HOW CAN WE CALCULATE THE RADIATION LOSS IN THE ELECTRON RING ACCELERATOR

J. D. LAWSON

Rutherford Laboratory, England

1. Introduction

One of the more important problems to be solved in assessing the potentialities of the electron ring accelerator (ERA) is that of estimating the energy radiated by the ring in passing through the accelerating cavities. For a useful device, this must necessarily be less than that gained from the accelerating field. Unfortunately, however, this radiation is very difficult to calculate, and estimates have varied over a wide range; some suggest that the effect is not too serious whereas others imply a severe limitation to the performance which would prevent very high energies from being reached.

In this paper an attempt is made to define the problem, and to examine the relation between the various models in the light of a general consideration of energy balance. No firm conclusion is reached, except that further work is needed to clarify the situation.

2. Definition of the Problem

The present suggestion is for an accelerating structure consisting of an array of separately fed cavities joined by drift tubes, as shown in the figures 1 and 2. An actual system would be almost periodic, but not quite so, since the energy of the particle increases continuously. At high values of $\gamma=(1-\beta^2)^{-\frac{1}{2}}$ the structure itself might be strictly periodic, but the fields associated with the charge would change continuously as γ increases.

It is premature to consider the behaviour of almost periodic systems, however, while serious doubt still remains about what happens in single cavities on the one hand and infinite periodic structures on the other.

In all calculations the radiating charge has been assumed to be rigid and infinitely massive, moving at a constant normalized velocity β. The energy of the charge on entry to a cavity is denoted by U, and the net change of energy in passing the gap by ΔU, so that

$$\begin{aligned} \Delta U &= Q\int E dx - BQ^2 \\ &\approx QEG - BQ^2 \end{aligned} \tag{1}$$

where Q is the charge, G is the gap and E is the accelerating field in the cavity in the absence of the charge [1]. B is a constant of dimensions L^{-1} which is a function of γ and the cavity geometry. The problem is to find B. In this paper attention is confined to the two geometries shown in the figures. The single cavity can be considered as the limit of these structures as $D \rightarrow \infty$

3. Energy Balance

Before examining explicit calculations of B, a simple physical argument leading to a lower limit in a cylindrical structure will be presented.

In the special case when the energy radiated by the charge just balances that received from the accelerating field, $\Delta U = 0$, so that from equation (1) it follows that $E/Q = B/C$. The energy radiated in this case may be related to that stored in the cavity before the charge arrives by setting

$$\begin{aligned} BQ^2 &= \pi R^2 G \cdot f \cdot E^2/8\pi \\ &= f^2 R^2 GE/8 \end{aligned} \tag{2}$$

where f is a constant, and $E^2R^2G/8$ is approximately the stored energy in the shaded volume indicated in the first figure. For a relativistic particle on the axis f might be expected to be of order unity, it can hardly be much greater since a light signal cannot reach the deeper recesses of the cavity in time to "tell" the energy to flow into the charge [2].

Substituting $E/Q = B/G$ in equation (2) yields

$$B = 8G/fR^2. \tag{3}$$

Formally this equation just expresses B in terms of another constant f. With the plausible assumption that $f \lesssim 1$ this becomes

$$B \gtrsim 8G/R^2. \tag{4}$$

(In ref. [1] a similar analysis using the theory of ref. [3] suggested the very satisfactory value for f of $1/\pi$; unfortunately however a 4π, lost when converting from MKS to gaussian units, converts this to 4. Using the analysis above, this figure becomes 16, an unlikely value; this point will be discussed further in section 4.

For planar geometry a somewhat different notation and criterion is relevant. The radiation loss per unit length in a direction perpendicular to the paper is bq^2, where q is the charge per unit length and b a dimensionless quantity. If, instead of a valume $\pi R^2 G$ in the cylindrical geometry a volume G^2 per unit length is taken, as shown in the second figure, then the criterion corresponding to equation (3) becomes

$$b=8\pi/f. \quad (5)$$

It might be argued that f could reasonably be expected to be greater than unity in this case so that the condition $b \gtrsim 8\pi$ corresponding to equation (4) is too restrictive.

Before studying specific models, some general observations on the behaviour of single cavities and infinite periodic systems will be made.

4. Physical Description of the Radiation Process

There are several valid ways of visualizing the radiation process; which one is used is somewhat a matter of personal preference, but care must be taken not to mix these viewpoints. The most convenient for a general discussion is probably the more formal description in terms of a manifold of field oscillators, whose structure is determined by the boundary conditions, and which are excited into forced oscillations by the charge. The problem then is formally to characterize these oscillators, calculate the coupling constants, and put in free oscillations to accord with the spatial and temporal boundary conditions.

First, a single lossless cavity with entry and exit pipes ($D \to \infty$ in fig. 1), will be considered. Such a cavity can support a finite set of trapped modes, corresponding to frequencies below the cut off frequency of the pipe, given by $\frac{\lambda_c}{2\pi} \sim R$. If a charge passes through such a cavity it excites these trapped modes, but it also gives rise to untrapped radiation of shorter wavelength $\frac{\lambda_c}{2\pi} < R$ which ultimately scatters out of the cavity and flows in both directions down the pipe. Such radiation covers a continuum of frequencies, and does not show the line spectrum of the trapped modes. Indeed, the two processes are very similar to excitation to bound levels and ionization respectively of an atom by a moving charge.

For a classical point charge all frequencies are present; fourier analysis of the field shows however that frequency components of wavelength λ die away in a distance of order $\gamma\lambda/2\pi$ from the charge[4].Radiation from a point charge on the axis into frequencies such that $\frac{\lambda}{2\pi} < R/\gamma$ is therefore small. The limiting frequencies of importance increase with γ, and one might for this reason expect some increase of B with γ also.

For an array of cavities with $DG \gg 1$ it is fairly obvious that the energy in the trapped modes for N cavities is just N times the trapped energy in one cavity. The continuum radiation however interferes coherently in a multi-cavity system and it is not immediately evident that the same relation holds. In a periodic system, the frequencies at which loss occurs, form a closely spaced line spectrum, and it is perhaps not unreasonable to expect that the envelope of the spectrum is similar in shape to that of the continuous spectrum appropriate to a single cavity, and that the power lost per cavity is the same in both cases.

Before cxamining the general interaction between a point charge and an infinite cavity array, it is instructive to see what happens in a non dispersive structure which supports a single mode. (This discussion, leading to consideration of a dispersive structure, is based on that given by Pierce[5] to which the reader should refer for a more thorough and detailed treatment). As before, an infinitely massive particle and lossless circuit are assumed. If the particle velocity is not equal to the velocity of free waves on the structure it causes a localized disturbance which travels with the particle, and (for a non dissipative structure) there is no interchange of energy between the particle and structure. If on the other hand the phase velocities are equal, the wave amplitude varies linearly along the line, and energy flows from charge to the field or vice versa. Since the amplitude of the wave changes linearly, the rate of energy loss also varies linearly with time. This behaviour corresponds to that of a linear oscillator; if there is dissipation the amplitude likewise limits to a constant value at which the dissipation equals the work done by the charge against the electric field associated with the travelling disturbance.

The cavity array in the figure on the other hand represents a dispersive lossless line and this behaves somewhat differently. The disturbance generated by the charge spreads out. Regarding the system as a continuum of lossless oscillators of spectral density which is approximately uniform in the neighbourhood of a resonant mode, it may readily be shown that in an initially field free structure energy is absorbed by the system as a whole at a constant rate. The central resonant oscillator behaves as in a non dispersive circuit and absords energy at a linearly increasing rate; nearly resonant oscillators absorb energy for $v/4\Delta v$ cycles, where Δv is the difference of phase velocity between and almost resonant wave, they then fall out of step and subsequently beat up and down. the resonant The linearly increasing rate of energy gain of the resonant oscillators is compensated for by the fact that the bandwidth of effectively resonant oscillators decreases inversely with time, so that the overall rate of energy absorption is constant. In his paper Pierce shows how the uniform loss of Cherenkov radiation can be described in this way.

In a dispersive system the amplitude of each oscillator settles down to a constant value, and the beating up and down characteristic of a lossless system dies out. The total rate of energy transfer to the assembly of oscillators however is greater than for a corresponding lossless system.

When calculating the radiation loss into infinite dispersive but non dissipative systems it is of course important to ensure that the free oscilation amplitudes of the oscillators are correctly chosen.

5. Comparison of Specific Models. (a) Closed Cavity Models

A number of calculations of losses in single cavities have been made, but in few of these have specific values of B been quoted. It is not the present intention to discuss the merits of these models in detail, but merely to quote the results and list some possible weaknesses in Table 1.

Table 1

Model	Reference	Value of B	Comments
Modal Analysis	[3]	$G/2R^2$	Neglects wavelength less than $2\pi R$, (trapped modes only are included).
Diffraction	[6]	$0.6G^{\frac{1}{2}}\gamma^{\frac{1}{2}}/R^{3/2}$ ($\gamma \gg 1$)	Approximations poor except perhap in high frequency limit, ($\frac{\lambda}{2\pi} < R/\gamma$).
Wave Matching	[7]	$\sqrt{2}G^{\frac{1}{2}}\gamma^{\frac{1}{2}}/R^{\frac{3}{2}}$ ($\gamma \gg 1$)	Only one component of field is matched.

The inequality (4) is only satisfied by the diffraction and wave matching models, and only then if γ is large enough. This suggests that the modal analysis model is always an underestimate, and the other models are underestimates at low values of γ.

Although radiation into chains of resonators has often been studied there appears to be only one published result which gives B explicity [8]. In this analysis $B=G^2/2R^2D$; the calculation includes a cut-off at $\lambda/2\pi=R$, and is therefore again likely to be an underestimate. It is identical to the single cavity result when $D=G$.

A related calculation, the result of which is known to be exact, is that of the energy radiated by a charge passing through a hole of radius R in a conducting plate. This gives[3] $B=\gamma/R$ when $\gamma \gg 1$. The radiation loss is proportional to γ, as one might expect by analogy with bremsstrahlung. This result might appear at first sight to conflict with the results of the diffraction and wave matching models, which predict a $\gamma^{\frac{1}{2}}$ dependence. If the plates are spaced so closely however that the field associated with the charge does not have time to readjust to free space conditions, the radiation is reduced; this can be estimated quite simply by finding the condition that the sideways diffraction of shortest wavelengths is large compared with the hole radius. Such a calculation yields the criterion $G \gg \gamma R$ for the screens to be independent; this will not be satisfied in any practical arrangement.

5 (b) Two Dimensional Open Models

The radiation from the open structure shown in fig. 1 for D=G can be calculated exactly, and it is therefore of interest to enquire whether the result is relevant to the closed cavity situation. Such a calculation has been performed by Bolotovsky and Voskresensky [9] and extended by Sessler[10].

The first point of interest is the very different values of b given by the two dimensional version of the diffraction and wave-matching calculations for a single cavity on the one hand and the accurate calculation for an infinite system on the other. In table 2 the value of b is given for the diffraction model and accurate calculations; also shown is $f=8\pi/b$ for a typical particular case:

Table 2

Model	Reference	b	f for the special case of $D/H=4, \gamma=100$
Diffraction model	[6]	$2.8\left(\gamma \frac{D}{H}\right)^{\frac{1}{2}}$	0.45
Accurate calculation	[10]	$0.8\left(\frac{D^3}{\gamma H^3}\right)^{\frac{1}{2}}$	40

The ratio between the results from the two models is of order γ. Although one can accept that f can be greater than unity, values as large as 40 (and increasing with γ) are at first sight difficult to understand, and are associated with the fact that the system is open and infinite. Since the system is open, energy can flow in a direction perpendicular to the structure, giving at least some modes which are effectively dissipative; since it is infinite, the resonant and near resonant dissipative modes limit to a constant amplitude as described at the end of section 4.

6. Characteristics of a Practical System

The accelerating system of an actual accelerator would, as remarked earlier, be equivalent to something between a single cavity and an infinite periodic chain. It is important to establish whether the loss per cavity in an infinite chain differs substantially from the loss in a single cavity; if it does not, then attention can be focused on a single cavity, if on the other hand there is a large difference then it will be necessary to find criteria for the transition between periodic and almost periodic structures, and also to take into account losses in the structure at high frequencies. The problem of irregularities and tolerances would become important at high energies where very high frequencies are significant.

A further assumption which has been made in the analysis is that the ring is a rigid body. This is not so, and it may be necessary to make some estimate of the nature of the radiation reaction force to ensure that this does not cause any distortion of the ring. Such distortion could supply or absorb energy and invalidate equation(1).

7. Some Numerical Estimates

It is impossible to make any general parameter studies of the ERA until the radiation problem is solved. It is perhaps not unreasonable however as a first guess to take a value of B equal to the minimum suggested by the energy balance argument. For a reasonable design the radiation loss should not exceed half the energy gained from the electric field; inserting this condition in equation (1) yields $EG > 2BQ$. Expressing B in terms of f from equation(4)

$$E > 8Q/fR^2. \tag{6}$$

This equation has a very simple physical interpretation, namely that the accelerating field must exceed 8/f time the field which would be produced at the gap by a stationary point charge Q placed on the axis. For such a charge containing N electrons the field at R cms is 1.4×10^{-13} N/R^2 Mev/cm, so that equation (6) may be written

$$E > 1.12 \times 10^{-12} N/fR^2 \text{ Mev/cm}. \tag{7}$$

Taking an optimistic figure of 4 for f, and $N = 10^{14}$, $R = 5$ this gives 1.1 Mev/cm, quite a high figure.

It is of interest to note also that the effective radius R_0 from which a uniformly accelerated charge in a uniform electric field extracts energy is given by

$$R_0 = (E/8Q)^{\frac{1}{2}} \tag{8}$$

This is identical with equation (7), with $R = R_0$ and $f = 1$.

8. Summary and Conclusion

A simple energy balance argument applicable to a relativistic particle moving through either single cavity or a chain of cavities suggests a lower limit to B, the coefficient relating the radiation loss per cavity to the square of the charge. This limit is sufficiently high nevertheless to imply severe constraints to the parameters permissible in an ERA. The argument gives no indication of the dependence of B on γ, but by implication excludes a form $B \propto \gamma^{-n}$, since, for sufficiently high γ, such a form must violate the condition $B \geqslant B_{min}$.

The values of B for some specific models have been examined, two of these satisfy the criterion, but only for large γ.

Estimates based on an accurate calculation of the radiation from a line charge passing close to a semi-infinite array of plates severely violate the criterion, and also differ considerably from single cavity estimates with the same geometry. It would appear therefore that estimates based on infinite open geometry are not appropriate to the finite closed system which constitutes the ERA accelerating system.

Further work is needed to clarify the situation, indications are however that radiation loss in the ERA is likely to prove a severe limitation.

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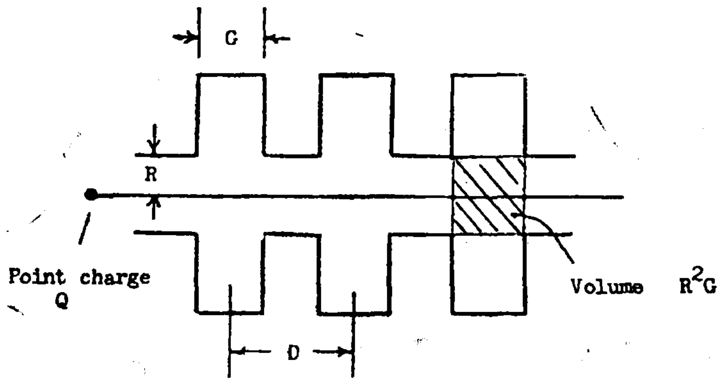

Fig. 1. Closed cylindrical geometry

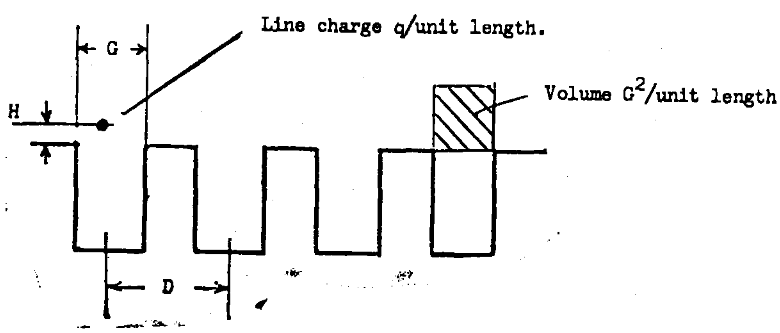

Fig. 2. Open planar geometry

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ДИСКУССИЯ

Рубин: В выступлении доктора ЛАУСОНА при оценке потерь на излучение использовался коэффициент пропорциональности f, которому довольно произвольно придавались числовые значения (например, f=4). Однако, само существо задачи заключается практически в определении этого коэффициента. Это можно показать хотя бы рассмотрев задачу о радиационных потерях при пролете заряда через отверстие в бесконечном экране. В данном случае имеются четыре размерных параметра; а-радиус отверстия, q-заряд, v_0-скорость заряда, с-скорость света. С помощью этих величин можно составить только одну комбинацию, имеющую размерность энергии $U \sim \frac{q^2}{a} f$, где f-функция от безразмерного параметра $\beta = \frac{v_0}{c}$ (или γ)

Решение задачи ДНЕСТРОВСКИМ и КОСТОМАРОВЫМ заключалось в определении этой функции, которая оказалась пропорциональной γ, т. е, может быть $\gg 4$ Повидимому, аналогия имеет место при пролете заряда около любого одиночного препятствия, причем роль а будет играть некоторый характерный размер (прицельный параметр), а остальные линейные параметры в виде безразмерных комбинаций войдут в функцию f.

Совсем другая ситуация получается, когда имеется бесконечная периодическая структура. Как оказывается, в этом случае нет сильной зависимости от γ, а численные оценки, если результаты привести в той же форме, как у д-ра ЛАУСОНА, дают значение для $f \ll 4$.