

# Relative and Absolute Time

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**Abstract.** In this paper, we derive a new identity that relates the proper time and the observer time as a contact transformation on configuration space. This leads to a consistent relativistic extension of the special theory to include Newtonian mechanics. The basic conclusions of this paper are:

- (i) There always exist two unique inertial frames for any given system of particles: one with a unique speed of light and time relative and one with unique simultaneity and the speed of light relative.
- (ii) If the ratio of the total energy of the universe to the total mass energy of the universe is constant, then a unique definition of Newtonian time exists for the universe with zero set at the big bang.
- (iii) There are two mathematically consistent representations for the same physical system.
- (iv) Data collected from distant objects in the universe does not have a unique physical interpretation.

We also use the isotopic dual of  $\mathbb{R}$  to provide a basic improvement in the big bang model. This leads to a model with conservation of energy, momentum, angular momentum, charge, a natural arrow for time and a natural explanation for the lack of antimatter in our universe. We predict that protons and antiprotons (respectively electrons and positrons) are gravitationally repulsive.

## 1. Background and History

In the beginning of the 1900's, a major problem in physics was to reconcile the group transformation problem between the theories of Newton and Maxwell. At that time, it was unthinkable that Maxwell's theory had any serious flaws. Lorentz [1, 2] had recently shown that all of the macroscopic phenomena of electrodynamics and optics could be accounted for based on an analysis of the microscopic behavior of electrons and ions.

Independent of Lorentz and Poincaré, Einstein took a deeper look at the problem. He derived the Lorentz transformations from kinematical arguments, as opposed to the symmetry properties of Maxwell's equations. As Brown points out [3], Einstein took this approach because he was not sure that Maxwell's theory would survive the existence of photons. Einstein believed that the Lorentz transformations were fundamental and would survive any failures in Maxwell's theory.



(At that time, Maxwell's theory was still not accepted by all, see [4] and references therein. )

Einstein [5] observed that the constant  $c$  appears in Maxwell's equations for all inertial observers. At that time experimental information about the speed of light was meager, being restricted to macroscopic and astronomical studies. Einstein was the first to realized that a formal postulate on the velocity of light was necessary. His proposal was that all physical theories should satisfy the (now well-known) postulates of special relativity:

- (1) The physical laws of nature and the results of all experiments are independent of the particular inertial frame of the observer (in which the experiment is performed).
- (2) The speed of light in empty space is constant and is independent of the motion of the source or receiver.

Like those before him, Einstein realized that the concept of absolute space was neither knowable nor necessary. The first postulate abandons this idea. Einstein's second postulate forces the abandonment of the idea of absolute time.

Poincaré was singular among first class physicists, who were equally well trained and accomplished in engineering, mathematics, physics and the philosophy of science. (E. T. Bell called Poincaré "The Last Universalist", because he excelled in all these fields as they existed in his lifetime [6].) Poincaré discovered an error in Lorentz's analysis, realized that after correction the transformations form a group, which he named for Lorentz [7]. As early as 1906, Poincaré recognized that the Lorentz group could be treated as a rotation in four-dimensional space if time is treated as an imaginary coordinate and introduced the metric (proper-time) now attributed to Minkowski (see [8]). However, his insight and understanding of the difference between mathematics and physics helped him to resist the temptation to use a "physically unjustified" mathematical observation as a (necessary) tool for the representation of physical reality. This leap was later made (without physical justification and much philosophical fanfare) by Minkowski, a well-known number theorist with few accomplishments in physics and a strong belief in Hilbert's program to geometrize physics [9]. A complete discussion of Minkowski's motivation, his prior knowledge of Poincaré's work and his background in physics can be found in Walters [10]. *Thus, we make explicit Minkowski's unacknowledged postulate for the special theory of relativity:*

- (3) The correct implementation of the first two postulates requires that time be treated as a fourth coordinate, and the relationship between components so constrained as to satisfy the natural invariance induced by the Lorentz group of electrodynamics, (Minkowski space).

### 1.1. Newtonian Mechanics

The four-geometry postulate became very popular, but Einstein, Lorentz, Poincaré, Ritz other important thinkers maintained that space and time have distinct physical properties. Einstein was the first to oppose Minkowski's postulate openly. In 1908, Einstein and Laub published two

papers which offered a different approach, that was simpler and did not depend on the spacetime formalism (see [11], [12]). They argued that the spacetime formalism was too complicated and did not add any new physics.

Sommerfeld later simplified Minkowski's complicated formulation, making it easy for physicists to understand. The theory also became attractive to the mathematics community, which made it even more popular. In the excitement, no one seem to notice that the theory did not work for more than one particle and thus was far from a complete extension of Newton's theory. By the time problems in attempts to merge the special theory of relativity with quantum theory forced researchers to take a deeper look at the foundations of classical electrodynamics, the Minkowski postulate had already become sacred. By the time Einstein considered the extension of the special to the general theory, the only approach of interest was one that extended the Minkowski postulate (see Pais [13] and [14]).

Once it was accepted that the proper Newtonian theory should be invariant under the Lorentz group, work on this problem was actually ignored until after World War Two when it was realized that quantum theory did not solve the problems left open by the classical theory. A canonical center of mass is fundamental for the extension of the special theory to include a many-particle Newtonian or a many-particle quantum theory.

In classical electrodynamics, Dirac partially by-passed many of the problems by replacing particles with fields (see [15]). However, this approach led to the first example of a divergent theory (infinite self-energy). This divergency was the motivation for the Wheeler-Feynman approach to classical electrodynamics (see [16]). Their theory solved the divergency problem, but could not be used as the foundations for quantum theory. The first study of the center of mass problem was conducted by Pryce [17]. This led to the implication that the canonical center-of-mass cannot be the three-vector part of a four-vector. (This will be made explicit later.) After the investigations of Pryce, Bakamjian and Thomas [18] showed that they could construct a global interacting quantizable many-particle theory that satisfied the first two postulates of Einstein (but not Minkowski's postulate). They conjectured that Minkowski's postulate would only be compatible with free particles.

*1.1.1. No-Interaction* There are a two major no-interaction theorems: the first was due to Haag [19] and applies to the foundations of quantum field theory. It is often confused with the one of interest here, first proved by Currie et al [20] and shows that the Bakamjian-Thomas conjecture was correct. The theorem has since been extended to an arbitrary number of particles by Leutwyler [21]. We present the general form.

**Theorem 1.1.** (No-Interaction Theorem) *Let  $n$  particles have a representation  $(\mathbf{x}_i, \mathbf{p}_i)$  on  $6n$ -dimensional phase space. Supposed that the following is satisfied:*

- (i) The system has a Hamiltonian representation.*
- (ii) The system has a canonical representation of the Poincare group.*
- (iii) The each coordinate  $\mathbf{x}_i$  is the vector part of a four-vector.*

*Then these assumptions are only compatible with free particles.*

All attempts to keep Minkowski's postulate, avoid The No-Interaction Theorem and merge the special theory with quantum mechanics have failed.

### 1.2. The 2.7 °K MBR and Mach's Principle

Penzias and Wilson discovered, the 2.7 °K microwave background radiation 1965. We know that this radiation represents a unique preferred frame of rest, which exists throughout the universe and is available to all observers (see [22]). This radiation is highly isotropic with anisotropy limits set at 0.001%. Furthermore, direct measurements have been made of the velocity of our Solar System and Galaxy (370 and 600 *km/sec* respectively, see Peebles [23] ). One can only speculate as to what impact this information would have had on the thinking of the founding fathers, Einstein, Lorentz, Poincaré, Ritz and the many other serious investigators of the early 1900's who were concerned with truly understanding the foundations of electrodynamics and mechanics.

Peebles has conjectured that the special theory is valid with or without a preferred frame of reference, so that the MBR does not violate the special theory (see [23]). This statement is not obvious, in addition general relativity predicts that at each point we can adjust our acceleration locally to find a freely falling frame where the special theory holds. In this frame, all observers with constant velocity are equivalent. Thus, according to the general theory we have an infinite family of freely falling frames. Within this context, the Penzias and Wilson findings show that, we can actually set the acceleration equal to zero.

Mach's principle was originally conceived to show that there is no difference between the rotation of the earth with respect to the fixed stars or the fixed stars with respect to the earth. The MBR shows that the fixed stars are not needed and Newton's bucket experiment does give the correct view.

## 2. Global World View

For the global world view, we begin with an interacting system of  $n$ -particles, observed in an inertial frame by  $O$ . We assume that  $O$  is able to identify the  $i^{\text{th}}$  particle with coordinates  $(\mathbf{x}_i, t)$ . We assume that the natural position variable representing the system as a whole is the center of mass position vector. Pryce, found that there are three possible definitions for this position vector. However, only one of them is canonical and independent of the frame in which it is defined. This is the natural and necessary choice if we want a theory that provides the same physics to all observers and is compatible with quantum mechanics. Before discussing this vector, we want to construct a unique clock for each particle and one for the system as a whole.

*2.0.1. One-Particle Clock* If  $\mathbf{w}_i$  is the velocity of particle  $i$ , let  $\gamma^{-1}(\mathbf{w}_i) = \sqrt{1 - \mathbf{w}_i^2/c^2}$ . The proper time of the  $i^{\text{th}}$  particle is defined by:

$$d\tau_i = \gamma^{-1}(\mathbf{w}_i)dt, \quad \mathbf{w}_i = \frac{d\mathbf{x}_i}{dt}, \quad d\tau_i^2 = dt^2 - \frac{1}{c^2}d\mathbf{x}_i^2. \quad (2.1)$$

We can rewrite the last term to get:

$$dt^2 = d\tau_i^2 + \frac{1}{c^2} d\mathbf{x}_i^2, \Rightarrow cdt = \left( \sqrt{\mathbf{u}_i^2 + c^2} \right) d\tau_i, \quad \mathbf{u}_i = \frac{d\mathbf{x}_i}{d\tau_i} = \gamma(\mathbf{w}_i)\mathbf{w}_i. \quad (2.2)$$

If we let  $b_i = \sqrt{\mathbf{u}_i^2 + c^2}$ , the second term in equation (2.2) becomes  $cdt = b_i d\tau_i$ . This leads to our first identity:

$$\frac{1}{c} \frac{d}{dt} \equiv \frac{1}{b_i} \frac{d}{d\tau_i} \quad (2.3)$$

This identity provides the correct way to define the relationship between the proper time and the observer time. This represents a contact transformation on configuration space, also known as a tangency transformation ([24]). We show in [4], that it is a canonical transformation which leaves phase space invariant. We note that, while the left hand-side of (2.3) is only defined in an inertial frame, the right hand side is well-defined when  $\dot{u}_i \neq 0$ . If we apply the identity to  $\mathbf{x}_i$ , we obtain our second identity:

$$\frac{\mathbf{w}_i}{c} = \frac{1}{c} \frac{d\mathbf{x}_i}{dt} \equiv \frac{1}{b_i} \frac{d\mathbf{x}_i}{d\tau_i} = \frac{\mathbf{u}_i}{b_i}.$$

The particle coordinates transform from  $(\mathbf{x}_i, t) \rightarrow (\mathbf{x}_i, \tau_i)$ , while configuration space transforms from  $(\mathbf{x}_i, \mathbf{w}_i) \rightarrow (\mathbf{x}_i, \mathbf{u}_i)$ . In the new representation,  $\mathbf{x}_i$  is uniquely defined relative to  $O$ , while  $\tau_i$  is uniquely defined relative to the  $i^{\text{th}}$  particle. Using  $\gamma(\mathbf{w}_i) = H_i/m_i c^2$ , we can also write  $d\tau_i = (m_i c^2 / H_i) dt$ . The  $i^{\text{th}}$  particle momentum remains invariant, since  $\mathbf{p}_i = m_i \gamma(\mathbf{w}_i) \mathbf{w}_i = m_i \mathbf{u}_i$ , where  $m_i$  is the rest mass. (Thus, phase space is left invariant.)

*2.0.2. Many-Particle Clock* To construct the many-particle clock, we assume the system is closed,  $n > 1$  with particle proper clocks  $\tau_i$  and with obvious notation, Hamiltonians  $H_i = \sqrt{c^2(\mathbf{p} - \frac{e}{c}\mathbf{A})^2 + m_i^2 c^4} + V_i(\mathbf{x}_i)$ . We defined the total Hamiltonian by  $H = \sum_{i=1}^n H_i$ , define the effective mass  $M$  and total momentum  $\mathbf{P}$  by

$$Mc^2 = \sqrt{H^2 - c^2 \mathbf{P}^2}, \quad \mathbf{P} = \sum_{i=1}^n \mathbf{p}_i.$$

We can then represent  $H$  as  $H = \sqrt{c^2 \mathbf{P}^2 + M^2 c^4}$ .

The canonical position of the center of mass  $\mathbf{X}$ , for a system of particles in the  $O$  frame is defined by (see [25]):

$$\mathbf{X} = \frac{1}{H} \sum_{i=1}^n H_i \mathbf{x}_i + \frac{c^2 (\mathbf{S} \times \mathbf{P})}{H (Mc^2 + H)}, \quad (2.4)$$

where  $\mathbf{S}$  is the global spin of the system of particles relative to  $O$ . (It is clear that (2.4) cannot represent the vector part of a four-vector.) If there is no interaction,  $S, H$  and  $M$  are constant, with no implicit dependence on the  $\{\mathbf{x}_i\}$  variables, so that:

$$\{X_i, X_j\} = \sum_{k=1}^n \frac{\partial X_i}{\partial \mathbf{p}_k} \cdot \frac{\partial X_j}{\partial \mathbf{x}_k} - \frac{\partial X_j}{\partial \mathbf{p}_k} \cdot \frac{\partial X_i}{\partial \mathbf{x}_k} \equiv 0.$$

However, when interaction is present,  $S, H$  and  $M$  may all depend implicitly on the  $\{\mathbf{x}_i\}$  variables, so that in general  $\{X_i, X_j\} \neq 0$ . Since  $\mathbf{X}$  is the canonical conjugate of  $\mathbf{P}$ , it precisely what we need for a consistent merge with quantum mechanics.

Let  $\mathbf{V}$  be the velocity of  $\mathbf{X}$  with respect to  $O$ . It follows that  $H$  also has the representation  $H = Mc^2\gamma(\mathbf{V})$ , so that  $\gamma(\mathbf{V})^{-1} = (Mc^2/H)$ . In this representation, we see that  $d\tau = \gamma(\mathbf{V})^{-1}dt = (Mc^2/H)dt$  does not depend on the number of particles in the system. It follows that, as long as  $Mc^2/H$  is fixed,  $\tau$  is invariant, so that the number of particles  $n$ , can increase or decrease without changing  $\tau$ . (This means that number  $n$  is not conserved and, in some cases of physical interest, may even be a integer-valued random variable).

From  $dt^2 = d\tau^2 + d\mathbf{X}^2/c^2$ , we see that ( $\mathbf{U} = d\mathbf{X}/d\tau$ )

$$c^2 dt^2 = (c^2 + \mathbf{U}^2) d\tau^2 \quad \Rightarrow \quad c dt = \left( \sqrt{c^2 + \mathbf{U}^2} \right) d\tau.$$

It is easy to see that  $\mathbf{U} = \gamma(\mathbf{V})\mathbf{V}$ , so that  $\mathbf{U}$  is constant. If we define  $b = \sqrt{\mathbf{U}^2 + c^2}$ , we can write  $c dt = b d\tau$ . Since  $b$  is constant we have:  $ct = b\tau$ .

For any other observer  $O'$  the same system has center of mass velocity  $\mathbf{V}'$  and, by the same calculations, we obtain  $ct' = b'\tau$ , where  $b' = \sqrt{\mathbf{U}'^2 + c^2}$ .

**Theorem 2.1.** *Given any closed system of interacting particles, there always exists two distinct sets of inertial frame coordinates for each observer, to describe the particles in the system and the system as a whole. The following is true:*

- (i) *In one frame, the speed of light is an invariant upper bound and time is relative, while in the other, time is invariant and the speed of light  $b$ , is relative with no upper bound.*
- (ii) *For the whole system and for each particle, the equations of motion are mathematically equivalent.*

*Proof.* Observer  $O$  has  $(\{\mathbf{x}_i\}_{i=1}^n, t)$  (respectively  $(\mathbf{X}, t)$ ) and, since  $b$  is constant, he also has  $(\{\mathbf{x}_i\}_{i=1}^n, \tau)$  (respectively  $(\mathbf{X}, \tau)$ ) to study the same system of particles. The same is also true for observer  $O'$ , since  $b'$  is constant. This proves the first part. To prove (1), we need only note that, in general,  $b \neq b'$  and each is  $\geq c$ . A proof of (2) can be found in [4].  $\square$

This theorem shows that a unique (operational) measure of time is available to all observers. From the definition of  $\tau$ , we obtain our third identity:

$$\frac{1}{c} \frac{d}{dt} \equiv \frac{1}{b} \frac{d}{d\tau} \equiv \frac{1}{b_i} \frac{d}{d\tau_i} \quad (2.5)$$

Let  $\mathbf{W}$  be the relative velocity between observer  $O$  and  $O'$ . Since  $\tau$  is the same for both we only need the relationship between the two scale factors  $b$  and  $b'$  to satisfy the first postulate. Starting with  $t' = \frac{b'}{c}\tau = \gamma(\mathbf{W}) \left( \frac{b}{c}\tau - \mathbf{X} \cdot \mathbf{V}/c^2 \right)$ , we see that, since  $\mathbf{U} = (\mathbf{X}/\tau)$ , we get:

$$b' = \gamma(\mathbf{W}) [b - (\mathbf{X}/\tau) \cdot (\mathbf{W}/c)] = \gamma(\mathbf{W}) (b - \mathbf{U} \cdot \mathbf{W}/c).$$

A similar calculation shows that  $b = \gamma(\mathbf{W}) (b' + \mathbf{U}' \cdot \mathbf{W}/c)$ . This shows that each observer can have direct access to all information available to any other observer once they know their relative velocities. (Thus the first postulate of Einstein is satisfied.)

**Definition 2.2.** *A theory is said to be Einsteinian if at least one representation exists, which satisfies the two postulates of the special theory.*

This definition allows each observer an additional degree of freedom, while the difference between either choice is a constant change of scale.

**Remark 2.3.** *For many experiments (e. g., high energy particle studies) the center of mass is the natural frame of choice. In this case,  $t = \tau$  and one has a constant speed of light for all events associated with the system of (interacting) particles.*

*This distinction may also prove important in the future, because there continues to appear research in cosmology, applied physics and engineering, suggesting that the constant  $c$  is not an upper bound in all cases (see for example [26, 27, 28, 29]).*

**Theorem 2.4.** *Any closed system of particles is Einsteinian and independent of the Minkowski postulate.*

Since we do not require the particle coordinates to transform as four-vectors (manifest Lorentz invariance), the no-interaction theorem does not apply.

*2.0.3. Canonical Hamiltonian* If we let  $\mathbf{L}$  be the boost (generator of pure Lorentz transformations) and define the total angular momentum  $\mathbf{J}$  by

$$\mathbf{J} = \sum_{i=1}^n \mathbf{x}_i \times \mathbf{p}_i,$$

we then have the following Poisson Bracket relations characteristic of the Lie algebra for the Poincaré group (when we use the observer clock):

$$\begin{aligned} \frac{d\mathbf{P}}{dt} = \{H, \mathbf{P}\} = \mathbf{0} & \quad \frac{d\mathbf{J}}{dt} = \{H, \mathbf{J}\} = \mathbf{0} & \quad \{P_i, P_j\} = 0 \\ \{J_i, P_j\} = \varepsilon_{ijk} P_k & \quad \{J_i, J_j\} = \varepsilon_{ijk} J_k & \quad \{J_i, L_j\} = \varepsilon_{ijk} L_k \\ \frac{d\mathbf{L}}{dt} = \{H, \mathbf{L}\} = -\mathbf{P} & \quad \{P_i, L_j\} = -\delta_{ij} H/c^2, & \quad \{L_i, L_j\} = -\varepsilon_{ijk} J_k/c^2. \end{aligned}$$

It is easy to see that  $M$  commutes with  $H$ ,  $\mathbf{P}$ , and  $\mathbf{J}$ , and to show that  $M$  commutes with  $\mathbf{L}$ .

Since we desire complete compatibility with quantum theory, it is natural to require that any change from the observer clock to the local clock of the observed system be a canonical change of variables. The key concept to our approach may be seen by examining the time evolution of a dynamical parameter  $W(\mathbf{X}, \mathbf{P})$ , via the standard formulation of classical mechanics, described in terms of the Poisson brackets:

$$\frac{dW}{dt} = \{H, W\}. \quad (2.6)$$

We can also represent the dynamics using the proper (or local) time of the system by using the representation  $d\tau = (1/\gamma)dt = (Mc^2/H)dt$ , so that:

$$\frac{dW}{d\tau} = \frac{dt}{d\tau} \frac{dW}{dt} = \frac{H}{Mc^2} \{H, W\}.$$

Since  $Mc^2$  is a well-defined (invariant) rest energy for the system, we determine the canonical proper-time Hamiltonian  $K$  such that:

$$\{K, W\} = \frac{H}{Mc^2} \{H, W\}, \quad K|_{\mathbf{p}=0} = H|_{\mathbf{p}=0} = Mc^2.$$

Using

$$\begin{aligned} \{K, W\} &= \left[ \frac{H}{Mc^2} \frac{\partial H}{\partial \mathbf{P}} \right] \frac{\partial W}{\partial \mathbf{X}} - \left[ \frac{H}{Mc^2} \frac{\partial H}{\partial \mathbf{X}} \right] \frac{\partial W}{\partial \mathbf{P}} \\ &= \frac{\partial}{\partial \mathbf{p}} \left[ \frac{H^2}{2Mc^2} + a \right] \frac{\partial W}{\partial \mathbf{X}} - \frac{\partial}{\partial \mathbf{X}} \left[ \frac{H^2}{2Mc^2} + a' \right] \frac{\partial W}{\partial \mathbf{P}}, \end{aligned}$$

we get that  $a = a' = \frac{1}{2}Mc^2$ , so that (assuming no explicit time dependence)

$$K = \frac{H^2}{2Mc^2} + \frac{Mc^2}{2}, \quad \text{and} \quad \frac{dW}{d\tau} = \{K, W\}.$$

Thus, we can use the same definitions for  $\mathbf{P}$ ,  $\mathbf{J}$ , and  $\mathbf{L}$  to obtain our new commutation relations:

$$\begin{aligned} \frac{d\mathbf{P}}{d\tau} &= \{K, \mathbf{P}\} = \mathbf{0}, \quad \frac{d\mathbf{J}}{d\tau} = \{K, \mathbf{J}\} = \mathbf{0}, \quad \{P_i, P_j\} = 0, \\ \{J_i, P_j\} &= \varepsilon_{ijk} P_k, \quad \{J_i, J_j\} = \varepsilon_{ijk} J_k, \quad \{J_i, L_j\} = \varepsilon_{ijk} L_k, \\ \frac{d\mathbf{L}}{d\tau} &= \{K, \mathbf{L}\} = \frac{-H}{Mc^2} \mathbf{P}, \quad \{P_i, L_j\} = -\delta_{ij} H/c^2, \quad \{L_i, L_j\} = -\varepsilon_{ijk} J_k/c^2. \end{aligned}$$

It follows that, except for a constant scale change, the inhomogeneous proper-time group is generated by the same Lie algebra as the Poincaré group. This result is not surprising given the close relation between the two groups. Thus, the form of  $K$  is fully relativistic.

### 3. Applications

#### 3.1. Newtonian Time

In his work, Newton assumed there exists a unique observer-independent measure of time for the universe.

**Theorem 3.1.** *Suppose that the observable universe is representable in the sense that the observed ratio of mass energy to total energy is constant and independent of our observed portion of the universe. Then the universe has a unique clock that is available to all observers.*

*Proof.* Let  $O$  be any observer anywhere in the universe. From his vantage point,  $H/Mc^2$  is constant, so that he can define  $\tau$  via  $ct = b\tau$ . Since all other observers can also define  $\tau$ , the proof follows.  $\square$

**Remark 3.2.** *The assumption of Theorem 3.1 is equivalent to the standard assumption that the universe is homogenous and isotropic.*

*Since all observers can transform to a frame at rest with respect to the  $2.7^\circ K$  mbr, if they did, all clocks would run at the same rate. In this case, any two observers view of the universe would differ by (at most) a translation and rotation. If in addition, the big bang is true, it defines a zero point for time, making the clock attached to the  $2.7^\circ K$  mbr unique.*

**Theorem 3.3.** (*Peebles*) *Suppose all observers choose a frame that is at rest with respect to the  $2.7^\circ\text{K}$  microwave background radiation, then all the laws of physics will be invariant and not just covariant with respect to Lorentz transformations.*

In the study of physical systems one is sometimes not interested in the behavior of the global system, but only in some subsystem. The cluster decomposition property is a requirement of any theory purporting to be a possible representation of the real world. Basically this is the property that, if any two or more subsystems become widely separated, then they may be treated as independent systems (clusters).

**Theorem 3.4.** *Suppose the system of particles can be decomposed into two or more clusters. Then there exists a unique (local) clock and corresponding canonical Hamiltonian for each cluster.*

*Proof.* We assume that the subsystems are sufficiently separated that all observers can agree that they are distinct. In this case, each observer can identify effective masses  $M_1$ ,  $M_2$  and Hamiltonians  $H_1$ ,  $H_2$ . It follows that  $d\tau_1 = [(M_1c^2)/H_1]dt$  and  $d\tau_2 = [(M_2c^2)/H_2]dt$ , so that each observer can construct a local-time theory for each cluster.  $\square$

Actually, the theorem is true without the assumption that the systems are weakly interacting. This makes the theorem less difficult to apply than the various phenomenological approaches, which require both model justification and consistency analysis prior to use. This theorem also allows us to prove a weaker version of Theorem (3.1), in the sense that we replace the assumption of homogeneity and isotropy of the energy and mass density for a possible infinite universe by finite energy and mass density for a possibly inhomogeneous universe.

**Theorem 3.5.** *Suppose the universe has finite mass and energy density and that each observer can choose a local inertial frame from which his/her region of the universe is at rest relative to the observed system. Then there exists a unique proper clock for the universe.*

*Proof.* Applying the cluster decomposition theorem, our observer can identify masses  $M_1$  for his/her region of the universe and  $M_2$  for the complement region, along with Hamiltonians  $H_1$  and  $H_2$ . It follows that  $H = H_1 + H_2$ ,  $M = M_1 + M_2$  and  $d\tau = [(Mc^2)/H]dt$  define the total mass, Hamiltonian and proper clock for the universe. We can now construct our canonical proper-time Hamiltonian  $K$ . Since  $M$  and  $H$  are fixed, and invariant for all observers, we see that both  $K$  and  $\tau$  are unique and invariant for all observers.  $\square$

**Remark 3.6.** *It should be remarked that  $M_i$  and  $H_i$ ,  $i = 1, 2$ , will vary with observers, reflecting the non-uniqueness of inertial frames.*

### 3.2. The Big Bang

The currently accepted cosmological model for the universe assumes that our universe began around 13.8 billion years ago with a big bang. From a minimalist point of view, this model should be consistent with our current experimentally obtained information about the universe:

- (i) Whenever an antiparticle is observed in experiment, we always find that it is also accompanied by a particle.

- (ii) Whenever an interaction is observed, a complete analysis always shows that momentum, angular momentum, charge and energy are conserved.

In this section, we suggest a slight alteration of the beginning, which makes the big bang event consistent our experimental understanding of the world.

We first revisit our conceptual view of the real numbers and their representation. Recall that a field is a set  $A$  that has two binary operations  $\oplus$  and  $\odot$  that satisfies all our common experience with real numbers. Formally:

**Definition 3.7.** *The real numbers is a triplet  $(\mathbb{R}, +, \cdot)$ , which is a field, with 0 as the additive identity (i.e.,  $a + 0 = a$  for all  $a \in \mathbb{R}$ ) and 1 as the multiplicative identity (i.e.,  $a \cdot 1 = a$  for all  $a \in \mathbb{R}$ ).*

This structure was developed by mathematicians without regard to its possible use in physics. As a consequence the asymmetry in this structure went unnoticed and physicists used it without investigation until Santilli [30] defined the isodual number field. His definition is more general. For our purpose, we only need the following.

**Definition 3.8.** *The isodual real numbers  $(\hat{\mathbb{R}}, +, *)$  is a field, with  $0 = \hat{0}$  as the additive identity (i.e.,  $\hat{a} + \hat{0} = \hat{a}$  for all  $-\hat{a} = \hat{a} \in \hat{\mathbb{R}}$ ) and  $-1 = \hat{1}$  as the multiplicative identity (i.e.,  $\hat{a} * \hat{1} = \hat{a}$  for all  $\hat{a} \in \hat{\mathbb{R}}$ ).*

**Example 3.9.** *A simple example from quantum theory is the following: the evolution of particle is defined on a Hilbert space  $\mathcal{H}$  over the complex numbers  $\mathbb{C} = \mathbb{R} + i\mathbb{R}$ , with Hamiltonian  $H$  by the equation:*

$$i\hbar \frac{\partial \psi}{\partial t} = H\psi.$$

The conjugate equation is:

$$-i\hbar \frac{\partial \psi^*}{\partial t} = H\psi^*.$$

If we use  $\hat{\mathbb{C}}$  as our number field, we can write the above equation as:

$$i * \hat{\hbar} * \frac{\partial \psi^*}{\partial \hat{t}} = \hat{H} * \psi^*$$

Thus, this approach allows us to naturally view anti-particles as time reversed particles, with their evolution define on  $\mathcal{H}^*$  over  $\hat{\mathbb{C}}$ .

We note that we can obtain the isodual of any physical quantity  $\hat{A}$  from the equation  $A + \hat{A} = 0$ .

**Remark 3.10.** *Santilli [30] has shown that charge conjugation and isoduality are equivalent for the particle-antiparticle symmetry operation. However, use of isoduality allows us to view the existence of antimatter and charge conservation as fundamental aspects of the universe, while also explaining why large amounts of antimatter is not found in this universe.*

*If our view of the beginning is correct, we would predict that the proton and antiproton (respectively electron and positron) should be gravitationally repulsive.*

In the diagram below, we provide a new picture of the big bang beginning. In this case, two universes are created, one going forward in  $\tau_N$  and one going backward in  $\tau_N$  (Newtonian time), relative to our reality.

$$\begin{array}{c} \nwarrow \nearrow \\ \hat{\tau}_N \quad \tau_N=0 \\ \swarrow \searrow \quad \tau_N \end{array} .$$

This view has the following advantages: we obtain

- (i) a natural arrow for time, with a zero initial point.
- (ii) a natural explanation for the lack of antimatter in this universe.
- (iii) antiparticles as particles moving backward in (Newtonian) proper time.
- (iv) conservation of charge, energy, linear and angular momentum.

It is important to be clear that our assumption does not imply that there are any other symmetries or necessary similarities between the two universes.

## Conclusion

We have shown that the correct relationship between the proper time and observer time is a contact transformation on configuration space. This approach is consistent with the first two postulates of Einstein, Peebles conjecture and Newton's universal time. We have also shown that, by introducing a symmetric view of the number line, we are able to introduce an improved version of the big bang, which explains the lack antimatter in the universe, introduces an the arrow for time and, provides conservation of energy, linear and angular momentum.

The most important conclusion from this investigation is embodied in Theorem 2.1: "given any closed system of interacting particles, there always exists two distinct sets of coordinates for each observer to describe the particles in the system and the system as a whole". Thus, the physical interpretation of experimental data is not unique. In order to make this above statement explicit in a very powerful manner, recall that many measurements are based on the dimensionless ratio  $\beta = \mathbf{w}/c$ . However,  $\mathbf{w}/c \equiv \mathbf{u}/b$ , so we see that measurements of velocity and the speed of light for distant objects are totally ambiguous. For example, ([31], pp. 556-561), the red shift factor  $z$ , used to determined distances in astronomy, is defined by:

$$z = \sqrt{\frac{1 + \frac{\mathbf{w}}{c}}{1 - \frac{\mathbf{w}}{c}}} - 1 \equiv \sqrt{\frac{1 + \frac{\mathbf{u}}{b}}{1 - \frac{\mathbf{u}}{b}}} - 1.$$

We thus conclude that distant objects may have much higher velocities and light may have velocities higher then  $c$ , from the same data, without any contradiction.

A known problem with the 2.7 °K MBR is that there is no known way for the early universe to instantly reach equilibrium following the big bang. One solution is to introduce the inflationary hypothesis. It has many critics and other approaches have been suggested. The varying speed of light assumption introduced by Moffet [32] is consistent with the use of  $b = \sqrt{U^2 + c^2}$  in the beginning (see also Magueijo [33] and references there in). As time progresses and after equilibrium,  $U$  can slow down to zero, while  $b$  reduces to  $c$ .

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