

# Cosmology of multigravity

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## Abstract

We have constructed a nonlinear multi-graviton theory. An application of this theory to cosmology is discussed. We found that scale factors in a solution for this theory repeat acceleration and deceleration.

## 1 Introduction

We have constructed a nonlinear multi-graviton theory [1] so far. The features of our model are following: (i) Gravitons as the fluctuation from Minkowski spacetime have a Fierz-Pauli (FP) type mass [2]. (ii) This model based on dimensional deconstruction method. So, we can tune the mass spectrum more easily than Kaluza-Klein theory. (iii) The mass term has a reflection symmetry at each vertex and a exchange symmetry at each edge of a graph.

In this article, beginning with graph theory, dimensional deconstruction [3, 4] and our model are reviewed (see also [5, 6]). Next, we consider the vacuum cosmological solutions of the case with the four-site star graph and the four-site path graph. Finally, we summarize our work and remark about the outlook.

## 2 A review of graph theory and dimensional deconstruction

### 2.1 Graph theory

We consider the matrix representation of graph theory.<sup>4</sup> A graph  $G$  is a pair of  $V$  and  $E$ , where  $V$  is a set of vertices while  $E$  is a set of edges. An edge connects two vertices; two vertices located at the ends of an edge  $e$  are denoted as  $o(e)$  and  $t(e)$ . Then, we introduce two matrices, an incidence matrix and a graph Laplacian, associated with a specific graph. The incidence matrix represents the condition of connection or structure of a graph, and the graph Laplacian  $\Delta$  can be obtained by  $EE^T$ . By use of these matrices, a quadratic form of vectors  $a^T \Delta a (= a^T EE^T a)$  can be written as a sum of  $(a_i \Delta_{ij} a_j)$ . If all  $a_i (i = 1, 2, \dots, \#V)$ , components of  $a$ , take the same value,  $E^T a = 0$  and then  $a = 0$ .

### 2.2 Dimensional deconstruction

It is assumed that we put fields on vertices or edges. An idea that there are four dimensional fields on the sites (vertices) and links (edges), dubbed as dimensional deconstruction, is introduced by Arkani-Hamed *et al.* In this scheme, the square of mass matrix is proportional to the Laplacian of the associated graph. In the case of a cycle graph (a ‘closed circuit’) with  $N$  sites ( $C_N$ ), when  $N$  becomes large, the model on the graph corresponds to the five-dimensional theory with  $S^1$  compactification. In other words, the mass scale of the model  $f$  over  $N$  corresponds to the inverse of the compactification radius  $L/(2\pi)$ :

$$M_\ell^2 = 4f^2(\sin \pi\ell/N)^2 \quad \rightarrow \quad M_\ell^2 = (2\pi\ell/L)^2 \quad (f/N \rightarrow 1/L).$$

For a cycle graph, the linear graviton model presented in the previous work [1] coincides with the FP model proposed in [2]. The model is a most general linear graviton theory on a generic graph.

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<sup>4</sup>Please see [7] for a review of application of graph theory to field theory.

### 3 Nonlinear multi-graviton theory on a graph

We consider a nonlinear multigravity on a graph. Following Nibbelink *et al.* [8, 9], we introduce the important ‘tool’:

$$\langle ABCD \rangle \equiv -\varepsilon_{abcd}\varepsilon^{\mu\nu\rho\sigma} A_\mu^a B_\nu^b C_\rho^c D_\sigma^d,$$

where  $\varepsilon$  is the totally antisymmetric tensor. Using this tool, we have the Einstein-Hilbert term replacing  $A$  and  $B$  by vierbeins and  $C$  and  $D$  by a curvature 2-form. In addition, because fourth power of vierbein in the angle brackets is equal to the determinant of vierbeins, the use of this tool illuminates that the Einstein-Hilbert term and the cosmological term have the same structure.

We assume the following term for each edge of a graph form;

$$\langle (e_1 e_1 - e_2 e_2)^2 \rangle,$$

where  $e_1$  and  $e_2$  are vierbeins at two ends of one edge. This term has a reflection symmetry  $e \leftrightarrow -e$  at each vertex and an exchange symmetry  $e_1 \leftrightarrow e_2$  at each edge.

In the weak-field limit,  $e_1 = \eta + f_1, e_2 = \eta + f_2$ ,

$$\langle (e_1 e_1 - e_2 e_2)^2 \rangle = 8 \left( ([f_1] - [f_2])^2 - [(f_1 - f_2)^2] \right) + O(f^3),$$

where  $\eta$  is Minkowski metric, and  $[f] = \text{tr} f$  for notational simplicity. This quadratic term corresponds to the FP mass term<sup>5</sup>. On the other hand,  $\frac{1}{2}|e|R$  contains the kinetic terms of a graviton in the leading order.

Therefore, in the case of the tree graph, we have the nonlinear Lagrangian of multi-graviton theory without higher derivative and non-local terms,

$$L_0 = \frac{1}{2} \exp \Phi \sum_{v \in V} |e^v| R^v + M^2 \sum_{e \in E} \langle (e_{o(e)} e_{o(e)} - e_{t(e)} e_{t(e)})^2 \rangle,$$

where  $M^2 \equiv m^2/16$  and we assume  $\phi_1 = \phi_2 = \dots = \phi_N = \Phi$ .

### 4 Vacuum cosmological solution

Now we consider two vacuum cosmological models, based on the four-site star graph and the four-site path graph. We assume the following metric;

$$g_{\mu\nu} dx^\mu dx^\nu = -e^{\Phi(t)} dt^2 + e^{\Phi(t)+2a_i(t)} (dr^2 + r^2 d\Omega^2)$$

where  $\Phi(t)$  is a scalar field and  $a_i (i = 1, \dots, 4)$  are scale factors. Then,

$$\langle (e_i e_i - e_j e_j)^2 \rangle = e^{-2\Phi(t)} (e^{2a_i(t)} - e^{2a_j(t)})^2.$$

In the case of the star graph, the Lagrangian is following;

$$L = \frac{1}{2} \exp \Phi(t) \sum_{i=1}^4 |e^i| R^i + M^2 \sum_{i=2}^4 \langle (e_1 e_1 - e_i e_i)^2 \rangle,$$

where,  $a_1$  is on the center of the graph. On the other hand, the Lagrangian of the case of the path graph is

$$L = \frac{1}{2} \exp \Phi(t) \sum_{i=1}^4 |e^i| R^i + M^2 \sum_{i < j} \langle (e_i e_i - e_j e_j)^2 \rangle,$$

where,  $a_1$  and  $a_4$  are on each end of the graph.

We show the results of numerical calculations of the two models on the same initial conditions in Figure 1 and Figure 2. Both scalar fields behave similarly and each scale factor repeats the increase and

<sup>5</sup>It is known that the asymmetric part of  $f$  can be omitted. [10]

the decrease. However, the scalar field in the star graph case changes slightly slower than the other. The oscillation of the scale factors in the path graph case include more modes of different frequencies than that of the scale factors in the star graph case.

The star graph model has more symmetries than the path graph model. A lot of modes in the star graph case degenerate. In the path graph case, increase of the number of sites gives the more complicated behaviors of the scale factors. On the other hand, in the star graph case, the symmetries are preserved if the number of sites are increased. Therefore, the behaviors of the scale factors are the same as the four-site model, essentially.

## 5 Summary and prospects

We considered the four-site star and path graph model and found that vacuum cosmological solutions with the scale factors show the repeated accelerating and decelerating expansions. The differences between these two models were discussed from a viewpoint about symmetries.

As the future works, we should investigate the case that the matter fields exist. We also should investigate the models based on arbitrary tree graphs.

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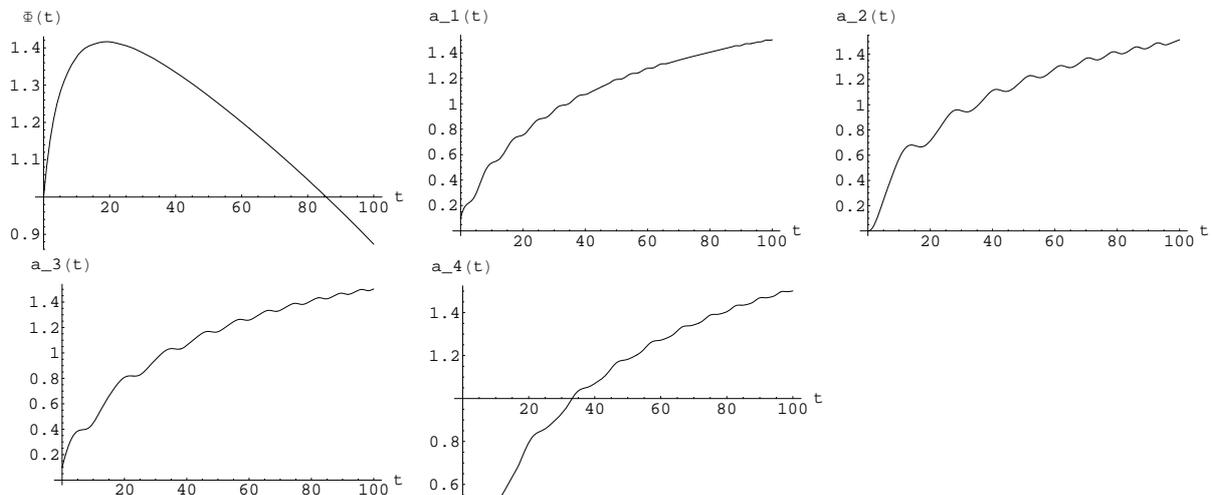


Figure 1: A numerical solutions of the scale factors in the case of the four-site star graph.

## References

- [1] T. Hanada, K. Shinoda and K. Shiraishi, to appear in Proceedings of the Seventeenth Workshop on General Relativity and Gravitation (Nagoya University, Nagoya, December 2-6, 2007) arXiv:0801.2641[gr-qc].
- [2] M. Fierz and W. Pauli, Proc. Roy. Soc. Lond. **A173** (1939) 211.
- [3] N. Arkani-Hamed, A. G. Cohen and H. Georgi, Phys. Rev. Lett. **86** (2001) 4757; Phys. Lett. **B513** (2001) 232.

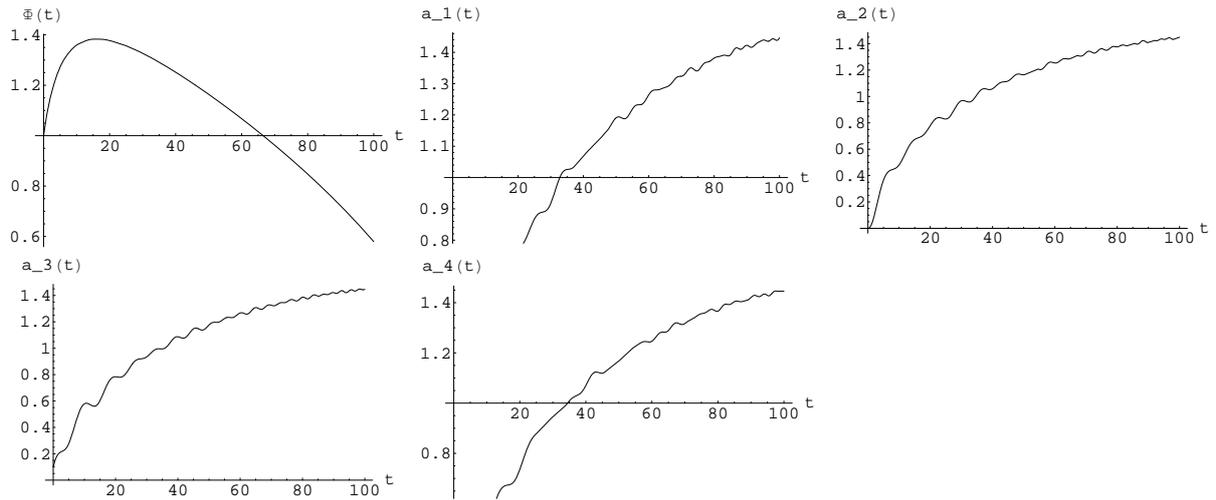


Figure 2: A numerical solutions of the scale factors the case of the four-site path graph.

- [4] C. T. Hill, S. Pokorski and J. Wang, *Phys. Rev.* **D64** (2001) 1050050.
- [5] S. Hamamoto, *Prog. Theor. Phys.* **97** (1997) 327, hep-th/9611141.
- [6] N. Kan and K. Shiraishi, *Class. Quant. Grav.* **20** (2003) 4965, gr-qc/0212113.
- [7] N. Kan and K. Shiraishi, *J. Math. Phys.* **46** (2005) 112301.
- [8] S. G. Nibbelink, M. Peloso and M. Sexton, *Eur. Phys. J.* **C51** (2007) 741, hep-th/0610169.
- [9] S. G. Nibbelink and M. Peloso, *Class. Quant. Grav.* **22** (2005) 1313, hep-th/0411184.
- [10] C. Bizdadea et al., *JHEP* **02** (2005) 016; C. Bizdadea et al., *Eur. Phys. J.* **C48** (2006) 265.