CMB Anisotropy Generated by Nonlinear Structures

— Effect of Λ —

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Abstract

We study the cosmic microwave background (CMB) anisotropy generated by nonlinear structures in a flat universe with a cosmological constant. We model a spherical compensated void/lump by a family of Lemaitre-Tolman-Bondi spacetimes, and numerically solve the null geodesic equations together with the Einstein equations. We find that voids redshift CMB photons regardless of Ω (or z), while lumps blueshift CMB photons if Ω (or z) is small. Those nonlinear structures could be observed as cold/hot spots in the CMB sky map.

1 Introduction

Recently it has been argued [1, 2] that the anomalies of the cosmic microwave background (CMB) such as octopole planarity and the alignment between quadrupole and octopole components [3], anomalously cold spots on angular scales $\sim 10°$ [4], and asymmetry in the large-angle power between opposite hemispheres [5] could be explained by the Rees-Sciama (RS) effect [6] of nonlinear large-scale structures.

To test such a conjecture, we study the RS effect due to nonlinear structures in a flat universe with a cosmological constant Λ. We model a spherical compensated void/lump by a family of Lemaitre-Tolman-Bondi (LTB) spacetimes, and numerically solve the null geodesic equations together with the Einstein equations. In the literature [7] the CMB signature of voids/lumps has been extensively studied, using LTB spacetimes; however, $\Lambda = 0$ has been assumed in all the papers. In this paper, we consider large voids/lumps (> 100Mpc) in a flat universe with $\Lambda > 0$, as suggested by recent observations.

2 Model and Basic Equations

Consider a family of spherically symmetric spacetimes with dust and a cosmological constant. Their general solutions are represented by the LTB metric,

$$ds^2 = -dt^2 + \frac{R'^2(t,r)}{1+f(r)}dr^2 + R^2(t,r)(d\theta^2 + \sin^2\theta d\varphi^2), \tag{1}$$

which satisfies the Einstein equations,

$$\dot{R}^2 = \frac{2Gm(r)}{R} + \frac{\Lambda}{3}R^2 + f(r), \quad \rho(t,r) = \frac{m'(r)}{4\pi R^2 R'}, \tag{2}$$

where $' \equiv \partial/\partial r$ and $\dot{} \equiv \partial/\partial t$. ρ is energy density of matter, and $m(r)$ and $f(r)$ are arbitrary functions, which should be fixed by initial conditions.

Our model of a void/lump is composed of three regions: the outer flat Friedmann-Robertson-Walker (FRW) spacetime ($r > r_+$), the inner open FRW spacetime ($r < r_-$) and the intermediate shell region ($r_- < r < r_+$). Hereafter we denote quantities in $r > r_+$ and in $r < r_-$ by subscripts $+$ and $-$,

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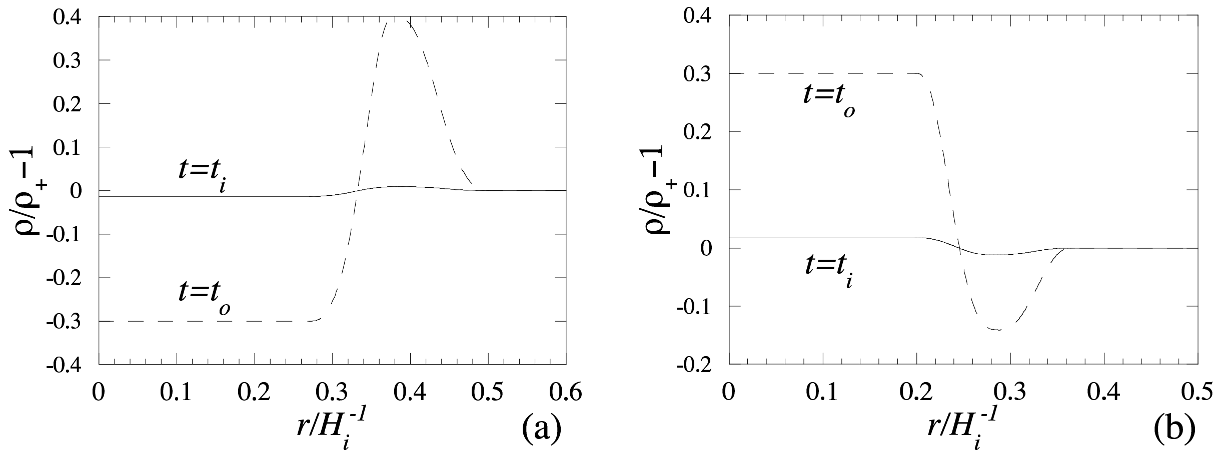

Figure 1: Examples of initial and evolved profiles of $\rho(t,r)$. (a) and (b) represent a void ($\delta<0$) and a lump ($\delta>0$), respectively.

respectively. At the initial time $t=t_i$ we give small perturbations on ρ in such a way that $\rho_-(t_i)=\rho_+(t_i)(1+\delta_i)$ with $\delta_i \ll 1$ and in the shell,

$$\rho(t_i,r)=\begin{cases}\rho_- & \text{for} \quad r\le r_-,\\ \dfrac{\rho_c-\rho_-}{16}(3X_-^5-10X_-^3+15X_-+8)+\rho_- & \text{for} \quad r_-\le r\le r_c,\\ \dfrac{\rho_+-\rho_c}{16}(3X_+^5-10X_+^3+15X_++8)+\rho_c & \text{for} \quad r_c\le r\le r_+,\\ \rho_+ & \text{for} \quad r\ge r_+,\end{cases} \tag{3}$$

$$\text{where} \quad r_c\equiv\frac{r_++r_-}{2}, \quad w\equiv\frac{r_+-r_-}{2}, \quad X_\pm\equiv\frac{r-r_c\mp w/2}{w/2}, \tag{4}$$

and $\rho_c\equiv\rho(r_c)$ is determined by the boundary condition at $r=r_+$. Examples of initial and evolved configurations of $\rho(t,r)$ are shown in Fig. 1.

As for initial values of $H(t,r)\equiv\dot{R}/R$, we assume $H(t_i,r)=H_+(t_i)=H_-(t_i)$. We fix the gauge of the radial coordinate as $r=R(t_i,r)$. In this model there are four dimensionless parameters,

$$\Omega\equiv\frac{8\pi G\rho_+}{3H_+^2}, \quad \delta\equiv\frac{\rho_-}{\rho_+}-1, \quad \frac{R(r_c)}{H_+^{-1}}, \quad \frac{w}{r_c}, \tag{5}$$

which should be fixed at a certain time.

Let us consider a photon which passes the center, $r=0$. The geodesic equations with the metric (1) are given by

$$\frac{dt}{d\lambda}=k^t, \quad \frac{dr}{d\lambda}=k^r, \quad k^\theta=k^\varphi=0, \quad k^r=\epsilon\frac{\sqrt{1+f}}{R'}k^t, \quad \epsilon\equiv\mathrm{sign}\left(\frac{dr}{dt}\right), \tag{6}$$

$$\frac{dk^t}{d\lambda}=-\frac{\dot{g_{rr}}}{2}(k^r)^2, \quad \frac{d}{d\lambda}(g_{rr}k^r)=\frac{{g_{rr}}'}{2}(k^r)^2, \quad g_{rr}\equiv\frac{(R')^2}{1+f} \tag{7}$$

By numerical integration of the null geodesic equations (6) and (7) together with the Einstein equation (2), we evaluate temperature fluctuations,

$$\frac{\Delta T}{T}=\frac{k^t}{k^t_+}-1, \tag{8}$$

where k^μ_+ is the null vector of another photon which passes the homogeneous region, and given by $k^t_+\propto 1/a_+$.

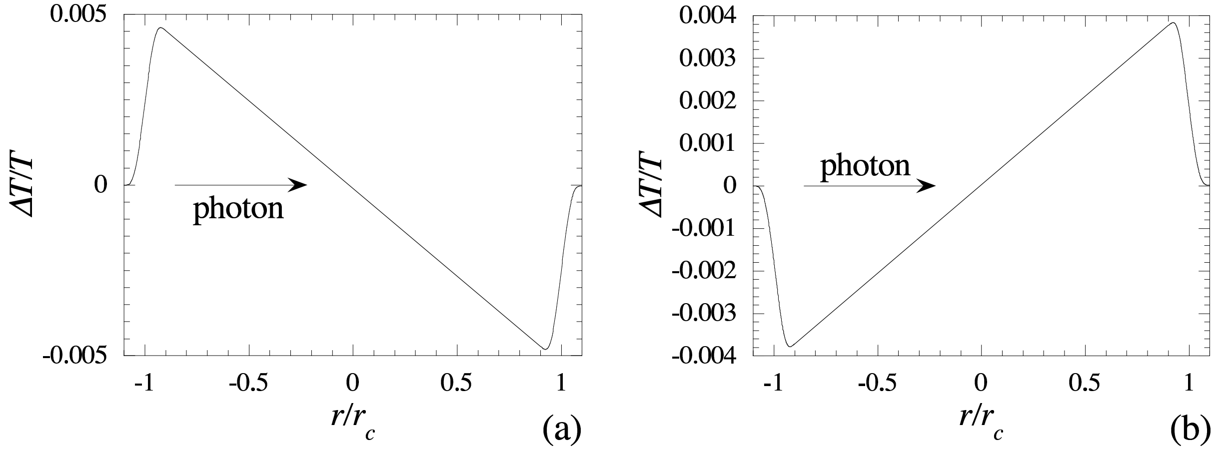

Figure 2: Temperature fluctuations of photons passing through a void (a) and a lump (b). We put $\delta_o = \mp 0.3$, $\Omega_o = 0.24$, $R_o(r_c) = 0.1 H_o^{-1}$ and $w/r_c = 0.1$. The arrow indicates the traveling direction of a photon.

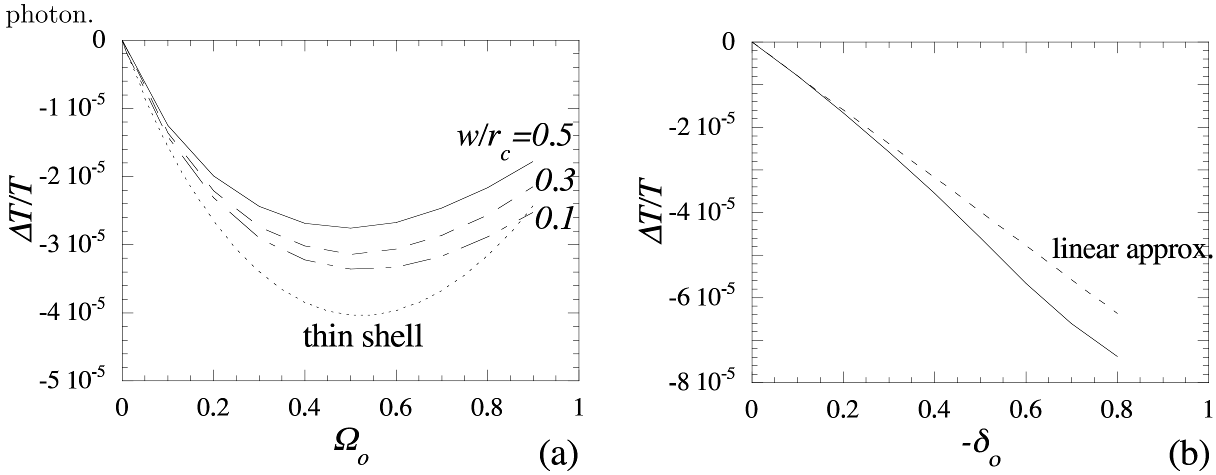

Figure 3: Temperature fluctuations for a large void with $R_o(r_c) = 0.1 H_o^{-1}$. (a) shows $\Delta T/T$ versus Ω_o with $\delta_o = -0.3$. The dotted line indicated by "thin shell" shows $\Delta T/T$ for the thin-shell model [2]. (b) shows $\Delta T/T$ versus δ_o with $\Omega_o = 0.24$ and $w/r_c = 0.3$ The dashed line indicated by "linear approx." shows a linear extrapolation from the values for $|\delta_o| \leq 0.1$.

3 Results and Discussions

Figure 2 shows temperature fluctuations of photons passing through a void/lump. The subscript o denotes quantities at the time t_o when a photon comes out of a void/lump. Although $\Delta T/T$ temporarily becomes $\sim 10^{-3}$, it finally reduces to $\sim 10^{-5}$ because of mass compensation of a void/lump.

In what follows we discuss only the eventual values of $\Delta T/T$ measured outside a void/lump. Figure 3 shows $\Delta T/T$ for a large void. (a) indicates how $\Delta T/T$ depends on Ω_o and the width of the shell w/r_c. We find our result is consistent with that for the thin-shell model [2], and that $\Delta T/T$ decreases as w/r_c increases. (b) shows that the nonlinear effects enhance $\Delta T/T$.

In Fig. 4 we plot $\Delta T/T$ for a large lump. According to Martínez-González and Silk [8], lumps redshift CMB photons in the Einstein-de Sitter universe ($\Omega = 1,\ \Lambda = 0$), just like voids. In contrast, we find in (a) that lumps blueshift CMB photons in low-Ω universes. That is, large lumps at high-z and at low-z have opposite effects on the CMB anisotropy. We also see that our result is consistent with that for the top-hat model calculated by the second-order perturbation [9]. (b) shows that the nonlinear effects reduce $\Delta T/T$, in contrast to those for a void.

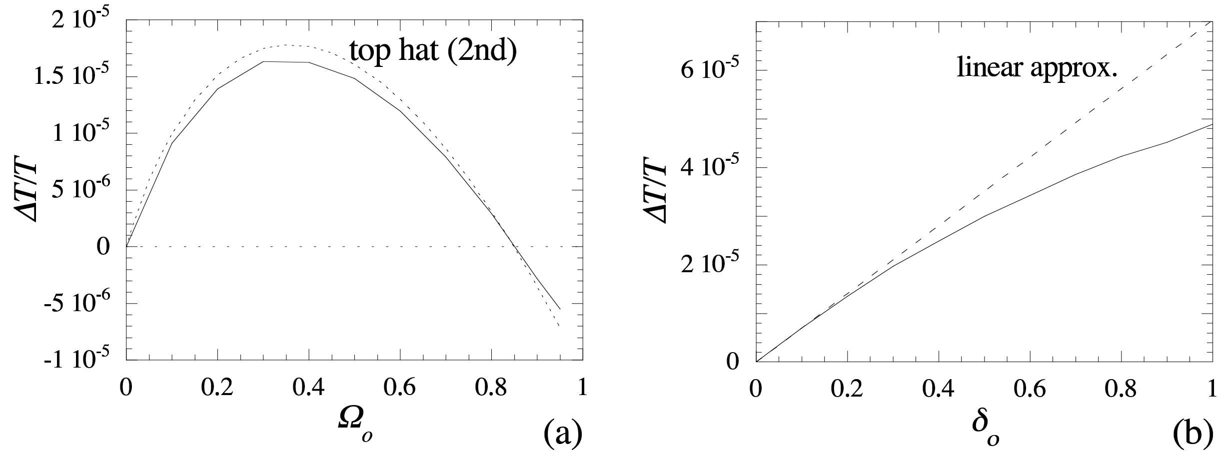

Figure 4: Temperature fluctuations for a large lump. (a) shows $\Delta T/T$ versus Ω_o with $\delta_o = 0.3$, $R_o(r_c) = 0.09H_o^{-1}$ and $w/r_c = 0.1$. The dotted line indicated by "top hat (2nd)" shows $\Delta T/T$ for the top-hat model calculated by the second-order perturbation [9]. (b) shows $\Delta T/T$ versus δ_o with $\Omega_o = 0.24$, $R_o(r_c) = 0.1H_o^{-1}$ and $w/r_c = 0.2$.

Our results indicate that, if quasi-linear ($|\delta| \sim 0.3$) and extra-large ($R \sim 0.1H^{-1}$) voids/lumps exist, they could be observed as cold/hot spots in the CMB sky map. Furthermore, with such observations we could estimate the quantity of dark matter in voids.

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