DETERMINATION OF THE GLUEBALL MASS SPECTRUM WITH THE SPIN-ORBIT INTERACTION IN NONPERTURBATIVE QCD-APPROACH

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Abstract

The mass spectrum of the bound state is analytically derived. The mechanism for arising of the constituent mass of the bound state forming particles is explained. Change of the bound state mass and of the constituent mass of particles is analyzed by the varying the coupling constant. The mass spectrum of the two-gluon glueball is calculated taking into account spin-orbit and spin-spin interactions.

1. Introduction

For the last three decades theorists have been trying to derive the effective potential of the quark interaction, starting from the basic principle of the QCD which explains the requirement of the confinement and deconfinement of the constituent particles [1]. There is a lot of potential models of quarks which are constructed on the basis of non-relativistic picture of confinement. These models are mainly applicable to physics of hadrons consisting of heavy quarks. The properties of hadrons consisting of light quarks are dominated by the relativistic character of the interactions, which requires additional efforts to incorporate relativistic effects. It is known (for instance, [2]) that relativistic effects in the bound state formation in the quantum field systems can be taking into account as small corrections only in the weak coupling regime, but the strongly coupled systems like hadrons in QCD ultimately require fully relativistic consideration which can be adequately realised only by means of genuine field theoretical methods. One of the most powerful method of this kind is based on the Bethe-Salpeter equation. Application of Bethe-Salpeter equation in QCD assumes an appropriate approximate choice of the kernel, which is usually chosen on the basis of physical assumptions about nonperturbative QCD vacuum (see, for example [3, 4]).

Another field theoretical method based on the Fock-Feynman-Schwinger representation was suggested in [5]. This method successfully applied [6] for the description of the hadron and glueball mass spectra. The keynote of this approach is the presentation of the polarized loop function as a functional integral and the main problem is its integration. Of course, this integral is not evaluated in general but only in certain physical assumptions. One of the alternative methods of the functional integral evaluation and determination of the glueball mass with taking into account the nonperturbative and relativistic character of the interaction is suggested in the [7]. In this work, we was present one of the alternative methods of the bound state mass determination. The bound state mass determined in the form

$$M = \sqrt{m_1^2 - 2\mu^2 E'(\mu)} + \sqrt{m_2^2 - 2\mu^2 E'(\mu)} + \mu E'(\mu) + E(\mu). \tag{1}$$

The parameter μ can be determined from the equation

$$\frac{1}{\mu} = \frac{1}{\mu_1} + \frac{1}{\mu_2} = \frac{1}{\sqrt{m_1^2 - 2\mu^2 \cdot E'(\mu)}} + \frac{1}{\sqrt{m_2^2 - 2\mu^2 \cdot E'(\mu)}} , \tag{2}$$

where the following notation is used:

$$E'(\mu) = \partial E(\mu)/\partial\mu. \tag{3}$$

We will consider the parameters μ_1 and μ_2 as masses of the constituent particles in the bound state. These masses differ from m_1 and m_2 which represent the masses of a free state. To describe the mass spectrum of the relativistic bound state, the constituent mass, which differs from the mass of the initially free particle. Particularly, when describing the hadron mass spectrum, the masses of the valence and current quarks are introduced. On the other hand, if the bound state consists of two gluons, then the constituent mass of gluons is nonzero, according to (2). In this case, one can identify the two-gluon bound state with the pomeron which is broadly used in describing the mechanism of the inelastic scattering of particles. The quantity $E(\mu)$ is defined as eigenvalues of the interaction Hamiltonian with the nonperturbative correction. The nonperturbative correction to the interaction Hamiltonian represented as(the detail see [7])

$$\begin{aligned} H^0 &= \frac{1}{2\mu} \cdot \vec{P}^2 - \frac{4}{3}\frac{\alpha_s}{r} + V(0) ; \\ \triangle H^0_{nonper} &= -\frac{4}{3}\frac{\alpha_s}{r} \cdot \left[\frac{1}{\sqrt{1 + \ell(\ell+1)/(c^2 r^2 \mu^2)}} - 1 \right] . \end{aligned} \tag{4}$$

2. Calculation of the glueball mass spectrum taking into account the spin-orbit interaction

Let us determine the mass spectrum of the two-gluon bound state when all effects of the gluon-gluon interaction such as the one-gluon exchange, nonperturbative character, and spin-orbit corrections are taken into account. The total Hamiltonian can be written as a sum of two parts. The first one is the central Hamiltonian which corresponds to the conditions of the one-gluon exchange and nonperturbative character of interaction and also to the confinement. The second one is the Hamiltonian of the spin-orbit interaction

$$H = H_c + H_{spin} , \tag{5}$$

where H_c is the central part

$$H_c = \frac{1}{2\mu}\vec{P}^2 + \sigma_{ad} r - \frac{4}{3}\frac{\alpha_s}{r} - \frac{4}{3}\frac{\alpha_s}{r} \left[\frac{1}{\sqrt{1 + \ell(\ell+1)/(r^2\mu^2)}} - 1 \right] . \tag{6}$$

The second part of the Hamiltonian is defined in the standard form

$$H_{spin} = H_{SS} + H_{LS} + H_{TT} . \tag{7}$$

where H_{SS} is the spin-spin interaction Hamiltonian

$$H_{SS} = \frac{(\mathbf{S_1 S_2})}{\mu^2} \Delta V_v \ , \tag{8}$$

and also the spin-orbit interaction Hamiltonian

$$H_{LS} = \frac{(\mathbf{LS})}{8\mu^2} \left[\frac{3}{r}\frac{\partial}{\partial r} V_v - \frac{1}{r}\frac{\partial}{\partial r} V_s \right] \ , \tag{9}$$

and, at last, the tensor interaction Hamiltonian

$$H_{TT} = \frac{S_{12}}{48\mu^2} \left[\frac{1}{r}\frac{\partial}{\partial r} V_v - \frac{\partial^2}{\partial r^2} V_v \right] \ . \tag{10}$$

Here V_v is the vector potential corresponding to the one-gluon exchange

$$V_v = -\frac{4\alpha_s}{3} \frac{1}{\sqrt{r^2 + \ell(\ell+1)/\mu^2}} \ ; \tag{11}$$

V_s is the confinement potential

$$V_s = r\sigma_{ad} \ ; \tag{12}$$

and also the following notation is used:

$$\begin{aligned} &\mathbf{S} = \mathbf{S_1} + \mathbf{S_2} \\ &S_{12} = \frac{4}{(2\ell+3)(2\ell-1)} \left[\mathbf{L}^2\mathbf{S}^2 - \frac{3}{2}(\mathbf{LS}) - 3(\mathbf{LS})^2 \right] \ . \end{aligned} \tag{13}$$

Using the explicit form of the Hamiltonian introduced in equations (7)-(13), let us start to determine the mass of the glueball with the spin-spin interactions when $\ell = 0$. In this case, for the energy spectrum we have

$$\varepsilon_0(E) = \varepsilon_0^C(E) + \varepsilon_0^{SS}(E) \ , \tag{14}$$

where $\varepsilon_0^C(E)$ and $\varepsilon_0^{SS}(E)$ are the contributions of the Hamiltonians of the central and spin-spin interactions, respectively:

$$\begin{aligned} \varepsilon_0^C(E) &= \frac{(1+\rho)\omega}{2} + \frac{4\rho^2\mu\sigma_{ad}}{\omega^{3\rho-1}} \frac{\Gamma(4\rho)}{\Gamma(1+\rho)} - \frac{4\rho^2\mu E}{\omega^{2\rho-1}} \frac{\Gamma(3\rho)}{\Gamma(1+\rho)} - \\ &- \frac{16\alpha_s\rho^2\mu\omega^{1+\rho}}{3\Gamma(1+\rho)} \int_0^\infty du \frac{u^{3\rho-1}e^{-u\omega}}{\sqrt{u^{2\rho} + \ell(\ell+1)/\mu^2}} \ ; \\ \varepsilon_0^{SS} &= \frac{\alpha_s\rho(\mathbf{S_1 S_2})}{36\mu} \frac{\omega^{1+\rho}}{\Gamma(1+\rho)} \ , \end{aligned} \tag{15}$$

After some simplifications we obtain for the energy spectrum

$$E(\mu) = \sqrt{\sigma_{ad}} \cdot \min_{\{\rho, Z\}} \left[xA + \frac{1}{x}B \right] \ , \tag{16}$$

where the following notation is used:

$$A = \frac{Z^2}{8\rho^2}\frac{\Gamma(2+\rho)}{\Gamma(3\rho)} - \frac{4Z\alpha_s}{3}\frac{\Gamma(2\rho)}{\Gamma(3\rho)} - \frac{\alpha_s\rho(\mathbf{S}_1\mathbf{S}_2)}{144\rho^2}\frac{Z^3}{\Gamma(3\rho)}\,; \tag{17}$$

$$B = \frac{1}{Z}\frac{\Gamma(4\rho)}{\Gamma(3\rho)}\,; \qquad x = \frac{\mu}{\sqrt{\sigma_{ad}}}\,,$$

and the x parameter is derived from (1), (2). Using this parameter for the glueball mass we have

$$M = \sqrt{\sigma_{ad}}\left[2x + \frac{E(\mu)}{\sqrt{\sigma_{ad}}}\right]\,. \tag{18}$$

In this case, the mass glueball corresponding to the following states, is determined

$$J^{PC} = 0^{++}\,,\quad 1^{+-}\,,\quad 2^{++}\,.$$

Our numerical results are presented in Table 1.

Table 1. The mass spectrum of the glueball with taking into account the nonperturbative character of interaction and spin-spin interaction for the case of $\ell = 0$. In GeV units. $\sigma_{ad} = 0.45\ GeV^2$, $\alpha_s = 0.3$

J^{PC}	*our result*	*lattice data*	*Exp.*	*other works*
0^{++}	1.64	1.73 [8] 1.63 [9] 1.61 [10] 1.75 [11]	1.50 [12] 2.11 [12] 2.32 [13]	1.98 [14] 2.69 [15]
2^{++}	1.97	2.40 [17] 2.35 [9] 2.26 [10] 2.42 [11]	2.02 [16]	2.42 [14] 2.70 [15]

From Table 1, we can see that our results are in good agreement with the results of other authors. Let us now consider the general case when $\ell \neq 0$. We obtain the energy spectrum $E(\mu)$ for the total Hamiltonian from SE

$$E = E^{(C)} + E^{(SS)} + E^{(LS)} + E^{(TT)}\,. \tag{19}$$

Here $E^{(C)}$ is the contribution of the central interaction Hamiltonian

$$\begin{aligned} E^{(C)} &= \frac{x^2\sqrt{\sigma_{ad}}}{8\rho^2}\frac{\Gamma(2+\rho+2\rho\ell)}{\Gamma(3\rho+2\rho\ell)} + \frac{\sqrt{\sigma_{ad}}}{xz}\frac{\Gamma(4\rho+2\rho\ell)}{\Gamma(3\rho+2\rho\ell)} - \\ &- \frac{4\alpha_s xz\sqrt{\sigma_{ad}}}{3\Gamma(3\rho+2\rho\ell)}\int\limits_0^\infty du\frac{u^{3\rho+2\rho\ell-1}e^{-u}}{\sqrt{u^{2\rho}+z^2\ell(\ell+1)}}\,; \end{aligned} \tag{20}$$

$E^{(LS)}$ is the spin-orbit interaction contribution

$$E^{(LS)} = \frac{z^2\sqrt{\sigma_{ad}}(\mathbf{LS})}{8\Gamma(3\rho+2\rho\ell)}\left\{-\frac{\Gamma(2\rho+2\rho\ell)}{xz} + 4\alpha_s xz\int\limits_0^\infty du\frac{u^{3\rho+2\rho\ell-1}e^{-u}}{[u^{2\rho}+z^2\ell(\ell+1)]^{3/2}}\right\}\,; \tag{21}$$

$E^{(TT)}$ is the inclusion for the tensor interaction

$$E^{(TT)} = \frac{\alpha_s x z^3 \sqrt{\sigma_{ad}} S_{12}}{12\Gamma(3\rho + 2\rho\ell)} \int_0^\infty du \frac{u^{5\rho+2\rho\ell-1} e^{-u}}{[u^{2\rho} + z^2\ell(\ell+1)]^{5/2}} ; \tag{22}$$

and $E^{(SS)}$ is the contribution of the spin-spin interaction

$$\begin{aligned} E^{(SS)} &= \frac{\alpha_s \ell \sqrt{\sigma_{ad}} (\mathbf{S_1 S_2})}{18\Gamma(3\rho + 2\rho\ell)} \cdot x z^3 \rho^2 \int_0^\infty du \frac{u^{3\rho+2\rho\ell-1} e^{-u}}{[u^{2\rho} + z^2\ell(\ell+1)]^{5/2}} \\ &\times \left[u^{2\rho} + \frac{z^2}{2}(3+2\ell)(1+\ell) \right] . \end{aligned} \tag{23}$$

The parameter x is derived from the equation

$$2 + \frac{1}{\sqrt{\sigma_{ad}}} \frac{\partial E}{\partial x} - 0 , \tag{24}$$

and then the energy spectrum is determined in the following form:

$$E(\mu) = \min_{\{\rho, Z\}} [E(x,\ \rho,\ z)] . \tag{25}$$

The numerical results are in Table 2.

Table 2. The mass spectrum of the glueball for the general case. In *GeV* units. $\sigma_{ad} = 0.45\ GeV^2$, $\alpha_s = 0.3$

	$\ell = 1$			$\ell = 2$			$\ell = 3$		
	J^{PC}	*our result*	*other works*	J^{PC}	*our result*	*other works*	J^{PC}	*our result*	*other works*
$S = 0$	0^{--}	2.95		0^{++}	3.39	1.72[8]	0^{--}	3.95	
	1^{--}	2.99		1^{++}	3.42		1^{--}	3.97	3.81 [8]
				2^{++}	3.47	3.50 [11]	2^{--}	4.00	3.90 [8]
							3^{--}	4.05	4.10 [8]
$S = 1$	0^{-+}	2.92	2.59 [17]	0^{+-}	3.36	4.82 [8]	0^{-+}	3.90	3.64 [10]
	1^{-+}	2.95		1^{+-}	3.39	2.95 [8]	1^{-+}	3.95	
	2^{-+}	3.02	3.10 [17]	2^{+-}	3.44	4.10 [8]	2^{-+}	3.99	3.89 [10]
				3^{+-}	3.52	3.53 [8]	3^{-+}	4.03	
							4^{-+}	4.10	
$S = 2$	0^{--}	2.86		0^{++}	3.31	2.67[8]	0^{--}	3.90	
	1^{--}	2.89		1^{++}	3.33		1^{--}	3.92	
	2^{--}	2.95		2^{++}	3.38	2.38 [8]	2^{--}	3.95	
	3^{--}	3.05		3^{++}	3.46	3.69 [10]	3^{--}	4.00	

The mass spectrum of the two-gluon glueball is calculated taking into account spin-orbit and spin-spin interactions.

Our approach allows a unified description of the mass spectrum of glueball for various states with various spin and orbit quantum numbers. Further, we will apply our approach for the description of the glueball mass spectrum taking radial excitation into consideration and for determination of the hadron mass spectrum.

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VII.

APPLIED USE OF RELATIVISTIC BEAMS