.6
.4
.2
H(Q)/m² x 10²
0
.1
.2
.3
.4
.5
Q² [(BeV/c)²]



Perfect focussing
divide 2 for real focus.
20 m³ fiducial volume
up to 5×10¹³ p/pulse
per pulse - up to 3×10¹³)
Neutrinos/GeV·m²·10⁵ incident protons
in target (thin target)
SEMI-LOGARITHMIC
3 CYCLES X 70 DIVISIONS
KEUFFEL & ESSER CO.
0
50
100
Ep (GeV)
7