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COOLING FLOWS AND SZ-EFFECT

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We will first present the dynamics of cooling flows for a homogeneous model by solving the corresponding Navier-Stokes equations for a spherical symmetric steady inflow. As a result we get the velocity profiles. The presence of a cooling flow modifies the electronic gas pressure on the lines of sight through the central cluster regions. As a second point, we outline a modified Kompaneets equation which takes into account the additional effect of the accelerated electron media in the cooling flow inside the cluster frame. This effect is different from the kinematic SZ-effect. We find a slight change in the spectral form of the thermal SZ-effect.

1 Introduction

We briefly recall the standard thermal SZ-effect. [12,11,2] The inverse Compton scattering of the CMB photons off the hot intracluster plasma (electrons) is described by the Kompaneets [7] equation, which gives the change in time of the CMB photon occupation number n:

$$\frac{\partial n}{\partial t} = \frac{kT}{m_e c}\frac{n_e \sigma_T}{x^2}\frac{\partial}{\partial x}\left[x^4 (n + n^2 + \frac{T_e}{T}\frac{\partial n}{\partial x})\right], \tag{1}$$

where $x = \frac{h\nu}{kT}$ is the dimensionless frequency, T_e the electron temperature, T the CMB temperature, n_e the electron density and σ_T the Thomson scattering cross section.
Due to only weak scattering, the photon occupation number n is still of Planckian form. Integration along the line of sight over the cluster dimension, assuming $\frac{\partial I}{\partial l} \simeq \frac{1}{c}\frac{\partial I}{\partial t}$ where the photon field intensity I is related to n through $I = i_0\, x^3 n$, $i_0 = \frac{2(kT)^3}{(hc)^2}$, gives the frequency dependent intensity change due to the scattering off the electrons:

$$\Delta I(x) = i_0\, g(x) \int\limits_{cluster} \left(\frac{kT_e}{m_e c^2}\right) \sigma_T n_e \, dl_{cluster}, \tag{2}$$

where the integral is the Comptonization parameter y describing the cluster properties and the function $g(x)$ defines the spectral shape of the thermal SZ-effect for $T_e \gg T$:

$$g(x) = \frac{x^4 \exp x}{(\exp x - 1)^2}\left(\frac{x(\exp x + 1)}{\exp x - 1} - 4\right). \tag{3}$$

In some clusters there is evidence for the existence of cooling flows [1,3,4,5] in the central regions. The aim of this note is to examine their influence on the thermal SZ-effect. Indeed, one would expect a contribution due to the moving electron media in the cooling flow. As it is pointed out in section 3, this effect has clearly to be distinguished from the kinematic SZ-effect.

This note is organised as follows: In section 2 we briefly outline the dynamics of a simple homogeneous cooling flow model. In section 3 we describe the modification to the standard Kompaneets equation (1) due to the inclusion of a cooling flow and in section 4 we present our preliminary results.

2 A homogeneous cooling flow model

We present a simple steady homogeneous cooling flow model to study its dynamics. In order to quantify the influence on the SZ-effect we shall mainly be interested in the velocity profiles. The cluster is expected to be in a relaxed state, so that hydrostatic equilibrium allows us to use an isothermal β- model. For spherical symmetry the Euler equations read:

$$4\pi r^2 \rho(r) v(r) = \dot{m} = const \tag{4}$$

$$v\frac{dv}{dr} + \frac{1}{\rho}\frac{d(\rho\theta)}{dr} + \frac{G\mathcal{M}(r)}{r^2} = 0 \tag{5}$$

$$\frac{3}{2}v\frac{d\theta}{dr} - \frac{\theta v}{\rho}\frac{d\rho}{dr} = \frac{\rho H(\theta)}{m_H}, \tag{6}$$

where $\mathcal{M}(r)$ is the cluster mass inside the radius r, $v(r)$ the velocity of the inward directed cooling flow and $\dot{m}$ the mass deposition rate which enters as a parameter in our model. $H(\theta)$ is an analytical approximation to the optically thin cooling function [10], which describes the overall energy loss.
θ is the square of the isothermal sound speed c_s which is introduced by the ideal gas law as follows:

$$\frac{P}{\rho} = \frac{kT_e}{\mu m_H} = c_s^2 := \theta, \tag{7}$$

where μ is the mean molecular weight and m_H the hydrogen mass.
By eliminating the density ρ in the equations (4)-(6), we end up with the following system of two ordinary coupled differential equations: [8]

$$\frac{dv}{dr} = v\left(3G\mathcal{M} - 10r\theta + \frac{\dot{m}}{2\pi}\frac{H(\theta)}{(vm_H)^2}\right) \Big/ \left(r^2(5\theta - 3v^2)\right) \tag{8}$$

$$\frac{d\theta}{dr} = 2\left(\theta(2v^2 r - G\mathcal{M}) - (v^2-\theta)\frac{\dot{m}}{4\pi}\frac{H(\theta)}{(vm_H)^2}\right) \Big/ \left(r^2(5\theta - 3v^2)\right). \tag{9}$$

Both equations have singularities at the sonic radius r_s, where $5\theta(r_s) = 3v^2(r_s)$. Fig.1 illustrates the dynamics [6] for a mass deposition rate $\dot{m} = 100M_\odot$. The velocity $v(r)$ and the isothermal sound speed $c_s(r)$ are plotted as a function of the radius r.

3 SZ-effect from a modified Kompaneets equation with cooling flow contribution

It is important to note that the Kompaneets equation (1) describes a *static scatterer*, assuming that in the average the electrons are macroscopically at rest. This is no longer true for the electrons in an accelerated cooling flow, where the macroscopic velocity $v(r) \neq 0$ from equation (8) describes the dynamics.
The Kompaneets equation (1) has to be modified in such a way, that the (macroscopic) bulk velocity of the moving electron media is explicitly taken into account. Psaltis et al. (1997) gave a very detailed analysis of Compton scattering by static and moving media. They made

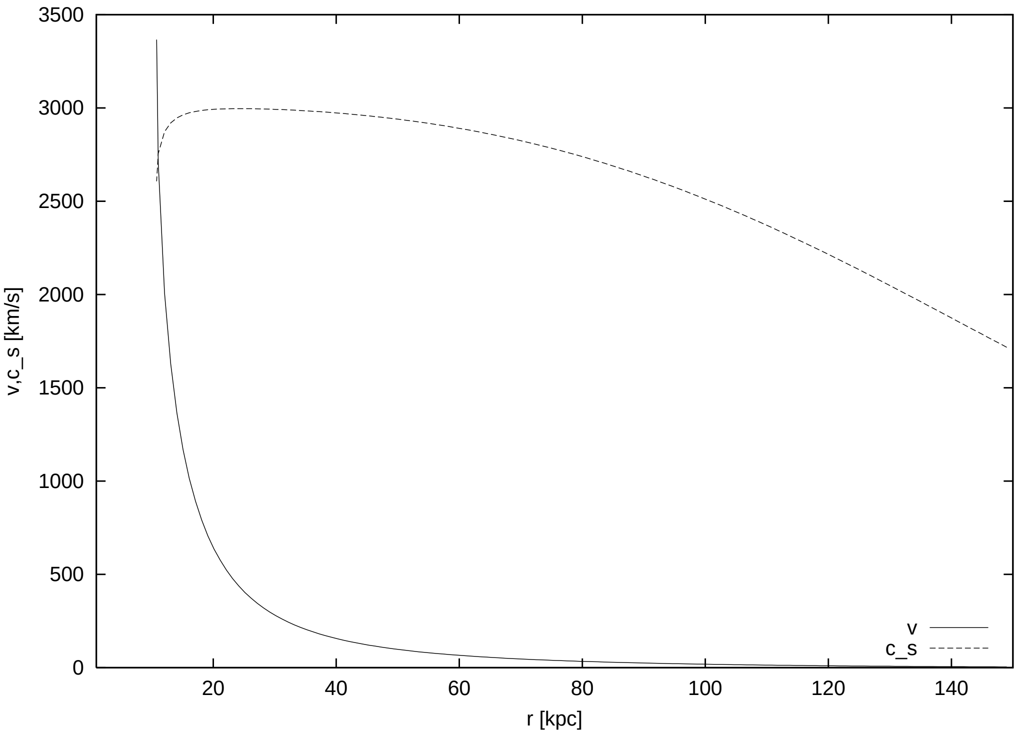

Figure 1: The velocity $v(r)$ and the isothermal sound speed $c_s(r)$ of a cooling flow with $\dot{m} = 100\, M_{\odot}$, plotted as a function of the radius r.

a careful distinction between the electron rest frame, the fluid frame (comoving with the fluid) and the system frame. Moreover, they showed that it is necessary to use the correct relativistic differential scattering cross section in order to obtain a photon kinetic equation that is correct to first order in ϵ/m_e, T_e/m_e and V, where ϵ is the photon energy, T_e and m_e are the electron temperature and rest mass, and V is the electron bulk velocity in units of the speed of light measured in the system frame. Moreover, they avoided the use of the diffusion approximation because this may lead to an inaccuracy.

Starting from the Boltzmann equation in the system frame, introducing the proper Lorentz transformations, expanding to the appropriate orders and assuming that the velocity distribution in the fluid frame is a relativistic Maxwellian, they end up with the zeroth moment of the radiative transfer equation with emission and absorption included. [9]

If the radiation field (CMB) is isotropic in the system frame and if there is no bulk velocity of the electrons, no absorption and emission, their equation reduces to:

$$\frac{1}{n_e \sigma_T} \frac{\partial I}{\partial t} = \frac{\epsilon}{m_e} \frac{\partial}{\partial \epsilon} (\epsilon I) - \frac{4T_e}{m_e} \epsilon \frac{\partial I}{\partial \epsilon} + \frac{T_e}{m_e} \epsilon \frac{\partial^2}{\partial \epsilon^2} (\epsilon I) + \frac{\epsilon}{m_e} (\epsilon \frac{\partial I}{\partial \epsilon} - \epsilon) \frac{I}{\epsilon^3}, \tag{10}$$

where the first two terms on the right side describe the effect of systematic down-scattering and up-scattering of the photons by electrons. The third term describes the diffusion in energy produced by the thermal motion of the electrons and the last one the induced Compton scattering. Equation (10) reduces to the Kompaneets equation (1) by neglecting higher order terms.

Allowing for a bulk velocity $V \equiv v/c$ the equation becomes: [9]

$$\frac{1}{n_e \sigma_T} \frac{\partial I}{\partial t} = \frac{\epsilon}{m_e} \frac{\partial}{\partial \epsilon} (\epsilon I) - \frac{4T_e}{m_e} \epsilon \frac{\partial I}{\partial \epsilon} + \frac{T_e}{m_e} \epsilon \frac{\partial^2}{\partial \epsilon^2} (\epsilon I) + \frac{V^2}{3} \left[-4\epsilon \frac{\partial I}{\partial \epsilon} + \epsilon \frac{\partial^2}{\partial \epsilon^2} (\epsilon I) \right], \tag{11}$$

where we neglected the term of induced Compton scattering from equation (10) and kept only

first order terms in ϵ to describe the effect of the bulk velocity.
Equation (11) shows that if the radiation field is isotropic in the system frame, Comptonization of *the bulk motion of the electrons inside the cluster* is described entirely by second order terms in V. All first order terms in V vanish identically. The effect is clearly different from *the kinematic SZ-effect, where the cluster as a whole moves through the CMB radiation.*

We express the cooling flow (CF) contribution to the SZ-effect as follows:

$$\frac{\partial n}{\partial t} = \left(\frac{\partial n}{\partial t}\right)_{Komp.} + \left(\frac{\partial n}{\partial t}\right)_{CF}, \tag{12}$$

where the first term on the right hand side is given by equation (1) and the second term is due to the moving electron media.
By changing to the variables used in equation (1) we find from equation (11) the cooling flow contribution:

$$\left(\frac{\partial n}{\partial t}\right)_{CF} = \frac{c n_e \sigma_T}{x^3} \frac{V^2}{3} \left[-4x\frac{\partial}{\partial x}(x^3 n) + x\frac{\partial^2}{\partial x^2}(x^4 n)\right]. \tag{13}$$

Integrating over the cluster cooling flow region (CF) gives:

$$\Delta I(x)_{CF} = i_0\, x^3 \frac{1}{c}\frac{1}{x^3} \left[-4x\frac{\partial}{\partial x}(x^3 n) + x\frac{\partial^2}{\partial x^2}(x^4 n)\right] \frac{1}{c} \int\limits_{CF} n_e \sigma_T \frac{v^2(r)}{3}\, dr_{CF}. \tag{14}$$

Assuming a Planckian photon field for n, we find:

$$\Delta I(x)_{CF} = i_0 \left[-\frac{x^4 \exp x(4+x)}{(\exp x - 1)^2} + \frac{2x^5 \exp 2x}{(\exp x - 1)^3}\right] \frac{1}{c^2} \int\limits_{CF} n_e \sigma_T \frac{v^2(r)}{3}\, dr_{CF}, \tag{15}$$

where we introduced again the macroscopic electron bulk velocity $v(r)$ from equation (8).

In order to simplify the computation, we assume that the velocity profiles for $v(r)$ from fig.1 can be analytically expressed as follows:

$$v(r) = \frac{a}{r}, \tag{16}$$

where the parameter a is given by $a = r_s\, v(r_s)$. This parameter is essentially determined by the initial value conditions for equation (8), which depend on the input values of $v(r_{cool})$, $c_s(r_{cool})$ and $\dot{m}$, r_{cool} being the radius of the cooling flow region.
Moreover, to get a first idea of the cooling flow contribution from equation (15) we allow n_e to be constant in the cooling flow region. We thus end up with the evaluation of the following integral:

$$2a^2 \int\limits_{r_s}^{r_{cool}} \frac{1}{r^2}\, dr, \tag{17}$$

where we limit the cooling flow region from r_{cool} to the sonic radius r_s along the line of sight through the cluster centre. Finally, assuming $r_{cool} \gg r_s$, we get the cooling flow contribution to the SZ-effect from equation (15):

$$\Delta I(x)_{CF} = i_0\, g_{CF}(x) \frac{2}{3} \frac{n_e \sigma_T}{c^2} r_s v^2(r_s), \tag{18}$$

where $g_{CF}(x)$ denotes the term in square brackets in equation (15).

4 Preliminary results

For a rough estimate of the effect produced by equation (15), we allow n_e to be constant in the cooling flow region. The parameters r_s and $v(r_s)$ in equation (18) depend on the mass deposition rate $\dot{m}$ and the initial value conditions in equation (8) and (9). We choose $r_s = 10\,kpc$ and $v(r_s) = 3000\,km/s$. In fig. 2 we compare the thermal SZ-effect from equation (2) with the cooling flow contribution from equation (18) for two different values of n_e. Adding up the cooling flow contribution as outlined in equation (12), yields the corresponding fig. 3.

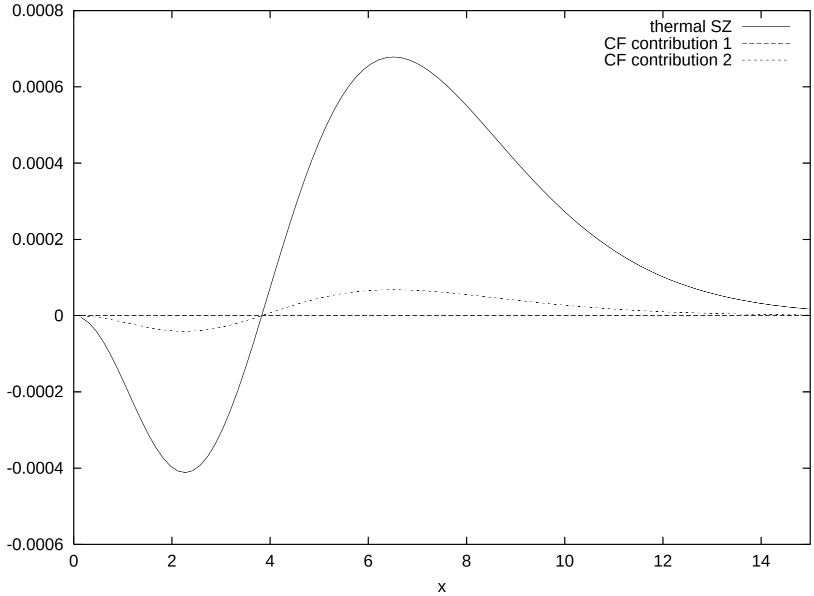

Figure 2: The thermal SZ-effect compared with the CF-contribution. CF contribution 1: $n_e = 10^4/m^3$, CF contribution 2: $n_e = 10^7/m^3$

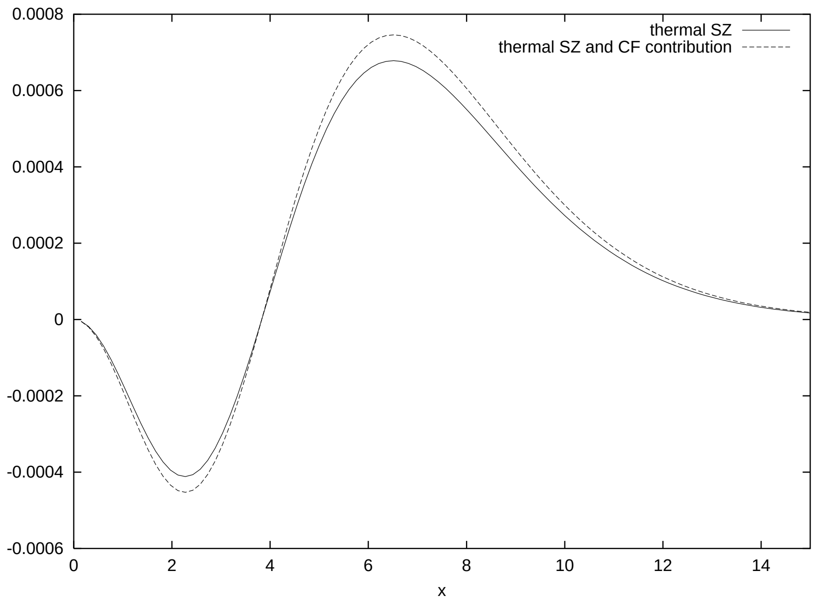

Figure 3: The total SZ-effect from equation (12) compared with the thermal SZ-effect for $n_e = 10^7/m^3$. The Comptonization parameter is $y = 10^{-4}$.

5 Summary and Outlook

We showed that the Kompaneets equation (1) can be modified to include the effect of a simple homogeneous cooling flow. This effect is different from the kinematic SZ-effect. The cooling flow contribution strongly depends on the specific dynamics of the cooling flow and its central cluster density.
Clumps on the line of sight or high density central region from the accumulated mass might give rise to an observable effect in the future. Moreover, higher infall velocities as for instance in a merging process might also affect the crossover frequency and lead to a measurable effect.
A more detailed analysis will be presented elsewhere.

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References

1. Allen, S.W., Fabian, A.C., Johnstone, R.M., White, D.A., Daines, S.J., Edge, A.C., Stewart, G,C., 1993, MNRAS, 262, 901
2. Birkinshaw, M., 1999, Phys. Rep. 310, 97
3. Fabian, A.C., Nulsen, P.E.J. and Canizares, C.R., 1991 Astr. Astrophys. Rev. 2, 191
4. Fabian, A.C. Ann.Rev.Astrophys. 1994, 32:277-318
5. Galactic and Cluster Cooling Flows, ASP conference Series, Vol.115, 1997
6. Koch, P.M., *Cooling flows in clusters of galaxies*, Diploma Thesis, 1999, University of Zürich
7. Kompaneets, A.S., Sov. Phys. JETP 4 (1957) 730
8. Mathews, W.G. and Bregman, J.N., 1978, ApJ, 224, 308-319
9. Psaltis, D., Lamb, F.K., 1997, ApJ, 488, 887
10. Raymond, J.C., Cox, D.P., and Smith, B.W., 1976, ApJ, 204, 290
11. Rephaeli, Y., 1995. Annu. Rev. Astron. Astrophys. 33, 541
12. Sunyaev, R.A., Zel'dovich, Ya.B., 1972. Comm. Astrophys. Space Phys. 4, 173