Ph.D. Thesis

Resonant Leptogenesis Based on Non-equilibrium Quantum Field Theory

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Abstract

Dynamics of matter fields on a classical space-time are believed to obey the principle of quantum field theory (QFT). In the early universe, the expansion of the universe originating from the inflation causes various non-equilibrium phenomena of quantum fields, and the most rigorous descriptions of such phenomena are obtained from the non-equilibrium QFT framework. However, in general, the first principle of QFT gives very complicated equations, and then, we need reasonable approximations to understand the phenomena.

In this thesis, as an example of non-equilibrium phenomena with significant quantum effects, we investigate the evolution of the lepton number asymmetry when the right-handed (RH) neutrinos have almost degenerate masses $|M_i - M_j| \ll M_i$. Because of the resonant oscillation resulting from the degenerate mass spectrum, the propagating process of RH neutrino is no longer classical process and the conventional Boltzmann equation fails to take into account the enhancement of CP asymmetry in the decay of the RH neutrino N_i. To describe the quantum propagating process, we rely on the Schwinger-Dyson (SD) equation from the non-equilibrium QFT with two levels of approximation.

The first one is the Kadanoff-Baym (KB) equation as the evolution equation of full propagators as a systematic truncation of the SD equation. By solving the KB equation of the RH neutrino directly, the resonantly enhanced CP-violating parameter ε_i associated with the quantum propagating process and decay of the RH neutrino N_i is obtained. It is proportional to an enhancement factor $(M_i^2 - M_j^2)M_iM_j/((M_i^2 - M_j^2)^2 + R_{ij}^2)$ with the regulator $R_{ij} = |M_i\Gamma_i + M_j\Gamma_j|$. This regulator differs from the one derived by the calculation based on the equilibrium quantum field theory $R_{ij} = |M_i\Gamma_i - M_j\Gamma_j|$. By focusing on the origin of the difference, we clarify what is missed in the conventional approach.

The second level of the approximation is to derive the evolution equation of the so-called density matrix of RH neutrino, which is usually used to describe the baryogenesis through the RH neutrino flavor oscillation, from the KB equation. Instead of solving the KB equation directly, we obtain the analytic solution of the density matrix equation under the assumption that the deviation from thermal equilibrium is small where the differential equation is reduced to a linear algebraic equation. Again we obtain the CP violating parameter ε in the thermal resonant leptogenesis, with the regulator consistent with the previous analysis.

Through these analyses, we see the importance of the non-equilibrium QFT as the starting point for the approximations, for the reason that it describes all the processes (propagation and collision) in the single frame work.

This thesis is based on our works [90, 91].

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1 Introduction and overview

Origin of the baryon asymmetry in the universe is one of the mysteries, that cannot be explained within the combination of today's standard model of particle physics (SM) and standard cosmology. The baryon asymmetry is parametrized as the baryon to photon ratio

$$\eta \equiv \left.\frac{n_B - n_{\overline{B}}}{n_\gamma}\right|_0 \approx 6.03 \times 10^{-10} \times \left(\frac{\Omega_b h^2}{0.022}\right) \tag{1.1}$$

where n_B, $n_{\overline{B}}$, n_γ are the number densities of baryon, anti-baryon and photon, respectively, and the subscript 0 implies the present values. At the second equality, it's rewritten in terms of the baryonic fraction of the critical density $\Omega_B \equiv \rho_B/\rho_{\rm cr}$ with the present Hubble parameter $h \equiv H_0/100\text{km s}^{-1}\text{Mpc}^{-1} = 0.673 \pm 0.010$ [1]. The value of baryon asymmetry is extracted from the two independent observations. The first one is the measurement of the primordial abundances of the light elements [2]. Big bang neucleosynthesis (BBN) scenario leads to the baryon to photon ratio corresponding to the value of the cosmological parameter

$$\Omega_B h^2 = 0.02202 \pm 0.00046 \qquad \text{(BBN)} . \tag{1.2}$$

The second one is the measurement of the cosmic microwave background (CMB). *Plank* 2015 results [1] give the cosmological parameter

$$\Omega_B h^2 = 0.02222^{+0.00045}_{-0.00043} \qquad \text{(CMB)} . \tag{1.3}$$

The yield value of the baryon asymmetry

$$Y_B \equiv \left.\frac{n_B - n_{\overline{B}}}{s}\right|_0 \approx \frac{\eta}{7.04} \tag{1.4}$$

is another parametrization of the baryon asymmetry, which is convenient in calculations because the entropy density $s = g_*(2\pi^2/45)T^3$ is conserved in the expansion of the universe, where $g_* = g_*(T)$ is the number of degrees of freedom in the plasma with temperature T. $s_0/n_{\gamma,0} \approx 7.04$ is used at the second equality. Therefore, we have to explain the observed value of the yield $Y_B \approx 8.56 \times 10^{-11}$ by adding some new degrees of freedom to the SM.

Three necessary conditions to generate the baryon asymmetry is known as the Sakharov's conditions [3]. Although the SM satisfies the Sakharov's conditions, the observed number of baryon asymmetry cannot be produced due to the smallness of the CP-asymmetry in the CKM matrices and the modest electroweek phase transition following the Higgs boson mass $m_h \approx 126$GeV. The simplest and reasonable extension of the SM is to introduce the three right-handed (RH) neutrinos N_i with large Majorana masses M_i, which gives not only the successful baryogenesis through leptogenesis [4] but also explanation of the neutrino oscillation via seesaw mechanism [5], see [6] for a comprehensive review. In this scenario, RH neutrinos are produced thermally through the interaction with the SM particles' thermal bath or the reheating process after inflation. As temperature decreases with the expansion of the universe down to the Majorana mass scale, RH neutrinos become out of thermal equilibrium and their CP-asymmetric decay into the SM leptons and the Higgs produce lepton number asymmetry in the universe. The lepton number asymmetry is then converted into the baryon number asymmetry through the non-perturbative $B+L$ -violating process of sphalerons in the SM [8].

If the Majorana masses of the RH neutrinos have a hierarchical structure, the lightest Majorana mass must satisfy the Davidson-Ibarra(DI) bound [11], $M \gtrsim 10^9$GeV in order to produce sufficient lepton number asymmetry. This lower bound requires high reheating temperature $T_{\rm rh} \gtrsim 10^9$GeV. In general, such a high reheat temperature is not favored because of the production of unwanted particles with rather long lifetime, which could spoil the BBN. The famous example is the so-called gravitino problem[12] arising in supersymmetric models. Hence, finding the way to escape the DI bound seems to be an important task. One of well-studied directions for this purpose is to take into account the lepton flavor effects [26, 27, 28]. However, even though such flavor effects are fully considered, the reheating temperature cannot be lowered considerably in the leptogenesis with hierarchical (or non-degenerate) mass spectrum of RH neutrinos. (As mentioned below, the successful baryogengesis scenarios based on the ARS mechanism [56] prefer GeV scale RH neutrinos. In this thesis, however, we focus on the usual leptogenesis scenario where the RH neutrino decay process is responsible for the final baryon asymmetry.)

The solution to lower the temperature is highly-degenerate mass spectrum of the RH neutrinos [39, 40, 41]. When at least two of the RH neutrinos are degenerate in their masses, the DI bound can be evaded. In this case, quantum oscillation of almost degenerate RH neutrinos resonantly enhance the CP-violating decay and hence, lepton number asymmetry can be produced sufficiently even for RH neutrino masses smaller than TeV scale. This scenario is known as the resonant leptogenesis. Such light RH neutrinos have attracted attention in light of the lepton flavor violation, neutrinoless double β decay and high energy collider experiments, for example [43]–[47]. It's also interesting possibility in the theoretical perspective because light RH neutrinos with $M \lesssim$ TeV do not give large radiative corrections, and then, they could be indispensable peaces of the theories connecting the EW and Planck scale directly [48]–[51].

In the resonant case, the CP-asymmetry in the decay of N_i mainly comes from an interference of the tree and the self-energy one-loop diagrams. It is expressed by the CP-violating parameter

$$\varepsilon_i \equiv \frac{\Gamma_{N_i\to\ell\phi} - \Gamma_{N_i\to\overline{\ell\phi}}}{\Gamma_{N_i\to\ell\phi} + \Gamma_{N_i\to\overline{\ell\phi}}} = \sum_{j(\neq i)} \frac{\Im(h^\dagger h)_{ij}^2}{(h^\dagger h)_{ii}(h^\dagger h)_{jj}} \frac{(M_i^2 - M_j^2)M_i\Gamma_j}{(M_i^2 - M_j^2)^2 + R_{ij}^2} \tag{1.5}$$

where h is the neutrino Yukawa coupling and $\Gamma_i \simeq (h^\dagger h)_{ii} M_i/8\pi$ is the decay width of N_i. The resonant enhancement of the CP-violating parameter was discussed in [38]. Systematic considerations were performed by Pilaftsis [39], and he found that the regulator in the denominator is given by $R_{ij} = M_i\Gamma_j$. If the mass difference is larger than the decay width, we have $|M_i^2 - M_j^2| \gg R_{ij}$, and ε_i is suppressed by $\Gamma_i/M \sim (h^\dagger h)_{ii}$. However, in the degenerate case, $|M_i - M_j| \sim \Gamma$ and ε can be enhanced to $\mathcal{O}((h^\dagger h)^0) \sim 1$. Hence the determination of the regulator R_{ij} is essential for a precise prediction of the lepton number asymmetry in the resonant leptogenesis. The authors [19][42] calculated the resummed propagator of the RH neutrinos and obtained a different regulator $R_{ij} = |M_i\Gamma_i - M_j\Gamma_j|$. By using their result, the enhancement factor becomes much larger. Since the scale of the leptogenesis is sensitive to the form of the regulator, it's important to systematically obtain the correct form of the regulator.

Conventionally, leptogenesis is often calculated based on the classical Boltzmann equation [80] which describes the time evolution of the phase space distribution function of on-shell particles. In the Boltzmann equation, the interactions between particles are taken into account through the collision terms that comprise the S-matrix elements calculated separately in the framework of (equilibrium) quantum field theory. The authors [58] applied the non-equilibrium Green's function method with the Kadanoff-Baym (KB) equations developed in studies of the transport phenomena [81, 82] and derived the full-quantum evolution equation for the lepton number in the hierarchical mass case. Using this method, one can systematically take into account quantum interference, finite temperature and finite density effects. The method was intensively used in the leptogenesis in various papers [61]–[78]. In the resonant leptogenesis, since the quantum interference effect is crucial to the evaluation of the CP-violating parameter, we can expect importance of such a full-quantum mechanical formulation based on the KB equations. In [59], the authors used the method to obtain an oscillating CP-violating parameter in the flat space-time. Then applying it to the Boltzmann equation in the expanding universe, they calculated the lepton number asymmetry. In the strong washout regime, the oscillation is averaged out and the lepton number asymmetry is expressed with an effective CP-violating parameter. Then the maximal value agrees with the case of $R_{ij} = M_i\Gamma_j$ [60].

Recently Garny et al. [71] systematically investigated generation of the lepton asymmetry in the resonant leptogenesis. In the investigation, they considered a non-equilibrium initial condition in a time-independent background and calculated the final lepton number asymmetry. Starting from the vacuum initial state for the RH neutrinos, they read off the CP-violating parameter from the generated lepton asymmetry. The effective regulator they derived is $R_{ij} = M_i\Gamma_i + M_j\Gamma_j$, which differs from the previous results, $R_{ij} = M_i\Gamma_j$ by [39] or $R_{ij} = |M_i\Gamma_i - M_j\Gamma_j|$ by [19, 42]. In our work [90], half of this thesis is based on, the validity of the regulator $R_{ij} = M_i\Gamma_i + M_j\Gamma_j$ is confirmed also in the thermal leptogenesis caused by the expansion of the universe.

In the νMSM [52, 53], the RH neutrinos are responsible for baryogenesis. However, baryon asymmetry is generated in the different way from the thermal leptogenesis, based on the ARS mechanism [56] but with the helicity asymmetry in RH neutrinos. Two of RH neutrinos have degenerate Majorana mass spectrum around $M_i \sim$ GeV, and then they start to decay after the electroweak sphaleron shutoff. (The lightest RH neutrino with keV scale Majorana mass becomes warm dark matter.) Therefore, only the flavor asymmetry generated through RH neutrino production process are relevant for the final baryon asymmetry (total lepton asymmetry is almost zero because the helicity-flipping process through the Majorana mass is negligible for $T \gg M_i$). And high reheating temperature $T_{\rm rh} \gtrsim 10^9$GeV is no longer required. In this scenario, to evaluate the baryon asymmetry generated during the production process of RH neutrinos, the density matrix formalism [57] is employed, which is a multi-flavor generalization of the Boltzmann equation and which can taken into account the quantum coherence in the (SM lepton and RH neutrino's) flavor oscillation. However, the most important mechanism to generate sufficient baryon number is the same as in the resonant leptogenesis, namely, resonant oscillation between different flavors of RH neutrino.[1]

If the density matrix formalism is a correct method to take the coherent flavor oscillation, it must be derived from KB equation under some appropriate assumptions, and lead to the consistent result with the one directly obtained from the KB equation [71]. The authors of [70] showed that the equation of the density matrix can be obtained from the KB equation, and applied to the analysis of the resonant leptogenesis in the same situation as [71], namely, the non-equilibrium initial condition in the flat space-time. In our work [91], the latter part of this thesis is based on, it's shown that the results obtained [71] and [70] are consistent with each other, by using the approximation which is valid in the typical case of resonant leptogenesis. Moreover, we have obtained the general form of the CP-violating parameter in the resonant leptogenesis.

The purpose of this thesis is to see (i) how useful the non-equilibrium QFT is in order to correctly understand the quantum effects in non-equilibrium situation, and (ii) how it's reduced more simple and practical framework like the density matrix formalism, through the detailed investigation of the resonant leptogenesis.

This thesis is organized as follows.

In section 2, we give a brief review of the leptogenesis scenario. After introducing the RH neutrinos and their interaction, we see the conventional derivation of the CP-violating parameter and the kinetic equation of the lepton number. This approach needs an artificial treatment, called real intermediate state subtraction, which relates to the insufficiency of the conventional approach. And we see the lower bound on the lightest RH neutrino mass in the simplest model

[1] In some parameter region, the RH neutrinos are not required to degenerate in their mass [54]. However, instead of the tuning in their masses, the tuning in the neutrino Yukawa coupling is necessary [55] to make the large interaction rate compatible with the light neutrino spectrum through the seesaw mechanism.

of leptogenesis and the effects of lepton and RH neutrino's flavor effects.

In section 3, we focus on how the counterpart of the collision term in the usual Boltzmann equation is expressed in the non-equilibrium QFT. In section 3.1 and 3.2, we summarize the basic properties of various Green functions and the Kadanoff-Baym (KB) equations that must be satisfied by them. Then we derive the evolution equation of the lepton number in the expanding universe in section 3.3, The evolution equation is written in terms of the propagators of the RH neutrinos, the SM leptons and the Higgs boson. In section 3.4 we explain how the KB equation is reduced to the ordinary Boltzmann equation. In section 3.5, it's mentioned that the most important ingredient to get the evolution equation for the lepton number is the Wightman propagators of the RH neutrinos. The flavor diagonal component is directly related to the distribution function, but more important for the lepton asymmetry is its off-diagonal component.

In section 4, we investigate how the expansion of the universe affects various propagators of RH neutrino. We derive the deviation of off-diagonal components of the Wightman propagator of RH neutrino by solving the KB equation directly. Plugging the solution into the evolution equation of lepton number obtained in section 3, we get the Boltzmann equation-like form with the modified CP-violating parameter. In section 4.1, we focus on the resonant oscillations in the thermal equilibrium. We study the properties of the retarded and advanced propagators in which the information of the spectrum is encoded, and the Wightman functions including the information of the distribution functions. In section 4.2, we scrutinize the behavior of Green functions out of equilibrium. By considering the KB equations, it gets to be clear how the deviation from the equilibrium distribution function is generated. And we show that the deviations of the flavor off-diagonal Wightman functions behave differently from the retarded and advanced Green functions. In section 4.3, we apply the calculated deviations of the Wightman functions of the RH neutrinos into the evolution equation derived in section 3, and obtain the quantum Boltzmann equation for the lepton number asymmetry. We read off the CP-violating parameter ε and show that the regulator is given by $R_{ij} = M_i\Gamma_i + M_j\Gamma_j$. In section 4.4, we give a physical interpretation why the regulator $R_{ij} = M_i\Gamma_i + M_j\Gamma_j$ appears instead of $R_{ij} = M_i\Gamma_i - M_j\Gamma_j$. In particular, we show that if we *neglect* a part of quantum effects (off-shell contributions), the regulator is erroneously given by $R_{ij} = M_i\Gamma_i - M_j\Gamma_j$. In section 4.5, we summarize our results following from the method in which the KB equation of RH neutrino is directly solved.

In section 5, instead of solving the KB equation directly, we reduce it into the evolution equation of the density matrix of RH neutrinos. Plugging the analytic solution of the equation into the evolution equation of the lepton number, we get the lepton number Boltzmann equation with the CP-violating parameter which is consistent with the one obtained in section 4. In section 5.1, by taking the multi-flavor generalization of quasi-particle ansatz for the Wightman propagator of RH neutrino, we derive the evolution equation of the density matrix of RH neutrino. In this derivation, well-known Kramers-Moyal expansion is employed and only the lowest order of the expansion is kept. As a result, the collision term is written by the Fourier transform of the self-energy of RH neutrino.

In section 5.2, we obtain the explicit form of the collision term by applying the quasi-particle ansatz to all the propagators in the imaginary part of self-energies. Then in manner of the optical theorem, the multi-flavor generalization of the squared amplitudes of decay and scattering process are obtained. The real part of self-energy is identified as the quantum correction to the energy of RH neutrino and contributes to flavor oscillation of RH neutrinos. In section 5.3, assuming the smallness of deviation from thermal equilibrium, we get the analytic solution of the equation. Plugging it into the equation of lepton number, we read off the CP-violating parameter. In general, it's shown that the regulator of the resonant enhancement includes the annihilation rate as well as the decay rate. In section 5.4, we discuss more practical definition of the CP-violating parameter convenient to obtain the final lepton asymmetry. The back reaction from generated lepton number is also taken into account. In section 5.5, we summarize the observations obtained from the method in which the KB equation of RH neutrino is reduced to the evolution equation of the density matrix of RH neutrino.

Finally, section 6 is devoted to the conclusion of this thesis.

In appendix A, we give a brief introduction to the closed time path (CTP) formalism and the KB equations. In appendix B, we derive the self-energies for the RH neutrinos and the SM leptons based on the 2PI formalism. In appendix C, we give anther derivation of the off-diagonal component of the Wightman functions out of equilibrium. The calculation explains why the regulator $R_{ij} = M_i\Gamma_i + M_j\Gamma_j$ naturally appears. In appendix D, we give details of the derivation of kinetic term of the evolution equation of the density matrix. In appendix E, the explicit forms of the analytic solution of the density matrix are shown.

2 Baryogenesis through Leptogenesis

In this section, the leptogenesis scenario is briefly reviewed. The interaction between the RH neutrinos and the SM lepton and Higgs boson gives the CP-asymmetric decay of the RH neutrino. In the simplest model of the leptogenesis, in which only the lightest RH neutrino is responsible for the lepton number generation, the lower bound of the lightest Majorana mass $M_1 \gtrsim 10^9$GeV appears in order for the sufficient CP asymmetry in the decay process, and rather high reheating temperature $T_{\rm rh} \gtrsim 10^9$GeV is required. Lepton and RH neutrino flavor effects cannot lower this bound on $T_{\rm rh}$ considerably. However, it can be evaded by considering the degenerate mass spectrum of the RH neutrinos. In this case, the quantum effects become more important, and the conventional calculations, reviewed in this section, must be replaced with the calculation based on the non-equilibrium QFT as we discuss in the later sections.

Before introducing the concrete model, let us look at the conditions to dynamically generate the baryon number [3]. (i) Baryon number should not be conserved. This is required in order to generate the baryon asymmetry from the initial state with $B = 0$. Even if the initial state of the universe has a baryon number, the inflation dilutes it significantly. (ii) C and CP symmetries must be broken. If either C or CP were unbroken, the processes involving baryons have their C or CP conjugate processes involving anti-baryons, and their interaction rates are the same. Then, no net baryon number is generated. (iii) Out of equilibrium dynamics are necessary. If the state is in thermal equilibrium, it must be characterized by a few parameters such as the temperature and the conserved charges, and then non-conserved quantities must be zero. All of these conditions are satisfied in the SM. But two of them are too small to reproduce the observed baryon asymmetry: (i) Non-conservation of the baryon number is provided by the non-perturbative effect which changes the Chern-Simons number of the $SU(2)$ gauge field in the early universe [8]. It leads to the processes involving nine left-handed quarks and three left-handed leptons, and violates the $B + L$ number [7]. (ii) C and CP are violated by the electroweak (EW) interactions and the complex phase in the CKM matrix respectively. However, the magnitude of CP-violation is too small and new CP-violating sources are needed. (iii) The significant deviation from equilibrium could be generated if the EW phase transition were strongly first order. But the observed Higgs mass 126GeV is too heavy to make the phase transition first order [10]. This also requires some new physics beyond the SM.

In the leptogenesis scenario [4], the RH neutrinos, which are introduced via the seesaw mechanism [5], can be responsible for the successful baryogenesis. Their Yukawa couplings provide the new source of the CP violation. Their large Majorana masses not only violate the lepton number as the seed of the baryon number, but also make their equilibrium number densities behave differently from those of the SM particles which construct the thermal bath, and cause the significant deviation from equilibrium.

2.1 Right-handed neutrino and its interaction

The model we consider is an extension of the SM with RH neutrinos $\nu_{R,i}$. i is the flavor index, $i = 1, 2, 3$. We set $N_i = \nu_{R,i} + \nu^c_{R,i}$. The Lagrangian is given by

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \frac{1}{2}\overline{N}^i(i\not\nabla - M_i)N^i + \mathcal{L}_{int} \ , \tag{2.1}$$

$$\mathcal{L}_{int} \equiv -h_{\alpha i}(\overline{\ell}^{\alpha}_a \epsilon_{ab}\phi^*_b)P_R N^i + h^\dagger_{i\alpha}\overline{N}^i P_L(\phi_b \epsilon_{ba}\ell^\alpha_a) \tag{2.2}$$

where $\alpha, \beta = 1, 2, 3$ and $a, b = 1, 2$ are flavor indices of the SM leptons ℓ^α_a and isospin $SU(2)_L$ indices respectively. M_i is the Majorana mass of N_i. $h_{i\alpha}$ is complex matrix as the Yukawa coupling of N^i, ℓ^α_a and the Higgs ϕ_a doublet. $P_{R/L}$ are chiral projections on right/left-handed fermions. It's convenient to assign the lepton number ± 1 to the chirality ± 1 component $P_{R/L}N$, so that, if the Majorana mass term were set to be zero, the lepton number would conserved at the classical level. The SM Lagrangian $\mathcal{L}_{\rm SM}$ includes the interaction term $-\lambda_\alpha(\overline{\ell}^\alpha_a \phi_a)e^\alpha_R + h.c.$ for the charged leptons e^α_R, with the diagonal and real Yukawa coupling λ. Although M and λ are complex matrices in general, the basis for the N^i, ℓ^α and e^β_R have been chosen so that they can be reduced to the diagonal and real matrices. In this basis, the neutrino Yukawa coupling h is left as a general complex matrix. By the phase redefinitions of ℓ and e_R, three phases can be removed and hence, h has 15 physical parameters. There are in total 21 physical parameters in the lepton sector of the theory.

Introducing the right-handed neutrino with the Majorana mass M leads to the mass matrix for the left-handed neutrinos

$$\mathcal{L}_{m_\nu} = \frac{1}{2}\nu^\dagger_L m_\nu \nu^c_L + h.c. \tag{2.3}$$

in the low energy effective theory. Hence it can explain the flavor oscillation in fluxes of solar, atmospheric, reactor and accelerator neutrinos, that requires the two mass-squared differences and mixing angles [13]

$$\Delta m^2_{\rm sol} \equiv m^2_{\nu_2} - m^2_{\nu_1} = (7.50^{+0.59}_{-0.48}) \times 10^{-5}{\rm eV} \ , \tag{2.4}$$

$$\Delta m^2_{\rm atm,NO} \equiv m^2_{\nu_1} - m^2_{\nu_1} = (2.457^{+0.150}_{-0.140}) \times 10^{-3}{\rm eV} \ , \tag{2.5}$$

$$\Delta m^2_{\rm atm,IO} \equiv m^2_{\nu_2} - m^2_{\nu_3} = (2.449^{+0.141}_{-0.142}) \times 10^{-3}{\rm eV} \ , \tag{2.6}$$

$$s^2_{12} \equiv \sin^2\theta_{12} = 0.304^{+0.040}_{-0.034} \ , \tag{2.7}$$

$$s^2_{13,\rm NO} \equiv \sin^2\theta_{12,\rm NO} = 0.0218^{+0.0032}_{-0.0032} \ , \tag{2.8}$$

$$s^2_{13,\rm IO} \equiv \sin^2\theta_{12,\rm IO} = 0.0219^{+0.0032}_{-0.0031} \tag{2.9}$$

where the subscripts NO and IO stands for the normal ordering case ($m_{\nu_1} < m_{\nu_2} < m_{\nu_3}$) and the inverted ordering case ($m_{\nu_3} < m_{\nu_1} < m_{\nu_2}$), respectively.

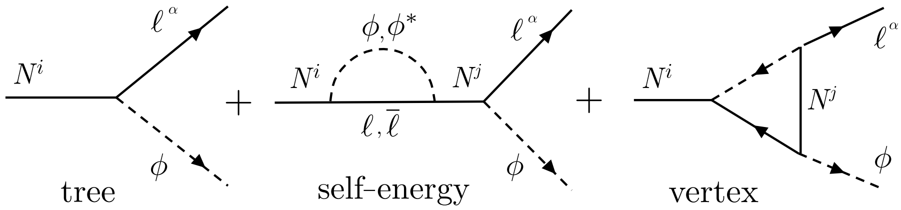

Figure 1: Tree and one-loop diagrams of the RH neutrino decay into the SM lepton and Higgs boson.

The Majorana mass term (2.3) originates from the dimension five operator $(\ell_\alpha\phi)(\ell_\beta\phi)$ which appears after integration of the heavy right-handed neutrinos with the Majorana masses M_i. Hence, the mass matrix for the left-handed neutrinos is given as

$$m_\nu = hM^{-1}h^T v^2 \tag{2.10}$$

where $v = 174$GeV is the vacuum expectation value of the Higgs boson.

2.1.1 *CP*-asymmetry in Majorana neutrino decay

Due to the complex phases in the neutrino Yukawa h, the decay process of the RH-neutrino into the SM lepton and Higgs boson becomes CP-asymmetric. At the moment, we focus on the case with *hierarchical* mass spectrum of the RH neutrinos. We are interested in the radiation-dominated era with the high temperature $T > T_{EW}$, then the $SU(2)$ symmetry is not broken and all of the SM particles are massless.[2]

The matrix elements of the tree diagram in Fig.(1) and its CP-conjugate process are written as

$$i\mathcal{M}^{\rm tree}_{N_{i,s}\to\ell^\alpha_a\phi_b} = -ih_{\alpha i}\overline{u}_\alpha(p)P_R u_{i,s}(q)\epsilon_{ab}\ , \tag{2.11}$$

$$i\mathcal{M}^{\rm tree}_{N_{i,s}\to\overline{\ell}^\alpha_a\phi^*_b} = -ih^\dagger_{i\alpha}\overline{v}_{i,s}(q)P_L v_\alpha(p)\epsilon_{ba} \tag{2.12}$$

where s represents the spin degrees of freedom of the RH neutrino. Summing the spin of the RH neutrino and $SU(2)$ and flavor indices, we get the squared amplitude

$$|\mathcal{M}^{\rm tree}_{N_i\to\ell\phi}|^2 \equiv \sum_{s,a,b,\alpha}|\mathcal{M}^{\rm tree}_{N_{i,s}\to\ell^\alpha_a\phi_b}|^2 = \sum_{s,a,b,\alpha}|\mathcal{M}^{\rm tree}_{N_{i,s}\to\overline{\ell}^\alpha_a\phi^*_b}|^2 \tag{2.13}$$

$$= g_w(h^\dagger h)_{ii}(2q\cdot p) = g_w(h^\dagger h)_{ii}M_i^2 \tag{2.14}$$

[2]In this section, we just omit the thermal masses of the SM particles $\sim gT$ for simplicity.

where $g_w = 2$ is the $SU(2)$ degrees of freedom. Therefore, there is no CP-asymmetry at the lowest order of $(h^\dagger h)$. The decay rates of the RH neutrino into the SM (anti-)particles is given as

$$\Gamma_{N_i \to \ell\phi} = \Gamma_{N_i \to \overline{\ell\phi}} \tag{2.15}$$
$$= \int d\Pi_p^\ell d\Pi_k^\phi (2\pi)^4 \delta^4(q-p-k) \sum_{s,a,\alpha} |\mathcal{M}^{\text{tree}}_{N_{i,s}\to \ell_a^\alpha \phi_b}|^2$$
$$= \frac{g_w (h^\dagger h)_{ii}}{32\pi} M_i \ . \tag{2.16}$$

The total decay width of the i-th RH neutrino is obtained as

$$\Gamma_{N_i} = \Gamma_{N_i \to \ell\phi} + \Gamma_{N_i \to \overline{\ell\phi}} = \frac{(h^\dagger h)_{ii}}{8\pi} M_i \ . \tag{2.17}$$

The CP-asymmetric contributions come from the interferences between the tree and one-loop diagrams. The matrix elements of the "vertex" diagram in Fig.(1) and its CP-conjugate process are given as

$$i\mathcal{M}^{\text{vertex}}_{N_{i,s}\to \ell_a^\alpha \phi_b} = +ih_{\alpha j} M_j b_j(q^2)(h^\dagger h)^*_{ji} \overline{u}_\alpha(p) P_R \not{q} u_{i,s}(q)\epsilon_{ab} \ , \tag{2.18}$$

$$i\mathcal{M}^{\text{vertex}}_{N_{i,s}\to \overline{\ell}_a^\alpha \phi_b^*} = +i(h^\dagger h)^*_{ij} M_j b_j(q^2) h^\dagger_{j\alpha} \overline{v}_{i,s}(q) \not{q} P_L v_\alpha(p)\epsilon_{ba} \ . \tag{2.19}$$

Although b_j is defined by the one-loop integral, its imaginary part

$$\Im[b_j(q^2)] = \frac{1}{16\pi\sqrt{q^2} M_j} f\left(\frac{M_j^2}{q^2}\right)\Theta(q^2) \ , \tag{2.20}$$
$$f(x) = \sqrt{x}\left(1-(1+x)\ln\left(\frac{1+x}{x}\right)\right) \ , \tag{2.21}$$

is finite and contributes to the CP-asymmetry in the decay process as seen below. From the "self-energy" diagram in Fig.(1), we get

$$i\mathcal{M}^{\text{self-energy}}_{N_{i,s}\to \ell_a^\alpha \phi_b} = -ih_{\alpha j} \overline{u}_\alpha(p) P_R G^{jj}(q) \Pi^{ji}(q) u_{i,s}(q) \epsilon_{ab} \tag{2.22}$$
$$= -ih_{\alpha j}\left(M_i (h^\dagger h)_{ji} + M_j (h^\dagger h)^*_{ji}\right) \frac{M_i a(q^2)}{q^2 - M_j^2} \overline{u}_\alpha(p) P_R u_{i,s}(q)\epsilon_{ab} \ ,$$

$$i\mathcal{M}^{\text{self-energy}}_{N_{i,s}\to \overline{\ell}_a^\alpha \phi_b^*} = -ih^\dagger_{j\alpha} \overline{v}_{i,s}(q) \Pi^{ij}(\overline{q}) G^{jj}(\overline{q}) P_L v_\alpha(p)\epsilon_{ba} \tag{2.23}$$
$$= -ih^\dagger_{j\alpha}\left(M_i (h^\dagger h)_{ij} + M_j (h^\dagger h)^*_{ij}\right) \frac{M_i a(q^2)}{q^2 - M_j^2} \overline{v}_{i,s}(q) P_R v_\alpha(p)\epsilon_{ba} \ .$$

$G^{jj} = i(\not{q} - M + i\epsilon)^{-1}$ is the standard bare Feynman propagator of the j-th RH neutrino. Π is the self-energy of the RH neutrino (B.3), with the bare propagators of the SM lepton and Higgs boson. It can be separated into two parts as

$$\Pi^{ji}(q) = -i\left(\not{q}P_R(h^\dagger h)_{ji} + \not{q}P_L(h^\dagger h)^*_{ji}\right)a(q^2) \tag{2.24}$$

where a comes from the loop integral, and after the renormalization, it's written as

$$a(q^2) = \frac{1}{16\pi^2}\left(\ln\frac{|q^2|}{\mu^2} - 2 - i\pi\Theta(q^2)\right) . \tag{2.25}$$

The first term of (2.24) represents the self-energy diagram in which the lepton number flows in the same direction as the 4-momentum q, and it gives the first terms of the matrix elements (2.22) and (2.23). Note that M_i^2's in these expressions come from the on-shell momentum $q^2 = M_i^2$. For theses terms, the decay amplitudes don't pick up the scalar component of the RH neutrino propagator G which is proportional to the Majorana mass violating the lepton number as mentioned below the Lagrangian (2.1). Hence, they correspond to the lepton number conserving processes. On the other hands, The second term of (2.24) represents the self-energy diagram in which the lepton number flows in the direction opposite to the 4-momentum. it gives the second terms of the matrix elements (2.22) and (2.23) as the lepton number violating contributions.

The leading order CP-asymmetry in the N_i decay processes appears from interferences between these one-loop diagrams and the tree level ones. The difference between the squared amplitudes of $N_i \to \ell^\alpha\phi$ and $N_i \to \overline{\ell}^\alpha\overline{\phi}$ processes is given as

$$\begin{aligned}
\Delta|\mathcal{M}|^2_{i,\alpha} \equiv & \sum_{s,a,b}\left(\left|\mathcal{M}_{N_{i,s}\to\ell^\alpha_a\phi_b}\right|^2 - \left|\mathcal{M}_{N_{i,s}\to\overline{\ell}^\alpha_a\phi^*_b}\right|^2\right) \\
= & -2g_w M_i^3 M_j \sum_{j(\neq i)}\left(\Re\left[h^\dagger_{i\alpha}h_{\alpha j}(h^\dagger h)_{ij}b_j(M_i^2)\right]\right. \\
& \left. - \Re\left[h^\dagger_{j\alpha}h_{\alpha i}(h^\dagger h)_{ji}b_j(M_i^2)\right]\right) \\
& + \frac{2g_w M_i^3}{M_i^2 - M_j^2}\sum_{j(\neq i)}\left(\Re\left[a(M_i^2)h^\dagger_{i\alpha}h_{\alpha j}\left\{(h^\dagger h)_{ij}M_j + (h^\dagger h)_{ji}M_i\right\}\right]\right. \\
& \left. - \Re\left[a(M_i^2)h^\dagger_{j\alpha}h_{\alpha i}\left\{(h^\dagger h)_{ji}M_j + (h^\dagger h)_{ij}M_i\right\}\right]\right) \\
= & \frac{g_w M_i^2}{4\pi}\sum_{j(\neq i)}\left(\Im[h^\dagger_{i\alpha}h_{\alpha j}(h^\dagger h)_{ij}]\left\{f\left(\frac{M_j^2}{M_i^2}\right) + \frac{M_iM_j}{M_i^2 - M_j^2}\right\}\right. \\
& \left. + \Im[h^\dagger_{i\alpha}h_{\alpha j}(h^\dagger h)_{ji}]\frac{M_i^2}{M_i^2 - M_j^2}\right) .
\end{aligned} \tag{2.26}$$

The first term proportional to f comes from the vertex corrections (2.18) and (2.19). The second term comes from the lepton number violating parts of the self-energy corrections (2.22) and (2.23). The lepton number conserving part of the self-energy gives the last term of (2.26). Note that this term vanishes when the lepton flavor index α is summed.

Finally, we get the CP-violating parameter of the decay process $N_i \to \ell^\alpha \phi$:

$$\varepsilon_{i\alpha} \equiv \frac{\Gamma_{N_i \to \ell^\alpha \phi} - \Gamma_{N_i \to \bar{\ell}^\alpha \phi^*}}{\sum_\alpha \left(\Gamma_{N_i \to \ell^\alpha \phi} + \Gamma_{N_i \to \bar{\ell}^\alpha \phi^*} \right)} \tag{2.27}$$

$$= \frac{\Delta |\mathcal{M}|^2_{i,\alpha}}{2 |\mathcal{M}^{\text{tree}}_{N_i \to \ell\phi}|^2} + \mathcal{O}(h^\dagger h)^2$$

$$= \sum_{j(\neq i)} \left\{ \frac{\Im[h^\dagger_{i\alpha} h_{\alpha j} (h^\dagger h)_{ij}]}{8\pi (h^\dagger h)_{ii}} g(x_{ji}) + \frac{\Im[h^\dagger_{i\alpha} h_{\alpha j} (h^\dagger h)_{ji}]}{8\pi (h^\dagger h)_{ii}} \frac{1}{1 - x_{ji}} \right\} , \tag{2.28}$$

where

$$x_{ji} = \frac{M_j^2}{M_i^2} \ , \quad g(x) = \sqrt{x} \left(1 - (1+x) \ln \left(\frac{1+x}{x} \right) + \frac{1}{1-x} \right) \ . \tag{2.29}$$

$g(x)$ behaves as $-x^{-1/2} \times 3/2$ in the limit of $x \gg 1$. In the limit of $x_{ji} = M_j^2/M_i^2 \to 1$, this expression becomes infinity and not applicable. Such a case with the almost degenerate mass spectrum is the main interest of this thesis.

2.1.2 Kinematic equation for the lepton number

In the conventional approach, the SM lepton number is calculated by using the Boltzmann equation. The Boltzmann equation is a classical equation which describes the time evolution of the one-particle distribution function $f(X, p)$ in the phase space:

$$p \cdot \mathcal{D} f(X, p) = \mathcal{C}(X, p) \ . \tag{2.30}$$

p is the on-shell 4-momentum of the particle and $\mathcal{D}_\mu = \partial_{X^\mu} + \Gamma^\rho_{\mu\nu} p_\rho \partial_{p_\nu}$ is the (Wigner transformed) covariant derivative. The r.h.s. is called as the collision term. The Boltzmann equation is a Markovian equation, that is, the collision term $C(X, p)$ depends only on the physical quantities at X and there is no memory integral. The quantum effects are taken into account only in the collision term. In the spatially flat universe we are interested in, the derivative operator in the r.h.s. is given as $p \cdot \mathcal{D} = \omega_p \partial_t - H(|\mathbf{p}|^2/a^2) \partial_{\omega_p} \equiv \omega_p d_t$ where $a = a(t)$ is the scale factor and $\mathbf{p}/a$ is the spatial physical momentum. Then the time evolution of the distribution function $f_{\ell^\alpha_a, p} = f_{\ell^\alpha_a}(t, p)$ of the SM lepton ℓ^α_a is described by

$$\omega_p d_t f_{\ell^\alpha_a, p} = \mathcal{C}_{\mathrm{D}, \ell^\alpha_a} + \mathcal{C}_{\mathrm{scatt}, \ell^\alpha_a} \ , \tag{2.31}$$

$$\mathcal{C}_{\mathrm{D},\ell_a^\alpha}/\omega_p = \frac{1}{2\omega_p}\sum_{i,s,b}\int d\Pi_q^{N_i} d\Pi_k^{\phi_b}(2\pi)^4\delta(q-p-k) \tag{2.32}$$
$$\times\Big\{|\mathcal{M}_{N_{i,s}\to\ell_a^\alpha\phi}|^2(1-f_{\ell_a^\alpha,p})(1+f_{\phi_b,k})f_{N_{i,s},q}$$
$$-|\mathcal{M}_{\ell_a^\alpha\phi_b}\to N_{i,s}|^2 f_{\ell_a^\alpha,p}f_{\phi_b,k}(1-f_{N_{i,s},q})\Big\}\ ,$$

$$\mathcal{C}_{\mathrm{scatt},\ell_a^\alpha}/\omega_p = \frac{1}{2\omega_p}\sum_{i,b,a',b',s}\int d\Pi_{p'}^{\overline{\ell}_{a'}^\beta} d\Pi_k^{\phi_b} d\Pi_{k'}^{\overline{\phi}_{b'}}(2\pi)^4\delta(p+k-p'-k') \tag{2.33}$$
$$\times\Big\{|\mathcal{M}_{\overline{\ell}_{a'}^\beta\overline{\phi}_{b'}\to\ell_a^\alpha\phi_b}|^2(1-f_{\ell_a^\alpha,p})(1+f_{\phi_b,k})f_{\overline{\ell}_{a'}^\beta,p'}f_{\overline{\phi}_{b'},k'}$$
$$-|\mathcal{M}_{\ell_a^\alpha\phi_b\to\overline{\ell}_{a'}^\beta\overline{\phi}_{b'}}|^2 f_{\ell_a^\alpha,p}f_{\phi_b,k}(1-f_{\overline{\ell}_{a'}^\beta,p'})(1+f_{\overline{\phi}_{b'},k'})\Big\}$$

where $f_{\phi_b,k}$, $f_{\overline{\ell}_{a'}^\beta,p'}$, $f_{\overline{\phi}_{b'},k'}$ and $f_{N_{i,s},q}$ are the distribution functions of ϕ_b, $\overline{\ell}_{a'}^\beta$, $\overline{\phi}_{b'}$, $N_{i,s}$ with momenta k, p', k', q' respectively.[3] For the anti-lepton $\overline{\ell}$, $\mathcal{C}_{\mathrm{D},\overline{\ell}_a^\alpha}$ and $\mathcal{C}_{\mathrm{scatt},\overline{\ell}_a^\alpha}$ are obtained just by replacing ℓ with $\overline{\ell}$ in (2.32) and (2.33). For the distribution function $f_{N_{i,s},q}$ of the RH neutrino $N_{i,s}$ with the momentum q, we get the similar equation:

$$d_t f_{N_{i,s},q} = \frac{1}{2\omega_p}\sum_{\alpha,a,b}\int d\Pi_p^{\ell_a^\alpha} d\Pi_k^{\phi_b}(2\pi)^4\delta(q-p-k) \tag{2.34}$$
$$\times\Big\{|\mathcal{M}_{\ell_a^\alpha\phi_b\to N_{i,s}}|^2 f_{\ell_a^\alpha,p}f_{\phi_b,k}(1-f_{N_{i,s},q})$$
$$-|\mathcal{M}_{N_{i,s}\to\ell_a^\alpha\phi}|^2(1-f_{\ell_a^\alpha,p})(1+f_{\phi_b,k})f_{N_{i,s},q}\Big\}$$
$$+\frac{1}{2\omega_p}\sum_{\alpha,a,b}\int d\Pi_p^{\overline{\ell}_a^\alpha} d\Pi_k^{\overline{\phi}_b}(2\pi)^4\delta(q-p-k)$$
$$\times\Big\{|\mathcal{M}_{\overline{\ell}_a^\alpha\overline{\phi}_b\to N_{i,s}}|^2 f_{\overline{\ell}_a^\alpha,p}f_{\overline{\phi}_b,k}(1-f_{N_{i,s},q})$$
$$-|\mathcal{M}_{N_{i,s}\to\overline{\ell}_a^\alpha\overline{\phi}}|^2(1-f_{\overline{\ell}_a^\alpha,p})(1+f_{\overline{\phi}_b,k})f_{N_{i,s},q}\Big\}\ .$$

These Boltzmann equations construct the system of infinite number of differential equations. However, we can integrate the momenta by taking some assumptions, and then the number of differential equations to be solved simultaneously is reduced to the finite number. The distribution functions are assumed to depend on the momenta in the following way:

$$f_p = \frac{n}{n^{eq}}\times f_p^{\mathrm{MB}}\ ,\quad n^{eq} = \int\frac{d^3p}{(2\pi)^3}f_p^{\mathrm{MB}} = \frac{1}{a^3}\int\frac{d^3\mathbf{p}}{(2\pi)^3}f_p^{\mathrm{MB}} \tag{2.35}$$

[3]Note that the processes mediated by the SM gauge interactions are very fast and their effects are taken into account by assuming that the SM lepton and Higgs boson are in kinematic equilibrium, that is, their distribution functions can be well described by the Fermi-Dirac and Bose-Einstein distribution function with time dependent chemical potentials.

where n is a number density and $f_p^{\rm MB}$ is the Maxwell-Boltzmann distribution function:

$$f_p^{\rm MB} = e^{-\omega_p/T} \ , \quad \omega_p = \sqrt{m^2 + |\mathbf{p}|^2/a^2} \ . \tag{2.36}$$

Using Maxwell-Boltzmann instead of the Bose-Einstein or Fermi-Dirac statistics makes a difference in n^{eq} of order 10% at $T = m$. For consistency, the quantum statistical factors $(1 - f_\ell)$, $(1 - f_N)$ and $(1 + f_\phi)$, representing the induced emission and Pauli blocking, must be neglected. The temperature region we are interested in is higher than the electro-weak temperature $T_{\rm EW}$, then the system is $SU(2)$ symmetric and the distribution functions of the SM lepton and Higgs boson does not depend on the $SU(2)$ indices: $f_{\ell^\alpha_a} = f_{\ell^\alpha}$, $f_{\phi_b} = f_\phi$. And we assume that the distribution function of the RH neutrino does not depend on the spin degrees of freedom: $f_{N_{i,s}} = f_{N_i}$. Then the number densities are defined as

$$n_{\ell^\alpha} = g_w \int \frac{d^3p}{(2\pi)^3} f_{\ell^\alpha,p}, \quad n_{\overline{\ell}^\alpha} = g_w \int \frac{d^3p}{(2\pi)^3} f_{\overline{\ell}^\alpha,p} \ , \tag{2.37}$$

$$n_\phi = g_w \int \frac{d^3k}{(2\pi)^3} f_{\phi,k} \ , \quad n_{\overline{\phi}} = g_w \int \frac{d^3k}{(2\pi)^3} f_{\overline{\phi},k} \ , \tag{2.38}$$

$$n_{N_i} = g_N \int \frac{d^3q}{(2\pi)^3} f_{N_i,q} \tag{2.39}$$

where $g_N = 2$ is the number of spin degrees of freedom. By integrating the momenta in (2.31) and (2.34) without quantum statistical factors, we obtain the evolution equations for these number densities:

$$\begin{aligned} d_t n_{\ell^\alpha} + 3H n_\ell =& \sum_i \left(\gamma_{N_i \to \ell^\alpha \phi} \frac{n_{N_i}}{n_{N_i}^{eq}} - \gamma_{\ell^\alpha \phi \to N_i} \frac{n_{\ell^\alpha}}{n_{\ell^\alpha}^{eq}} \frac{n_\phi}{n_\phi^{eq}} \right) \\ & + (\text{scattering process}) \ , \end{aligned} \tag{2.40}$$

$$\begin{aligned} d_t n_{\overline{\ell}^\alpha} + 3H n_{\overline{\ell}} =& \sum_i \left(\gamma_{N_i \to \overline{\ell}^\alpha \overline{\phi}} \frac{n_{N_i}}{n_{N_i}^{eq}} - \gamma_{\overline{\ell}^\alpha \overline{\phi} \to N_i} \frac{n_{\overline{\ell}^\alpha}}{n_{\overline{\ell}^\alpha}^{eq}} \frac{n_{\overline{\phi}}}{n_{\overline{\phi}}^{eq}} \right) \\ & + (\text{scattering process}) \ , \end{aligned} \tag{2.41}$$

$$\begin{aligned} d_t n_{N_i} + 3H n_{N_i} =& \sum_\alpha \left(\gamma_{\ell^\alpha \phi \to N_i} \frac{n_{\ell^\alpha}}{n_{\ell^\alpha}^{eq}} \frac{n_\phi}{n_\phi^{eq}} - \gamma_{N_i \to \ell^\alpha \phi} \frac{n_{N_i}}{n_{N_i}^{eq}} \right) \\ & + \sum_\alpha \left(\gamma_{\overline{\ell}^\alpha \overline{\phi} \to N_i} \frac{n_{\overline{\ell}^\alpha}}{n_{\overline{\ell}^\alpha}^{eq}} \frac{n_{\overline{\phi}}}{n_{\overline{\phi}}^{eq}} - \gamma_{N_i \to \overline{\ell}^\alpha \overline{\phi}} \frac{n_{N_i}}{n_{N_i}^{eq}} \right) \ , \end{aligned} \tag{2.42}$$

$$
\begin{aligned}
\gamma_{N_i\to\ell^\alpha\phi} &= \gamma_{\overline{\ell}^\alpha\overline{\phi}\to N_i} && (2.43)\\
&= \int d\Pi_q^{N_i} d\Pi_p^{\ell^\alpha} d\Pi_k^{\phi}(2\pi)^4\delta(q-p-k)\sum_{s,a,b}|\mathcal{M}_{N_{i,s}\to\ell^\alpha_a\phi_b}|^2 f_{N_i}^{\mathrm{MB}}\ , \\
\gamma_{\ell^\alpha\phi\to N_i} &= \gamma_{N_i\to\overline{\ell}^\alpha\overline{\phi}} && (2.44)\\
&= \int d\Pi_q^{N_i} d\Pi_p^{\ell^\alpha} d\Pi_k^{\phi}(2\pi)^4\delta(q-p-k)\sum_{s,a,b}|\mathcal{M}_{\ell^\alpha_a\phi_b\to N_{i,s}}|^2 f_{N_i}^{\mathrm{MB}}\ .
\end{aligned}
$$

In (2.43) and (2.44), we have used the relationship $|\mathcal{M}_{N\to\ell\phi}|^2 = |\mathcal{M}_{\overline{\ell\phi}\to N}|^2$ and $|\mathcal{M}_{\ell\phi\to N}|^2 = |\mathcal{M}_{N\to\overline{\ell\phi}}|^2$ following from the CPT invariance and the energy conservation $f_\ell^{\mathrm{MB}} f_\phi^{\mathrm{MB}} = f_N^{\mathrm{MB}}$. Finally, by taking the difference between (2.40) and (2.41), the evolution equation for the lepton number density

$$
n_{L^\alpha} = n_{\ell^\alpha} - n_{\overline{\ell}^\alpha} \tag{2.45}
$$

is obtained as

$$
\begin{aligned}
d_t n_{L^\alpha} + 3Hn_{L^\alpha} = &\sum_i \Bigg(\left(\gamma_{N_i\to\ell^\alpha\phi} - \gamma_{N_i\to\overline{\ell}^\alpha\overline{\phi}}\right)\frac{\Delta n_{N_i}}{n_{N_i}^{eq}} \\
&\qquad - \left(\gamma_{\ell^\alpha\phi\to N_i} + \gamma_{\overline{\ell}^\alpha\overline{\phi}\to N_i}\right)\frac{n_{L^\alpha}}{2n_{\ell^\alpha}^{eq}} \Bigg) \qquad (2.46)\\
&+ [d_t n_{L^\alpha} + 3Hn_{L^\alpha}]^{\mathrm{D}}_{thermal} \\
&+ (\text{scattering process})\ .
\end{aligned}
$$

Δn_N is the deviation from equilibrium number density of the RH neutrino. Therefore, the first line represents the fact that the lepton number is generated by the deviation from the equilibrium. On the second line, $n_\ell^{eq} = n_{\overline{\ell}}^{eq}$ has been used. And it's assumed that the deviation from the equilibrium value is given by the small chemical potential $\mu_\ell = -\mu_{\overline{\ell}}$ in the distribution function, then $n_\ell - n_\ell^{eq} = n_{\overline{\ell}} - n_{\overline{\ell}}^{eq}$. This term corresponds to the wash-out of the generated lepton number. For brevity, the contribution from the Higgs chemical potential has been omitted. The third line is given as

$$
\begin{aligned}
[d_t n_{L^\alpha} + 3Hn_{L^\alpha}]^{\mathrm{D}}_{thermal} &= \left(\gamma_{N_i\to\ell^\alpha\phi} - \gamma_{N_i\to\overline{\ell}^\alpha\overline{\phi}} - \gamma_{\ell^\alpha\phi\to N_i} + \gamma_{\overline{\ell}^\alpha\overline{\phi}\to N_i}\right) \\
&= 2(\gamma_{N_i\to\ell^\alpha\phi} - \gamma_{N_i\to\overline{\ell}^\alpha\overline{\phi}}) \propto \Delta|\mathcal{M}|^2_{i,\alpha}\ . \qquad (2.47)
\end{aligned}
$$

This term must be artificial because this means that the thermal equilibrium with $n_L = 0$ is unstable as long as there is the CP asymmetry. As seen in the next subsection, this is caused by "double counting" of the same diagrams. Just omitting this unphysical term, we get the evolution equation of the lepton number:

$$
\begin{aligned}
d_t n_{L^\alpha} + 3Hn_{L^\alpha} =& \sum_i \frac{TM_i^2}{\pi^2} K_1(M_i/T)\Gamma_{N_i} \left(\varepsilon_{i\alpha} \frac{\Delta n_{N_i}}{n_{N_i}^{eq}} - \frac{h_{i\alpha}^\dagger h_{\alpha i}}{(h^\dagger h)_{ii}} \frac{n_{L^\alpha}}{2n_\ell^{eq}} \right) \quad (2.48) \\
&+ (\text{scattering process}) \ . \\
=& \sum_i \Bigg(\varepsilon_{i,\alpha} \Gamma_{N_i} \frac{K_1(M_i/T)}{K_2(M_i/T)} \Delta n_{N_i} \\
&\qquad - \frac{h_{i\alpha}^\dagger h_{\alpha i}}{(h^\dagger h)_{ii}} \Gamma_{N_i} \frac{M_i^2}{4T^2} K_1(M_i/T) n_{L^\alpha} \Bigg) \quad (2.49) \\
&+ (\text{scattering process})
\end{aligned}
$$

where $K_i(z)$ is the modified Bessel function of the second kind. In the second equality,

$$
n_{N_i}^{eq} = \frac{g_N T M_i^2}{2\pi^2} K_2(M_i/T) \ , \quad n_\ell^{eq} = \frac{g_w T^3}{\pi^2} \tag{2.50}
$$

have been used. The CP violating parameter $\varepsilon_{i\alpha}$ and the total decay width Γ_{N_i} are given in (2.27) and (2.17).

2.1.3 RIS subtraction

We will see that the problematic term (2.47) is a consequence of the "double counting" of the on-shell state [80] of the RH neutrino, by looking at the scattering contributions to the kinetic equation. For simplicity, we restrict our analysis to the unflavored regime where the interaction rates of the scatterings which distinguish the SM lepton flavors are negligible compared to the Hubble expansion rate.

The s-channel $\times$ s-channel part of the squared amplitude of $\overline{\ell\phi} \to \ell\phi$ is given as

$$
\begin{aligned}
&\sum_{a,b,a',b',\alpha,\beta} |\mathcal{M}_{\overline{\ell}^\beta_{a'}, \overline{\phi}_{b'} \to \ell^\alpha_a \phi_b}|^2 \Big|_{\mathrm{s\times s}} \quad (2.51) \\
&\qquad = g_w^2 (p_1 \cdot p_2) \sum_{i,j} M_i M_j 2 (h_+^\dagger h_+)_{ij} (h_+^\dagger h_+)_{ij} D_i^*(q^2) D_j(q^2)
\end{aligned}
$$

where $D(q^2)$ is the Breit-Wigner propagator

$$
D_i(q^2) = \frac{1}{q^2 - M_i^2 + i\Theta(q^2) M_i \Gamma_i} \ , \tag{2.52}
$$

and h_+ is the one-loop effective Yukawa coupling [40] defined as

$$
h_{+\alpha i} = h_{\alpha i} - i h_{\alpha j} (h^\dagger h)^*_{ji} \frac{g(x_{ji})}{16\pi} \ . \tag{2.53}
$$

$g(x_j i)$ is defined as (2.29). The squared amplitude of the CP conjugate process $\ell\phi \to \overline{\ell\phi}$ is obtained by replacing h_+ with

$$h_{-\alpha i} = h^*_{\alpha i} - i h^*_{\alpha j}(h^\dagger h)_{ji}\frac{g(x_{ji})}{16\pi} \ . \tag{2.54}$$

The part mediated only by N_i in (2.51) is

$$\sum_{SU(2),\alpha,\beta} |\mathcal{M}_{\overline{\ell}^\beta_{a'}\overline{\phi}_{b'}\to N_i\to \ell^\alpha_a\phi_b}|^2 \equiv \frac{2g_w^2[(h_+^\dagger h_+)_{ij}]^2 M_i^2(p_1\cdot p_2)}{(q^2-M_i^2)^2+(M_i\Gamma_i)^2} \ , \tag{2.55}$$

in which two Breit-Wigner propagators cam be on-sell simultaneously. To consider the on-shell N_i limit, the replacement

$$\frac{2M_i\Gamma_i}{(q^2-M_i^2)^2+(M_i\Gamma_i)^2} \to 2\pi\delta(q^2-M_i^2) \ , \tag{2.56}$$

motivated by the relation $\lim_{\epsilon\to 0} 2\epsilon/(\omega^2+\epsilon^2) \to 2\pi\delta(\omega)$, can be employed. Then (2.55) becomes

$$\begin{aligned}&\sum_{SU(2),\alpha,\beta} |\mathcal{M}_{\overline{\ell}^\beta_{a'}\overline{\phi}_{b'}\to N_i\to \ell^\alpha_a\phi_b}|^2 \\ &\qquad \to [g_w(h_+^\dagger h_+)_{ii}M_i^2]^2\frac{\pi\delta(q^2-M_i^2)}{M_i\Gamma_i}\frac{2(p_1\cdot p_2)}{M_i^2} \ .\end{aligned} \tag{2.57}$$

First, let us see how this on-shell part contributes to the collision term in the thermal equilibrium. By replacing $|\mathcal{M}_{\overline{\ell}^\beta_{a'}\overline{\phi}_{b'}\to \ell^\alpha_a\phi_b}|^2$ with $|\mathcal{M}_{\overline{\ell}^\beta_{a'}\overline{\phi}_{b'}\to N_i\to \ell^\alpha_a\phi_b}|^2$ in the collision term for ℓ^α_a (2.33), and summing the $SU(2)$ and flavor indices, we get

$$\begin{aligned}\int\frac{d^3p}{(2\pi)^3}\frac{\mathcal{C}^{\mathrm{RIS},thermal}_{\mathrm{scatt},\ell}}{\omega_p} &= \sum_i\int d\Pi^\ell_p d\Pi^{\overline{\ell}}_{p'} d\Pi^\phi_k d\Pi^{\overline{\phi}}_{k'}(2\pi)^4\delta(p+k-p'-k') \\ &\quad\times\Big\{\sum_{SU(2),\alpha,\beta} |\mathcal{M}_{\overline{\ell}^\beta_{a'}\overline{\phi}_{b'}\to N_i\to \ell^\alpha_a\phi_b}|^2 \\ &\qquad - \sum_{SU(2),\alpha,\beta} |\mathcal{M}_{\ell^\alpha_a\phi_b\to N_i\to\overline{\ell}^\beta_{a'}\overline{\phi}_{b'}}|^2\Big\} f^{\mathrm{MB}}_{\ell,p} f^{\mathrm{MB}}_{\phi,k} \qquad (2.58) \\ &= \sum_{i,\alpha}\varepsilon_{i\alpha}\Gamma_{N_i}K_1(M_i/T)\frac{TM_i^2}{\pi^2} \\ &= \sum_{i,\alpha}\left(\gamma_{N_i\to\ell^\alpha\phi} - \gamma_{N_i\to\overline{\ell}^\alpha\overline{\phi}}\right) \ . \qquad (2.59)\end{aligned}$$

This is the half of (2.47). Since $(h_\pm^\dagger h_\pm)_{ii} \approx (h^\dagger h)_{ii}(1\pm\varepsilon_i)$, the collision term for $\overline{\ell}$ gives the same magnitude but with a negative sign. Therefore, the on-shell part of collision term for the lepton number n_L gives

$$\int \frac{d^3p}{(2\pi)^3}\Big[\frac{\mathcal{C}^{\mathrm{RIS},thermal}_{\mathrm{scatt},\ell}}{\omega_p} - \frac{\mathcal{C}^{\mathrm{RIS},thermal}_{\mathrm{scatt},\overline{\ell}}}{\omega_p}\Big] \tag{2.60}$$
$$= 2\times\sum_{i,\alpha}\Big(\gamma_{N_i\to\ell^\alpha\phi} - \gamma_{N_i\to\overline{\ell}^\alpha\overline{\phi}}\Big) = [d_t n_L + 3Hn_L]^D_{thermal}\ .$$
$$= \int \frac{d^3p}{(2\pi)^3}\Big[\frac{\mathcal{C}^{thermal}_{\mathrm{D},\ell}}{\omega_p} - \frac{\mathcal{C}^{thermal}_{\mathrm{D},\overline{\ell}}}{\omega_p}\Big] \tag{2.61}$$

This correspondence means that, by taking into account both of the decay and scattering processes, the evolution equation (2.46) counts twice the same process including the on-shell RH neutrino, and by subtracting the real intermediate state (RIS) of the RH neutrino, that is, replacing $|\mathcal{M}_{\overline{\ell\phi}\to\ell\phi}|^2$ and $|\mathcal{M}_{\ell\phi\to\overline{\ell\phi}}|^2$ with $|\mathcal{M}_{\overline{\ell\phi}\to\ell\phi}|^2 - |\mathcal{M}_{\overline{\ell\phi}\to N_i\to\ell\phi}|^2$ and $|\mathcal{M}_{\overline{\ell\phi}\to\ell\phi}|^2 - |\mathcal{M}_{\overline{\ell\phi}\to N_i\to\ell\phi}|^2$ respectively in the collision terms, we can correct the double counting and solve the problem which appeared in (2.46).

In order to take into account the contributions mediated by "off-shell" N_i's as $\Delta L = 2$ scattering processes, it's convenient to introduce the RIS-subtracted propagator [72]

$$\mathcal{D}^2_{ii}(q^2) = \frac{(q^2-M_i^2)-(M_i\Gamma_i)^2}{[(q^2-M_i^2)+(M_i\Gamma_i)^2]^2} \tag{2.62}$$

based on the observation that (2.62) can be rewritten as

$$\mathcal{D}^2_{ii}(q^2) = \frac{1}{(q^2-M_i^2)+(M_i\Gamma_i)^2} - \frac{4(M_i\Gamma_i)^3}{[(q^2-M_i^2)+(M_i\Gamma_i)^2]^2}\frac{1}{2(M_i\Gamma_i)}$$
$$\equiv D_i^*(q^2)D_i(q^2) - \widetilde{\mathcal{D}}_i^2(q^2)\ , \tag{2.63}$$

where "extended quasi-particle" [83, 84, 85] spectral density $2(M_i\Gamma_i)\times\widetilde{\mathcal{D}}_i^2$ approaches the Dirac delta function in the narrow width limit:

$$2(M_i\Gamma_i)\times\widetilde{\mathcal{D}}_i^2(q^2) = \frac{4(M_i\Gamma_i)^3}{[(q^2-M_i^2)+(M_i\Gamma_i)^2]^2} \to 2\pi\delta(q^2-M_i^2) \tag{2.64}$$

faster than the ordinary Breit-Wigner form. This corresponds to splitting the Breit-Wigner propagator $D(q^2)$ into two parts as depicted in Fig.2. The second term in (2.63) is supposed to cancel with the unphysical term (2.47).[4] With the

[4]In the derivation of the Boltzmann equation from the KB equation [72], the detailed balance is automatically satisfied, and problematic terms like (2.47) don't appear. The s-channel × s-channel part of the scattering and the (inverse) decay contributions are obtained from the same diagram Fig.9. However, the degrees of freedom, whose distribution function evolve in the Boltzmann equation, are classical particles, and hence, on-shell and off-shell part in a propagator must be discriminated to obtain the Boltzmann equation. By extracting the "extended quasi-particle" spectral density (2.64) in the full propagator as a classical on-shell particle to obtain the (inverse) decay contribution, the RIS-subtracted propagator (2.62) is naturally left behind and contributes as the scattering process.

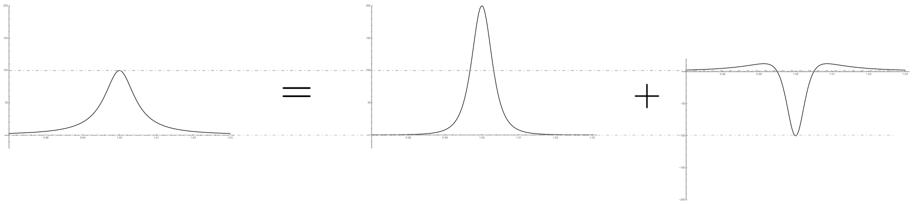

Figure 2: Breit-Wigner propagator (2.52) (left hand side) is separated into "extended quasi-particle" spectral density (2.64) plus RIS-subtracted propagator (2.62) (right hand side). The above shows $M\Gamma \times |D|^2 = M\Gamma \times \widetilde{\mathcal{D}}^2 + M\Gamma \times \mathcal{D}^2$.

definition of the off-diagonal parts of $\mathcal{D}^2$

$$\mathcal{D}^2_{i\neq j}(q^2) = D_i^*(q^2)D_j(q^2) \ , \tag{2.65}$$

the RIS subtraction corresponds to rewrite the squared amplitudes of scattering processes as

$$\begin{aligned} &\sum_{SU(2),\alpha,\beta} |\mathcal{M}_{\overline{\ell}^\beta_{a'}\overline{\phi}_{b'}\to\ell^\alpha_a\phi_b}|^2\Big|_{\mathrm{s\times s}} \\ &\xrightarrow{\text{RIS subtraction}} g_w^2(p_1\cdot p_2)\sum_{i,j} M_iM_j2(h_+^\dagger h_+)_{ij}(h_+^\dagger h_+)_{ij}\mathcal{D}^2_{ij}(q^2) \ , \end{aligned} \tag{2.66}$$

$$\begin{aligned} &\sum_{SU(2),\alpha,\beta} |\mathcal{M}_{\ell^\alpha_a\phi_b\to\overline{\ell}^\beta_{a'}\overline{\phi}_{b'}}|^2\Big|_{\mathrm{s\times s}} \\ &\xrightarrow{\text{RIS subtraction}} g_w^2(p_1\cdot p_2)\sum_{i,j} M_iM_j2(h_-^\dagger h_-)_{ij}(h_-^\dagger h_-)_{ij}\mathcal{D}^2_{ij}(q^2) \ . \end{aligned} \tag{2.67}$$

After this procedure, we get the evolution equation of the lepton number

$$\begin{aligned} d_tn_L + 3Hn_L = &\sum_{i,\alpha}\Bigg(\varepsilon_{i\alpha}\Gamma_{N_i}\frac{K_1(M_i/T)}{K_2(M_i/T)}\Delta n_{N_i} \\ &\qquad - \Gamma_{N_i}\frac{M_i^2}{4T^2}K_1(M_i/T)n_{L^\alpha}\Bigg) \\ &+ (\text{RIS subtracted scattering process}) \ . \end{aligned} \tag{2.68}$$

Because the resonances are subtracted, (2.66) and (2.67) are identical to each other in the leading order of the CP-asymmetry $\mathcal{O}((h^\dagger h)^2)$ and contribute to (2.68) only as the sub-leading wash-out term of the generated lepton number.

Remember that, in the above discussion, the RH neutrino's propagator mediating the process is assumed to be equilibrium one without finite density

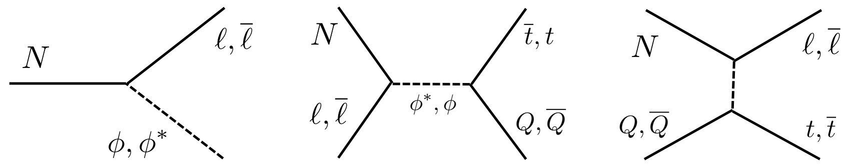

Figure 3: Decay and scattering processes. Although the former is basically responsible for the final lepton asymmetry, the latter mainly contributes to the production of RH neutrino from the zero-initial abundance.

effects. In a situation with slowly varying space-time, non-equilibrium contributions are expected to appear as a non-equilibrium distribution function in a propagator. Such non-equilibrium contributions are also expected to satisfy the same property as the equilibrium on-shell part, that is, the result from (inverse) decay diagram corresponds to the one from s-channel $\times$ s-channel part of the scattering. However, in the main part of this thesis with non-equilibrium QFT frame work, it turns out that the CP-violating parameter from the conventional method (which is supposed to contribute to non-equilibrium processes) doesn't match its counterpart from the s-channel $\times$ s-channel part of the scattering in the non-equilibrium state, especially, for the degenerate RH neutrino spectrum.

2.2 The simplest model and the Davidson-Ibarra bound

In this subsection, we consider the simplest (toy) model of the leptogenesis. The three RH neutrinos have a hierarchical mass spectrum, and only the lightest RH neutrino N_1 with the Majorana mass M_1 is responsible for the generation of the lepton number. The effects of the lepton's flavor and the RIS-subtracted scattering ($\ell\phi \leftrightarrow \bar{\ell}\phi^*$) process, which relates to the upper bound on the light neutrino mass and M_1, are omitted for simplicity.

As seen above, the evolution equations of the number densities of N_1 and lepton number are given as

$$d_t n_{N_1} + 3Hn_{N_1} = -\,\Gamma_{N_i}\frac{K_1(M_i/T)}{K_2(M_i/T)}\Delta n_{N_i}\ , \tag{2.69}$$

$$d_t n_L + 3Hn_L = \left(\varepsilon_1 \Gamma_{N_1}\frac{K_1(M_1/T)}{K_2(M_1/T)}\Delta n_{N_1} - \Gamma_{N_1}\frac{M_1^2}{4T^2}K_1(M_1/T)n_L\right)\ . \tag{2.70}$$

For the moment, we just omit the scattering contributions in Fig.3 for brevity. The heavier RH neutrinos $N_{2,3}$ appears only in the one-loop diagrams, and give the CP-violating parameter

$$\varepsilon_1 = \sum_\alpha \varepsilon_{1\alpha} = \sum_{j\neq 1} \frac{\Im[(h^\dagger h)_{1j}]^2}{8\pi (h^\dagger h)_{11}} g(x_{j1})$$
$$\xrightarrow{x_{j1}\gg 1} -\frac{3}{16\pi} \sum_{j\neq 1} \frac{\Im[(h^\dagger h)_{1j}]^2}{(h^\dagger h)_{11}} \frac{M_1}{M_j} \ . \tag{2.71}$$

On the second line, the asymptotic form of $g(x)$ for the hierarchical spectrum $x_{j1} = M_j^2/M_1^2 \gg 1$ has been used.

Hereafter, let us use the yield value

$$Y_{N_1} = \frac{n_{N_1}}{s} \ , \quad Y_L = \frac{n_L}{s} \ , \tag{2.72}$$

where

$$s = \frac{g_* 2\pi^2}{45} T^3 \tag{2.73}$$

is the entropy density with the number of the effectively massless degrees of freedom $g_*(T \gtrsim T_{\rm EW}) \sim 10^2$ in the SM. Using the time variable $z = M_1/T$, the evolution equations (2.69) and (2.70) are written as

$$\frac{dY_{N_1}}{dz} = -\mathcal{K}_1 \frac{zK_1(z)}{K_2(2)} (Y_{N_1} - Y_{N_1}^{eq}) \ , \tag{2.74}$$

$$\frac{dY_L}{dz} = +\varepsilon_1 \mathcal{K}_1 \frac{zK_1(z)}{K_2(2)} (Y_{N_1} - Y_{N_1}^{eq}) - \frac{\mathcal{K}_1}{4} z^3 K_1(z) Y_L \ . \tag{2.75}$$

Here, we have defined

$$\mathcal{K}_1 \equiv \frac{\Gamma_{N_1}}{H(T = M_1)} \ . \tag{2.76}$$

The decay width $\Gamma_{N_1} = (h^\dagger h)_{ii} M_i / 8\pi$ gives the time scale of the relaxation to the thermal equilibrium. On the other hands, the Hubble expansion leads to the departure from the equilibrium. Therefore, $\mathcal{K}_1$ quantifies how close to the equilibrium the RH neutrino N_1 is at $T \approx M_1$.

Analytic solutions of Y_{N_1} and Y_L are obtained in the following way [23]. Here, let us write the evolution equations (2.74) and (2.75) as

$$\frac{dY_{N_1}}{dz} = -D(Y_{N_1} - Y_{N_1}^{eq}) \ , \tag{2.77}$$

$$\frac{dY_L}{dz} = +\varepsilon_1 D(Y_{N_1} - Y_{N_1}^{eq}) - \beta W Y_L \tag{2.78}$$

where

$$D = \mathcal{K}_1 \frac{zK_1(z)}{K_2(2)} \ , \quad W = \frac{\mathcal{K}_1}{4} z^3 K_1(z) \ . \tag{2.79}$$

The coefficient β has been introduced to see how the washout term affects the final results. In the case here we are focusing on, β corresponds to unity. (2.78) can be solved as

$$Y_L(z) = Y_L(z_i) e^{-\int_{z_i}^{z} dz' W(z')} + \varepsilon_1 \times \eta(z) \times Y_N^{eq}(0) \tag{2.80}$$

where

$$\eta(z) = \frac{1}{Y_{N_1}^{eq}(0)} \int_{z_i}^{z} dz' D(Y_{N_1} - Y_{N_1}^{eq}) e^{-\int_{z'}^{z} dz'' \beta W(z'')} \tag{2.81}$$

$$= -\frac{1}{Y_{N_1}^{eq}(0)} \int_{z_i}^{z} dz' \frac{dY_{N_1}}{dz} e^{-\int_{z'}^{z} dz'' \beta W(z'')} \tag{2.82}$$

is called *efficiency factor* which represent the suppression coming from the washout of the generated lepton asymmetry by the inverse decay and scattering processes. Hence, its maximum value is one. On the second line of (2.82), the evolution equation of N_1 (2.77) has been used. We are now interested in the zero initial abundance of the lepton number $Y_L(z_i) = 0$. And the normalization factor $Y_N(0)$ is the high temperature limit of the equilibrium yield value of N_1:

$$Y_{N_1}^{eq}(z) = Y_{N_1}^{eq}(0) \frac{z^2 K_2(z)}{2} \ , \quad Y_{N_1}^{eq}(0) = \frac{45}{g_* \pi^4} \sim 5 \times 10^{-3} \ . \tag{2.83}$$

2.2.1 Weak washout regime

First, let us consider the case of $\mathcal{K}_1 \ll 1$ with zero initial abundance $Y_{N_1}(z_i) = 0$. Then, we have the two periods. In the early period, RH neutrinos are produced from the SM thermal bath and gradually populated. In the latter period, they are decaying. The whole time evolution is divided into these two periods by the dimensionless time z_{eq} defined as $Y_{N_1}(z_{eq}) = Y_{N_1}^{eq}(z_{eq})$. From the equation (2.77), we can get an approximate value of the Yield at z_{eq} as $Y_{N_1}(z_{eq}) = (3\pi\mathcal{K}_1/4) Y_N^{eq}(0)$. As we can see in Fig.4, the yield value of RH neutrino cannot follow the equilibrium value $Y_N^{eq}(z)$ because the smallness of $\mathcal{K}_1$ which corresponds to the weakness of the interaction with the SM thermal bath.

The final value of the efficiency factor (2.82) is obtained as a sum of two contributions:

$$\eta_f \approx -\frac{1}{Y_{N_1}^{eq}(0)} \left(\int_{z_i}^{z_{eq}} dz' + \int_{z_{eq}}^{\infty} dz' \right) \frac{dY_{N_1}}{dz} e^{-\int_{z'}^{z} dz'' W(z'')} \equiv \eta_f^- + \eta_f^+ \ . \tag{2.84}$$

Although η_f^+ comes from the latter period of RH neutrino decay ($z > z_{eq}$) and gives a positive contribution, η_f^- leads to a negative contribution because it

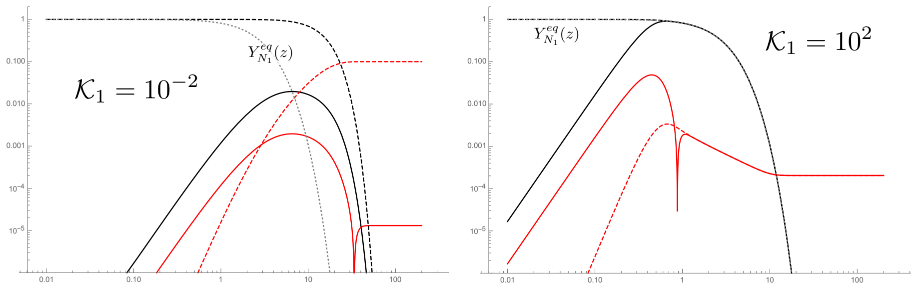

Figure 4: Solutions of the Boltzmann equation (2.77) and (2.78) without the scatterings. The horizontal axis is the dimensionless time $z = M_1/T$. The black solid (dashed) line is the yield value of RH neutrino Y_{N_1} normalized by $Y^{eq}_{N_1}(0)$ for the zero (thermal) initial abundance. In the weak washout case with $\mathcal{K}_1 = 10^-2$ (left panel), Y_{N_1} cannot follow the equilibrium value $Y^{eq}_{N_1}(z)$ (dotted gray line). On the other hand, in the strong washout case with $\mathcal{K}_1 = 10^2$ (tight panel), the difference between Y_{N_1} and the equilibrium value $Y^{eq}_{N_1}$ is too small to tell them apart for $z > z_{eq}$. With the thermal initial abundance, Y_{N_1} is almost coincident with $Y^{eq}_{N_1}$ all the time. The red solid (dashed) line is the absolute value of the generated lepton asymmetry Y_L divided by $10 \times \varepsilon_1$. In the weak washout case, the final value of the lepton number highly depends on the initial condition, in contrast with the strong washout case.

comes from the early period of RH neutrino production ($z < z_{eq}$) and then the net lepton number generated in this period has an opposite sign against that in the latter period. They can be evaluated as

$$\eta_f^- \approx -\frac{2}{\beta}\left(1 - e^{-\beta Y_{N_1}(z_{eq})/(2Y^{eq}_N(0))}\right) , \tag{2.85}$$

$$\eta_f^+ \approx -\frac{1}{Y^{eq}_N(0)}\left(Y_{N_1}(z) - Y_{N_1}(z_{eq})\right)_{z\to\infty} = \frac{Y_{N_1}(z_{eq})}{Y^{eq}_N(0)} . \tag{2.86}$$

Note that η_f^+ does not depend on β. This follows from that the washout term with $\mathcal{K}_1$ is negligible for larger $z > z_{eq}$. Eventually, we have

$$\begin{aligned} \eta_f &\approx -\frac{2}{\beta}\left(+\frac{\beta Y_{N_1}(z_{eq})}{2Y^{eq}_{N_1}(0)} - \frac{1}{2}\left(\frac{\beta Y_{N_1}(z_{eq})}{2Y^{eq}_{N_1}(0)}\right)^2\right) + \frac{Y_{N_1}(z_{eq})}{Y^{eq}_{N_1}(0)} && (2.87) \\ &= \beta\left(\frac{3\pi}{8}\mathcal{K}_1\right)^2 . && (2.88) \end{aligned}$$

This means that the first order negative contribution $\mathcal{O}(\mathcal{K}_1)$ generated in the early period is canceled by the positive contribution generated in the latter period, and only the second order part $\mathcal{O}(\mathcal{K}_1^2)$ is left as the final lepton asymmetry.

Note that, if the washout term were not included ($\beta = 0$), the cancellation between the two period would be perfect in this analytic method. The final lepton asymmetry is a consequence of the partial washout in the early period.

For the initial thermal abundance $Y_{N_1}(z_i) = Y^{eq}_{N_1}(z_i)$ with $z_{eq} \approx z_i$, basically there is no negative contribution η^- from the production process of RH neutrino. Therefore, the efficiency factor is given as $\eta_f = \eta^+_f = Y_{N_1}(z_{eq})/Y^{eq}_{N_1}(0) \approx 1$, and relatively large lepton asymmetry can be generated as seen in the left panel of Fig.4.

2.2.2 Strong washout regime

Let us see the case with $\mathcal{K}_1 \gg 1$. Even if we assume the zero initial abundance of RH neutrino, RH neutrino get populated rapidly and its yield value almost follows the equilibrium value, see Fig.4, because of the rapid interaction between RH neutrino and the SM thermal bath. In this case, the negative sign lepton asymmetry generated in the N_1 production period is completely washed out before the yield of N_1 starts to decay. This means that the initial thermal abundance $Y_{N_1}(z_i) = Y^{eq}_{N_1}(z_i)$ gives the same result of the final lepton asymmetry as the zero initial abundance $Y_{N_1}(z_i) = 0$.

Therefore, we can evaluate the final value of the efficiency factor only by considering the latter period:

$$\eta_f \approx -\frac{1}{Y^{eq}_{N_1}(0)} \int_{z_{eq}}^{\infty} dz' \frac{dY_{N_1}}{dz} e^{-\int_{z'}^{z} dz'' W(z'')} \, . \tag{2.89}$$

Now, the deviation $\Delta Y_{N_1} = Y_{N_1} - Y^{eq}_{N_1}$ is small $\sim \mathcal{O}(1/\mathcal{K}_1)$ and then, we get the perturbative solution of (2.77) as

$$\Delta Y_{N_1}(z) \approx -\frac{1}{D}\frac{dY^{eq}_{N_1}}{dz} = \frac{Y^{eq}_{N_1}(0)}{D}\frac{2W}{\mathcal{K}_1 z} \tag{2.90}$$

at the leading order of $1/\mathcal{K}_1$. Although this deviation is too small to be recognized in Fig.4, it can generate the sufficient lepton asymmetry with the efficiency factor:

$$\eta_f(\beta\mathcal{K}_1) \approx \frac{2}{\beta\mathcal{K}_1 z_B(\beta\mathcal{K}_1)}\left(1 - e^{-\beta\mathcal{K}_1 z_B(\beta\mathcal{K}_1)/2}\right) \tag{2.91}$$

where

$$z_B(\beta\mathcal{K}_1) \approx 2 + 4(\beta\mathcal{K}_1)^{0.13} e^{-\frac{2.5}{\beta\mathcal{K}_1}} \tag{2.92}$$

corresponds to the time when the washout term begin to be negligible $W < 1$ and the freeze-out of the produced lepton asymmetry occurs. In the expression (2.91), we can see β appearing as $\beta\mathcal{K}_1$. The final result can be evaluated only from the strength of washout, and it becomes smaller for stronger washout. Note that this follows from the almost perfect washout of the asymmetry produced in the early period.

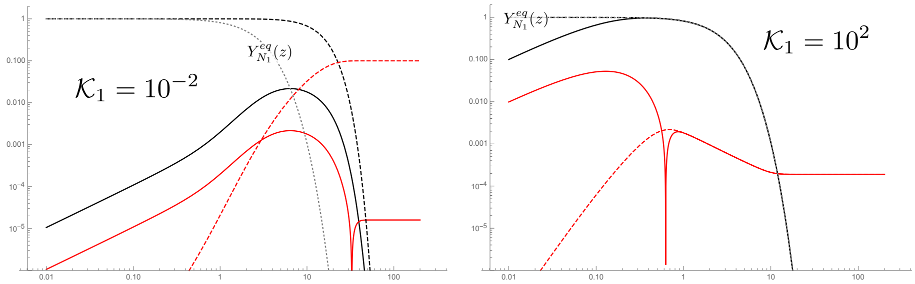

Figure 5: Solutions of the Boltzmann equation (2.77) and (2.78) without the scatterings. The black solid (dashed) line is the yield value of RH neutrino Y_{N_1} normalized by $Y_{N_1}^{eq}(0)$ for the zero (thermal) initial abundance. The red solid (dashed) line is the absolute value of the generated lepton asymmetry Y_L divided by $10 \times \varepsilon_1$. Because of the scattering contributions, they have significant value in early stage, compared with Fig.4 considering only the (inverse) decay process. However, the final lepton asymmetry is scarcely affected by the inclusion of the scatterings.

Interpolation

An interpolation between the weak and strong washout regime is given as

$$\eta_f^-(\mathcal{K}_1) \approx -2e^{-3\pi\mathcal{K}_1/8}\left(e^{x(\mathcal{K}_1)/2} - 1\right) \ , \tag{2.93}$$

$$\eta_f^+(\mathcal{K}_1) \approx \frac{2}{\mathcal{K}_1 z_B(\mathcal{K}_1)}\left(1 - e^{-\mathcal{K}_1 z_B(\mathcal{K}_1) x(\mathcal{K}_1)/2}\right) \tag{2.94}$$

where

$$x(\mathcal{K}_1) = \frac{3\pi\mathcal{K}_1}{4}\left(1 + \sqrt{\frac{3\pi\mathcal{K}_1}{4}}\right)^{-2} \tag{2.95}$$

is used to reproduce the expressions (2.88) and (2.88) in the weak and strong washout limit, respectively. Here, we set $\beta = 1$. For the thermal initial abundance of N_1, we can use the expression (2.91):

$$\eta_f(\mathcal{K}_1) \approx \frac{2}{\mathcal{K}_1 z_B(\mathcal{K}_1)}\left(1 - e^{-\mathcal{K}_1 z_B(\mathcal{K}_1)/2}\right) \ . \tag{2.96}$$

Including the scattering process

The scattering contributions in Fig.3 can be taken into account by analyzing the Boltzmann equations [29]

$$\frac{dY_{N_1}}{dz} = -jD(Y_{N_1} - Y_{N_1}^{eq}) \ , \tag{2.97}$$

$$\frac{dY_L}{dz} = +\varepsilon_1 j D(Y_{N_1} - Y_{N_1}^{eq}) - j_W W Y_L \tag{2.98}$$

where

$$j = \frac{D+S}{D} \approx \left[\frac{z}{a}\ln\left(1+\frac{a}{z}\right) + \frac{r_S}{z}\right]\left(1+\frac{15}{8z}\right) , \tag{2.99}$$

$$j_W = \frac{W+W_S}{W} \approx \left[\frac{z}{a}\ln\left(1+\frac{a}{z}\right) + y(z)\frac{r_S}{z}\right]\left(1+\frac{15}{8z}\right) \tag{2.100}$$

are responsible for taking the scattering contributions S and W_S. The difference between j and j_W is only the factor $y(z)$ in j_W. In the early period $z < z_{eq}$, the negligible amount of the RH neutrino gives $y(z) \approx 2/3$. However, for $z > z_{eq}$, $y(z)$ can generally become larger than unity. In the strong washout case, the abundance of the RH neutrino is almost equal to the thermal abundance $Y_{N_1}^{eq}(z)$, then $y(z) \approx 1$ holds. r_S is defined as the ratio of strength of the scattering process to strength of the decay process:

$$r_S = \frac{9m_t^2}{8\pi^2 v^2} \approx 0.1 . \tag{2.101}$$

And a is given by

$$a = \frac{1}{r_S \ln(M_1/M_h)} \tag{2.102}$$

with the effective mass of the Higgs boson M_h as a IR cut-off of the momentum integrals in the scattering amplitude. Then, for the efficiency factor

$$\eta(z) = \frac{1}{Y_{N_1}^{eq}(0)} \int_{z_i}^{z} dz' j D(Y_{N_1} - Y_{N_1}^{eq}) e^{-\int_{z'}^{z} dz'' j_W W(z'')} \tag{2.103}$$

$$= -\frac{1}{Y_{N_1}^{eq}(0)} \int_{z_i}^{z} dz' \frac{dY_{N_1}}{dz} e^{-\int_{z'}^{z} dz'' j_W W(z'')} , \tag{2.104}$$

we can estimate the final value of the efficiency factor by following expressions for the zero initial abundance of N_1:

$$\eta_f(\mathcal{K}_1) = \eta_f^-(\mathcal{K}_1) + \eta_f^+(\mathcal{K}_1) \tag{2.105}$$

$$\eta_f^-(\mathcal{K}_1) \approx -2e^{-3\pi\mathcal{K}_1/8}\left(e^{x(\mathcal{K}_1)/2} - 1\right) , \tag{2.106}$$

$$\eta_f^+(\mathcal{K}_1) \approx \frac{2}{\mathcal{K}_1 z_B(\mathcal{K}_1) j(z_B)}\left(1 - e^{-\mathcal{K}_1 z_B(\mathcal{K}_1) j(z_B) x(\mathcal{K}_1)/2}\right) \tag{2.107}$$

with (2.95). And for the thermal initial abundance of N_1, we have

$$\eta_f(\mathcal{K}_1) \approx \frac{2}{\mathcal{K}_1 z_B(\mathcal{K}_1) j(z_B)}\left(1 - e^{-\mathcal{K}_1 z_B(\mathcal{K}_1) j(z_B)/2}\right) \tag{2.108}$$

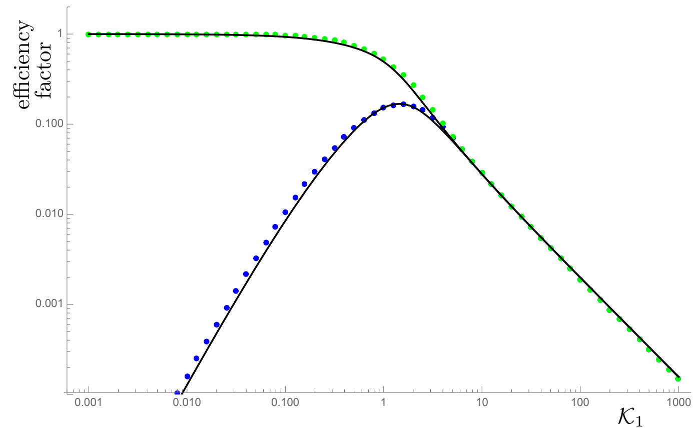

Figure 6: $\mathcal{K}_1$ dependence of the efficiency factor η_f. The blue (green) points are the numerical value of η_f obtained by solving the Boltzmann equations (2.97) and (2.98) for the zero (thermal) initial abundance of N_1 with corresponding value of $\mathcal{K}_1$. In the numerical calculation, the Higgs effective mass is assumed to be the temperature dependent thermal mass $M_h = 0.4 \times T = 0.4 \times M_1/z$. The black lines are the analytic expressions (2.105) and (2.108) with $M_1/M_h = 100$ in the numerical factor (2.102).

Since the inverse decay process is dominant at the freeze-out time, z_B is still given by the (2.92).

For the weak washout regime, $z_{eq} > 1$ and $x_S \to 3\pi\mathcal{K}_1/4$ hold. Again, we get

$$\eta_f^-(\mathcal{K}_1) \approx -\mathcal{K}_1 \frac{3\pi}{4} + \mathcal{K}_1^2 \left(\frac{3\pi}{8}\right)^2 , \tag{2.109}$$

$$\eta_f^+(\mathcal{K}_1) \approx +\mathcal{K}_1 \frac{3\pi}{4} . \tag{2.110}$$

The $\mathcal{O}(\mathcal{K}_1)$ terms are canceled between η_f^- and η_f^+, but the $\mathcal{O}(\mathcal{K}_1^2)$ contribution is left. In spite of the inclusion of the scattering contributions, we have the same result as the case without scatterings because, although the scattering processes washout the negative lepton asymmetry, they also contribute to increase the negative asymmetry through the production process of RH neutrino. For the strong washout case, we have

$$\eta_f^-(\mathcal{K}_1) \approx 0, \tag{2.111}$$

$$\eta_f^+(\mathcal{K}_1) \approx \frac{2}{\mathcal{K}_1 z_B(\mathcal{K}_1) j(z_B)} \left(1 - e^{-\mathcal{O}(\mathcal{K}_1)}\right) . \tag{2.112}$$

The difference from the case without the scattering (2.91) is only the $\mathcal{O}(1)$ factor $j(z_B)$ in the denominator.

As can be seen in Fig.6, it obeys the simple power law [23]

$$\eta_f(\mathcal{K}_1 \gg 1) \approx (3 \pm 1) \times 10^{-2} \left(\frac{0.01\text{eV}}{\widetilde{m}_1} \right)^{1.1 \pm 0.1} \tag{2.113}$$

where we used the effective neutrino mass [18] (cf. the seesaw relation (2.10))

$$\widetilde{m}_1 = \frac{v^2}{M_1} (h^\dagger h)_{11} = \Gamma_{N_1} \frac{8\pi v^2}{M_1^2} \ , \tag{2.114}$$

which is shown to be larger than the lightest SM neutrino mass: $\widetilde{m}_1 \geq m_{\nu,\text{min}}$ [22] and related to the parameter $\mathcal{K}_1$ as

$$\mathcal{K}_1 \equiv \frac{\Gamma_{N_1}}{H(T = M_1)} = \frac{\widetilde{m}_1}{m_*} \ , \quad m_* \equiv \frac{16\pi^{5/2}\sqrt{g_*}}{3\sqrt{5}} \frac{v^2}{M_{\text{Pl}}} \approx 1.08 \times 10^{-3}\text{eV}. \tag{2.115}$$

The efficient factor (provided zero-initial abundance of RH neutrino) takes the maximum value when $\widetilde{m}_1$ is of the same order as the equilibrium neutrino mass m_*.

Lower bound on RH neutrino mass

Below the temperature $T \sim 10^{12}$GeV, the electroweak sphaleron process becomes in equilibrium, and causes non-conservation of the $B+L$ number. Therefore, the above equations for Y_L, which does not take into account the sphaleron processes, should be regarded as the equation for $(-1) \times Y_{B-L}$.

Using the efficiency factor, we get the final $B - L$ number as

$$Y_{B-L} \approx \varepsilon_1 Y_{N_1}^{eq}(0) \times \eta_f(\mathcal{K}_1) \ . \tag{2.116}$$

The $B - L$ asymmetry is partially transferred to the B asymmetry through the EW sphaleron processes to satisfy the relation $Y_B = (12/37)Y_{B-L}$.[5] Then, the final baryon asymmetry can be written as $Y_B = (12/37)Y_N^{eq}(0) \times \varepsilon_1 \times \eta_f(\mathcal{K}_1)$.

To obtain some constraints on the high energy scale parameters such as Γ_{N_1} and M_1, it's needed to connect them to the low energy parameters such as the neutrino masses m_ν and the neutrino mixing matrix U_ν. The Casas-Ibarra parametrization [15] of the neutrino Yukawa h is convenient to do that.

[5] The coefficient to relate Y_{B-L} to Y_B depends on which interactions are in equilibrium at each temperature regime. Using the chemical equilibrium and charge conservation, one gets the relation [9]

$$Y_B = Y_{B-L} \times \begin{cases} \frac{24+4m}{66+13m} & \text{for } T > T_{\text{EWPT}} \\ \frac{32+4m}{98+13m} & \text{for } T < T_{\text{EWPT}} \end{cases} \tag{2.117}$$

where m is the number of the Higgs doublets. Therefore, setting $m = 1$ and assuming that the sphaleron shut off occurs below T_{EWPT}, we obtain the coefficient 12/37.

The unitary matrix U_ν [14] is defined as the diagonalization matrix of the light neutrino mass (2.10) :

$$m_\nu = U D_m U^T \ . \tag{2.118}$$

By introducing the complex orthogonal matrix R, h can be rewritten as

$$h_{\alpha i} = \frac{1}{v}\left(U_\nu D_m^{1/2} R^T M^{1/2}\right)_{\alpha i} \ . \tag{2.119}$$

In this parametrization, 15 physical parameters in h are divided into 3 neutrino mass eigenvalue $D_m = \mathrm{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3})$, 3 angles and 3 physical (1 Dirac and 2 Majorana) phases in U_ν, and 3 complex angles in R. Then, the CP-violating parameter (2.71) becomes

$$\varepsilon_1 = \frac{3}{16\pi}\frac{M_1}{v^2}\frac{\sum_i m_{\nu_i}^2 \Im[R_{1i}^2]}{\sum_i m_{\nu_i}|R_{1i}|^2} \ . \tag{2.120}$$

Using the orthogonality condition $\sum_i R_{1i}^2 = 1$, we obtain the Davidson-Ibarra (DI) bound [11] in the limit of $m_{\nu,\min}/\widetilde{m}_1 \to 0$:

$$|\varepsilon_1| \le \frac{3}{16\pi}\frac{M_1}{v^2}\sqrt{\Delta m_{\mathrm{atm}}^2} \equiv \varepsilon_{\max} \ . \tag{2.121}$$

Due to this upper bound depending on the lightest Majorana mass M_1, we get the lower bound on M_1 to reproduce the observed baryon asymmetry $Y_B \sim 6 \times 10^{-10}$:

$$M_1 > \frac{6 \times 10^8 \mathrm{GeV}}{\eta_f(\mathcal{K}_1)}. \tag{2.122}$$

Since the maximum value of the efficiency factor η_f is $\mathcal{O}(0.1)$, the lightest Majorana mass M_1 must be larger than 10^9GeV. Note this bound is derived by using the expression (2.71) for the *hierarchical mass spectrum* of the RH neutrinos and assuming that the lepton flavor effects and the heavier RH neutrinos $N_{2,3}$ are negligible. However, the lepton flavor and the heavier RH neutrinos' effects hardly modify this bound itself. Because of the reason briefly mentioned in the next subsection, these effects affect the lower bound on M_1 in the less hierarchical light neutrino spectrum region. The largest impact on the lower bound of the Majorana mass is obtained by assuming the degenerate mass spectrum of the RH neutrinos, which is the mechanism investigated in this thesis with the non-equilibrium QFT frame work.

2.2.3 Lepton flavor effects

As the temperature drops, the interactions through the charged lepton Yukawa couplings y, which discriminate the lepton flavors, become faster than the expansion of the universe: $H < \Gamma_\alpha \sim 5 \times 10^{-3} y_\alpha^2 T$ [25]. While the unflavored approximation is valid for $T \gtrsim 10^{12}$GeV, two-flavored calculation in which only the τ-flavor is treated separately is required for $10^{12}\text{GeV} \gtrsim T \gtrsim 10^9\text{GeV}$. Below $T \sim 10^9\text{GeV}$, all the flavors must be distinguished in the calculation. In other words, although leptons are generated through N_i's decay as a state of coherent superposition $|\ell_i\rangle$ of different flavors, leptons are "measured" by the existing thermal bath and the decoherence into the flavor eigenstate $|\ell_\alpha\rangle$ takes place soon after their production. The role of flavor effect were first discussed in [26] and highlighted in [27, 28] (see [36] for a review). By using the partial decay rate $\Gamma_{N_i \to \ell^\alpha \phi}$ of RH neutrino N_i to the lepton ℓ^α, the ratio of the number of α-flavor lepton to the total number of the lepton produced through N_i's decay is written as

$$P_{i\alpha} = |\langle \ell_\alpha | \ell_i \rangle|^2 = \frac{\Gamma_{N_i \to \ell^\alpha \phi}}{\sum_\alpha \Gamma_{N_i \to \ell^\alpha \phi}} \approx \frac{\Gamma_{N_i \to \ell^\alpha \phi}}{\Gamma_{N_i}/2} \tag{2.123}$$

For the case of the anti-lepton, we have

$$\overline{P}_{i\alpha} = |\langle \overline{\ell}_\alpha | \overline{\ell}_i \rangle|^2 = \frac{\Gamma_{N_i \to \overline{\ell}^\alpha \phi^*}}{\sum_\alpha \Gamma_{N_i \to \overline{\ell}^\alpha \phi^*}} \approx \frac{\Gamma_{N_i \to \overline{\ell}^\alpha \phi^*}}{\Gamma_{N_i}/2} \ . \tag{2.124}$$

Note that, because of the CP-violation, $|\overline{\ell}_i\rangle \neq \mathcal{CP}|\ell_i\rangle$. The strength of the washout of each lepton flavor is quantified by $\mathcal{K}_{i\alpha} = P^0_{i\alpha}\mathcal{K}_i$ with the tree-level projection given by

$$P^0_{i\alpha} = \frac{P_{i\alpha} + \overline{P}_{i\alpha}}{2} \approx \frac{\Gamma^{\text{tree}}_{N_i \to \ell^\alpha \phi}}{\Gamma_{N_i}/2} = \frac{\Gamma^{\text{tree}}_{N_i \to \overline{\ell}^\alpha \phi^*}}{\Gamma_{N_i}/2} \ . \tag{2.125}$$

Another characteristic quantity to the flavored case is the difference between the coefficient of the superposition:

$$\Delta P_{i\alpha} = P_{i\alpha} - \overline{P}_{i\alpha} \ . \tag{2.126}$$

This satisfies $\sum_\alpha \Delta P_{i\alpha}$ by definition. Then, the CP-violating parameter (2.28) is rewritten as

$$\varepsilon_{i\alpha} = \varepsilon_i P^0_{i\alpha} + \frac{\Delta P_{i\alpha}}{2} \tag{2.127}$$

where ε_i is the CP-violating parameter (2.71) in the unflavored case. From this expression, one can see the possibility that the flavor, in which a large part of the asymmetry is produced through heavier RH neutrinos' decay, can be different from the flavor which is mainly washed out by the inverse process. By suppressing the washout effects for a particular lepton flavor, the lower bound on the lightest RH neutrino N_1 is considerably lowered only for the case of less hierarchical light neutrino spectrum, see Figure 9 in [31] or Figure 6 in [32]. This means that the flavor effect can be significant only for the *strong washout* regime [27, 28]. And also, it should be emphasized that, even if the total asymmetry in RH neutrino's decay vanishes: $\varepsilon_i = \sum_\alpha \varepsilon_{i\alpha} = 0$, the successful leptogenesis is possible due to the second term of (2.127).

Aside from the charged lepton Yukawa interactions, as temperature decreases, we have to consider all the chemical equilibriums which result from the interactions much faster than Hubble expansion and leptogenesis processes. They are called spectator processes, which is relevant even in the unflavored case [21, 24], including the quark Yukawa interactions and QCD sphaleron process as well as EW sphaleron process. In the flavored leptogenesis, such processes can be taken into account by connecting the lepton asymmetry Y_{L^α} and the $B/3 - L^\beta$ asymmetry Y_{Δ^β} with A matrix [26]–[29] as $Y_{L^\alpha} = \sum_\beta A_{\alpha\beta} Y_{\Delta^\beta}$ in the Boltzmann equations. A is 2×2 (3×3) matrix in the two (three) flavor regime and the components have temperature dependence because interactions with smaller coupling get to equilibrate in lower temperature.

In the transient regime between one and two flavor regime ($T \sim 10^{12}$GeV) and two and three flavor regime ($T \sim 10^9$GeV), we have to employ the so-called density formalism, where the distribution functions of the leptons f_{ℓ^α} are regarded as diagonal components of the matrix valued extension of the distribution function of the SM lepton doublets [26, 27, 30, 37]

$$f_{\ell,p} = \begin{pmatrix} \langle a^\dagger_{1,\mathbf{p}} a_{1,\mathbf{p}} \rangle & \langle a^\dagger_{1,\mathbf{p}} a_{2,\mathbf{p}} \rangle \\ \langle a^\dagger_{2,\mathbf{p}} a_{1,\mathbf{p}} \rangle & \langle a^\dagger_{2,\mathbf{p}} a_{2,\mathbf{p}} \rangle \end{pmatrix} \tag{2.128}$$

for the transient regime $T \sim 10^{12}$GeV, where $a_\mathbf{p}$ is the annihilation operator of the lepton with the spatial momentum $\mathbf{p}$, and the subscripts 2 and 1 stand for the τ flavor and the direction orthogonal to τ, respectively.[6] 3×3 matrix is needed to describe the regime $T \sim 10^9$GeV. Then, evolution equation is also extended to have the matrix valued collision term and the mass difference matrix causing the oscillation between the components. This kind of extension was discussed by the authors of [57], and it has been applied intensively to the early universe neutrino and core collapse supernovae neutrino physics as well as the leptogenesis and EW baryogenesis scenarios. Systematic derivations from the first principle of QFT have been developed in [70][75]–[79][87]. In section 5, we apply this framework to the *RH neutrino* to understand the mechanism of the resonant enhancement of the CP-violation in the RH neutrinos' decay.

There are several cases where heavier RH neutrinos $N_{2,3}$ become important. The simplest case is that the lightest RH neutrino is almost decoupled [33], and then, the lepton asymmetries produced by the heavier RH neutrinos are hardly washed out by the subsequent processes including the lightest RH neutrino N_1. Once flavor effects are taken into account, other interesting possibilities arise. The first one is attributed to the so-called heavy neutrino flavor projection effect [35], which can be realized when the lightest RH neutrino has the Majorana mass $M_1 \gtrsim 10^9$GeV, that is, the washout of the asymmetry from heavier ones occurs before all the lepton flavor get to be distinguishable. This temperature region guarantee that $|\ell_1\rangle$ has a orthogonal direction to $|\ell_2\rangle$ unless the flavor structure is significantly tuned so that they become parallel. And then, the orthogonal

[6] Because the e and μ flavor are not distinguished yet for the temperature $T \gg 10^9$GeV, the orthogonal direction to the τ flavor is effectively one-dimensional.

component of $|\ell_2\rangle$ escapes the washout, even though N_1 couples strongly with all three lepton flavors ($\mathcal{K}_{1\alpha} \gg 1$). The second possibility is realized when the lightest RH neutrino has the Majorana mass $M_1 \lesssim 10^{12}$GeV and at least one of the washout strength is small: $\mathcal{K}_{1\beta} \lesssim 1$ [34]. Then, the asymmetries generated from heavier RH neutrinos undergo the washout through the interaction with N_1 in the flavored regime $T \lesssim 10^{12}$GeV, and the direction $|\ell_\beta\rangle$ weakly coupling with N_1 escapes the washout. Because of these possibilities, it can be realized that the asymmetries originating from the heavier RH neutrinos dominate the final lepton asymmetry, depending of the flavor structure of the RH neutrinos and leptons. Such N_2-dominated scenarios may allow the lightest Majorana mass to be small $M_1 < 10^9$GeV. However, the heavier ones still have to be large $M_{2,3} \gtrsim 10^9$GeV to obtain the sufficient lepton asymmetry, and hence, the reheating temperature cannot be lowered.

2.3 Resonant enhancement of CP-violating parameter

As we have already seen, for the almost same Majorana masses $M_i \approx M_j$, the CP-violating parameter (2.28) appears to become infinity. That divergent term comes from the self-energy one-loop diagrams (See Fig.1). In the following, we review the conventional computation [19, 42] of the regulated CP-violating parameter ε. The analysis is done in the similar way as in the study of the RIS subtraction (section 2.1.3), however, there is crucial difference: the CP-violating parameter in the decay process is derived from the matrix element of the s-channel $\times$ s-channel part of the scattering process, but not from the squared amplitude.

From the derivation of (2.28), it's obvious that the problem is caused by the use of the bare propagator for the internal line of N_j. The RH neutrinos have their decay width, and then their resummed propagator must have the Breit-Wigner like form (2.52). To implement the resummation, let us consider the $2 \to 2$ scattering diagrams Fig.(7). Their matrix elements are given as

$$
\begin{aligned}
i\mathcal{M}_{\ell_a^\alpha \phi_b \to \bar{\ell}_c^\beta \phi_d^*} &= + i\epsilon_{ab}\epsilon_{cd} h^\dagger_{j\beta} h^\dagger_{i\alpha} (CP_L v_\beta)^T G^{LL}_{ji} P_L u_\alpha \ , && (2.129)\\
i\mathcal{M}_{\bar{\ell}_a^\alpha \phi_b^* \to \ell_c^\beta \phi_d} &= + i\epsilon_{ab}\epsilon_{cd} h^T_{j\beta} h^T_{i\alpha} \overline{u}_\beta P_R G^{RR}_{ji} (\overline{v}_\alpha P_R C)^T \ , && (2.130)\\
i\mathcal{M}_{\ell_a^\alpha \phi_b \to \ell_c^\beta \phi_d} &= - i\epsilon_{ab}\epsilon_{cd} h^T_{j\beta} h^\dagger_{i\alpha} \overline{u}_\beta P_R G^{RL}_{ji} \not{q} P_L u_\alpha \ , && (2.131)\\
i\mathcal{M}_{\bar{\ell}_a^\alpha \phi_b^* \to \bar{\ell}_c^\beta \phi_d^*} &= - i\epsilon_{ab}\epsilon_{cd} h^\dagger_{j\beta} h^T_{i\alpha} (CP_L v_\beta)^T G^{LR}_{ji} \not{q} (\overline{v}_\alpha P_R C)^T \ , && (2.132)
\end{aligned}
$$

where the Feynman propagator[7] $G(q)$ is the solution of the Fourier transformed Schwinger-Dyson (SD) equation

$$[\not{q} - M - i\Pi(q)]\, G(q) = i \ . \qquad (2.133)$$

[7]In terms of the closed time path formalism reviewed in Appendix A.1, the Feynman propagator corresponds to the $(++)$ component of (A.21). The KMS relation (3.8) means $G_F^{(eq)}(q) = -\frac{i}{2}\tanh\left(\frac{q_0}{2T}\right) G_\rho^{(eq)}(q) \to -\frac{i}{2}\mathrm{sign}(q_0) G_\rho^{(eq)}(q)$ in the zero temperature limit. Therefore, now the Feynman propagator is given as $-i(G_R + G_A)$ using the retarded and advanced Green functions.

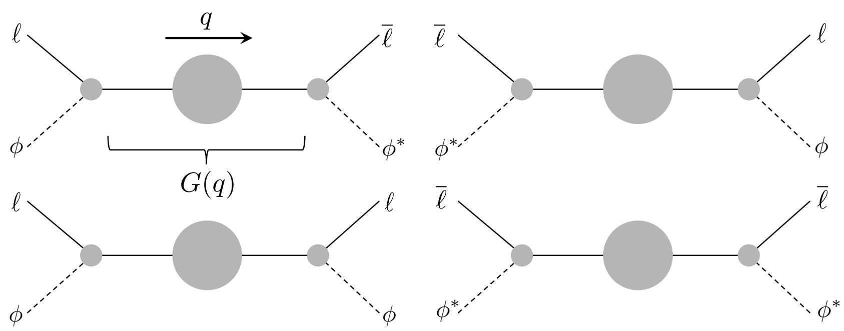

Figure 7: The s-channel contributions to the lepton-Higgs scattering. The internal line with the blob represents the full propagator of the RH neutrino. The vertex corrections (small blob) are negligible when we consider the resonant case.

Π is the 1PI self-energy. At the one-loop level, it can be written as (2.24) and then (2.133) becomes

$$\left[\not{q}\left(\left(1-ia(q^2)(h^\dagger h)\right)P_R+\left(1-ia(q^2)(h^\dagger h)^* P_L\right)\right)-M\right]G(q)=i\ . \tag{2.134}$$

Each superscript of G corresponds to a component of the Lorentz decomposition of the propagator:

$$G(q)=P_R G^{RR}(q^2)+P_L G^{LL}(q^2)+P_L\not{q}G^{LR}(q^2)+P_R\not{q}G^{RL}(q^2)\ . \tag{2.135}$$

In the conventional approach, by rewriting the matrix elements (2.129)-(2.132) into the forms of

$$i\mathcal{M}_{\ell^\alpha_a\phi_b\to\bar{\ell}^\beta_c\phi^*_d}=\sum_I i\mathcal{M}_{\ell^\alpha_a\phi_b\to N_I}\times iD_I(q^2)\times i\mathcal{M}_{N_I\to\bar{\ell}^\beta_c\phi^*_d}\ , \tag{2.136}$$

$$i\mathcal{M}_{\bar{\ell}^\alpha_a\phi^*_b\to\ell^\beta_c\phi_d}=\sum_I i\mathcal{M}_{\ell^\alpha_a\phi_b\to N_I}\times iD_I(q^2)\times i\mathcal{M}_{N_I\to\bar{\ell}^\beta_c\phi^*_d}\ , \tag{2.137}$$

$$i\mathcal{M}_{\ell^\alpha_a\phi_b\to\ell^\beta_c\phi_d}=\sum_I i\mathcal{M}_{\ell^\alpha_a\phi_b\to N_I}\times iD_I(q^2)\times i\mathcal{M}_{N_I\to\bar{\ell}^\beta_c\phi^*_d}\ , \tag{2.138}$$

$$i\mathcal{M}_{\bar{\ell}^\alpha_a\phi^*_b\to\bar{\ell}^\beta_c\phi^*_d}=\sum_I i\mathcal{M}_{\ell^\alpha_a\phi_b\to N_I}\times iD_I(q^2)\times i\mathcal{M}_{N_I\to\bar{\ell}^\beta_c\phi^*_d}\ , \tag{2.139}$$

the matrix elements of the decay process of the *mass eigenstates* N_I of the RH neutrino are read off. As soon mentioned, D is the Breit-Wigner propagator of the mass eigenstate. Note that, in the rest part of this section, it's assumed that the off-diagonal components $(h^\dagger h)'$ of $h^\dagger h$ is smaller than the diagonal parts.

Let us focus on the lepton number violating-scattering process (2.129) and

(2.130). The SD equation (2.133) gives the expressions

$$G^{LL}(q^2) = i\left[\left(1 - ia(q^2)(h^\dagger h)\right)\frac{q^2}{M}\left(1 - ia(q^2)(h^\dagger h)^*\right)\right]^{-1} , \tag{2.140}$$

$$G^{RR}(q^2) = i\left[\left(1 - ia(q^2)(h^\dagger h)^*\right)\frac{q^2}{M}\left(1 - ia(q^2)(h^\dagger h)\right)\right]^{-1} . \tag{2.141}$$

There are two methods to tell apart the contributions from different mass eigenstates. The first one is to diagonalize the propagator. Since G^{LL} and G^{RR} are the complex symmetric matrices in the flavor space, they can be diagonalized by the complex orthogonal matrices U and V:

$$G^{LL}(q^2) = iV^T(q^2)MD(q^2)V(q^2) \ , \quad G^{RR}(q^2) = iU^T(q^2)MD(q^2)U(q^2) \ , \tag{2.142}$$

where the eigenvalue D can be written as

$$D_I(q^2) \approx \frac{Z_I}{q^2 - (M_I^{\text{eff}})^2 + iM_I^{\text{eff}}\Gamma_I} \ , \quad Z_I = 1 + \frac{(h^\dagger h)_{ii}}{8\pi^2}\left(\ln\frac{M_i^2}{\mu^2} - 1\right)\Bigg|_{i=I} \tag{2.143}$$

which is the Breit-Wigner propagator of the mass eigenstate with the effective mass and decay width

$$(M_I^{\text{eff}})^2 = M_{i=I}^2\left[1 + \frac{(h^\dagger h)_{ii}}{8\pi^2}\left(\ln\frac{M_i^2}{\mu^2} - 2\right)\right]_{i=I} \ , \quad \Gamma_I = \frac{(h^\dagger h)_{ii}}{8\pi}M_i\Bigg|_{i=I} . \tag{2.144}$$

Here, we rewrite the subscripts as $i \to I$ to indicate the mass eigenstate, just for the notational consistency. Then, the matrix elements of the N_I decay processes in (2.136) and (2.137) are found out to be

$$i\mathcal{M}_{N_I\to\ell_a^\alpha\phi_b} = -i(hU^T(q^2 = (M_I^{\text{eff}})^2 - iM_I^{\text{eff}}\Gamma_I))_{\alpha I}\overline{u}_\alpha P_R u_{i=I}\epsilon_{ab} \ , \tag{2.145}$$

$$i\mathcal{M}_{N_I\to\overline{\ell}_a^\alpha\phi_b^*} = -i(V(q^2 = (M_I^{\text{eff}})^2 - iM_I^{\text{eff}}\Gamma_I)h^\dagger)_{I\alpha}\overline{v}_{i=I}P_L v_\alpha\epsilon_{ba} \tag{2.146}$$

with the effective couplings $(hU^T)_{\alpha i}$ and $(h^\dagger V)_{\alpha i}$. The second method is the pole expansion of the propagator. We can decompose $G(q)$ into partial fractions as

$$G(q)^{LL} = \sum_I X_I^{LL}D_I(q^2) \ , \quad G(q)^{RR} = \sum_I X_I^{RR}D_I(q^2) \ . \tag{2.147}$$

Note that the matrices X_I have the form

$$X_I \propto D_{J\neq I}(q^2)\big|_{q^2=(M_I^{\text{eff}})^2 - iM_I^{\text{eff}}\Gamma_I} \propto \frac{1}{((M_J^{\text{eff}})^2 - (M_I^{\text{eff}})^2) - i(M_J^{\text{eff}}\Gamma_J - M_I^{\text{eff}}\Gamma_I)} \tag{2.148}$$

because of the partial fraction expansion of the propagator $\propto (q^2 - s_1)^{-1}(q^2 - s_2)^{-1} = (s_1 - s_2)^{-1}\{(q^2 - s_1)^{-1} - (q^2 - s_2)^{-1}\}$ which reflects the superposition of the different mass eigenstates (cf. Eq.(4.27)). To leading order of the coupling constant, the complex symmetric matrices X_I are written as $X_I = iM_I^{\text{eff}} x_I x_I^T$ using the complex vector $x_I^{L(R)}$ in the flavor space.

$$i\mathcal{M}_{N_I \to \ell_a^\alpha \phi_b} = -i(h \cdot x_I)_\alpha \overline{u}_\alpha P_R u_{i=I} \epsilon_{ab} \ , \tag{2.149}$$

$$i\mathcal{M}_{N_I \to \overline{\ell}_a^\alpha \phi_b^*} = -i(x_I^T \cdot h^\dagger)_\alpha \overline{v}_{i=I} P_L v_\alpha \epsilon_{ba} \tag{2.150}$$

with the effective couplings $(h \cdot x_I)_\alpha$ and $(x_I^T \cdot h^\dagger)_\alpha$. Both of the sets of the effective couplings, $(hU^T)_{\alpha I}$, $(Vh^\dagger)_{I\alpha}$ and $(h \cdot x_I)_\alpha$, $(x_I^T \cdot h^\dagger)_\alpha$, are known to give a consistent result. We get the CP-violating parameter

$$\begin{aligned} \varepsilon_i &= \frac{1}{8\pi} \sum_{j(\neq i)} \frac{\Im(h^\dagger h)_{ij}^2}{(h^\dagger h)_{ii}} \frac{(M_i^2 - M_j^2) M_i M_j}{(M_i^2 - M_j^2)^2 + R_{ij}^2} \\ &= \frac{1}{8\pi} \sum_{j(\neq i)} \frac{\Im(h^\dagger h)_{ij}^2}{(h^\dagger h)_{ii} (h^\dagger h)_{jj}} \frac{(M_i^2 - M_j^2) M_i \Gamma_j}{(M_i^2 - M_j^2)^2 + R_{ij}^2} \end{aligned} \tag{2.151}$$

with the *regulator* $R_{ij} = |M_i \Gamma_i - M_j \Gamma_j|$. We can get the same conclusion from the lepton-number conserving processes (2.131) and (2.132) with the components G^{LR} and G^{RL}. In this expression, we used lower-case character in the subscripts and dropped the quantum correction to the Majorana mass. (2.151) should be regarded as the CP-violating parameter of the i-th mass eigenstate decay process.

It's clear that the $R_{ij} = |M_i \Gamma_i - M_j \Gamma_j|$ comes from the factor (2.148). However, note that this form of regulator becomes zero in the doubly degenerate limit $M_i = M_j$ and $\Gamma_i = \Gamma_j$, hence it does not seem to be the correct form of the regulator.

Systematic considerations were first performed by Pilaftsis [39], and he found that the regulator in the denominator is given by $R_{ij} = M_i \Gamma_j$. Then, in the degenerate case $|M_i - M_j| \sim \Gamma$, ε can be enhanced to $\mathcal{O}((h^\dagger h)^0) \sim 1$. After that, the authors [19, 42] gave the above calculation and obtained a regulator $R_{ij} = |M_i \Gamma_i - M_j \Gamma_j|$. By using their result, the enhancement factor can become much larger.

2.4 Summary and comments

We have seen that the RH neutrinos, which are introduced via the see-saw mechanism, provide the CP asymmetric process. The magnitude of the CP asymmetry in the decay process $N_i \to \ell, \phi$ is quantified by the CP-violating parameter (2.27). The time evolution of a set of the classical distribution functions of the RH neutrinos and the SM leptons is expected to be described by the Boltzmann equations (2.31), (2.34). However, the artificial separation of the collision term into the decay and scattering processes leads to the unphysical evolution equation of the lepton number (2.46), and we have to make the

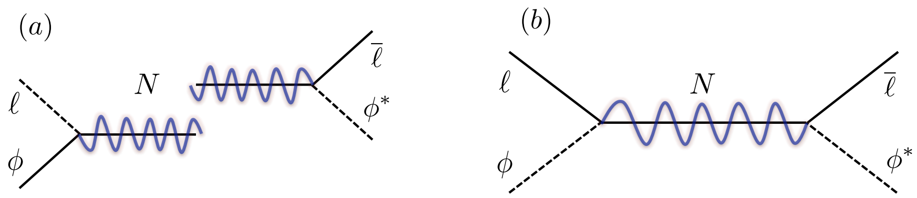

Figure 8: When we take only the decay and inverse decay processes of RH neutrino in the collision term, then the incoming and outgoing state of RH neutrino must be described as quantum states (a). If we consider only the scattering process, the internal line of RH neutrino mediating the process should have the information of the non-equilibrium quantum state of RH neutrino because this means that the RH neutrinos, which cause the system to be out of equilibrium, appear only as the internal line of scattering diagram (b).

procedure of the RIS-subtraction to get a physically acceptable equation (2.68). This means that the "(inverse) decay" process of the RH neutrinos are already included in the $2 \to 2$ scattering processes as the contributions mediated by the real intermediated states (RIS) of the RH neutrinos. The reason why we need such a separation is that the Boltzmann equations are *Markovian* evolution equations of *on-shell* classical particles' distribution functions: If we consider only the decay and inverse decay processes, we need evolution equations of the off-shell RH neutrinos, that is, time evolution of the "distribution function" of non-equilibrium quantum state must be considered (Fig.8 (a)). And if we consider only the scattering processes of the SM lepton and Higgs boson, we have to take into account the time evolution of the quantum state mediating the scattering processes (Fig.8 (b)), then the corresponding collision term is necessarily non-local in time. As we see in the later sections, the Kadanoff-Baym (KB) equation describes them in a consistent way. In section 4, solving the KB equation of RH neutrino in advance, we correctly understand the resonant leptogenesis from the view point of (b). On the other hand, in section 5, we rewrite the KB equation into the equation of "distribution function of quantum state" describing the picture (a).

The lower bound of the Majorana mass of the lightest RH neutrino (2.122) is obtained by using the expression (2.71) of the CP-violating parameter in the hierarchical mass spectrum case. Therefore, it can be evaded in the degenerate mass spectrum case. In the conventional calculation, the CP-violating parameter with the degenerate mass spectrum is read off from the $2 \to 2$ scattering processes to do the re-summation of the self-energy diagram of the RH neutrino, as seen in section 2.3. However, note that, in that calculation, the squared amplitudes of the scattering processes are not considered. One of the motivation for such a calculation could be said to be the cancellation between the "on-shell" and "off-shell" terms, that occurs in *equilibrium* case as a natural consequence of the unitarily and CPT invariance [20] and explicitly checked in [19]. In section 4, it's shown that, *with the non-equilibrium propagator of the RH neutrino*, the contributions of the interferences of the different mass eigenstates, which are dropped in the conventional calculation, become crucial to get the correct form of the CP-violating parameter in the degenerate mass spectrum case.

3 Evolution equations of lepton numbers from non-equilibrium QFT

A systematic method to investigate the evolution of lepton asymmetry is the Kadanoff-Baym (KB) equations. The KB equation corresponds to the time evolution equation of *full* two-point function which does not distinguish on-shell and off-shell states. Accordingly it can take into account quantum coherence of the system. And derived from the first principle of the quantum field theory, the KB equation cannot be local (consisting only of physical quantities at the moment) like the Boltzmann equation. The KB equation can be reduced to the classical Boltzmann equation only in special cases where quantum and memory effects can be neglected, and then, the double counting problem can be systematically resolved (see [72] and references therein).

Time-evolution of a quantum system is determined by the Hamiltonian of the system and the initial density operator $\hat{\rho}$ at the initial time $t = t_i$. All of the information of the system is encoded in the time-dependent density operator $\hat{\rho}(t)$, or instead, a set of all the n-point Green functions. Although the equations for all the n-point functions are known as the Schwinger-Dyson equations, it is practically impossible to study the evolution equations containing all the n-point functions. We need to select an important set of observables. In the classical approach based on the Boltzmann equation, one-particle distribution function on the phase space is selected to describe the system. In the KB approach, two-point Green functions are selected.

In this section, we summarize notations of various Green functions and their basic properties in the thermal equilibrium. We also summarize the non-equilibrium evolution equation (KB equation) for the Green functions. After brief reviews in section 3.1 and 3.2, we derive the evolution equation of the lepton number in section 3.3 and 3.4.

3.1 Green functions and KMS relations

Various Green functions are introduced in field theories (see also Appendix A). Consider a fermion field ψ. The statistical propagator G_F and the spectral density G_ρ are defined as

$$G_F(x,y) = \frac{1}{2}\langle[\hat{\psi}(x), \overline{\hat{\psi}}(y)]\rangle \ , \tag{3.1}$$

$$G_\rho(x,y) = i\langle\{\hat{\psi}(x), \overline{\hat{\psi}}(y)\}\rangle \tag{3.2}$$

where $\langle\cdots\rangle$ is defined as

$$\langle\hat{\mathcal{O}}(x)\rangle \equiv \mathrm{Tr}\{\hat{\rho}(t_i)\hat{\mathcal{O}}(x)\} \ . \tag{3.3}$$

The statistical propagator G_F contains information of the particle density of the state on which operators are evaluated. On the other hand, the spectral density G_ρ gives information of spectrum, such as particle's mass and decay width. Because of the anti-commutator, $\gamma^0 G_\rho(x^0, y^0)$ becomes proportional to the spatial delta function $\delta^3(\mathbf{x}-\mathbf{y})$ at the equal time $x^0 = y^0$:

$$\gamma_0 G_\rho(x,y) = i\delta^3(\mathbf{x}-\mathbf{y})\mathbf{1} \tag{3.4}$$

where $\mathbf{1}$ is an identity matrix in the flavor and the spinor indices.

Other useful Green functions are the Wightman functions

$$G_>(x,y) = G_F(x,y) - \frac{i}{2}G_\rho(x,y) = \langle \hat{\psi}(x)\overline{\hat{\psi}}(y)\rangle \ , \tag{3.5}$$

$$G_<(x,y) = G_F(x,y) + \frac{i}{2}G_\rho(x,y) = -\langle \overline{\hat{\psi}}(y)\hat{\psi}(x)\rangle \tag{3.6}$$

and the retarded and advanced Green functions are given by

$$G_{R/A}(x,y) = \pm\,\Theta(\pm(x^0-y^0))G_\rho(x,y) \ . \tag{3.7}$$

The spectral function can be written as $G_\rho = G_R - G_A = i(G_> - G_<)$.

In this thesis, we assume homogeneity along the spatial directions so that we can always use the Fourier transform in the 3-dimensional space. If the state is described by the thermal equilibrium state, we can further Fourier transform in the time direction.[8] In the thermal equilibrium at temperature T, the Green functions $G(x,y)$ are anti-periodic in the time direction with an imaginary period $i\beta = i/T$ and their Fourier transforms satisfy the KMS (Kubo Martin Schwinger) relation

$$G^{(eq)}_{\gtrless}(q) = -i\begin{Bmatrix} 1-f(q_0) \\ -f(q_0)\end{Bmatrix} G^{(eq)}_\rho(q) \ , \quad G^{(eq)}_F(q) = -i\left(\frac{1}{2} - f(q_0)\right) G^{(eq)}_\rho(q) \ . \tag{3.8}$$

Here $f(q_0)$ is the Fermi-Dirac distribution function $f(q_0) = 1/(e^{q_0/T}+1)$. In presence of the chemical potential μ, q_0 is replaced by $q_0 - \mu$. Since the relation relates the fluctuation described by the Wightman function to the dissipation described by the retarded Green function, it is also called the fluctuation-dissipation relation. By this relation, the spectrum of the system determines all the Green functions. When the system becomes out of equilibrium, the KMS relation is violated. The violation plays an important role in the leptogenesis.

As a final remark in this section, let us recall that the explicit forms of the Wightman functions of free charged fermions (bosons) are given by

$$\begin{aligned} G^{\rm free}_{\gtrless}(x,y) = &\int \frac{d^3q}{(2\pi)^3} e^{+i\mathbf{q}\cdot(\mathbf{x}-\mathbf{y})}\frac{1}{2\omega_q} \\ &\times\left[e^{-i\omega_q(x^0-y^0)}\begin{Bmatrix} 1-\eta f_{\mathbf{q}} \\ -\eta f_{\mathbf{q}}\end{Bmatrix}\hat{g}_+ + e^{+i\omega_q(x^0-y^0)}\begin{Bmatrix} -\eta \bar{f}_{-\mathbf{q}} \\ 1-\eta\bar{f}_{-\mathbf{q}}\end{Bmatrix}\hat{g}_-\right] \end{aligned} \tag{3.9}$$

[8] We often use the Fourier transform in the time direction when the system is in the thermal equilibrium at the local temperature $T(t)$ at time t. Then the Green functions in the four-momentum representation depends on time t through the local temperature.

where ω_q is the energy of the on-shell particle, and $\hat{g}_\pm = (\pm\omega_q\gamma^0 - \mathbf{q}\cdot\boldsymbol{\gamma} + m)$, $\eta = +1$ for Dirac fermions with their mass m and $\hat{g}_\pm = 1$, $\eta = -1$ for bosons. $f_\mathbf{q} \equiv \langle \hat{N}_\mathbf{q} \rangle$ and $\bar{f}_\mathbf{q} \equiv \langle \hat{\bar{N}}_\mathbf{q} \rangle$ are given as the expectation values of the number operator of on-shell particles and anti-particles respectively. In general, they are different from the equilibrium distribution functions.

3.2 Kadanoff-Baym equations

If the system is out of equilibrium and the state is time-dependent, we cannot use the ordinary perturbative method based on the so-called in-out formalism. A general formalism is given by the closed-time-path (CTP) formalism in which perturbative vertices are inserted on the closed-time-path $\mathcal{C} = \mathcal{C}_+ + \mathcal{C}_-$. See appendix A.1 for brief review and Figure 17 there. In this formalism, we can obtain the exact relations between correlation functions, i.e. Schwinger-Dyson (SD) equations, which has all the information of the system. However, SD equations are not closed in the sense that the equation for a given n-point function involves information about $m(> n)$-point functions. Therefore, some approximation suitable to describe a system are needed. One of the self-consistent approximation of the SD equations in the CTP formalism is called Kadanoff-Baym (KB) equation. Derivation of the KB equations is given in appendix A.2 and A.3.

The equations for the retarded and advanced Green functions are

$$iG^{-1}_{0(x)} G_{R/A}(x,y) - \int_{t_{int}}^{\infty} d^4z_g \ \Pi_{R/A}(x,z) G_{R/A}(z,y) = -\delta^g(x-y) \ . \tag{3.10}$$

Here d^4z_g is an abbreviation of $d^4z\sqrt{-g(z)}$ and $\delta^g(x-y) = \delta^4(x-y)/\sqrt{-g}$. $G^{-1}_{0(x)}$ is the free kinetic operator whose derivatives act on a field at x. $\Pi_{R/A}$ is the self-energy and defined in eq.(A.26). They have the same properties as $G_{R/A}$, e.g., $\Pi_R(x,y) = 0$ for $x^0 < y^0$ is satisfied. Note that the integration range in (3.10) is constrained between x_0 and y_0:

$$\int_{y^0(x^0)}^{x^0(y^0)} d^4z_g \ \Pi_{R/A}(x,z) G_{R/A}(z,y) \tag{3.11}$$

because of the step functions in $\Pi_{R/A}$ and $G_{R/A}$. Therefore $G_{R/A}(x,y)$ is determined by the local information between x_0 and y_0. Namely $G_{R/A}$ does not depend on the information of the system in the past: there is no memory effect for $G_{R/A}$.

Other Green functions G_* ($* = F, \rho, \lessgtr$) satisfy

$$\begin{aligned} iG^{-1}_{0(x)} G_*(x,y) = & \int_{t_{int}}^{\infty} d^4z_g \ \Pi_R(x,z) G_*(z,y) + \int_{t_{int}}^{\infty} d^4z_g \ \Pi_*(x,z) G_A(z,y) \\ = & \int_{t_{int}}^{x^0} d^4z_g \ \Pi_R(x,z) G_*(z,y) + \int_{t_{int}}^{y^0} d^4z_g \ \Pi_*(x,z) G_A(z,y) \ . \end{aligned} \tag{3.12}$$

In the second equality, we have used the properties of R/A functions. By using eq.(3.10), this equation can be solved formally in terms of the self-energy function and the R/A Green functions as

$$
\begin{aligned}
G_*(x,y) &= -\int_{t_{int}}^{\infty} d^4z_g d^4w_g G_R(x,z)\Pi_*(z,w)G_A(w,y) \\
&\equiv -(G_R * \Pi_* * G_A)(x,y) \ . \qquad (3.13)
\end{aligned}
$$

In the last line $*$-operation denotes the convolution operation.

Let us see the memory effect of G_*. Generally speaking, the integrals in (3.12) over z are performed from the past at the initial time t_{int} to x^0 or y^0. This makes Green functions dependent on the state of the system in the past before x^0 or y^0. This is indeed the case for G_F and $G_{\lessgtr}$, but for the spectral density G_ρ, there is no memory effect (only the information between x^0 and y^0 is needed). It can be seen by using $\Pi_\rho = \Pi_R - \Pi_A$. Then the integral of (3.12) can be rewritten as

$$
iG^{-1}_{0(x)}G_\rho(x,y) = \int_{y^0}^{x^0} d^4z_g \ \Pi_\rho(x,z)G_\rho(z,y) \ . \qquad (3.14)
$$

Or it can be directly seen from the relation $G_\rho = G_R - G_A$. The relation $G_\rho = -G_R\Pi_\rho G_A = G_R - G_A$ is equivalent to $\Pi_\rho = \Pi_R - \Pi_A = G_R^{-1} - G_A^{-1}$.

In the thermal equilibrium, since the system is translationally invariant, (3.13) can be Fourier transformed as

$$
G_*^{(eq)}(p) = -G_R^{(eq)}(p)\Pi_*^{(eq)}(p)G_A^{(eq)}(p) \ . \qquad (3.15)
$$

These equations (3.10), (3.12) are not closed within the two-point Green functions because the self-energy Π contains also $n(>2)$-point functions.[9] Hence, in order to solve them explicitly, we need to make an approximation to express $n(>2)$-point functions in terms of the two-point functions.[10] 2PI effective action method is one of the simplest and self-consistent methods. (See appendix A.3 for brief explanation.) By using it, the self-energies Π in the above equations (3.10) (3.12) are represented as a sum of 1PI diagrams made of full propagators, and consequently these equations can be interpreted as simultaneous equations for various propagators in the system. These self-consistent equations among the propagators are especially called the Kadanoff-Baym equations.

[9]When we describe the evolution of scalar fields, also the one-point function should be considered (see [86] for a review and references therein). Then, we have to solve the coupled equations of one- and two-point functions.

[10]As a result, all of the vertices in the self-energy is bare ones. In some case, by implementing the loop expansion, the equation fail to take into account important non-perturbative effects which make full vertices very different from bare ones. In the 2PI formalism, such non-perturbative effects could be considered by the resummation of selected diagrams. Otherwise, we have to employ so-called nPI formalism to obtain the consistent evolution equations of the full $k(<n)$ point functions.

3.3 Evolution of lepton number in the expanding universe

Now we investigate the KB equations of lepton numbers in the expanding universe. The theory we consider is (2.1). We first define Green functions, G, S and Δ for the RH neutrinos, the SM lepton doublet and the Higgs doublet respectively:

$$G^{ij}_{>}(x,y) = \langle \hat{N}^i(x)\overline{\hat{N}}^j(y)\rangle \ , \ \ G^{ij}_{<}(x,y) = -\langle \overline{\hat{N}}^j(y)\hat{N}^i(x)\rangle \ , \tag{3.16}$$

$$S^{\alpha\beta}_{ab>}(x,y) = \langle \hat{\ell}^\alpha_a(x)\bar{\hat{\ell}}^\beta_b(y)\rangle \ , \ \ S^{\alpha\beta}_{ab<}(x,y) = -\langle \bar{\hat{\ell}}^\beta_b(y)\hat{\ell}^\alpha_a(x)\rangle \ , \tag{3.17}$$

$$\Delta_{ab>}(x,y) = \langle \hat{\phi}_a(x)\hat{\phi}^\dagger_b(y)\rangle \ , \ \ \Delta_{ab<}(x,y) = +\langle \hat{\phi}^\dagger_b(y)\hat{\phi}_a(x)\rangle \ . \tag{3.18}$$

The classical inverse propagators are given by

$$iG_0^{-1\ ij}(x,y) =(i\nabla\!\!\!\!/_x - M_i)\delta^{ij}\delta^g(x-y) \ , \tag{3.19}$$

$$iS_0^{-1\ \alpha\beta}{}_{ab}(x,y) =i\nabla\!\!\!\!/_x P_L\delta^{\alpha\beta}\delta_{ab}\delta^g(x-y) \ , \tag{3.20}$$

$$i\Delta_0^{-1}{}_{ab}(x,y) =-\nabla^2_x\delta_{ab}\delta^g(x-y) \ . \tag{3.21}$$

In this thesis, we consider the spatially flat space-time with the scale factor $a(t)$:

$$ds^2 = dt^2 - a^2(t)d\mathbf{x}\cdot d\mathbf{x} \ . \tag{3.22}$$

We use $\tilde{\mu},\tilde{\nu},\dots$ as the space-time indices and $\mu,\nu,\dots$ as the local Lorentz indices. γ matrices are written as $\gamma^{\tilde{\mu}}(t) = \gamma^\mu e_\mu^{\ \tilde{\mu}}$ where the vier-bein field $e_\mu^{\ \tilde{\mu}}$ satisfies $e_\mu^{\ \tilde{\mu}}e_\nu^{\ \tilde{\nu}}g_{\tilde{\mu}\tilde{\nu}} = \eta_{\mu\nu}$. In the following we mainly use t-independent $\gamma^\mu = (\gamma^0,\boldsymbol{\gamma})$ instead of t-dependent $\gamma^{\tilde{\mu}}(t)$. The delta-function becomes $\delta^g(x-y) = \delta^4(x-y)/a^3(x^0)$.

In the background, the covariant derivative (3.21) becomes

$$\nabla_{x^{\tilde{\mu}}} = \partial_{\tilde{\mu}} + 3H(x^0)\delta^0_{\tilde{\mu}} \ . \tag{3.23}$$

Since the spin connection is given by $\Omega_{\tilde{\mu}} = aH[\gamma_\mu,\gamma_0]/4$, the covariant derivative for spinors in (3.19), (3.20) is given by

$$\nabla\!\!\!\!/_x =\gamma^{\tilde{\mu}}(x)(\partial_{\tilde{\mu}} + \Omega_{\tilde{\mu}}) = \gamma^0\left(\partial_{x^0} + \frac{3}{2}H(x^0)\right) - \frac{\boldsymbol{\gamma}\cdot\partial_{\mathbf{x}}}{a(x^0)} \ . \tag{3.24}$$

Here the Hubble parameter is defined by $H(t) = \dot{a}/a$. In the radiation dominant universe, it is given by

$$H(t) = 1.66\sqrt{g_*}\frac{T^2}{M_{pl}} \sim \frac{T^2}{10^{18}\mathrm{GeV}} \ . \tag{3.25}$$

Lepton number density n_L is a matrix with flavor indices α,β and isospin indices a,b. It is given by the $\tilde{\mu}=0$ component of the lepton number current

$$\begin{aligned} j^{\tilde{\mu}\beta\alpha}_{Lba}(x) \equiv&\langle \bar{\hat{\ell}}^\beta_b(x)\gamma^{\tilde{\mu}}(x)\hat{\ell}^\alpha_a(x)\rangle \\ =&-\ \mathrm{tr}\{\gamma^{\tilde{\mu}}(x)S^{\alpha\beta}_{ab>}(x,y)\}\Big|_{y=x} \\ =&-\ \mathrm{tr}\{\gamma^{\tilde{\mu}}(x)S^{\alpha\beta}_{ab<}(x,y)\}\Big|_{y=x} \ . \end{aligned} \tag{3.26}$$

Here $\mathrm{tr}\{\cdots\}$ is the trace of the spinors. Because of the spatial homogeneity, divergence of the current j_L is equal to

$$\nabla_{\tilde{\mu}}\ j_L^{\tilde{\mu}}(x) = \frac{dn_L}{dt} + 3H(t)n_L\ . \tag{3.27}$$

On the other hand, it can be rewritten as[11]

$$\begin{aligned}
\nabla_{\tilde{\mu}}\ j_L^{\tilde{\mu}}(x) &= -\ \mathrm{tr}\{\not{\nabla}_x S_{\gtrless}(x,y) - S_{\gtrless}(x,y)\overleftarrow{\not{\nabla}}_y\}\Big|_{y=x} \\
&= i\int d^4z_g\ \mathrm{tr}\{iS_0^{-1}(x,z)S_{\gtrless}(z,x) - S_{\gtrless}(x,z)iS_0^{-1}(z,x)\}\ .
\end{aligned}\tag{3.28}$$

In the second equality, we have used the definition of $S_0^{-1}(x,z)$ in (3.20).

By using the KB equation of (3.12) for the SM lepton Green function $S_{\gtrless}$, we have

$$\begin{aligned}
\int d^4z_g\ iS_0^{-1}(x,z)S_{\gtrless}(z,x) &= \int_{t_{int}}^{x_0} d^4z_g\ \Big(\Sigma_R(x,z)S_{\gtrless}(z,x) + \Sigma_{\gtrless}(x,z)S_A(z,x)\Big) \\
&= -i\int_{t_{int}}^{x^0} d^4z_g\ \Big(\Sigma_<(x,z)S_>(z,x) - \Sigma_>(x,z)S_<(z,x)\Big)
\end{aligned}\tag{3.29}$$

where Σ is the self-energy of the SM lepton. The second equality is obtained by using the relations (A.14) and (A.15).

Acting iS_0^{-1} from the right, a similar equation can be derived:

$$\int d^4z_g\ S_{\gtrless}(x,z)iS_0^{-1}(z,x) = -i\int_{t_{int}}^{x^0} d^4z_g\ \Big(S_<(x,z)\Sigma_>(z,x) - S_>(x,z)\Sigma_<(z,x)\Big)\ . \tag{3.30}$$

By using these equation, (3.27) becomes

$$\begin{aligned}
\frac{dn_L}{dt} + 3H(t)n_L = \int_{t_{int}}^{x^0} d^4z_g\ \mathrm{tr}\Big\{&\Sigma_<(x,z)S_>(z,x) - \Sigma_>(x,z)S_<(z,x) \\
&-S_<(x,z)\Sigma_>(z,x) + S_>(x,z)\Sigma_<(z,x)\Big\}\ .
\end{aligned}\tag{3.31}$$

This is the evolution equation for the lepton numbers in the expanding universe.

The right hand side (r.h.s.) is written as an integral of the full propagator S of the SM lepton and its self-energy Σ. Since the self-energy Σ contains various diagrams, some systematic simplification of Σ is necessary for practical calculations. A well-known approach is to use the 2-particles-irreducible (2PI)

[11] Here, we have defined the derivative operator $\overleftarrow{\not{\nabla}}_y$ as

$$S(x,y)\overleftarrow{\not{\nabla}}_y \equiv -\nabla_{\tilde{\mu}}\left[S(x,y)\gamma^{\tilde{\mu}}(y)\right] + S(x,y)\gamma^{\tilde{\mu}}(y)\Omega_{\tilde{\mu}} = \left(-\partial_{y^\mu} - \frac{3}{2}H(y^0)\right)S(x,y)\gamma^\mu.$$

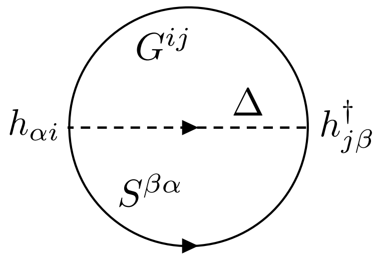

Figure 9: An example of 2PI diagrams for the Lagrangian (2.2) with Yukawa interactions. Each line represents a full propagator of the SM lepton, Higgs and the RH neutrino. By taking a functional derivative with respect to each propagator, we can obtain the self-energy for the corresponding particle.

formalism briefly reviewed in appendix A.3. In the 2PI formalism, the self-energy diagrams are obtained by taking a variation of 2PI diagrams made of full propagators with respect to the full propagator.

In the leading approximation, the self-energy Σ is obtained from the simplest 2PI diagram of Figure 9. Note that each propagator represents a full propagator, and the self-energy of the SM lepton is obtained by cutting the propagator ℓ. The next simplest 2PI diagram is given by Figure 18 in appendix B, but in most of the present analysis, we consider only the contribution from Figure 9. It gives a good approximation if the RH neutrinos have almost degenerate masses.

The contribution to the lepton self-energy Σ from Figure 9 is written in terms of the full propagators:

$$\Sigma^{\alpha\beta}_{ab\,\gtrless}(x,y) = -\,\delta_{ab} h_{\alpha i} h^\dagger_{j\beta} P_{\mathrm{R}} G^{ij}_{\gtrless}(x,y) P_{\mathrm{L}} \Delta_{\lessgtr}(y,x) \equiv\ \delta_{ab} \Sigma^{\alpha\beta}_{\gtrless}(x,y)\ . \tag{3.32}$$

Recall that (i,j) are flavor indices of the RH neutrinos. Then, summing the lepton flavor α,β and $SU(2)_L$ isospin a,b indices, we have

$$\begin{aligned}\frac{dn_L}{dt} + 3Hn_L = -g_w h_{\alpha i} h^\dagger_{j\beta} \int_{t_{int}}^{x^0} d^4 z_g\ \Big[&\mathrm{tr}\Big\{ P_{\mathrm{R}} G^{ij}_{<}(z,x) P_{\mathrm{L}} S^{\beta\alpha}_{>}(x,z) \Big\} \Delta_{>}(x,z) \\ &-\mathrm{tr}\Big\{ P_{\mathrm{R}} G^{ij}_{>}(z,x) P_{\mathrm{L}} S^{\beta\alpha}_{<}(x,z) \Big\} \Delta_{<}(x,z) \\ &-\mathrm{tr}\Big\{ P_{\mathrm{R}} G^{ij}_{>}(x,z) P_{\mathrm{L}} S^{\beta\alpha}_{<}(z,x) \Big\} \Delta_{<}(z,x) \\ &+\mathrm{tr}\Big\{ P_{\mathrm{R}} G^{ij}_{<}(x,z) P_{\mathrm{L}} S^{\beta\alpha}_{>}(z,x) \Big\} \Delta_{>}(z,x) \Big]\ .\end{aligned} \tag{3.33}$$

Here we used the fact that the electroweak symmetry is restored at the temperature $T \gtrsim$ TeV we are in mind and hence the propagators are written in $SU(2)$ symmetric forms: $S^{\alpha\beta}_{ab} = S^{\alpha\beta}\delta_{ab}$, $\Delta_{ab} = \Delta\delta_{ab}$. $g_w = 2$ is the number of d.o.f. of $SU(2)_L$ doublets. Since the third and the fourth terms are complex conjugate to the second and the first terms, we can simplify the above equation as

$$\frac{dn_L}{dt} + 3Hn_L = 2\,\Re \int_{t_{int}}^{x^0} d\tau d^3\mathbf{z}_g\ h_{\alpha i} h^\dagger_{j\beta} \Big[\mathrm{tr}\Big\{ P_{\mathrm{R}} G^{ij}_{<}(x,z) P_{\mathrm{L}} \widetilde{\pi}^{\beta\alpha}_{>}(z,x) \Big\} \\ - \mathrm{tr}\Big\{ P_{\mathrm{R}} G^{ij}_{>}(x,z) P_{\mathrm{L}} \widetilde{\pi}^{\beta\alpha}_{<}(z,x) \Big\} \Big] \quad (3.34)$$

where we have defined $\tau = z^0$ and

$$\widetilde{\pi}^{\beta\alpha}_{\gtrless}(z,x) = -g_w S^{\beta\alpha}_{\gtrless}(z,x) \Delta_{\gtrless}(z,x)\ . \quad (3.35)$$

This is the equation we evaluate in the following investigations. As we mentioned above, the r.h.s. contains only the contribution from the simplest 2PI diagram of Figure 9. If we restrict our discussion to the almost on-shell part of the RH neutrino propagator, this corresponds to taking the processes of decay and inverse-decay of the RH neutrinos, and equivalently the resonant part of the s-cannel $\times$ s-cannel part of the scattering process $\ell\phi \leftrightarrow \bar{\ell}\phi^*$, which are the most important parts in the leading order calculation. The full contributions of scattering can be taken into account by considering the next simplest diagram of Figure 18. A systematical study of the KB equation including the scattering effects is given in [72][74].

3.4 Boltzmann equation for the lepton number

The evolution equation (3.34) of the lepton number is determined by the behavior of full propagators of the RH neutrinos G, the SM leptons S and the Higgs Δ. In sections 4.1 and 4.2, we investigate detailed properties of the propagator G of the RH neutrinos. In this section, we will see how an ordinary Boltzmann-type equation can be derived from eq.(3.34) by using the quasi-particle approximation for the SM particles described by S and Δ.

The quasi-particle approximation is an approximation to express the Green functions in terms of distribution functions of quasi-particles with a mass m and a width Γ. Hence the propagators in this approximation are obtained from the free Wightman function of eq.(3.9) by introducing the decay width Γ. For a moment, we neglect the time-dependence of the background. For the SM leptons, we have

$$\begin{aligned} S^{\beta\alpha}_{\gtrless}(x,y) &= \delta^{\alpha\beta} \int \frac{d^3p}{(2\pi)^3} \frac{1}{2\omega_p} e^{+i\mathbf{p}\cdot(\mathbf{x}-\mathbf{y})}\ e^{-|x^0-y^0|\Gamma_\ell/2} \\ &\quad \times \Big[e^{-i\omega_p(x^0-y^0)} \begin{Bmatrix} 1-f_{\ell p} \\ -f_{\ell p} \end{Bmatrix} P_{\mathrm{L}} \not{p}_+ P_{\mathrm{R}} + e^{+i\omega_p(x^0-y^0)} \begin{Bmatrix} -f_{\bar{\ell} p} \\ 1-f_{\bar{\ell} p} \end{Bmatrix} P_{\mathrm{L}} \not{p}_- P_{\mathrm{R}} \Big] \\ &= \delta^{\alpha\beta} \sum_{\epsilon_\ell=\pm} \int \frac{d^3p}{(2\pi)^3} \frac{1}{2\omega_p} e^{+i\mathbf{p}\cdot(\mathbf{x}-\mathbf{y})} e^{-i\epsilon_\ell\omega_p(x^0-y^0)-|x^0-y^0|\Gamma_\ell/2} \\ &\qquad \times (-1)^{\epsilon_\ell} \begin{Bmatrix} 1-f^{\epsilon_\ell}_{\ell p} \\ -f^{\epsilon_\ell}_{\ell p} \end{Bmatrix} P_{\mathrm{L}} \not{p}_{\epsilon_\ell} P_{\mathrm{R}} \end{aligned} \quad (3.36)$$

where $\omega_p = \sqrt{m_\ell^2 + |\mathbf{p}|^2/a^2}$ and $\not{p}_\pm \equiv \pm\omega_p\gamma^0 - \mathbf{p}\cdot\boldsymbol{\gamma}/a$. Here we assumed the flavor independence of the lepton propagators, $S^{\alpha\beta} \propto \delta^{\alpha\beta}$, for simplicity. Similarly the Wightman functions of the Higgs boson becomes

$$\begin{aligned}\Delta_{\gtrless}(x,y) &= \int \frac{d^3k}{(2\pi)^3}\frac{1}{2\omega_k} e^{+i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})} e^{-|x^0-y^0|\Gamma_\phi/2} \\ &\quad \times \left[e^{-i\omega_k(x^0-y^0)} \begin{Bmatrix} 1+f_{\phi k} \\ +f_{\phi k} \end{Bmatrix} + e^{+i\omega_k(x^0-y^0)} \begin{Bmatrix} +f_{\bar{\phi}k} \\ 1+f_{\bar{\phi}k} \end{Bmatrix} \right] \\ &= \sum_{\epsilon_\phi=\pm} \int \frac{d^3k}{(2\pi)^3}\frac{1}{2\omega_k} e^{+i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})} e^{-i\epsilon_\phi\omega_k(x^0-y^0)-|x^0-y^0|\Gamma_\phi/2} \\ &\qquad\qquad \times (-1)^{\epsilon_\phi} \begin{Bmatrix} 1+f_{\phi k}^{\epsilon_\phi} \\ +f_{\phi k}^{\epsilon_\phi} \end{Bmatrix} \end{aligned} \tag{3.37}$$

where $\omega_k = \sqrt{m_\phi^2 + |\mathbf{k}|^2/a^2}$.

The thermal mass and width are given by $m_{\ell,\phi} \sim gT$, $\Gamma_{\ell,\phi} \sim g^2T$ where g is the SM gauge coupling g. Especially, we exploit the largeness of $\Gamma_{\ell,\phi}$ as an important thermal effect.[12]

In these expressions we defined $(-1)^\epsilon = \pm 1$ for $\epsilon = \pm$ respectively. The distribution functions are assumed to be in the kinematical equilibrium and given by the Fermi-Dirac or the Bose-Einstein distributions at temperature T with a chemical potential:

$$f_{\ell p} = \frac{1}{e^{(\omega_p-\mu_\ell)/T}+1}\ ,\ f_{\phi k} = \frac{1}{e^{(\omega_k-\mu_\phi)/T}-1}\ . \tag{3.38}$$

For anti-particles, the signs of the chemical potentials are reversed and their distributions are given by

$$f_{\bar{\ell}p} = \frac{1}{e^{(\omega_p+\mu_\ell)/T}+1}\ ,\ \ f_{\bar{\phi}k} = \frac{1}{e^{(\omega_k+\mu_\phi)/T}-1}\ . \tag{3.39}$$

In the second equalities of eq. (3.36) and (3.37), we have defined

$$f_{\ell p}^\epsilon \equiv \frac{1}{e^{(\epsilon\omega_p-\mu_\ell)/T}+1}\ ,\ \ f_{\phi k}^\epsilon \equiv \frac{1}{e^{(\epsilon\omega_k-\mu_\phi)/T}-1} \tag{3.40}$$

which satisfy

$$f_{\ell p} = f_{\ell p}^+\ ,\ \ f_{\bar{\ell}p} = (1-f_{\ell p}^-)\ ,\ \ f_{\phi k} = f_{\phi k}^+\ ,\ \ f_{\bar{\phi}k} = -(1+f_{\phi k}^-)\ . \tag{3.41}$$

Now we come back to the time-dependence of the background. Since the scale factor $a(t)$ is time-dependent, temperature T, thermal mass and width are

[12] The effects of the thermal plasma play several roles, and are systematically investigated in [72][74]. For example, the thermal mass of the Higgs boson becomes larger than the RH neutrino masses at very high temperature, and then, the channel $\phi \to N\bar{\ell}$ get to be kinematically allowed.

dependent on the time t and we need to specify at which time these quantities in the quasi-particle approximation of eq. (3.36) and (3.37) are defined. If the temperature of the universe is sufficiently low (e.g., ~ 10 TeV), the decay width is much larger than the Hubble expansion rate:

$$\Gamma_{\ell,\phi} \sim g^2 T \gg H \sim \frac{T^2}{10^{18}\mathrm{GeV}} \tag{3.42}$$

and the propagators damp quickly at $|x^0 - y^0| \gg 1/\Gamma_{l,\phi}$. For such short period, time-dependence of the physical quantities such as the scale factor in the propagators (3.36) and (3.37) are suppressed by $H/\Gamma_{\ell,\phi}$, and we can approximate these quantities as being constant in the integration of τ in (3.34). Then the physical quantities can be evaluated at time $t = X_{xy} = (x^0 + y^0)/2$ as we see in (4.49).

By Fourier transforming in the spatial direction and using the above approximation, (3.34) becomes

$$\begin{aligned}\frac{dn_L}{dt} + 3Hn_L = 2\Re \int \frac{d^3q}{(2\pi)^3} \int_{-\infty}^{t} d\tau \; (h^\dagger h)_{ji} \Big[&\mathrm{tr}\Big\{ P_{\mathrm{R}} G^{ij}_{<}(t,\tau;\mathbf{q}) P_{\mathrm{L}} \widetilde{\pi}_{>}(\tau,t;\mathbf{q}) \Big\} \\ &- \mathrm{tr}\Big\{ P_{\mathrm{R}} G^{ij}_{>}(t,\tau;\mathbf{q}) P_{\mathrm{L}} \widetilde{\pi}_{<}(\tau,t;\mathbf{q}) \Big\} \Big]\end{aligned} \tag{3.43}$$

where $t = x^0$ and $d^3q = d^3\mathbf{q}/a^3(t)$. Using the quasi-particle approximations (3.36) and (3.37), $\widetilde{\pi}_{\gtrless}(\tau,t;\mathbf{q})$ are given by

$$\widetilde{\pi}_{\gtrless}(\tau,t;\mathbf{q}) = P_{\mathrm{L}} \pi_{\gtrless}(\tau,t;\mathbf{q}) P_{\mathrm{R}} \; , \tag{3.44}$$

$$\begin{aligned}\pi_{\gtrless}(\tau,t;\mathbf{q}) \equiv (-g_w) \sum_{\epsilon_\ell,\epsilon_\phi} \int \frac{d^3p}{(2\pi)^3 2\omega_p} \frac{d^3k}{(2\pi)^3 2\omega_k} \; (2\pi)^3 \delta^3(q-p-k) \\ \times \; \mathcal{D}^{\epsilon_\ell \epsilon_\phi}_{\gtrless(p,k)} \; \not{p}_{\epsilon_\ell} \; e^{-i(\epsilon_\ell \omega_p + \epsilon_\phi \omega_k)(\tau - t) - \Gamma_{\ell\phi}/2|\tau - t|}\end{aligned} \tag{3.45}$$

where $\Gamma_{\ell\phi} \equiv \Gamma_\ell + \Gamma_\phi$ and $\mathcal{D}^{\epsilon_\ell \epsilon_\phi}_{\gtrless(p,k)}$ is defined as

$$\mathcal{D}^{\epsilon_\ell \epsilon_\phi}_{\gtrless(p,k)} \equiv (-1)^{\epsilon_\ell} (-1)^{\epsilon_\phi} \begin{Bmatrix} (1 - f^{\epsilon_\ell}_{\ell p})(1 + f^{\epsilon_\phi}_{\phi k}) \\ (-f^{\epsilon_\ell}_{\ell p})(+f^{\epsilon_\phi}_{\phi k}) \end{Bmatrix} \; . \tag{3.46}$$

From (3.45) and (3.46), we can see that the term with $\widetilde{\pi}_{>}$ in (3.43) contains a factor $(1 - f_\ell)$ or $(1 - f^-_\ell) = f_{\bar{\ell}}$ and corresponds to gain in the lepton number while the other term with $\widetilde{\pi}_{<}$ contains a factor f_ℓ or $f^-_\ell = (1 - f_{\bar{\ell}})$ and corresponds to loss. Since the damping rates of the SM particles' propagators are very large (3.42) and the time integration in (3.43) is dominated around $\tau = t$, let us assume that the physical quantities, the lepton number evolution equation (3.43) is well approximated by the form

$$\frac{dn_L}{dt} + 3Hn_L \propto (h^\dagger h)_{ji} \Big\{ G^{ij}_{<}(t,\tau=t)(1 - f_\ell(t))(1 + f_\phi(t)) - G^{ij}_{>}(t,\tau=t) f_\ell(t) f_\phi(t) + \cdots \Big\} \; . \tag{3.47}$$

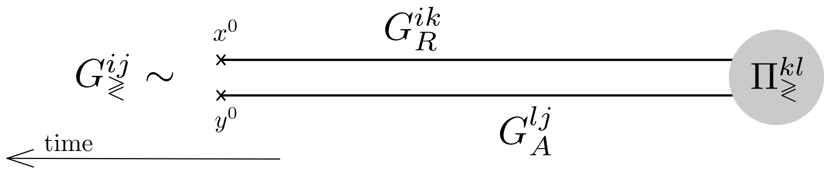

Figure 10: In the formal solution 3.48, information in past is taken in to count through the time integration. Therefore, the Wightman function having two points close to the time $x^0 \sim y^0 \sim t$ in (3.43) still remembers that the SM thermal bath composing the RH neutrino's self-energy Π had higher temperature and larger number density of the SM particles in past than those at the time t.

Hence the evolution equation (3.43) can be regarded as the Boltzmann-like equation for the lepton number.

3.5 Summary and comments

In the first two subsections, we have briefly introduce the property of two-point functions and the Kadanoff-Baym (KB) equation as the self-consistent evolution equation of two-point functions. Generally, two-point functions encode not only the information about distribution of classical on-shell particles, but also the one of quantum off-shell states in a system. From the KB equation of the SM lepton propagator, the evolution equation of the lepton number is obtained as (3.34) or its Fourier transform (3.43). By adopting the quasi-particle approximations of the SM lepton (3.36) and Higgs boson (3.37), the r.h.s. of (3.43) turns out to have the similar structure to the collision term of the ordinary Boltzmann equation (3.47). Although such a simple quasi-particle approximation is not applicable to the RH neutrino propagators G because they have the flavor off-diagonal component, now we have the formal solution of the Wightman propagator in (3.13):

$$G^{ij}_{\gtrless}(x,y) = -\sum_{k,l} \int_{-\infty}^{x^0} du^0 d^3\mathbf{u}_g \int_{-\infty}^{y^0} dv^0 d^3\mathbf{v}_g \; G^{ik}_R(x,u)\Pi^{kl}_{\gtrless}(u,v)G^{lj}_A(v,y) \; . \tag{3.48}$$

When this expression is inserted into the lepton number evolution equation (3.43), the full quantum effects are expected to be taken into account correctly.

Although (3.43) seems to be Markovian equation (3.47), once the above formal solution is plugged, the resulting equation becomes a non-Markovian equation with the time integration again, see Fig.10. As mentioned in section 2.4, such a non-Markovian equation enables us to describe the system without distinguishing on-shell and off-shell states, which can resolve the problems in the conventional calculation using the Boltzmann equation. Note that the reason why we can consider the formal solution (3.48) in the lepton number evolution equation (3.43) is that the KB equation as the starting point is the self-consistent equation of the full propagators.

In the next section 4, we estimate the formal solution (3.48) in the expanding universe and obtain the CP-violating parameter. In the section 5, we consider a generalization of the quasi-particle approximations as another approach.

4 Right-handed neutrino propagators and the enhancement of CP-asymmetry

In this section, by solving the KB equation directly, we investigate how the expansion of the universe affects various propagators of the RH neutrino. The equilibrium part and the deviation-from-equilibrium part of the propagators are calculated separately. With the degenerate mass spectrum of the RH neutrinos, the flavor off-diagonal components are much enhanced compared to the hierarchical spectrum case. We see that there are two types of the enhancement factor. One of them corresponds to the conventional form of the enhancement of the CP-violating parameter (2.151), but it turns out not to be relevant to the generation of the lepton number. Other one comes form the non-equilibrium propagator of RH neutrino and contribute to the time evolution of the lepton number.

In section 4.1, we solve the KB equation of RH neutrino propagator in equilibrium under the assumption that the off-diagonal components of the neutrino Yukawa coupling $(h^\dagger h)'$ is smaller than the diagonal part $(h^\dagger h)^d$. In section 4.2, restricting the discussion to the strong washout case, which is typical to the resonant leptogenesis, we obtain the small deviation from the equilibrium value of RH neutrino propagator. Plugging the solution into the evolution equation of lepton number (3.43), we get the Boltzmann equation-like form with the modified CP-violationg parameter in section 4.3. Finally, in section 4.4, we discuss the origin of the modification of the CP-violating parameter.

4.1 Resonant oscillation of RH neutrinos

In this subsection, we study how the RH neutrinos with almost degenerate masses behave in the thermal equilibrium. Deviation from the thermal equilibrium is investigated in the next section 4.2.

We consider two flavors $i = 1, 2$ whose masses are almost degenerate. The third flavor RH neutrino is assumed to have larger mass. In order to calculate the evolution of the lepton asymmetry in (3.34), we need to know the Wightman functions $G_{\gtrless}$ of the RH neutrinos. And, since the KB equation of $G^{ij}_{\gtrless}$ is formally solved by the convolution eq.(3.48), it is necessary to investigate the properties of the retarded (advanced) Green functions $G^{ij}_{R/A}$ first.

We first study both of the flavor diagonal ($i = j$) and off-diagonal ($i \neq j$) components of G^{ij}_R in the equilibrium. Then we will see the behaviors of the Wightman functions $G^{ij}_{\gtrless}$ in the thermal equilibrium. Throughout this thesis, G^d (also Π^d for the self-energy) and G' (Π') denote the flavor diagonal $i = j$ and off-diagonal $i \neq j$ components respectively:

$$\begin{aligned} G^d &\longleftrightarrow \text{flavor diagonal}\ , \\ G' &\longleftrightarrow \text{flavor off-diagonal}\ . \end{aligned} \tag{4.1}$$

4.1.1 Retarded/Advanced propagators

From (3.10) and (3.19), $G_{R/A}$ satisfies

$$(i\nabla\!\!\!/_x - M)G^{ij}_{R/A}(x,y) - \int_{t_{int}}^{\infty} dz^0 d^3\mathbf{z}\ a^3(z^0)\ \Pi^{ik}_{R/A}(x,z)G^{kj}_{R/A}(z,y) = -\delta_{ij}\delta^g(x-y)\ . \tag{4.2}$$

We first define the spatial Fourier transform of $G_{R/A}$ by

$$G^{ij}_{R/A}(x^0,y^0;\mathbf{q}) = \int d^3(\mathbf{x}-\mathbf{y})e^{-i\mathbf{q}\cdot(\mathbf{x}-\mathbf{y})}a^{3/2}(x^0)G^{ij}_{R/A}(x^0,y^0,\mathbf{x}-\mathbf{y})a^{3/2}(y^0)\ . \tag{4.3}$$

Similarly, for the self-energy, we define

$$\Pi^{ij}_{R/A}(x^0,y^0;\mathbf{q}) = \int d^3(\mathbf{x}-\mathbf{y})e^{-i\mathbf{q}\cdot(\mathbf{x}-\mathbf{y})}a^{3/2}(x^0)\Pi^{ij}_{R/A}(x^0,y^0,\mathbf{x}-\mathbf{y})a^{3/2}(y^0)\ . \tag{4.4}$$

Then using (3.24), the KB equation (4.2) becomes

$$\begin{aligned}&\Big\{i\gamma^0\partial_{x^0} - \frac{\boldsymbol{\gamma}\cdot\mathbf{q}}{a(x^0)} - M\Big\}G_{R/A}(x^0,y^0;\mathbf{q})\\ &\quad - \int_{t_{int}}^{\infty} dz^0\ \Pi_{R/A}(x^0,z^0;\mathbf{q})G_{R/A}(z^0,y^0;\mathbf{q}) = -\delta(x^0-y^0)\ .\end{aligned} \tag{4.5}$$

This is the basic equation for $G_{R/A}$.

We then decompose the propagator and the self-energy into flavor diagonal and off-diagonal parts:

$$\begin{aligned}G_{R/A}(x^0,y^0;\mathbf{q}) &\equiv G^d_{R/A}(x^0,y^0;\mathbf{q}) + G'_{R/A}(x^0,y^0;\mathbf{q})\ ,\\ \Pi_{R/A}(x^0,y^0;\mathbf{q}) &\equiv \Pi^d_{R/A}(x^0,y^0;\mathbf{q}) + \Pi'_{R/A}(x^0,y^0;\mathbf{q})\ .\end{aligned} \tag{4.6}$$

Using this decomposition, we solve the KB equation (4.5) iteratively.

First we define the differential-integral operator $D^d_{x^0}$ by

$$D^d_{x^0}f(x^0) \equiv \Big\{i\gamma^0\partial_{x^0} - \frac{\boldsymbol{\gamma}\cdot\mathbf{q}}{a(x^0)} - M\Big\}f(x^0) - \int_{t_{int}}^{\infty} dz^0\ \Pi^d_{R/A}(x^0,z^0;\mathbf{q})\ f(z^0)\ . \tag{4.7}$$

In terms of the operator, the flavor diagonal component of the KB equation (4.5) becomes

$$D^d_{x^0}G^d_{R/A}(x^0,y^0;\mathbf{q}) - \int_{t_{int}}^{\infty} dz^0\ \Pi'_{R/A}(x^0,z^0;\mathbf{q})G'_{R/A}(z^0,y^0;\mathbf{q}) = -\delta(x^0-y^0)\ . \tag{4.8}$$

Similarly the KB equation of the flavor off-diagonal component is written as

$$D^d_{x^0} G'_{R/A}(x^0, y^0; \mathbf{q}) = \int_{t_{int}}^{\infty} dz^0 \; \Pi'_{R/A}(x^0, z^0; \mathbf{q}) G^d_{R/A}(z^0, y^0; \mathbf{q}) \; . \tag{4.9}$$

We then introduce the kernel $G^{d(0)}_{R/A}$ of the operator $D^d_{x^0}$:

$$D^d_{x^0} G^{d(0)}_{R/A}(x^0, y^0; \mathbf{q}) \equiv -\delta(x^0 - y^0) \; . \tag{4.10}$$

with a retarded (advanced) boundary condition. Using $G^{d(0)}_{R/A}$, we can integrate the equations (4.8), (4.9) as

$$\begin{aligned} G^d_{R/A}(x^0, y^0; \mathbf{q}) = & \, G^{d(0)}_{R/A}(x^0, y^0; \mathbf{q}) \\ & - \int_{t_{int}}^{\infty} d\tau d\tau' \; G^{d(0)}_{R/A}(x^0, \tau; \mathbf{q}) \Pi^{'}_{R/A}(\tau, \tau'; \mathbf{q}) G'_{R/A}(\tau, y^0; \mathbf{q}) \; , \end{aligned} \tag{4.11}$$

$$G'_{R/A}(x^0, y^0; \mathbf{q}) = -\int_{t_{int}}^{\infty} d\tau d\tau' \; G^{d(0)}_{R/A}(x^0, \tau; \mathbf{q}) \Pi^{'}_{R/A}(\tau, \tau'; \mathbf{q}) G^d_{R/A}(\tau, y^0; \mathbf{q}) \; . \tag{4.12}$$

Then we can iteratively solve the above equations by expanding it with respect to the small off-diagonal component of the Yukawa coupling $(h^\dagger h)^{'}$ involved in $\Pi^{'}$:

$$\begin{aligned} G^d_{R/A} = & G^{d(0)}_{R/A} + G^{d(2)}_{R/A} + \cdots \; , \qquad (4.13) \\ & G^{d(2)}_{R/A} \equiv G^{d(0)}_{R/A} * \Pi^{'}_{R/A} * G^{d(0)}_{R/A} * \Pi^{'}_{R/A} * G^{d(0)}_{R/A} \; , \\ G'_{R/A} = & - G^{d(0)}_{R/A} * \Pi^{'}_{R/A} * G^{d(0)}_{R/A} + \cdots \; . \qquad (4.14) \end{aligned}$$

Here $*$ denotes a convolution in the time-direction. The second term $G^{d(2)}_{R/A}$ in the flavor diagonal propagator (4.13) is the second order of $(h^\dagger h)^{'}$ and smaller than $G^{d(0)}_{R/A}$ or $G'_{R/A}$. Hence we drop it and write $G^{d(0)}$ as G^d for notational simplicity in the following.

We note that the above integrals do not have the memory effect. This is because the convolution is written explicitly as, e.g.,

$$(G_R * \Pi_R * G_R)(x^0, y^0) = \int_{y^0}^{x^0} du \int_{y^0}^{u} dv \; G_R(x^0, u) \Pi_R(u, v) G_R(v, y^0) \tag{4.15}$$

and the integration region is limited between x^0 and y^0. Namely, the retarded (advanced) propagators are "local" functions of time during x^0 and y^0 and insensitive to the past ($t < x^0, y^0$). This is different from the convolution contained in the Wightman functions (3.48) in which the integration range of time is extended to the past.

4.1.2 Diagonal $G^d_{R/A}$ in thermal equilibrium

We will first look at the flavor diagonal component of the propagator $G^d_{R/A}(x^0, y^0; \mathbf{q})$ in the thermal equilibrium at temperature T. The scale factor a is also fixed at $a_0 = a(x^0) = a(y^0)$. Because of the translational invariance in the time direction, $G_{R/A}(x^0, y^0; \mathbf{q})$ can be further Fourier transformed:

$$G^{d(eq)}_{R/A}(q) = \int d(x^0 - y^0) e^{+iq_0(x^0-y^0)} G^{d(eq)}_{R/A}(x^0, y^0; \mathbf{q}) \ . \tag{4.16}$$

Then the KB equation (4.11) becomes

$$\Big\{\gamma^0 q_0 - \frac{\boldsymbol{\gamma}\cdot\mathbf{q}}{a_0} - M - \Pi^{d(eq)}_{R/A}(q_0, \mathbf{q})\Big\} G^{d(eq)}_{R/A}(q) = -1 \tag{4.17}$$

and can be solved

$$G^{d(eq)}_{R/A}(q) = -\Big(\not{q} - M - \Pi^{d(eq)}_{R/A}(q)\Big)^{-1} \ . \tag{4.18}$$

The real part of the self-energy gives the mass and wave-function renormalization. In the following we assume that they are already taken into account in the bare Lagrangian and focus only on the imaginary part $\Pi^d_\rho = \Pi^d_R - \Pi^d_A = 2i\Im(\Pi^d_R)$. The one-loop diagonal self-energy in the thermal equilibrium is expressed as $\Pi^d_\rho = \gamma^\mu \Pi^d_{\rho,\mu}$. From the imaginary part of the pole of the propagator $G^{d(eq)}_R(q)$, we see that the decay width Γ_q of the RH neutrino is given by

$$q \cdot \Pi^{d(eq)}_\rho(q)|_{q_0 = \pm\omega_{\mathbf{q}}} \equiv \mp i\omega_q \Gamma_q \ . \tag{4.19}$$

The i-th diagonal component $G^{d(eq)ii}_{R/A}(q)$ becomes

$$G^{d(eq)ii}_{R/A}(q) \simeq -\frac{\not{q} - \Pi^{d(eq)ii}_{R/A}(q) + M_i}{(q_0 \pm i\Gamma_{iq}/2)^2 - \omega^2_{iq}} \simeq \begin{cases} \displaystyle\sum_{\epsilon=\pm} \frac{iZ^i_\epsilon}{q_0 - \Omega_{\epsilon i}} \\ \displaystyle\sum_{\epsilon=\pm} \frac{iZ^i_\epsilon}{q_0 - \Omega^*_{\epsilon i}} \end{cases} \tag{4.20}$$

where

$$\Omega_{\epsilon i} \equiv \epsilon\omega_{iq} - i\Gamma_{iq}/2 \tag{4.21}$$

and

$$Z^i_\epsilon = \frac{i\epsilon}{2\omega_{iq}}(\not{q}_{\epsilon i} + M_i), \quad \not{q}_{\epsilon i} \equiv \epsilon\omega_{iq}\gamma^0 - \mathbf{q}\cdot\boldsymbol{\gamma}/a_0 \ . \tag{4.22}$$

In the real time representation, it becomes[13]

$$G_R^{d(eq)ii}(x^0, y^0; \mathbf{q}) = +\Theta(x^0 - y^0) \sum_{\epsilon=\pm} Z_\epsilon^i e^{-i\Omega_{\epsilon i}(x^0 - y^0)} \ ,$$
$$G_A^{d(eq)ii}(x^0, y^0; \mathbf{q}) = -\Theta(y^0 - x^0) \sum_{\epsilon=\pm} Z_\epsilon^i e^{-i\Omega^*_{\epsilon i}(x^0 - y^0)} \ . \tag{4.23}$$

Γ_q is multiplied by the Lorentz boost factor as $\Gamma_q \simeq (M/\omega_q) \times \Gamma$ where $\Gamma \equiv \Gamma_{q=0}$ is the decay width of the RH neutrino. In this thesis, we consider a situation that two RH neutrinos are almost degenerate in their masses

$$\Delta M \equiv |M_i - M_j| \simeq \Gamma \ . \tag{4.24}$$

In the following, we sometimes use the averages denoted by quantities without the flavor index i, j

$$M = \frac{M_i + M_j}{2} \ , \quad \omega_q = \frac{\omega_{iq} + \omega_{jq}}{2} \ , \quad \Omega_\epsilon = \frac{\Omega_{\epsilon i} + \Omega_{\epsilon j}}{2} \ , \quad etc. \tag{4.25}$$

4.1.3 Off-diagonal $G'_{R/A}$ in thermal equilibrium

We then study the behavior of the flavor off-diagonal component $G_{R/A}^{'(eq)}$ of the retarded (advanced) propagators in the thermal equilibrium. From (4.14), it is given by

$$G_{R/A}^{'(eq)ij}(x^0, y^0; \mathbf{q}) = -\int \frac{dq_0}{2\pi} e^{-iq_0(x^0 - y^0)} G_{R/A}^{d(eq)ii}(q) \Pi_{R/A}^{'(eq)ij}(q) G_{R/A}^{d(eq)jj}(q) \ . \tag{4.26}$$

The q_0 integration can be performed by summing residues of the poles. Eq. (4.20) shows that the retarded propagator $G_R^{d(eq)ii}$ has poles at $q_0 = \Omega_{\pm,i}$ and the advanced propagator $G_A^{d(eq)jj}$ has poles at $q_0 = \Omega^*_{\pm,j}$. The self-energy $\Pi_{R/A}$ consists of the SM lepton and the Higgs propagator, and hence it has poles at $q_0 = \epsilon_\ell \omega_p + \epsilon_\phi \omega_k \mp i\Gamma_{\ell\phi}/2$ with a large imaginary part. Because of this, the residues of the poles of the self-energy are suppressed by $\Gamma_i/\Gamma_{\ell\phi} \ll 1$. Noting the relation

$$\frac{1}{q_0 - \Omega_{\epsilon i}} \frac{1}{q_0 - \Omega_{\epsilon' j}} = \frac{1}{\Omega_{\epsilon i} - \Omega_{\epsilon' j}} \left(\frac{1}{q_0 - \Omega_{\epsilon i}} - \frac{1}{q_0 - \Omega_{\epsilon' j}} \right) \ , \tag{4.27}$$

we can see that the contribution $\epsilon = -\epsilon'$ is also suppressed by $\Delta M/M$ compared to the $\epsilon = \epsilon'$ contribution. Hence, dropping these suppressed contributions, we

[13] In the present analysis, we expand various quantities with respect to $(h^\dagger h)'$. Hence the propagator of i-th flavor is almost identified with the propagator of the i-th mass eigenstate up to higher order terms of $(h^\dagger h)'$. Propagations of a single N_i corresponds to propagations of a single mass eigenstate with mass M_i and width Γ_i.

have

$$G_R^{'(eq)ij}(x^0, y^0; \mathbf{q}) \simeq +\,\Theta(x^0 - y^0) \sum_\epsilon Z_\epsilon \Pi_R^{'(eq)ij}(\epsilon\omega_q) Z_\epsilon \frac{-i}{\Omega_{\epsilon i} - \Omega_{\epsilon j}} \times \left(e^{-i\Omega_{\epsilon i}(x^0-y^0)} - e^{-i\Omega_{\epsilon j}(x^0-y^0)}\right) \tag{4.28}$$

and

$$G_A^{'(eq)ij}(x^0, y^0; \mathbf{q}) \simeq -\,\Theta(y^0 - x^0) \sum_\epsilon Z_\epsilon \Pi_A^{'(eq)ij}(\epsilon\omega_q) Z_\epsilon \frac{-i}{\Omega^*_{\epsilon i} - \Omega^*_{\epsilon j}} \times \left(e^{-i\Omega^*_{\epsilon i}(x^0-y^0)} - e^{-i\Omega^*_{\epsilon j}(x^0-y^0)}\right) . \tag{4.29}$$

We also used the approximation $\Pi(\Omega_{\epsilon i}) \simeq \Pi(\Omega_{\epsilon j}) \simeq \Pi(\epsilon\omega_q)$ because $\Gamma_i \ll \Gamma_{\ell\phi}$.

The minus signs in the parentheses come from the relative minus sign of the residue in (4.27). Because of this, the off-diagonal Green functions vanish at $x^0 = y^0$:

$$G'_{R/A}(x, y)\big|_{x^0=y^0} = 0 \ . \tag{4.30}$$

This should generally hold by the definition of $G_{R/A}$ in (3.7) because $G_\rho^{ij}(x, y)$ is proportional to $\delta^{ij}\delta^3(\mathbf{x} - \mathbf{y})$ at equal time $x^0 = y^0$:

$$\gamma^0 G_R^{ij}(x, y)\big|_{x^0=y^0} = \Theta(x^0 - y^0)\gamma^0 G_\rho^{ij}(x, y)\big|_{x^0=y^0} = \frac{i}{2}\delta(\mathbf{x} - \mathbf{y})\delta^{ij} \ . \tag{4.31}$$

Note that the flavor off-diagonal components of the retarded (advanced) propagators are enhanced by the factor $1/(\Omega_i - \Omega_j)$ (or its complex conjugate). Such a large enhancement comes from the large mixing of the RH neutrinos with almost degenerate masses.

For the self-energies $\Pi_{R/A} = \Pi_h \pm \Pi_\rho/2$, if we use the vacuum value $\Pi'_\rho(\epsilon\omega_q) = -g_w \Re(h^\dagger h)' i\epsilon \not{q}_\epsilon/(16\pi)$ and $\Pi'_h(\epsilon\omega_q) = 0$ as in Appendix B, the following expressions [71] are reproduced:

$$\begin{aligned} G_R^{'(eq)ij}(x^0, y^0; \mathbf{q}) &\simeq +\,\Theta(x^0 - y^0) \sum_\epsilon \frac{\not{q}_\epsilon + M}{2\omega_q} \frac{g_w M^2 \Re(h^\dagger h)'/(16\pi)}{M_i^2 - M_j^2 - i\epsilon(M_i\Gamma_i - M_j\Gamma_j)} \\ &\quad \times \left(e^{-i\Omega_{\epsilon i}(x^0-y^0)} - e^{-i\Omega_{\epsilon j}(x^0-y^0)}\right) , \\ G_A^{'(eq)ij}(x^0, y^0; \mathbf{q}) &\simeq -\,\Theta(y^0 - x^0) \sum_\epsilon \frac{\not{q}_\epsilon + M}{2\omega_q} \frac{-g_w M^2 \Re(h^\dagger h)'/(16\pi)}{M_i^2 - M_j^2 + i\epsilon(M_i\Gamma_i - M_j\Gamma_j)} \\ &\quad \times \left(e^{-i\Omega^*_{\epsilon i}(x^0-y^0)} - e^{-i\Omega^*_{\epsilon j}(x^0-y^0)}\right) \end{aligned} \tag{4.32}$$

with the "regulator" $|M_i\Gamma_i - M_j\Gamma_j|$ Here we have used the relation (cf. Eq.(2.148))

$$\begin{aligned} \frac{\epsilon}{2\omega_q} \frac{1}{\Omega_{\epsilon i} - \Omega_{\epsilon j}} &\simeq \frac{1}{\omega_{iq}^2 - \omega_{jq}^2 - i\epsilon(\omega_{iq}\Gamma_{iq} - \omega_{jq}\Gamma_{jq})} \\ &\simeq \frac{1}{M_i^2 - M_j^2 - i\epsilon(M_i\Gamma_i - M_j\Gamma_j)} \end{aligned} \tag{4.33}$$

which is valid for $\omega_q \simeq \omega_{iq} \simeq \omega_{jq}$ and $\omega_q \Gamma_q \simeq M\Gamma$. As shown in section 4.1.6, the same enhancement factor, that is, the same regulator appears in the off-diagonal Wightman function in the thermal equilibrium. For the deviations of the off-diagonal Wightman functions out of equilibrium, however, we show in section 4.2.5 that the enhancement factor is changed to be $1/(\Omega_i - \Omega_j^*)$. This corresponds to the regulator $(M_i\Gamma_i + M_j\Gamma_j)$.

Finally we note the validity of the expansion with respect to the off-diagonal components of the Yukawa couplings $(h^\dagger h)'$. From the expressions (4.32), the iterative expansions (4.13) and (4.14) turn out to be valid when the real part of the off-diagonal components of Yukawa coupling $\Re(h^\dagger h)'$ is smaller than the mass difference $|M_i - M_j|/M \simeq \Gamma/M \sim (h^\dagger h)^d_{ii}$. Hence the expansion is understood as an expansion of the ratio $(h^\dagger h)'/(h^\dagger h)^d$.[14]

4.1.4 Wightman functions

The Wightman functions can be solved as (3.13) or (3.48). If we take terms up to the first order of $(h^\dagger h)'$, the flavor diagonal component is given by

$$G^{dii}_{\gtrless} = -\, G^{dii}_R * \Pi^{dii}_{\gtrless} * G^{dii}_A \ . \tag{4.34}$$

Similarly the flavor off-diagonal component is given by

$$G^{'ij}_{\gtrless} = -\, G^{'ij}_R * \Pi^{djj}_{\gtrless} * G^{djj}_A - G^{dii}_R * \Pi^{dii}_{\gtrless} * G^{'ij}_A - G^{dii}_R * \Pi^{'ij}_{\gtrless} * G^{djj}_A \ . \tag{4.35}$$

By using (4.14) and (4.34), (4.35) can be also rewritten as

$$G^{'ij}_{\gtrless} = -\, G^{dii}_R * \Pi^{'ij}_R * G^{djj}_{\gtrless} - G^{dii}_{\gtrless} * \Pi^{'ij}_A * G^{djj}_A - G^{dii}_R * \Pi^{'ij}_{\gtrless} * G^{djj}_A \tag{4.36}$$

which makes it clear that the off-diagonal part of the self-energy causes the flavor mixing of the RH neutrino.[15]

4.1.5 Diagonal Wightman $G^d_{\gtrless}$ in thermal equilibrium

In the thermal equilibrium, the Wightman function can be easily obtained by using the KMS relation. From (4.34), the diagonal component $G^{d(eq)}_{\gtrless}$ can be written as

$$G^{d(eq)}_{\gtrless}(x^0, y^0; \mathbf{q}) = -\int \frac{dq_0}{2\pi} e^{-iq_0(x^0-y^0)} G^{d(eq)}_R(q) \Pi^{d(eq)}_{\gtrless}(q) G^{d(eq)}_A(q) \ . \tag{4.37}$$

Let $f(q)$ be the thermal distribution function for the RH neutrinos. Note that $f(q)$ is a function of q^0, which is not equal to the on-shell energy $\omega_{\mathbf{q}}$. The KMS

[14] In [71], numerical analysis has been done beyond this parameter region.

[15] This form is convenient for the systematic derivation of the Boltzmann equation from the KB equation in the hierarchical mass spectrum [72], in which the diagonal components of the Wightman propagator are identified as the on-shell external line of the RH neutrinos. In this thesis, we are focusing on the resonant mass spectrum, and we use this form, without such an assumption, to solve the off-diagonal components of the Wightman propagator.

relation for the self-energy function is

$$\Pi_{\gtrless}^{(eq)}(q) = -i \begin{Bmatrix} 1 - f(q_0) \\ -f(q_0) \end{Bmatrix} \Pi_\rho^{(eq)}(q) \ . \tag{4.38}$$

Using the solution of the KB equation for the spectral density $G_\rho \equiv G_R - G_A = -G_R * \Pi_\rho * G_A$, we have

$$\begin{aligned} & G_{\gtrless}^{d(eq)}(x^0, y^0; \mathbf{q}) \\ & = -\int \frac{dq_0}{2\pi} e^{-iq_0(x^0-y^0)}(-i) \begin{Bmatrix} 1 - f(q_0) \\ -f(q_0) \end{Bmatrix} G_R^{d(eq)}(q) \Pi_\rho^{d(eq)}(q) G_A^{d(eq)}(q) \\ & = +\int \frac{dq_0}{2\pi} e^{-iq_0(x^0-y^0)}(-i) \begin{Bmatrix} 1 - f(q_0) \\ -f(q_0) \end{Bmatrix} \left[G_R^{d(eq)}(q) - G_A^{d(eq)}(q) \right] \ . \end{aligned} \tag{4.39}$$

It is nothing but the KMS relation (3.8) for the Green function.

Performing the q_0 integration, it becomes

$$\begin{aligned} G_{\gtrless}^{d(eq)ii}(x^0, y^0; \mathbf{q}) \simeq \sum_\epsilon \begin{Bmatrix} 1 - f_{iq}^\epsilon \\ -f_{iq}^\epsilon \end{Bmatrix} (-i) Z_\epsilon^i \Big(& \Theta(x^0 - y^0) e^{-i\Omega_{\epsilon i}(x^0-y^0)} \\ & + \Theta(y^0 - x^0) e^{-i\Omega_{\epsilon i}^*(x^0-y^0)} \Big) \ . \end{aligned} \tag{4.40}$$

Here we have dropped the contributions from poles of the distribution function $f(q_0)$ since they are suppressed by $\Gamma/T \ll 1$. Furthermore we used the distribution function

$$f_{ip}^\epsilon \equiv f(q_0 = \epsilon\omega_{iq}) = \frac{1}{e^{\epsilon\omega_{iq}/T} + 1} \tag{4.41}$$

by dropping the imaginary part of the pole $\Omega_{\epsilon i}$ in $f(q)$ because it is suppressed again by the factor $\Gamma \ll T$. Recall that it satisfies the relation $(1 - f_{ip}^\epsilon) = +f_{ip}^{-\epsilon}$.

4.1.6 Off-diagonal Wightman $G'_{\gtrless}$ in thermal equilibrium

Next we calculate the flavor off-diagonal component $G_{\gtrless}^{\prime(eq)}$ in the thermal equilibrium. The off-diagonal component also satisfies the KMS relation and we have

$$\begin{aligned} G_{\gtrless}^{\prime(eq)ij}(x^0, y^0; \mathbf{q}) &= +\int \frac{dq_0}{2\pi} e^{-iq_0(x^0-y^0)}(-i) \begin{Bmatrix} 1 - f(q_0) \\ -f(q_0) \end{Bmatrix} G_\rho^{\prime(eq)ij}(q) \\ &= +\int \frac{dq_0}{2\pi} e^{-iq_0(x^0-y^0)}(-i) \begin{Bmatrix} 1 - f(q_0) \\ -f(q_0) \end{Bmatrix} \left[G_R^{\prime(eq)ij}(q) - G_A^{\prime(eq)ij}(q) \right] \ . \end{aligned} \tag{4.42}$$

Performing q_0 integration, it becomes

$$\begin{aligned} G_{\gtrless}^{\prime(eq)ij}(x^0, y^0; \mathbf{q}) = & \sum_\epsilon Z_\epsilon \Pi_R^{\prime(eq)ij}(\epsilon\omega_q) Z_\epsilon \frac{-i}{\Omega_{\epsilon i} - \Omega_{\epsilon j}} \\ & \times (-i) \left[\begin{Bmatrix} 1 - f_{iq}^\epsilon \\ -f_{iq}^\epsilon \end{Bmatrix} e^{-i\Omega_{\epsilon i}(x^0-y^0)} - \begin{Bmatrix} 1 - f_{jq}^\epsilon \\ -f_{jq}^\epsilon \end{Bmatrix} e^{-i\Omega_{\epsilon j}(x^0-y^0)} \right] \end{aligned} \tag{4.43}$$

for $x^0 > y^0$. We have used similar approximations by dropping suppressed contributions by Γ/T and $\Gamma/\Gamma_{\ell\phi}$.

The off-diagonal component of the thermal Wightman functions are enhanced by the same factor $1/(\Omega_{\epsilon i}-\Omega_{\epsilon j})$ as in (4.28). Hence the flavor oscillation of the Wightman function in the thermal equilibrium is enhanced by a factor with the regulator $M_i\Gamma_i - M_j\Gamma_j$.

At the temperature $T \gg \Delta M$ we have in mind, f_i and f_j can be almost identified. Writing $f_i \simeq f_j \simeq f$, we have

$$\begin{aligned}
G_{\gtrless}^{\prime(eq)ij}(x^0,y^0;\mathbf{q}) =& \Theta(x^0-y^0)\sum_\epsilon Z_\epsilon \Pi_R^{\prime(eq)ij}(\epsilon\omega_q) Z_\epsilon \frac{-1}{\Omega_{\epsilon i}-\Omega_{\epsilon j}} \\
&\times \begin{Bmatrix} 1-f_q^\epsilon \\ -f_q^\epsilon \end{Bmatrix} \left(e^{-i\Omega_{\epsilon i}(x^0-y^0)} - e^{-i\Omega_{\epsilon j}(x^0-y^0)}\right) \\
&+\Theta(y^0-x^0)\sum_\epsilon Z_\epsilon \Pi_A^{\prime(eq)ij}(\epsilon\omega_q) Z_\epsilon \frac{-1}{\Omega^*_{\epsilon i}-\Omega^*_{\epsilon j}} \\
&\times \begin{Bmatrix} 1-f_q^\epsilon \\ -f_q^\epsilon \end{Bmatrix} \left(e^{-i\Omega^*_{\epsilon i}(x^0-y^0)} - e^{-i\Omega^*_{\epsilon j}(x^0-y^0)}\right) \ . \qquad (4.44)
\end{aligned}$$

The off-diagonal Wightman functions in the thermal equilibrium vanishes at the equal time $x^0 = y^0$:

$$\lim_{x_0\to y_0} G_{\gtrless}^{\prime(eq)ij}(x^0,y^0;\mathbf{q}) \propto (\Omega_i-\Omega_j)(x^0-y^0) \sim \Delta M(x^0-y^0) \to 0 \ . \qquad (4.45)$$

Later this property becomes very important to evaluate the deviation of the off-diagonal component of the Wightman function when the system is out of thermal equilibrium.

4.1.7 Short summary

In this section, we calculated various propagators of the RH neutrinos in the thermal equilibrium. We especially focused on the resonant enhancement of the flavor oscillation of N_i. Because of the assumption that the off-diagonal components of the neutrino Yukawa coupling $(h^\dagger h)'$ is smaller than the diagonal part $(h^\dagger h)^d$, Retarded or advanced propagators are composed of two propagating modes, i and j flavors. The flavor diagonal components are given by (4.20) or (4.23). Since their masses are almost degenerate, the flavor off-diagonal component is largely enhanced due to their oscillation as in (4.28) or (4.29). The enhancement factor is proportional to $1/(\Omega_i - \Omega_j)$ (or its complex conjugate) where $\Omega_i = \omega_i - i\Gamma_i/2$ and gives the regulator $R_{ij} = M_i\Gamma_i - M_j\Gamma_j$ to the enhancement factor. Similarly, the resonant enhancement of Wightman functions is calculated. In the thermal equilibrium, because of the KMS relation, the behavior of the Wightman functions is the same as the retarded (advanced) Green functions. The flavor diagonal component $G^d_{\gtrless}$ is given by (4.40) while the off-diagonal component $G'_{\gtrless}$ is given by (4.43). A very important property of $G'_{\gtrless}$ is that it vanishes at the equal time as (4.45).

4.2 Propagators out of equilibrium

Now we study effects of the expanding universe into account. First we summarize various time-scales in the system. An important time scale is given by the Hubble expansion rate H of the universe. Other scales are the decay widths of the SM particles Γ_ϕ, Γ_ℓ and of the RH neutrino Γ_i. Another important time scale in the resonant leptogenesis is given by the mass difference ΔM of the RH neutrinos because it gives the frequency of the flavor oscillation.

In type I see-saw model studied in this thesis, the decay width Γ_i of the RH neutrino is approximately given by $\Gamma_i \sim (h^\dagger h)_{ii} M_i / 8\pi$. The ratio of Γ_i to the Hubble parameter at temperature $T = M_i$ (2.76) is rewritten in terms of the effective neutrino mass (2.115)

$$\mathcal{K}_i = \frac{\Gamma_i}{H(M_i)} \approx \frac{\tilde{m}_i}{10^{-3}\mathrm{eV}} \ , \quad \widetilde{m}_i \equiv \frac{(h^\dagger h)_{ii} v^2}{M_i} \ . \tag{4.46}$$

where v is the Higgs vev. In the following, we consider only the *Strong washout* regime, that is, $\widetilde{m}_i > 10^{-3}$eV and $\mathcal{K}_i > 1$ hold.[16] As reviewed in section 2.2, the final baryon asymmetry has the simple power law dependence on $\widetilde{m}_i > 10^{-3}$eV through the efficiency factor (2.113) in the unflavored analysis. As shown later, the derivative expansion we employ here is justified only for large $\mathcal{K}_i$. It's equivalent to the validity of the perturbative solution of the number density of RH neutrino in the strong washout regime (2.90) and then, the close-to-equilibrium behavior $Y_N(z) \approx Y_N^{eq}(z)$ for $z > z_{eq}$ holds as seen in Fig.5. However, the Yukawa coupling itself is very small ($h \sim \sqrt{\widetilde{m}_i M}/v$), and we have the inequalities

$$\Gamma_\phi, \Gamma_\ell \gg \Gamma_i \gg H \ . \tag{4.47}$$

4.2.1 Deviation of self-energy from the thermal value

Under the condition (4.47), we can expand the scale factor as

$$a_{(X)} = a_{(t)} + a_{(t)} H_{(t)} (X - t) + \cdots \ . \tag{4.48}$$

The other physical quantities such as temperature are correlated with the change of the scale factor, and can be similarly expanded.

In order to calculate the out-of-equilibrium behavior of various Green functions in the expanding universe, we need to evaluate the change of the self-energies $\Pi(x,y)$. The self-energy of the RH neutrino is a rapidly decreasing function with the relative time as $\sim e^{-\Gamma_{\ell\phi}(x^0 - y^0)}$ due to the SM gauge interactions. So in the leading order approximation, the self-energy $\Pi(x,y)$ can be evaluated by the thermal value with the local temperature at the center-of-mass time $x^0 \sim y^0 \sim X_{xy}$. Therefore it is convenient to write the self-energy as

$$\Pi(x^0, y^0; \mathbf{q}) = \Pi(X_{xy}; s_{xy}; \mathbf{q}) \simeq \Pi^{(eq)}(X_{xy}; s_{xy}; \mathbf{q}) \ , \tag{4.49}$$

[16] Compared with the seesaw formula (2.10), $\widetilde{m}_i \sim 0.1$eV seems to be obtained from the neutrino mass scale ~ 0.1eV without any cancelation in $(h^\dagger h)_{ii}$.

where

$$X_{xy} \equiv \frac{x^0 + y^0}{2} \ , \quad s_{xy} \equiv x^0 - y^0 \ . \tag{4.50}$$

The first equation of of (4.49) is the definition of $\Pi(X; s; \mathbf{q})$. In the second equality, we replaced Π by its thermal value $\Pi^{(eq)}$ since the SM leptons and Higgs are in the thermal equilibrium and the self-energy of the RH neutrinos is well approximated by its thermal value. $\Pi^{(eq)}(X_{xy}; s)$ means the thermal self-energy in the thermal equilibrium evaluated at time X_{xy}.

In evaluating the Wightman function $G_{\gtrless}$ of the RH neutrinos, we need to know a difference of the self-energy $\Pi(u, v)$ from the thermal value at a later time t. For example, in (3.48), the difference of the self-energy $\Pi(X_{uv}; s)$ at X_{uv} and the thermal value $\Pi^{(eq)}(t; s)$ at $t = X_{xy}$ controls the behavior of $G^{ij}_{\gtrless}$. In this case, the time difference between X_{uv} and $t = X_{xy}$ is given by the inverse of the decay width Γ_i of the RH neutrino N_i. Since

$$\frac{1}{\Gamma_{\ell\phi}} \ll t - X_{uv} \sim \frac{1}{\Gamma_i} \ll \frac{1}{H} \ , \tag{4.51}$$

the derivative expansion of the self-energy around the thermal value is a good approximation:

$$\Pi(X_{uv}; s; \mathbf{q}) \simeq \Pi^{(eq)}(t; s; \mathbf{q}) + (X_{uv} - t)\partial_t \Pi^{(eq)}(t; s; \mathbf{q}) + \Delta_{\mu(X)}\Pi \ . \tag{4.52}$$

The second term is of order $\mathcal{O}(H/\Gamma_i)$ owing to (4.51). The third term comes from the chemical potential of leptons generated by CP-violating decay of the RH neutrinos. So it is the genuine deviation of the self-energy from the thermal value at the same time X_{uv}.

In this section, we mainly focus on the change of the physical quantities, namely the second term because the back reaction of the generated lepton asymmetry to the evolution of the number density of the RH neutrinos is very small. The effect of the chemical potential becomes important in the generation of the lepton asymmetry and is considered in section 4.3.

4.2.2 Notice for notations

As already used in (4.49), $\Pi(X; s)$ is the self-energy at the center-of-mass time X with the relative time s. For the thermal value $\Pi^{(eq)}(X; s)$, X is not necessarily at the center-of-mass time, but, more generally, denotes the reference time when it is evaluated. s is always the relative time. For the thermal value, we also use its Fourier transform

$$\Pi^{(eq)}(X; q) = \int ds \ \Pi^{(eq)}(X; s) e^{-iqs} \ . \tag{4.53}$$

In order to avoid complications of appearance, we use the same notations Π for $\Pi(X; s)$ and its Fourier transform $\Pi(X; q)$. They can be distinguished by their arguments, s or q, if necessary. We always use s for the relative time and q for its conjugate frequency. For the first argument (the reference time), we use X or t. The same notation is used for the thermal Green functions. We hope it does not cause any confusion to the readers.

4.2.3 Retarded propagator out of equilibrium ΔG_R

First we study how the retarded (advanced) propagators of the RH neutrinos deviate from the thermal value in the expanding universe. Consider the flavor diagonal component $G^d_{R/A}$ first. We write the deviation around the thermal value $G^{d(eq)}$ by ΔG^d:

$$G^d_{R/A}(X_{xy}; s_{xy}; \mathbf{q}) = G^{d(eq)}_{R/A}(t; s_{xy}; \mathbf{q}) + \Delta G^d_{R/A}(X_{xy}; s_{xy}; \mathbf{q}) \ . \tag{4.54}$$

Note that $\Delta G^d_{R/A}$ depends on the reference time t at which the equilibrium value is evaluated. It is calculated in the appendix G of [90] and given by

$$\begin{aligned}\Delta G^d_R(x^0, y^0; \mathbf{q}) \simeq \Theta(s_{xy}) \sum_\epsilon \Bigg[& \partial_t \left(Z_\epsilon e^{-i\Omega_\epsilon s_{xy}} \right) (X_{xy} - t) \\ & - i \frac{H_{(t)} M}{4\omega_q^2} \gamma^0 \frac{\boldsymbol{\gamma} \cdot \mathbf{q}}{a_{(t)}} \ s_{xy} \ e^{-i\Omega_{\epsilon q} s_{xy}} \Bigg] \ . \end{aligned} \tag{4.55}$$

The first term is the change of the physical parameters such as mass or width in Ω_ϵ and Z_ϵ. The second term represents a change of the spinor structure due to an expansion of the universe in the propagator during the propagation. The retarded (advanced) propagator does not have the memory effect, and the deviation is essentially determined by the change of the local temperature.

By taking a variation of (4.14), the deviation of the off-diagonal components $G'_{R/A}$ can be expressed in terms of the deviation of the diagonal components $G^d_{R/A}$ as

$$\begin{aligned}\Delta G'^{ij}_{R/A} = & - G^{d(eq)ii}_{R/A} * \Delta\Pi'^{(eq)ij}_{R/A} * G^{d(eq)jj}_{R/A} - \Delta G^{dii}_{R/A} * \Pi'^{(eq)ij}_{R/A} * G^{d(eq)jj}_{R/A} \\ & - G^{d(eq)ii}_{R/A} * \Pi'^{(eq)ij}_{R/A} * \Delta G^{djj}_{R/A} \ . \end{aligned} \tag{4.56}$$

The above formula is used to evaluate the deviation of the Wightman functions of the RH neutrinos in the latter section 4.2.5. Since the above relation (4.56) is sufficient for latter calculations of $\Delta G'_{\gtrless}$, we do not calculate an explicit form of $\Delta G'_R$ here. We note that, since the retarded (advanced) propagators do not have the memory effect, its deviation is essentially determined by the change of the local temperature. Also note that the enhancement factor is proportional to $1/(\Omega_i - \Omega_j)$ as the Green functions in the thermal equilibrium since there is no chance to mix G_R and G_A.

4.2.4 Diagonal Wightman out of equilibrium $\Delta G^{d}_{\gtrless}$

The deviation of the flavor diagonal Wightman function $\Delta G^{d}_{\gtrless}(x^0, y^0)$ can be calculated by taking a variation of (4.34):

$$\begin{aligned}\Delta G^{d}_{\gtrless} = &- \Delta G^{d}_{R} * \Pi^{d(eq)}_{\gtrless} * G^{d(eq)}_{A} - G^{d(eq)}_{R} * \Pi^{d(eq)}_{\gtrless} * \Delta G^{d}_{A} \\ &- G^{d(eq)}_{R} * \Delta\Pi^{d(eq)}_{\gtrless} * G^{d(eq)}_{A} \ . \end{aligned} \tag{4.57}$$

There are three terms. The first two terms are interpreted as the change of the spectrum in the expanding universe contained in $G_{R/A}$. On the other hand, the third term reflects the memory effect.

The third term is explicitly written[17] as

$$- \int_{-\infty}^{x^0} du \int_{-\infty}^{z^0} dv \ G^{d(eq)}_{R}(x,u) \ \Delta\Pi^{d(eq)}_{\gtrless}(u,v) \ G^{d(eq)}_{A}(v,z) \ . \tag{4.58}$$

This shows that the Wightman function is sensitive to the change of the background before x^0 and y^0 unlike the retarded or advanced Green functions. Writing the self-energy in terms of the center of mass coordinate $X_{uv} = (u+v)/2$ and the relative coordinate $s_{uv} = u - v$, its deviation from the thermal self-energy at time $t = x^0$ is written as

$$\begin{aligned}\Delta\Pi^{(eq)}_{\gtrless}(X_{uv}; s_{uv}; q) &= \int \frac{dq_0}{2\pi} e^{-iq_0 s_{uv}} \partial_X \Pi^{(eq)}_{\gtrless}(X;q)\big|_{X=t}(X_{uv} - t) \\ &\simeq \int \frac{dq_0}{2\pi} e^{-iq_0 s_{uv}} \partial_X \left[(-i) \begin{Bmatrix} 1 - f(q_0) \\ -f(q_0) \end{Bmatrix} \Pi^{(eq)}_{\rho}(X;q) \right]_{X=t} (X_{uv} - t) \ . \end{aligned} \tag{4.59}$$

Note that $|s_{uv}| \lesssim 1/\Gamma_{\ell\phi}$ due to the rapid damping of SM leptons and Higgs propagators. In the second equality the KMS relation for the thermal self-energy (4.38) is used. As explained in eq.(4.49), the self-energy function out of equilibrium can be approximated by the equilibrium self-energy $\Pi^{(eq)}$ of (4.38) at the local temperature. Note that the distribution function $f(q_0) = 1/(e^{q_0/T}+1)$ is time-dependent through the time-dependence of the temperature $T = T(X)$.

The calculation of the deviation of the diagonal Wightman function $\Delta G^{d}_{\gtrless}$ is performed in the appendix I of [90]. For $x^0 > y^0$, it is given by

$$\begin{aligned}\Delta G^{dii}_{\gtrless}(x^0, y^0; \mathbf{q}) \simeq (-i) \sum_{\epsilon} \Bigg[&\begin{Bmatrix} 1 - f^{\epsilon}_{iq} \\ -f^{\epsilon}_{iq} \end{Bmatrix} \Delta\hat{G}^{dii}_{R}(x^0, y^0; \epsilon, \mathbf{q}) \\ &+ d_t \begin{Bmatrix} 1 - f^{\epsilon}_{iq} \\ -f^{\epsilon}_{iq} \end{Bmatrix} \left(\frac{-1}{\Gamma_{iq}} + (X_{xy} - t - |s_{xy}|/2) \right) Z^{i}_{\epsilon} e^{-i\Omega_{\epsilon i}(x^0 - y^0)} \Bigg] \end{aligned} \tag{4.60}$$

where

$$d_t \equiv \frac{\partial T}{\partial t} \frac{\partial}{\partial T} + \frac{\partial \omega_q}{\partial t} \frac{\partial}{\partial \omega_q} \ . \tag{4.61}$$

[17] Since all quantities are already Fourier transformed in the spatial direction with momentum $\mathbf{q}$, we use u, v instead of u^0, v^0 to avoid complications.

Each term of (4.60) is classified into three types of terms.

The first term of $\Delta G^d_{\gtrless}$ in the square bracket reflects the change of the spectrum in the propagators G_R and related by the KMS relation (4.39). It reflects a change of the local temperature during the period x^0 and y^0.

The term proportional to $(X_{xy} - t)$ comes from a difference between the distribution function $f_q(t)$ at the reference time t and $f_q(X_{xy}) = f_q(t) + (X_{xy} - t)d_t f_q$ at time X_{xy}. The time-dependence of f_q comes from both of the local temperature and the physical frequency ω_q as shown in the definition of the derivative operator d_t. The term with s_{xy} is similar. If $x^0 \neq y^0$, the distribution function at X_{xy} is affected by the information in the past.

The most important part is the term proportional to $1/\Gamma_i$, which reflects the memory effect of the Wightman function. Since the Wightman function is written as a convolution $G^d_{\gtrless}(X_{xy}; s_{xy}) = -(G_R * \Pi_{\gtrless} * G_A)(X_{xy}; s_{xy})$, they depend on the information in the past at X_{uv} where $X_{xy} - X_{uv} \sim 1/\Gamma_i$ (see (4.58)). In the expanding universe, the temperature is higher in the past and the number density of leptons and Higgs are larger than the present density. Accordingly the number density of the RH neutrinos is also larger by an amount of

$$\Delta \begin{Bmatrix} 1 - f^\epsilon_{iq} \\ -f^\epsilon_{iq} \end{Bmatrix} \equiv d_t \begin{Bmatrix} 1 - f^\epsilon_{iq} \\ -f^\epsilon_{iq} \end{Bmatrix} \times \frac{-1}{\Gamma_{iq}} = \frac{d_t f^\epsilon_{iq}}{\Gamma_{iq}} \ . \tag{4.62}$$

Hence the term with $1/\Gamma_i$ is directly related to the memory effect of $G^d_{\gtrless}$.

In applying $\Delta G_{\gtrless}$ to the evolution equation of the lepton asymmetry, it always appears as a product with the propagators of the SM particles (leptons and Higgs) as in eq. (3.43). Since these propagators damp quickly with the decay widths $\Gamma_{\ell,\phi}$, we can drop all the terms in (4.60) except the term containing $1/\Gamma_i$. Furthermore, during the period $1/\Gamma_{\ell\phi}$, RH neutrinos are almost stable: $\Gamma_i \ll \Gamma_{\ell\phi}$. Hence we can replace the frequency Ω_i by its real part ω_i.

Let us write this simplified form of ΔG as $\Delta\mathcal{G}$:

$$\Delta\mathcal{G}^{dii}_{\gtrless}(x^0, y^0; \mathbf{q}) \equiv \sum_\epsilon (-i)\Delta \begin{Bmatrix} 1 - f^\epsilon_{iq} \\ -f^\epsilon_{iq} \end{Bmatrix} \times Z^i_\epsilon e^{-i\epsilon\omega_{iq}(x^0 - y^0)} \ . \tag{4.63}$$

The definition of Z^i_ϵ is given in (4.22). $\sum Z^i_\epsilon e^{-i\epsilon\omega_{iq}(x^0-y^0)}$ is nothing but $G^{dii}_\rho = G^{dii}_R - G^{dii}_A$ within the above simplification.

As a final remark in this section, we mention that the above simplified form is directly obtained from the classical Boltzmann equation as follows. The Boltzmann equation for the RH neutrino distribution function is given by (2.34)

$$\begin{aligned} d_t f_{iq} = \frac{2}{2\omega_{iq}} \int \frac{d^3p}{(2\pi)^3} \frac{1}{2\omega_p} \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_k} (2\pi)^4 \delta^4(q - p - k) \\ \times |\mathcal{M}|^2_{tree} \left[(1 - f_{iq}) f^{(eq)}_{\ell p} f^{(eq)}_{\phi k} - f_{iq}(1 - f^{(eq)}_{\ell p})(1 - f^{(eq)}_{\phi k}) \right] \ . \end{aligned} \tag{4.64}$$

All external momenta are on-shell. Leptons and Higgs are assumed to be in the thermal equilibrium. $|\mathcal{M}|^2_{tree} = g_w (h^\dagger h)_{ii} (q \cdot p)$ is the square of the tree-level decay amplitude of a RH neutrino into a lepton and a Higgs. The spin in the initial state is averaged and the isospin sum in the final state is performed. By using the relation $(1-f_{iq}^{(eq)})f_{\ell p}^{(eq)}f_{\phi k}^{(eq)} = f_{iq}^{(eq)}(1-f_{\ell p}^{(eq)})(1-f_{\phi k}^{(eq)})$, it is rewritten as

$$
\begin{aligned}
d_t f_{iq} &= -\frac{2}{2\omega_{iq}} \int \frac{d^3p}{(2\pi)^3} \frac{1}{2\omega_p} \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_k} (2\pi)^4 \delta^4(q-p-k) \\
&\quad \times |\mathcal{M}|^2_{tree} \left[1 - f_{\ell p}^{(eq)} + f_{\phi k}^{(eq)}\right] \left(f_{iq} - f_{iq}^{(eq)}\right) \\
&= -\Gamma_{iq} \left(f_{iq} - f_{iq}^{(eq)}\right) \ . \qquad (4.65)
\end{aligned}
$$

Here, we have used the definition of the decay width (4.19) with (B.9).[18] The solution of (4.65) is given by

$$
f_{iq}(t) \sim f_{iq}^{(eq)}(t) - \frac{1}{\Gamma_{iq}} d_t f_{iq}^{(eq)}(t) \qquad (4.66)
$$

and (4.63) is reproduced.

4.2.5 Off-diagonal Wightman out of equilibrium $\Delta G'_{\gtrless}$

We then investigate the deviation of the flavor off-diagonal Wightman function. It is most important for generating the lepton asymmetry. Since the flavor off-diagonal Wightman function is a sum of three terms as in (4.36), its variation contains 9 terms. Details of the calculations are given in the appendix J of [90]. 6 terms containing $\Delta G^d_{R/A}$ or $\Delta \Pi^{'(eq)}_{R/A}$ reflect the change of the spectrum $\Omega_\epsilon = \epsilon\omega_q \mp i\Gamma_q/2$ during the decay of N_i. The change of the distribution functions is contained in the 3 terms with $\Delta G^d_{\gtrless}$ and $\Delta \Pi^{'(eq)}_{\gtrless}$. In Appendix C, we give a different derivation of $\Delta G^d_{\gtrless}$ and $\Delta G'_{\gtrless}$.

After lengthy calculations,

$$
\begin{aligned}
&\Delta G^{'ij}_{\gtrless}(x^0, y^0; \mathbf{q}) \\
&\simeq \Bigg[\sum_\epsilon Z_\epsilon \Pi_R^{'(eq)ij}(\epsilon\omega_q) Z_\epsilon \Delta \begin{Bmatrix} 1 - f^\epsilon_{jq} \\ -f^\epsilon_{jq} \end{Bmatrix} \frac{1}{\Omega_{\epsilon i} - \Omega^*_{\epsilon j}} \ e^{-i\Omega_\epsilon s_{xy}} \\
&\quad - \sum_\epsilon Z_\epsilon \Pi_A^{'(eq)ij}(\epsilon\omega_q) Z_\epsilon \Delta \begin{Bmatrix} 1 - f^\epsilon_{iq} \\ -f^\epsilon_{iq} \end{Bmatrix} \frac{1}{\Omega_{\epsilon i} - \Omega^*_{\epsilon j}} \ e^{-i\Omega_\epsilon s_{xy}} \Bigg] \qquad (4.67)
\end{aligned}
$$

for $x^0 > y^0$. In this expression, we have assumed that the reference time t is very close to X_{xy}, and the conditions $|X_{xy} - t|, |s_{xy}| \lesssim 1/\Gamma_{\ell\phi}$ are satisfied. Such conditions appear when we use the Wightman functions in evaluating the

[18]The factor $[1 - f_{\ell p}^{(eq)} + f_{\phi k}^{(eq)}]$ represents the finite density effects, which depend only linearly on the distribution functions [61, 62, 68, 64, 66]. The RH neutrino interaction rate including all the relevant SM couplings was computed in [69].

evolution equation of the lepton number. We also took the leading order terms with respect to $\Gamma/\Gamma_{\ell\phi} \sim \Gamma/T$. (4.67) is of order (H/Γ).[19]

We have also identified $\Omega_i \simeq \Omega_j$ in $e^{-i\Omega_\epsilon s_{xy}}$ since the mass difference ΔM and the widths Γ_i are much smaller than the typical scale of $1/|s_{xy}| = \Gamma_{\ell\phi}$.

Here is an important comment. As discussed in (4.1.6), the off-diagonal components of the Wightman function in the thermal equilibrium (4.43) is enhanced by a large factor $1/(\Omega_i - \Omega_j)$ because of the resonant oscillation between flavors. But in the limit $x_0 \to y_0$ it vanishes as in (4.45). Both of these properties are related to the behavior of $G'_{R/A}$ through the KMS relation and the fact that $G'_{\gtrless}$ is separated into the retarded and advanced propagators as in (4.42).

The deviation $\Delta G'^{ij}_{\gtrless}$ does not satisfy either properties. First, the enhancement factor is replaced by $1/(\Omega_i - \Omega_j^*)$. Second, $\Delta G'^{ij}_{\gtrless}$ does not vanish in the limit $x^0 \to y^0$:

$$\lim_{x_0 \to y_0} \Delta G'^{ij}_{\gtrless}(x^0, y^0; \mathbf{q}) \neq 0 \ . \tag{4.68}$$

The replacement of the enhancement factor by $1/(\Omega_i - \Omega_j^*)$ reflects the mixing between the retarded and advanced propagators. Such mixing is naturally generated because the off-diagonal component of the Wightman function is solved as in (4.36) to contain both types of Green functions. Since the retarded and advanced propagators have poles at $q_0 = \Omega_{\epsilon i}$ and $q_0 = \Omega_{\epsilon j}^*$ respectively, the appearance of the term $1/(\Omega_{\epsilon i} - \Omega_{\epsilon j}^*)$ by q_0 integration can be naturally understood. In the equilibrium case, since the retarded and advanced propagators are decoupled by the KMS relation, such mixings of poles at $q_0 = \Omega_{\epsilon i}$ and at $q_0 = \Omega_{\epsilon j}^*$ disappear in the final result of $G'^{ij}_{\gtrless}$ so that the enhancement factor becomes $1/(\Omega_{\epsilon i} - \Omega_{\epsilon j})$ or $1/(\Omega_{\epsilon i}^* - \Omega_{\epsilon j}^*)$.

When we use $\Delta G'^{ij}_{\gtrless}(x^0, y^0)$ in the evolution equation of the lepton number, the arguments x^0, y^0 are restricted to the region $s_{xy} = x^0 - y^0 < 1/\Gamma_{\ell\phi} \sim 1/T$ as mentioned above. During such short period, the decay of N_i is neglected and we can safely replace $\Omega_{\epsilon i}$ in $e^{-i\Omega_\epsilon s_{xy}}$ by its real part ω_ϵ. We write the simplified version of $\Delta G'^{ij}_{\gtrless}$ as $\Delta \mathcal{G}'^{ij}_{\gtrless}$:

$$\begin{aligned}\Delta \mathcal{G}'^{ij}_{\gtrless}(x^0, y^0; \mathbf{q}) \simeq \sum_\epsilon e^{-i\epsilon\omega_q(x^0 - y^0)} & \frac{\omega_q \epsilon}{(M_i^2 - M_j^2) - i\epsilon(M_i\Gamma_i + M_j\Gamma_j)} \\ \times \Bigg\{ & Z_\epsilon \Pi'^{(eq)ij}_\rho(\epsilon\omega_q) Z_\epsilon \left[\Delta \begin{Bmatrix} 1 - f^\epsilon_{jq} \\ -f^\epsilon_{jq} \end{Bmatrix} + \Delta \begin{Bmatrix} 1 - f^\epsilon_{iq} \\ -f^\epsilon_{iq} \end{Bmatrix}\right] \\ & + 2 Z_\epsilon \Pi'^{(eq)ij}_h(\epsilon\omega_q) Z_\epsilon \left[\Delta \begin{Bmatrix} 1 - f^\epsilon_{jq} \\ -f^\epsilon_{jq} \end{Bmatrix} - \Delta \begin{Bmatrix} 1 - f^\epsilon_{iq} \\ -f^\epsilon_{iq} \end{Bmatrix}\right] \Bigg\} \ .\end{aligned} \tag{4.69}$$

[19] Higher order contributions in the gradient expansion are of order H/T as found in [65]. Since $H/T \ll H/\Gamma$, we do not consider such terms here.

The second term in the square bracket with the real part of the self-energy can be dropped by imposing $\Pi_h = 0$ by the mass renormalisation. If we include the effect of the temperature dependent mass, Π_h is not always zero.

4.2.6 Short summary

In this section, we studied the deviation of various Green functions from the thermal equilibrium. Because of the limited domain of time integration in the KB equation, the deviation of the retarded/advanced Green function $\Delta G_{R/A}$ is mainly caused by the local change of the physical quantities. On the other hand, the deviation of the Wightman function $\Delta G_{\gtrless}$ is caused by tracing the history in the integration. And the time integration contributes mainly for the time interval $\sim 1/\Gamma_i$ because of the exponential damping of the propagators. It's reflected in the expressions (4.63) and (4.69) as the factors which become smaller as the each decay rate Γ_i becomes larger, that is the reason why the off-diagonal component of the deviation from the equilibrium Wightman function have the enhancement factor $1/(\Omega_i - \Omega_j^*)$.

However, note there is the crucial difference between the diagonal and off-diagonal components of the Wightman function. Contrary to the diagonal components of the deviation (4.63) the off-diagonal components cannot be expressed as a change of the local equilibrium Green function $G_{\gtrless}^{'(eq)}$ in (4.44):

$$(\Delta G_{\gtrless})^{'ij} \neq \Delta(G_{\gtrless}^{'ij}) \ . \tag{4.70}$$

Eq. (4.69) and this property are the main results of this section. The property (4.70) becomes evident when we notice that $G_{\gtrless}^{'ij}$ vanishes in the leading order approximation at $x^0 = y^0$ as in (4.45) while $\Delta G_{\gtrless}^{'}$ is nonzero at the equal time, which produces the lepton asymmetry. This corresponds to the fact that the resonant enhancement of $\Delta G_{\gtrless}^{'}$ with the factor $1/(\Omega_i - \Omega_j^*)$ occurs through the memory effect, differently from the resonant oscillation of $G_{\gtrless}^{'(eq)}$ with $1/(\Omega_i - \Omega_j)$ which is controlled by the KMS relation in thermal equilibrium. We come back to this property in subsection 4.4.

4.3 Boltzmann eq. from Kadanoff-Baym eq.

The evolution equation of the lepton asymmetry is given by the KB equation (3.43). The r.h.s. is written as a functional of the Wightman functions of RH neutrinos, SM leptons and SM Higgs. Since the SM leptons and Higgs are almost in the thermal equilibrium, their distribution functions are approximated by the thermal values at the local temperature. But the RH neutrinos decay much slower, and furthermore the RH neutrinos with almost degenerate masses coherently oscillate between different flavors during their propagation. Hence the Wightman functions $G_{\gtrless}^{ij}$ of the RH neutrinos must be treated in a full quantum mechanical way by using the KB equation, not by the classical Boltzmann equation. Now that we have the explicit form of $G_{\gtrless}^{ij}$ calculated from the KB equation, we can obtain the correct evolution equation for the lepton asymmetry by inserting it into the r.h.s. of (3.43).

The evolution equation of lepton number is given by

$$\frac{dn_L}{dt} + 3Hn_L = 2\Re \sum_{i,j} \int \frac{d^3q}{(2\pi)^3} \int_{-\infty}^{t} d\tau \ (h^\dagger h)_{ji} \times \Big[\mathrm{tr}\Big\{ P_\mathrm{R} G^{ij}_{<}(t,\tau;\mathbf{q}) P_\mathrm{L} \pi_{>}(\tau,t;\mathbf{q}) \Big\} - \mathrm{tr}\Big\{ P_\mathrm{R} G^{ij}_{>}(t,\tau;\mathbf{q}) P_\mathrm{L} \pi_{<}(\tau,t;\mathbf{q}) \Big\} \Big] \tag{4.71}$$

where $P_\mathrm{L}\pi_{\gtrless}P_\mathrm{R} = \widetilde{\pi}_{\gtrless}$ as defined in (3.44). After Fourier transformation of the r.h.s., the frequencies q_0, p_0, k_0 of the Green functions, $G_{\gtrless}(q_0)$ and $S_{\gtrless}(p_0)$, $\Delta_{\gtrless}(k_0)$ in $\widetilde{\pi}_{\gtrless}$, satisfy the relation $q_0 = p_0 + k_0$. Furthermore, in the thermal equilibrium, the Wightman functions are related to the retarded (advanced) propagators through the KMS relation (3.8), (4.38) and (4.39). Then, by using the relation

$$f_N(q_0)(1 - f_\ell(p_0))(1 + f_\phi(k_0)) = (1 - f_N(q_0)) f_\ell(p_0) f_\phi(k_0) \ , \tag{4.72}$$

two terms in the square bracket cancel each other. Hence there is no generation of lepton asymmetry in the thermal equilibrium. In the following, we see that the deviations from the equilibrium propagator bring the non-zero collision term on the r.h.s. of (4.71) which give the production and washout of the lepton number.

4.3.1 Lepton asymmetry out of equilibrium

In the expanding universe, there are three sources for changing the lepton asymmetry, and the r.h.s. of (4.71) can be classified into the following three terms:

$$\frac{dn_L}{dt} + 3Hn_L = \sum_{K=d,\prime} \left(\mathcal{C}^K_{\Delta f} + \mathcal{C}^K_{W} + \mathcal{C}^K_{\mathrm{BR}} \right) \ . \tag{4.73}$$

Here we rewrite the sum over i, j into the flavor diagonal part $K = d$ and the off-diagonal part $K =\prime$. Namely $K = d$ corresponds to a summation of $i = j = 1$ and $i = j = 2$ while $K =\prime$ corresponds to a summation of $i = 1, j = 2$ and $i = 2, j = 1$.

The first term $\mathcal{C}^K_{\Delta f}$ comes from the deviation of the Wightman functions of the RH neutrinos (i.e., the distortion of the distribution function Δf) from the thermal value

$$(\Delta G_{\gtrless})' = G'_{\gtrless} - G'^{(eq)}_{\gtrless} \neq 0 \ , \tag{4.74}$$

and is given by

$$\mathcal{C}_{\Delta f}^{K} = 2\Re \int \frac{d^3q}{(2\pi)^3} \sum_{i,j\in K} (h^\dagger h)_{ji}$$
$$\times \int_{-\infty}^{t} d\tau \left[\mathrm{tr}\Big(P_\mathrm{R} \Delta\mathcal{G}_{<}^{Kij}(t,\tau;\mathbf{q}) P_\mathrm{L} \pi_{>}^{(eq)}(\tau,t;\mathbf{p}) \Big) \right.$$
$$\left. - \, \mathrm{tr}\Big(P_\mathrm{R} \Delta\mathcal{G}_{>}^{Kij}(t,\tau;\mathbf{q}) P_\mathrm{L} \pi_{<}^{(eq)}(\tau,t;\mathbf{p}) \Big) \right] . \tag{4.75}$$

This generates the lepton asymmetry in the expanding universe.

The second term comes from the deviation of $\pi_{\gtrless}$:

$$\Delta\pi_{\gtrless} = \pi_{\gtrless} - \pi_{\gtrless}^{(eq)} \tag{4.76}$$

which is caused by the deviation of the distribution functions of the SM leptons and Higgs. $\mathcal{C}_{W}^{K}$ is written as

$$\mathcal{C}_{W}^{K} = 2\Re \int \frac{d^3q}{(2\pi)^3} \sum_{i,j\in K} (h^\dagger h)_{ji}$$
$$\times \int_{-\infty}^{t} d\tau \left[\mathrm{tr}\Big(P_\mathrm{R} G_{<}^{K(eq)ij}(t,\tau;\mathbf{q}) P_\mathrm{L} \Delta\pi_{>}(\tau,t;\mathbf{p}) \Big) \right.$$
$$\left. - \, \mathrm{tr}\Big(P_\mathrm{R} G_{>}^{K(eq)ij}(t,\tau;\mathbf{q}) P_\mathrm{L} \Delta\pi_{<}(\tau,t;\mathbf{p}) \Big) \right] . \tag{4.77}$$

This gives washout effect of the lepton asymmetry.

The third term comes from the back reaction of the generated lepton asymmetry to $G_{\gtrless}'$, namely to the distribution function of the RH neutrinos. It is written as

$$\mathcal{C}_{\mathrm{BR}}^{K} = 2\Re \int \frac{d^3q}{(2\pi)^3} \sum_{i,j\in K} (h^\dagger h)_{ji}$$
$$\times \int_{-\infty}^{t} d\tau \left[\mathrm{tr}\Big\{ P_\mathrm{R} \Delta_\mu G_{<}^{Kij}(t,\tau;\mathbf{q}) P_\mathrm{L} \pi_{>}^{(eq)}(\tau,t;\mathbf{p}) \Big\} \right.$$
$$\left. - \, \mathrm{tr}\Big\{ P_\mathrm{R} \Delta_\mu G_{>}^{Kij}(t,\tau;\mathbf{q}) P_\mathrm{L} \pi_{<}^{(eq)}(\tau,t;\mathbf{p}) \Big\} \right] . \tag{4.78}$$

Here $\Delta_\mu G$ is defined as the back reaction of the generated chemical potential of the lepton and Higgs to the RH Wightman function.

4.3.2 Effect of $\Delta G_{\gtrless}$ on the lepton asymmetry: $\mathcal{C}_{\Delta f}$

The deviation of the Wightman function from the equilibrium value generates the lepton asymmetry out of equilibrium.

First let us look at the contribution of the flavor diagonal ($K = d$) part of $\mathcal{C}^K_{\Delta f}$. Inserting (4.63)[20] and (3.45) into (4.75), we have

$$
\begin{aligned}
\mathcal{C}^d_{\Delta f} = & \sum_i \int \frac{d^3p}{(2\pi)^3 2\omega_p} \frac{d^3k}{(2\pi)^3 2\omega_k} \frac{d^3q}{(2\pi)^3 \omega_q} \, (2\pi)^3 \delta^3(q-p-k) \\
& \frac{\Gamma_{\ell\phi}}{(\omega_q - \omega_p - \omega_k)^2 + \Gamma^2_{\ell\phi}/4} \, g_w (h^\dagger h)_{ii} (q \cdot p) \\
& \times \left\{ \Delta f_{iq} \left((1 - f_{\ell p})(1 + f_{\phi k}) - (1 - f_{\overline{\ell} p})(1 + f_{\overline{\phi} k}) \right) \right. \\
& \left. \qquad - \Delta(1 - f_{iq}) \left(f_{\ell p} f_{\phi k} - f_{\overline{\ell} p} f_{\overline{\phi} k} \right) \right\} \\
= & \; 0 \; .
\end{aligned} \tag{4.79}
$$

Here we took all the ϵ's, ϵ in (4.63) and $\epsilon_\ell, \epsilon_\phi$ in (3.45), the same $\epsilon = \epsilon_\ell = \epsilon_\phi$ because the temperature considered is not so high that a process like $\phi \to \ell + N$ does not occur. Hence the flavor diagonal component does not generate the asymmetry. In the last equality, we used the relation $f_\ell = f_{\overline{\ell}} = f^{(eq)}_\ell$, $f_\phi = f_{\overline{\phi}} = f^{(eq)}_\phi$ for the thermal distribution function.

Next we calculate the off-diagonal term $\mathcal{C}^{'}_{\Delta f}$ with $K ='$. Inserting (4.69) into (4.75), we have

$$
\begin{aligned}
\mathcal{C}^{'}_{\Delta f} = & \sum_{i,j (i \neq j)} \int \frac{d^3p}{(2\pi)^3 2\omega_p} \frac{d^3k}{(2\pi)^3 2\omega_k} \frac{d^3q}{(2\pi)^3 2\omega_q} \, \frac{(2\pi)^3 \delta^3(q-p-k) \Gamma_{\ell\phi}}{(\omega_q - \omega_p - \omega_k)^2 + \Gamma^2_{\ell\phi}/4} g_w \Im (h^\dagger h)^2_{ij} \\
& \left[\frac{(M_i^2 - M_j^2)/2}{(M_i^2 - M_j^2)^2 + (M_i \Gamma_i + M_j \Gamma_j)^2} \right. \\
& \left(4i (q \cdot \pi^{(eq)}_\rho(\omega_q))(q \cdot p) + 4i \left(-M^2 (p \cdot \pi^{(eq)}_\rho(\omega_q)) + (q \cdot \pi^{(eq)}_\rho(\omega_q))(q \cdot p) \right) \right) \\
& \left([\Delta f_{iq} + \Delta f_{jq}] (1 - f^{(eq)}_{\ell p})(1 + f^{(eq)}_{\phi k}) - [\Delta(1 - f_{iq}) + \Delta(1 - f_{jq})] f^{(eq)}_{\ell p} f^{(eq)}_{\phi k} \right) \\
& + \frac{M_i \Gamma_i + M_j \Gamma_j}{(M_i^2 - M_j^2)^2 + (M_i \Gamma_i + M_j \Gamma_j)^2} \\
& \left(4 (q \cdot \pi^{(eq)}_h(\omega_q))(q \cdot p) + 4 \left(-M^2 (p \cdot \pi^{(eq)}_h(\omega_q)) + (q \cdot \pi^{(eq)}_h(\omega_q))(q \cdot p) \right) \right) \\
& \left. \left([\Delta f_{iq} - \Delta f_{jq}] (1 - f^{(eq)}_{\ell p})(1 + f^{(eq)}_{\phi k}) - [\Delta(1 - f_{iq}) - \Delta(1 - f_{jq})] f^{(eq)}_{\ell p} f^{(eq)}_{\phi k} \right) \right] .
\end{aligned} \tag{4.80}
$$

Here, using the definition of $\pi_\gtrless$ in (3.45), we have defined $\pi_\rho = i(\pi_> - \pi_<) = (\pi_R - \pi_A)$, $\pi_h = (\pi_R + \pi_A)/2$ and their Fourier transform in the time direction, to

[20] We note again that $(X_{xy} - t)$ and s_{xy} of the arguments of $G_\gtrless(x^0, y^0)$ are smaller than $1/\Gamma_{\ell\phi}$ due to $\pi_\gtrless(\tau, t) \sim e^{-(t-\tau)\Gamma_{\ell\phi}/2}$. Hence the use of $\Delta\mathcal{G}$ is justified.

separate the self-energies $\Pi^{'(eq)}_{\rho/h}$ in (4.69) into the Yukawa coupling $(h^\dagger h)'$ and the equilibrium values of $\pi_{\rho/h}$ (see (B.8) and (B.14)). If we use the vacuum values for the self-energy calculated in Appendix B, i.e., $\pi_\rho(\epsilon\omega_q) = -g_w i\epsilon \not{q}_\epsilon/(16\pi)$ and $\pi_h(\epsilon\omega_q) = 0$, the second term in the square bracket is dropped and (4.80) is simplified as

$$\begin{aligned}\mathcal{C}'_{\Delta f} =& \sum_{i=1,2}\int \frac{d^3p}{(2\pi)^3 2\omega_p}\frac{d^3k}{(2\pi)^3 2\omega_k}\frac{d^3q}{(2\pi)^3\omega_q}\;\frac{(2\pi)^3\delta^3(q-p-k)\Gamma_{\ell\phi}}{(\omega_q-\omega_p-\omega_k)^2+\Gamma^2_{\ell\phi}/4}\\ &\times \delta|\mathcal{M}|^2\left(\Delta f_{iq}(1-f^{(eq)}_{\ell p})(1+f^{(eq)}_{\phi k}) - \Delta(1-f_{iq})f^{(eq)}_{\ell p}f^{(eq)}_{\phi k}\right)\end{aligned} \tag{4.81}$$

where

$$\delta|\mathcal{M}|^2 \equiv g_w \Im(h^\dagger h)^2_{ij}(q\cdot p)\frac{g_w M^2}{8\pi}\frac{M_i^2-M_j^2}{(M_i^2-M_j^2)^2+(M_i\Gamma_i+M_j\Gamma_j)^2}\;. \tag{4.82}$$

The factor $\delta|\mathcal{M}|^2$ can be interpreted as the CP-asymmetric part of the decay amplitudes, which gives the CP-asymmetry of the decay rates $\Gamma_{N_i\to\ell\phi}-\Gamma_{N_i\to\overline{\ell\phi}}$.

The term (4.81) produces the lepton asymmetry through the CP-asymmetric decay of the RH neutrinos that are out of the thermal equilibrium. The distortion of the distribution function is given in (4.62). An important point in (4.81) is that the enhancement factor of the CP-asymmetry is given by $(M_i^2-M_j^2)/((M_i^2-M_j^2)^2+(M_i\Gamma_i+M_j\Gamma_j)^2)$, and the regulator R_{ij} relevant to the CP-asymmetric decay of the RH neutrinos is given, not by $(M_i\Gamma_i-M_j\Gamma_j)$, but by $(M_i\Gamma_i+M_j\Gamma_j)$.

4.3.3 Washout effect on the lepton asymmetry: $\mathcal{C}_W$

The term $\mathcal{C}_W^K$ washes out the generated lepton asymmetry. In order to calculate $\Delta\pi$, we first perform the Fourier transform of $\pi_{\gtrless}(\tau,t;\mathbf{q})$ defined in (3.45):

$$\pi_{\gtrless}(q) = -\,g_w\sum_{\epsilon_\ell,\epsilon_\phi}\int \frac{d^3p}{(2\pi)^3 2\omega_p}\frac{d^3k}{(2\pi)^3 2\omega_k}\frac{(2\pi)^3\delta^3(q-p-k)\;\Gamma_{\ell\phi}}{(q_0-\epsilon_\ell\omega_p-\epsilon_\phi\omega_k)^2+\Gamma^2_{\ell\phi}/4}\;\not{p}_{\epsilon_\ell}\mathcal{D}^{\epsilon_\ell\epsilon_\phi}_{\gtrless(p,k)} \tag{4.83}$$

where $\mathcal{D}^{\epsilon_\ell\epsilon_\phi}_{\gtrless(p,k)}$ is defined in (3.46). Then $\Delta\pi_{\gtrless}$ is given by

$$\begin{aligned}\Delta\pi_{\gtrless}(q) \equiv& \;\pi_{\gtrless}(q) - \pi^{(eq)}_{\gtrless}(q)\\ =& -\,g_w\sum_{\epsilon_\ell,\epsilon_\phi}\int \frac{d^3p}{(2\pi)^3 2\omega_p}\frac{d^3k}{(2\pi)^3 2\omega_k}\;\frac{(2\pi)^3\delta^3(q-p-k)\Gamma_{\ell\phi}}{(q_0-\epsilon_\ell\omega_p-\epsilon_\phi\omega_k)^2+\Gamma^2_{\ell\phi}/4}\;\not{p}_{\epsilon_\ell}\Delta\mathcal{D}^{\epsilon_\ell\epsilon_\phi}_{\gtrless(p,k)}\end{aligned} \tag{4.84}$$

where $\Delta\mathcal{D}^{\epsilon_\ell\epsilon_\phi}_{\gtrless(p,k)} \equiv \mathcal{D}^{\epsilon_\ell\epsilon_\phi}_{\gtrless(p,k)} - \mathcal{D}^{\epsilon_\ell\epsilon_\phi(eq)}_{\gtrless(p,k)}$.

First consider the diagonal component $K = d$. Inserting (4.40) into (4.77), we have

$$
\begin{aligned}
\mathcal{C}_W^d = \sum_i \int & \frac{d^3p}{(2\pi)^3 2\omega_p} \frac{d^3k}{(2\pi)^3 2\omega_k} \frac{d^3q}{(2\pi)^3 \omega_q} \frac{(2\pi)^3\delta^3(q-p-k)\Gamma_{\ell\phi}}{(\omega_q-\omega_p-\omega_k)^2+\Gamma_{\ell\phi}^2/4} \\
& g_w(h^\dagger h)_{ii}(q\cdot p)\Big\{ f_{iq}^{(eq)}\Delta\left\{(1-f_{\ell p})(1+f_{\phi k})-(1-f_{\overline{\ell}p})(1+f_{\overline{\phi}k})\right\} \\
& \qquad -(1-f_{iq}^{(eq)})\Delta\left\{f_{\ell p}f_{\phi k}-f_{\overline{\ell}p}f_{\overline{\phi}k}\right\}\Big\} \ . \qquad (4.85)
\end{aligned}
$$

This gives a washout effect on the generated lepton asymmetry and it is physically interpreted as the inverse decay of the RH neutrinos.

Next let us see the flavor off-diagonal component, $K ='$. Because of the property (4.45), it vanishes in the leading order approximation:

$$
\mathcal{C}_W^{'} = 0 \ . \qquad (4.86)
$$

Hence only the diagonal component plays a role of washing out the generated lepton asymmetry.

4.3.4 Backreaction of the generated lepton asymmetry: $\mathcal{C}_{\rm BR}$

Finally let us see the back reaction of the generated lepton number asymmetry (i.e., the nonzero chemical potential of the SM leptons) to the Wightman functions of the RH neutrinos.

By using (B.6) and the flavor symmetry $S^{\alpha\beta} = \delta^{\alpha\beta}S$, the deviation of the self-energy in the presence of the chemical potential is written as

$$
\begin{aligned}
\Delta_{\mu_{(t)}}\Pi^{ij}_{\gtrless}(q) = & \int ds e^{+iq_0 s}\Delta_{\mu_{(t)}}\Pi^{ij}_{\gtrless}(X=t;s;\mathbf{q}) \\
= & (h^\dagger h)_{ij}P_{\rm L}\Delta\pi_{\gtrless}(q) + (h^\dagger h)^*_{ij}P_{\rm R}\Delta\overline{\pi}_{\gtrless}(q) \ . \qquad (4.87)
\end{aligned}
$$

$\Delta\overline{\pi}_{\gtrless}$ is the CP-conjugate of $\Delta\pi_{\gtrless}$ and obtained by changing the sign of the chemical potential of the SM leptons and the Higgs. $\Delta_\mu G^d_{\gtrless}(q)$ is given by replacing $\Pi^d_{\gtrless}$ in (4.34) by $i = j$ component of (4.87), and the contribution of the flavor diagonal component is shown to vanishes:

$$
\mathcal{C}^d_{\rm BR} = 0 \ . \qquad (4.88)
$$

Similarly the off-diagonal contribution becomes

$$\mathcal{C}'_{\mathrm{BR}} = \sum_i \int \frac{d^3p}{(2\pi)^3 2\omega_p} \frac{d^3k}{(2\pi)^3 2\omega_k} \frac{d^3q}{(2\pi)^3 \omega_q} \frac{(2\pi)^3 \delta^3(q-p-k)\Gamma_{\ell\phi}}{(\omega_q - \omega_p - \omega_k)^2 + \Gamma_{\ell\phi}^2/4}$$
$$g_w (q \cdot p)(-1) \frac{g_w M^2}{16\pi} (\Im(h^\dagger h)_{ij})^2 \frac{(M_i\Gamma_i + M_j\Gamma_j)}{(M_i^2 - M_j^2)^2 + (M_i\Gamma_i + M_j\Gamma_j)^2}$$
$$\times \left[f_{iq}^{(eq)} \Delta \left((1 - f_{\ell p})(1 + f_{\phi k}) - (1 - f_{\bar{\ell} p})(1 + f_{\bar{\phi} k}) \right) \right.$$
$$\left. - (1 - f_{iq}^{(eq)}) \Delta \left\{ f_{\ell p} f_{\phi k} - f_{\bar{\ell} p} f_{\bar{\phi} k} \right\} \right] . \tag{4.89}$$

Details of the calculations are given in the appendix L in [90]. In the above calculations, we took the weak coupling limit discussed in Appendix B. This term represents the effect of back reaction of the generated lepton asymmetry on the Wightman functions of the RH neutrinos. Such a term appears because we first solved the propagators of the RH neutrinos in the background of the SM leptons and the Higgs. The relative sign of the back reaction to the washout effect $\mathcal{C}_W^d$ in (4.85) is opposite so that the back reaction tends to reduce the washout of the generation of lepton asymmetry. If we solve the KB equations for the lepton asymmetry and the Wightman functions of the RH neutrinos simultaneously, the generated lepton asymmetry (namely the effect of the chemical potential) makes the RH neutrinos further away from the equilibrium. It is the reason why the back reaction reduces the washout.

4.3.5 CP-violating parameter

The CP-violating parameter can be read off from (4.81). $\delta|\mathcal{M}|^2$ of (4.82) gives the CP-asymmetry of the decay rates $\Gamma_{N_i \to \ell\phi} - \Gamma_{N_i \to \overline{\ell\phi}}$. Since the tree decay amplitude is given by $|\mathcal{M}|^2_{tree} = g_w (h^\dagger h)_{ii} (q \cdot p)$, the CP-violating parameter ε_i is given by

$$\begin{aligned}
\varepsilon_i &\equiv \frac{\Gamma_{N_i \to \ell\phi} - \Gamma_{N_i \to \overline{\ell\phi}}}{\Gamma_{N_i \to \ell\phi} + \Gamma_{N_i \to \overline{\ell\phi}}} \\
&= \frac{\sum_{j(\neq i)} g_w \Im(h^\dagger h)_{ij}^2 (q \cdot p) \frac{g_w M^2}{8\pi} \frac{M_i^2 - M_j^2}{(M_i^2 - M_j^2)^2 + (M_i\Gamma_i + M_j\Gamma_j)^2}}{2 \times g_w (h^\dagger h)_{ii} (q \cdot p)} \\
&= \sum_{j(\neq i)} \frac{\Im(h^\dagger h)_{ij}^2}{(h^\dagger h)_{ii}} \frac{g_w M^2}{16\pi} \frac{M_i^2 - M_j^2}{((M_i^2 - M_j^2))^2 + (M_i\Gamma_i + M_j\Gamma_j)^2} \\
&= \sum_{j(\neq i)} \frac{\Im(h^\dagger h)_{ij}^2}{(h^\dagger h)_{ii} (h^\dagger h)_{jj}} \frac{(M_i^2 - M_j^2) M_i \Gamma_j}{(M_i^2 - M_j^2)^2 + (M_i\Gamma_i + M_j\Gamma_j)^2} \times (1 + \mathcal{O}(\Delta M/M)) .
\end{aligned} \tag{4.90}$$

Hence the regulator discussed in the introduction is given by

$$R_{ij} = M_i \Gamma_i + M_j \Gamma_j . \tag{4.91}$$

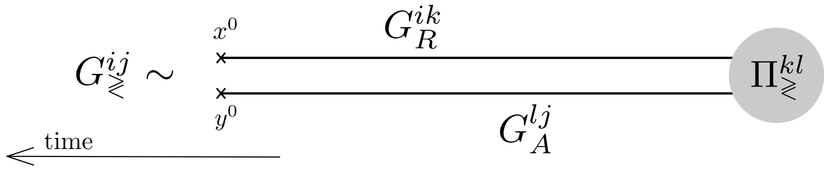

Figure 11: The information of the Wightman functions of the RH neutrinos are encoded in the self-energies $\Pi \gtrless$ in the past and transferred from the past to $t = x^0, y^0$ by the retarded and advanced Green functions.

The result is consistent with the result obtained in [71]. In the paper [71], the CP-violating parameter is obtained indirectly from the generated lepton asymmetry in a static background with an out-of-equilibrium initial condition. In our calculation, we directly obtained the same result in the expanding universe. It shows that the result obtained by Garny et al. is universal and can be applied to the thermal resonant leptogenesis.

4.3.6 Short summary

By using $\Delta G^{'ij}_{\gtrless}$ calculated in the previous section 4.2 in the r.h.s. of (4.71), we obtained the evolution equation (4.73) with three terms. $\mathcal{C}^{'}_{\Delta f}$ generates the lepton asymmetry and corresponds to the CP-asymmetric decay of the RH neutrinos. $\mathcal{C}_W$ gives the washout effects on the generated lepton numbers. $\mathcal{C}_{\mathrm{BR}}$ is the effect of the back reactions of the generated lepton asymmetry on the distribution functions of the RH neutrinos. From $\mathcal{C}^{'}_{\Delta f}$, we extracted the CP-asymmetric parameter ε_i given in (4.90). The enhancement factor due to the degenerate masses is regularized with an regulator $R_{ij} = M_i\Gamma_i + M_j\Gamma_j$, which reflects the enhancement factor of $\Delta G^{'}_{\gtrless}$.

4.4 Physical interpretation of the regulators

In this section, we give a physical interpretation of the appearance of the regulator $R_{ij} = |M_i\Gamma_i + M_j\Gamma_j|$ instead of $|M_i\Gamma_i - M_j\Gamma_j|$ in the flavor off-diagonal component of the Wightman function $\Delta G^{'}_{\gtrless}$.

The Wightman Green functions of the RH neutrinos are solved as in (4.34) and (4.35) in terms of the retarded, advanced propagators and the self-energies. These equations mean that the information of the distribution function of the RH neutrinos in the Wightman functions $G^{ij}_{\gtrless}$ are encoded in the self-energies $\Pi^{kl}_{\gtrless}$ in the past, and transferred from the past to the present $t = x^0, y^0$. The self-energies $\Pi^{kl}_{\gtrless}$ encode the information of the distributions functions of the SM leptons and the Higgs in the past (see Fig.11). In the flavor diagonal case of (4.34), all flavor indices of the RH neutrino propagators are the same in the leading order approximation. On the other hand, in the flavor off-diagonal case of (4.35), the flavor oscillation plays an important role.

Here we note that, as shown in (4.28) and (4.29), $G_{R/A}^{'ij}$ is a coherent sum of two terms, each of which corresponds to a propagation of the i-th (or j-th) flavor RH neutrino. We divide it as follows:

$$G_R^{'ij} = \left[G_R^{'ij}\right]_i + \left[G_R^{'ij}\right]_j . \tag{4.92}$$

4.4.1 On-shell and off-shell separation of $G_{\gtrless}^{'(eq)}$

Now let's investigate $G_{\gtrless}^{'ij}$. By looking at the first term of (4.35), it contains G_A^{djj} which describes the propagation of the j-th RH neutrino. The propagator $G_R^{'ij}$ in the first term contains both of the propagations of i-th and j-th flavor neutrinos. If the j-th neutrino propagates in $G_R^{'ij}$, only a single (j-th) neutrino propagates from the past, when the decay/inverse-decay represented by $\Pi_{\gtrless}$ takes place, to the present at $t = x^0, y^0$. We call this type of contributions the "on-shell" contributions.[21] These contributions are all taken into account in the classical Boltzmann equation.

On the contrary, if the i-th neutrino propagates in $G_R^{'ij}$, two different flavors propagate from the past to the present. This type of contributions are essentially "off-shell". In the classical Boltzmann equation, we first calculate the S-matrix elements of various processes and the external lines are taken to be on-shell. Hence this type of "off-shell" contributions are not taken into account by ordinary methods.[22] Separation of various Green functions, especially $\Delta G_{\gtrless}^{'ij}$, are calculated in the appendix M of [90].

For $G_{\gtrless}^{'}$ in eq.(4.35), on-shell contributions come from j-th propagation $\left[G_R^{'ij}\right]_j$ of $G_R^{'ij}$ in the first term and the i-th propagation $\left[G_A^{'ij}\right]_i$ of $G_A^{'ij}$ in the second term. All the other terms, i-th propagation $\left[G_R^{'ij}\right]_i$ of $G_R^{'ij}$ in the first term, the j-th propagation $\left[G_A^{'ij}\right]_j$ of $G_A^{'ij}$ in the second term give the off-shell contributions. The third term is off-shell since different mass eigenstates propagate in G_R^{dii} and G_A^{djj}. $G_R^{'(eq)}$ is separated into

$$G_R^{'(eq)} = \left[G_R^{'(eq)}\right]_{\text{on-shell}} + \left[G_R^{'(eq)}\right]_{\text{off-shell}} . \tag{4.93}$$

If we neglect the off-shell terms and take only the on-shell terms, $\left[G_R^{'(eq)}\right]_{\text{on-shell}}$

[21] See the footnote of the section 4.1.2. Propagations of a single N_i corresponds to propagations of a single mass eigenstate with mass M_i and width Γ_i. It is why we call this contrition as "on-shell".

[22] In the evolution equation of the lepton number, "off-shell" contributions can be interpreted as the interference terms in the (inverse)decay process of the superposition of different mass eigenstates.

becomes ($x^0 > y^0$)

$$
\begin{aligned}
&\left[G^{\prime(eq)ij}_{\gtrless}(x^0, y^0; \mathbf{q})\right]_{\text{on-shell}} \\
&\quad = \sum_{\epsilon} Z_\epsilon \Pi^{\prime(eq)ij}_{R}(\epsilon\omega_q) Z_\epsilon \frac{+i}{\Omega_{\epsilon i} - \Omega_{\epsilon j}} (-i) \left\{ \begin{matrix} 1 - f^{\epsilon}_{jq} \\ -f^{\epsilon}_{jq} \end{matrix} \right\} e^{-i\Omega_{\epsilon j}(x^0 - y^0)} \\
&\qquad + \sum_{\epsilon} Z_\epsilon \Pi^{\prime(eq)ij}_{A}(\epsilon\omega_q) Z_\epsilon \frac{-i}{\Omega^*_{\epsilon i} - \Omega^*_{\epsilon j}} (-i) \left\{ \begin{matrix} 1 - f^{\epsilon}_{iq} \\ -f^{\epsilon}_{iq} \end{matrix} \right\} e^{-i\Omega_{\epsilon i}(x^0 - y^0)} \ .
\end{aligned} \tag{4.94}
$$

Note that the sum of the on-shell contributions do not vanish even at $x^0 = y^0$ and $f_i \simeq f_j$:

$$
\lim_{x^0 \to y^0} \left[G^{\prime(eq)ij}_{\gtrless}(x^0, y^0; \mathbf{q})\right]_{\text{on-shell}} \neq 0 \ . \tag{4.95}
$$

It is different from the property of the full contributions given in (4.43).

4.4.2 On-shell and off-shell separation of $\Delta G^{\prime}_{\gtrless}$

We next investigate $\Delta G^{\prime}_{\gtrless}$. We show that neglecting the off-shell contribution in $\Delta G^{\prime}_{\gtrless}$, we get an enhancement factor for the CP-violating parameter with a regulator $|M_i \Gamma_i - M_j \Gamma_j|$.

In the appendix M.7 of [90], we separate $\Delta G^{\prime}_{\gtrless}$ into on-shell and off-shell contributions:[23]

$$
\Delta G^{\prime}_{\gtrless} = \left[\Delta G^{\prime ij}_{\gtrless}\right]_{\text{on-shell}} + \left[\Delta G^{\prime ij}_{\gtrless}\right]_{\text{off-shell}} \ . \tag{4.96}
$$

The on-shell contribution is given by (for $x^0 > y^0$)

$$
\begin{aligned}
&\left[\Delta G^{\prime ij}_{\gtrless}(x^0, y^0; \mathbf{q})\right]_{\text{on-shell}} \\
&\quad = \sum_{\epsilon} Z_\epsilon \Pi^{\prime(eq)ij}_{R}(\epsilon\omega_q) Z_\epsilon (-i) \Delta \left\{ \begin{matrix} 1 - f^{\epsilon}_{jq} \\ -f^{\epsilon}_{jq} \end{matrix} \right\} \frac{i}{\Omega_{\epsilon i} - \Omega_{\epsilon j}} \ e^{-i\Omega_\epsilon s_{xy}} \\
&\qquad + \sum_{\epsilon} Z_\epsilon \Pi^{\prime(eq)ij}_{A}(\epsilon\omega_q) Z_\epsilon (-i) \Delta \left\{ \begin{matrix} 1 - f^{\epsilon}_{iq} \\ -f^{\epsilon}_{iq} \end{matrix} \right\} \frac{-i}{\Omega^*_{\epsilon i} - \Omega^*_{\epsilon j}} \ e^{-i\Omega_\epsilon s_{xy}} \ .
\end{aligned} \tag{4.97}
$$

[23] This is analogous to the separation in [71], in which the authors emphasized an importance of the first principle calculation to keep the quantum coherence between the different flavor RH neutrinos. Calculating the evolution of the generated lepton number under a non-equilibrium initial condition in the flat space-time, they found two different behaviors of the generated lepton number. One is the ordinary term common in the conventional Boltzmann equation. The other term is specific to the quantum treatment by the quantum KB approach. The latter oscillates in time and reduces the eventual lepton number. "Off-shell" contribution here corresponds to the latter effect. However, note that in the present case the CP-violating parameter, and hence the resulting lepton number does not oscillate. the oscillatory behavior is averaged out because the deviation from the equilibrium is caused by the expansion of the universe, and its expansion rate H is much smaller than the oscillation scale $\Delta M \simeq \Gamma$. This averaging also occurs in the analysis by [60] in the strong washout regime.

This on-shell contribution has two important properties. First, it satisfies

$$\left[\Delta G^{\prime ij}_{\gtrless}\right]_{\text{on-shell}} = \Delta\left[G^{\prime ij}_{\gtrless}\right]_{\text{on-shell}} \tag{4.98}$$

where $\left[G^{\prime ij}_{\gtrless}\right]_{\text{on-shell}}$ is given in (4.94). The on-shell contribution (4.97) is simply obtained by replacing $f^{(eq)}$ by its variation Δf in (4.94). This replacing means that the process of the flavor oscillations and the process of taking a variation from the thermal values are commutative if we neglect the off-shell contributions. For full quantum calculations, (4.44) cannot be obtained by such a replacement from (4.43). This is because the flavor oscillations and the deviation from the thermal values are coherently mixed and these processes are not commutable. Namely, dropping the off-shell contributions corresponds to neglecting the interference between the flavor oscillations and the deviation of the distribution functions from the thermal equilibrium.

Second, compared with the full result (4.67), the enhancement factor $1/(\Omega_i - \Omega_j^*)$ is replaced by $1/(\Omega_i - \Omega_j)$. It is related to the above non-commutativity of taking Δ and flavor oscillation effects.

By inserting the on-shell formula (4.94) and (4.97) into (4.75), and supposing $\pi_\rho(\epsilon\omega_q) = -g_w i\epsilon \not{q}_\epsilon/(16\pi)$, $\pi_h(\epsilon\omega_q) = 0$, we have an on-shell approximation $\left[\mathcal{C}^{\prime}_{\Delta f}\right]_{\text{on-shell}}$ of $\mathcal{C}^{\prime}_{\Delta f}$:

$$\begin{aligned}\left[\mathcal{C}^{\prime}_{\Delta f}\right]_{\text{on-shell}} \simeq \sum_{i=1,2}\int &\frac{d^3p}{(2\pi)^3 2\omega_p}\frac{d^3k}{(2\pi)^3 2\omega_k}\frac{d^3q}{(2\pi)^3\omega_q}\,\frac{(2\pi)^3\delta^3(q-p-k)\Gamma_{\ell\phi}}{(\omega_q-\omega_p-\omega_k)^2+\Gamma_{\ell\phi}^2/4}\\ &g_w \Im(h^\dagger h)^2_{ij}(q\cdot p)\frac{g_w M^2}{8\pi}\,\frac{M_i^2-M_j^2}{(M_i^2-M_j^2)^2+(M_i\Gamma_i - M_j\Gamma_j)^2}\\ &\times\left\{\Delta f_{iq}(1-f^{(eq)}_{\ell p})(1+f^{(eq)}_{\phi k}) - \Delta(1-f_{iq})f^{(eq)}_{\ell p}f^{(eq)}_{\phi k}\right\}.\end{aligned} \tag{4.99}$$

Hence the regulator $|M_i\Gamma_i + M_j\Gamma_j|$ is replace by $|M_i\Gamma_i - M_j\Gamma_j|$ if we take only the on-shell terms. It is not valid in general, especially in the resonant leptogenesis. If the masses are hierarchical, it becomes identical with the correct value in (4.81).

4.4.3 Short summary

As emphasized above, if we neglect the off-shell contributions that are not included in the ordinary Boltzmann type analysis, we get a result (4.97) which is different from the correct one given in (4.67). The only difference is the enhancement factor, and if the mass difference is much larger than the width they coincide. But the difference is enlarged when the masses are almost degenerate. This reflects the fact that the flavor oscillation becomes important only for degenerate masses. Another important point is that the property of the non-commutativity (4.70) in the full result disappears if we take only the on-shell contributions as in (4.98). The noncommutativity is related to the vanishing of

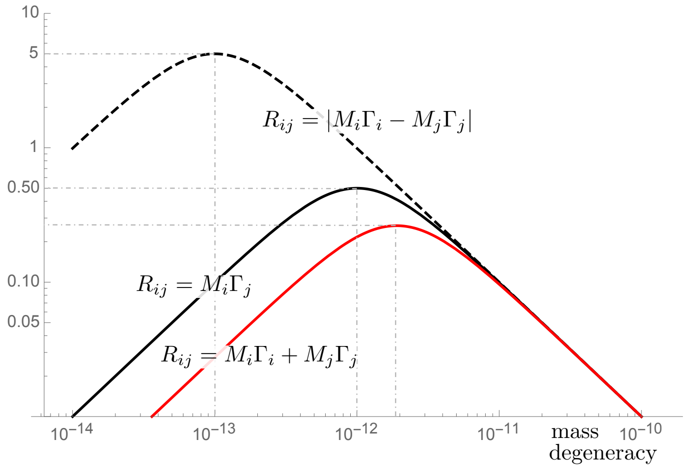

Figure 12: A comparison between the three types of regulator R_{ij}. The horizontal axis is the mass degeneracy $x \equiv \Delta M/M$ and the vertical axis is the value of the factor $|M_i^2 - M_j^2| M_i \Gamma_j/((M_i^2 - M_j^2)^2 + R_{ij}^2) = x(\Gamma_j/2M)/(x^2 + (R_{ij}/2M^2)^2)$ with the Yukawa couplings $\Gamma_j/2M = (h^\dagger h)_{jj}/16\pi = 1 \times 10^{-12}$ and $\Gamma_i/2M = (h^\dagger h)_{ii}/16\pi = 0.9 \times 10^{-12}$. The red line corresponds to the result obtained from the analysis using the KB equation, whose maximum value becomes about one half of the conventional one with $R_{ij} = M_i\Gamma_j$ (black solid line). And they can be many orders of magnitude less than the maximum value with $R_{ij} = |M_i\Gamma_i - M_j\Gamma_j|$ (black dashed line).

$G'_{\gtrless}$ at the equal time (4.45). For the on-shell contributions, $[G'_{\gtrless}]_{\text{on-shell}}$ does not vanish as shown in (4.95).

Based on this observation, we give another derivation of the properties of $\Delta G'_{\gtrless}$ in Appendix C by directly solving the KB equations. If we assume the vanishing condition (C.34) of $G'_{\gtrless}$ which is equivalent to (4.45), we show that the enhancement factor with a regulator $M_i\Gamma_i + M_j\Gamma_j$ appears as in (C.35). On the other hand, if we erroneously assume that it does not vanish, it leads to a much larger enhancement factor.

4.5 Summary and comments

In this section, by extending the analysis in the static background [71] to the thermal resonant leptogenesis in the expanding universe, we obtain an analytical expression of the evolution equation of the lepton number asymmetry. The CP-violating parameter is obtained as in (4.90). The regulator we obtained is consistent with the result [71] and it gives smaller CP-violating parameter than the conventional results (2.151), see Fig.12.

The difference between the regulator $R_{ij} = |M_i\Gamma_i + M_j\Gamma_j|$ and $R_{ij} = |M_i\Gamma_i - M_j\Gamma_j|$ comes from different forms of the enhancement factors of flavor off-diagonal components of the RH neutrino propagators. We show that the resonant oscillations between different flavors have two different types; one proportional to $1/(\Omega_i - \Omega_j^*)$ and the other proportional to $1/(\Omega_i - \Omega_j)$. Here $\Omega_i = \omega_{iq} - i\Gamma_{iq}/2$ is a position of the pole of the i-th RH neutrino. In the thermal equilibrium, the resonant oscillations in the flavor off-diagonal Green functions have the type $1/(\Omega_i - \Omega_j)$ (or its complex conjugate) as shown in (4.28) or in (4.44). Since $1/(\Omega_i - \Omega_j)$ is rewritten by (4.33), the enhancement of flavor oscillation corresponds to the regulator $R_{ij} = |M_i\Gamma_i - M_j\Gamma_j|$. However, the deviation of the off-diagonal components of out of equilibrium has a different enhancement factor $1/(\Omega_i - \Omega_j^*)$ as shown in (4.67), which corresponds to the regulator $R_{ij} = |M_i\Gamma_i + M_j\Gamma_j|$. Physical interpretation of the change of the regulator is given in Section 4.4. The off-shell contributions to the off-diagonal component to the Wightman functions are essential. they correspond to the contributions missed in the conventional calculation (see section 2.4).

In section 3.5, we mentioned that the lepton number evolution equation has the non-Markovian form after plugging the solution of the RH neutrino Wightman propagator (3.48). In this section, we have implemented the time integration in (3.48) by taking up to the first order of the derivative expansion in terms of $H/\Gamma = 1/\mathcal{K} \ll 1$ (4.47). The form of regulator $R_{ij} = |M_i\Gamma_i + M_j\Gamma_j|$ reflects the memory integral in the non-Markovian equation as mentioned in subsection 4.2.6. Such a memory effect is taken into account through the formal solution (3.48) which comprise the retarded and advanced propagators of the mass eigenstates[24] (4.20), and hence both of complex frequencies Ω_i and Ω_j^*.

The property (4.45) is also important for this change of the regulator. As we show in Appendix C, if we erroneously assume that the off-diagonal component of the Wightman function is non-vanishing and its deviation is given by the change of the local temperature, it leads to much more enhanced oscillation similar to the regulator $R_{ij} = |M_i\Gamma_i - M_j\Gamma_j|$.

The major differences from the conventional calculation in Section 2.3 are: (i) the interferences between the different mass eigenstates have been taken into account. (ii) the non-equilibrium Wightman propagator, which has the enhancement factor $1/(\Omega_i - \Omega_j^*)$, has been used. This is crucial because, by taking into account the effect (i), the equilibrium Wightman propagator disappears in the two-point coincidence limit (4.45). As a consequence, we have gotten the CP-violating parameter (4.90) with the regulator $R_{ij} = M_i\Gamma_i + M_j\Gamma_j$, and moreover, we never need the RIS-subtraction in thermal equilibrium.[25]

From the form of the Wightman propagator out of equilibrium (4.67), we find the validity of an approximation in which, although the spectral density G_ρ

[24]Note that, within the validity of the calculation in this section, the frequency of the i-th mass eigenstate's propagator almost coincides with the i-th bare propagator. See the footnote of the section 4.1.2.

[25]From this perspective, the reason why one needs the RIS-subtraction should not be understood as "double counting". It's because the conventional calculations just miss the off-shell contribution even in thermal equilibrium.

of RH neutrino has the form of the quasi-particle approximation, the statistical propagator G_F has the off-diagonal components in the flavor indices. In the next section 5, by using such an approximation, we take into account the important off-shell effects in the evaluation of the r.h.s. of (3.34).

5 Density matrix formalism and resonant leptogenesis

In the previous section, we have solved the KB equations of RH neutrino in advance, and plugging it into the KB equation of the SM lepton, we have gotten the evolution equation of the lepton number with the effective CP-violating parameter ε. However, in the analysis, we can notice the validity of another approximated form of the Wightman propagator of RH neutrinos: From the expression (4.69), it's found that the deviation of the thermal Wightman propagator of RH neutrinos can be written in the form of quasi-particle approximation with the non-diagonal distribution function in terms of flavor indices.

In this section, such a non-diagonal distribution function is identified as the so-called density matrix, and we reduce the KB equation of RH neutrino into the Markovian equation for the density matrix of RH neutrinos, corresponding the picture (a) in Fig.8 at the end of section 2. Moreover, by plugging the analytic solution of that equation into the evolution equation of lepton number, we will get the effective CP-violating parameter *without* the assumption of small off-diagonal components of Yukawa coupling.

5.1 From KB to density matrix evolution

In this subsection, we derive an evolution equation of the multi-flavour density matrix of the RH neutrinos N_i [57] starting from the Kadanoff-Baym equation. We extend the method established in the flat space-time by [70] to the case of the expanding universe. In [70], the evolution equation in the expanding universe was derived by replacing the physical time and the Majorana mass M by the conformal time and aM, where a is the scale factor. Our result in the following agrees with the result in [70], and give a justification of their method to obtain the evolution equation in the expanding universe. KB equation is derived from the Schwinger-Dyson equation on the closed-time-path, which is a fully systematic equation of the Green functions in a non-equilibrium setting. Deriving the kinetic equation for density matrix from the KB equation makes it clear under what conditions the density matrix equation is obtained and what kinds of diagrams contribute to various terms in the density matrix formalism, especially the resonantly enhanced CP-violating parameter and the *decay widths* Γ_i contained in the regulator of ε_i.

5.1.1 Kramers-Moyal expansion of the Kadanoff-Baym equation

Assuming the spatial homogeneity, The Kadanoff-Baym (KB) equation of the RH neutrinos in the expanding universe (3.12) is Fourier-transformed as

$$\left(i\gamma^0\partial_{x^0} - \frac{\mathbf{q}\cdot\boldsymbol{\gamma}}{a(x^0)} - \hat{M}\right) G_{\lessgtr}(x^0,y^0) - (\Pi_R * G_{\lessgtr})(x^0,y^0) = (\Pi_{\lessgtr} * G_A)(x^0,y^0). \tag{5.1}$$

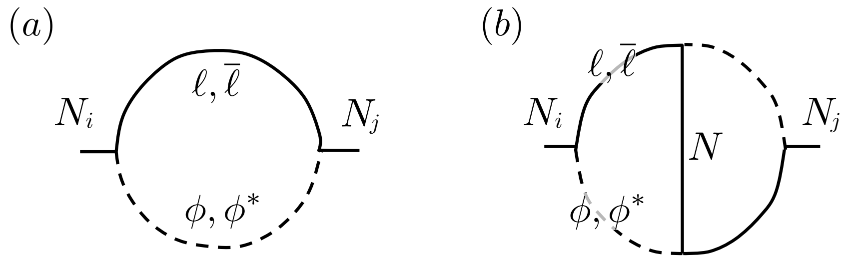

Figure 13: Self-energy diagrams of RH neutrino N_i. In the 2PI formalism, each internal line represents a full propagator while vertices are given by tree vertices. Tree-level decay width is generated from the left figure (a). The right figure (b) gives the so-called direct CP-violating parameter of the RH neutrino, an interference between the tree and the one-loop vertex corrections.

where $\mathbf{q}$ is the comoving momentum and we omitted the argument $\mathbf{q}$ of $G_{\lessgtr}(x^0, y^0; \mathbf{q})$. $*$ represents the convolution in the time coordinate. Symbolically we write it as

$$iG_0^{-1}G_{\lessgtr} - \Pi_R G_{\lessgtr} = \Pi_{\lessgtr} G_A. \tag{5.2}$$

The 1PI self-energy function Π of RH neutrino is obtained by cutting a (full) propagator of 2PI diagrams. In the 2PI formalism, all internal lines represent full propagators while vertices are tree. For more details, see appendix A.3 and appendix B. Figure 13 are examples of self-energy diagrams. In deriving the KB equation, Fig. 13(a) gives the decay width at tree level while Fig. 13(b) gives an interference between the tree and the one-loop vertex diagrams [72]. Hence the direct CP-violating parameter is contained in Fig. 13 (b). If we include Z' gauge boson or a scalar field coupled with the RH neutrinos, other self-energy diagrams in Fig. (18) contribute to Π.

By taking the Fourier transform with respect to the relative time coordinate $s = x^0 - y^0$, eq. (5.1) becomes

$$e^{-i\diamondsuit}\left\{\gamma^0 q_0 - \frac{\mathbf{q}\cdot\boldsymbol{\gamma}}{a(X)} - \hat{M} - \Pi_R(X;q_0)\right\}\left\{G_{\lessgtr}(X;q_0)\right\} = e^{-i\diamondsuit}\left\{\Pi_{\lessgtr}(X;q_0)\right\}\left\{G_A(X;q_0)\right\}. \tag{5.3}$$

$X = (x^0 + y^0)/2$ is the center-of-mass time coordinate. Here we used the Moyal-Weyl bracket defined by

$$e^{-i\diamondsuit}\{f(X;q_0)\}\{g(X;q_0)\} = e^{\frac{i}{2}(\partial^f_{q_0}\partial^g_X - \partial^f_X\partial^g_{q_0})} f(X;q_0)g(X;q_0). \tag{5.4}$$

In the expanding universe with the Hubble parameter H, X derivative is often estimated as $\partial_X \sim \mathcal{O}(H)$. On the other hand, derivative with respect to the relative momentum q_0 is estimated as $\partial_{q_0} f \sim \mathcal{O}(1/\Gamma_f)$ where Γ_f is the decay width of the function $f(X,s) \sim e^{-\Gamma_f s}$. In (5.3), Γ for G_* ($* = \lessgtr, A, R, , ,$) is given by the decay width Γ_N of the RH neutrinos. In the strong washout regime, we have an inequality $H \ll \Gamma_N$. Since the dominant contribution to the self-energy Π comes from the diagram in Fig. 13 (a), Γ for Π_* is given by the decay widths of the charged lepton and Higgs $\Gamma_{l,\phi}$ propagating in the internal lines. They are much larger than Γ_N. An expansion with respect to $\diamondsuit$ is given by $H/\Gamma_{N,\ell,\phi}$ and hence justified by (4.47).

Taking up to the first order of the derivative expansion of $\diamondsuit$, we have

$$\begin{aligned}\left(\gamma^0 q_0 - \frac{\mathbf{q}\cdot\boldsymbol{\gamma}}{a} - \hat{M} - \Pi_R\right) G_{\lessgtr} - i\diamondsuit\left\{\gamma^0 q_0 - \frac{\mathbf{q}\cdot\boldsymbol{\gamma}}{a} - \hat{M} - \Pi_R\right\}\{G_{\lessgtr}\} \\ = \Pi_{\lessgtr} G_A - i\diamondsuit\{\Pi_{\lessgtr}\}\{G_A\}\end{aligned} \tag{5.5}$$

The spectral function G_ρ satisfies a similar equation in which $\gtrless$ (of G and Π) is replaced by ρ.

5.1.2 KB equation for small deviation from $G^{eq}_{\lessgtr}$

As we have seen in section 3.1, in thermal equilibrium, Fourier transform of the propagators satisfy the Kubo-Martin-Schwinger (KMS) relation

$$G^{eq}_{\gtrless}(q) = -i\begin{Bmatrix} 1 - f^{eq}(q) \\ -f^{eq}(q)\end{Bmatrix} G^{eq}_\rho(q), \tag{5.6}$$

where f^{eq} is the Fermi distribution function $f^{eq}(q) = 1/(e^{q_0/T}+1)$. Especially, as shown in subsection 4.1.6, the off-diagonal component of the Wightman functions $G'^{eq}_{\gtrless}(x^0, y^0)$ vanishes in the limit of $x^0 \to y^0$. It directly follows from the KMS relation together with the equal-time anti-commutation relation of the fields N_i. When the system is out of equilibrium, it deviates from zero whose imaginary part gives the CP-violating source for the lepton number asymmetry.

If the system is slightly deviated from the local equilibrium, KMS relation indicates that the deviation is written as

$$\delta G_{\gtrless}(q) = -i\delta\begin{Bmatrix} 1 - f(q) \\ -f(q)\end{Bmatrix} G_\rho(q) - i\begin{Bmatrix} 1 - f^{eq}(q) \\ -f^{eq}(q)\end{Bmatrix}\delta G_\rho(q) \tag{5.7}$$

where $\delta f(q)$ stands for the deviation of the distribution function from the equilibrium value $f^{eq}(q)$. We then define

$$\widetilde{\delta G}_{\lessgtr} \equiv \delta G_{\lessgtr} + i\begin{bmatrix} -f \\ 1-f\end{bmatrix}\delta G_\rho = \delta G_F + i\left(\tfrac{1}{2} - f\right)\delta G_\rho. \tag{5.8}$$

which represents a deviation of the distribution function $\widetilde{\delta G}_{\lessgtr} \sim i(\delta f) G_\rho$. For the notational simplicity, we omit the superscript "*eq*" from equilibrium distribution function $f^{eq}(q)$. Note that, here, "equilibrium" means *local* equilibrium with the *time-dependent*temperature of the SM thermal bath, and δf is the deviation from the local equilibrium.

We now derive the KB equation for a small deviation from the local equilibrium. Taking a variation in (5.5) and picking up to the first order terms of δ, we have

$$
\begin{aligned}
&\left(\gamma^0 q_0-\frac{\mathbf{q}\cdot\boldsymbol{\gamma}}{a}-\hat{M}-\Pi_R^{eq}\right)\delta G_{\lessgtr}-\delta\Pi_R G_{\lessgtr}^{eq}-i\diamondsuit\left\{\gamma^0 q_0-\frac{\mathbf{q}\cdot\boldsymbol{\gamma}}{a}-\hat{M}-\Pi_R^{eq}\right\}\{\delta G_{\lessgtr}\}\\
&-i\diamondsuit\left\{\gamma^0 q_0-\frac{\mathbf{q}\cdot\boldsymbol{\gamma}}{a}-\hat{M}-\Pi_R^{eq}-\delta\Pi_R\right\}\left\{G_{\lessgtr}^{eq}\right\}\\
&=\Pi_{\lessgtr}^{eq}\delta G_A+\delta\Pi_{\lessgtr}G_A^{eq}-i\diamondsuit\left\{\Pi_{\lessgtr}^{eq}\right\}\{G_A^{eq}+\delta G_A\}-i\diamondsuit\{\delta\Pi_{\lessgtr}\}\{G_A^{eq}\}\,. \qquad (5.9)
\end{aligned}
$$

We can obtain the same equation for G_ρ by replacing $\lessgtr$ by ρ. By combining these equations and using the KMS relation, some terms are cancelled and we have

$$
\begin{aligned}
&\left(\gamma^0 q_0-\frac{\mathbf{q}\cdot\boldsymbol{\gamma}}{a}-\hat{M}-\Pi_R^{eq}\right)\left(\delta G_{\lessgtr}+i\begin{bmatrix}1-f\\-f\end{bmatrix}\delta G_\rho\right)\\
&-i\diamondsuit\left\{\gamma^0 q_0-\frac{\mathbf{q}\cdot\boldsymbol{\gamma}}{a}-\hat{M}-\Pi_R^{eq}\right\}\left(\{\delta G_{\lessgtr}\}+i\begin{bmatrix}1-f\\-f\end{bmatrix}\{\delta G_\rho\}\right)\\
&-i\diamondsuit\left\{\gamma^0 q_0-\frac{\mathbf{q}\cdot\boldsymbol{\gamma}}{a}-\hat{M}-\Pi_R^{eq}-\delta\Pi_R\right\}\left\{\begin{bmatrix}1-f\\-f\end{bmatrix}\right\}G_\rho^{eq}(-i)\\
&=\left(\delta\Pi_{\lessgtr}+i\begin{bmatrix}1-f\\-f\end{bmatrix}\delta\Pi_\rho\right)G_A^{eq}-i(-i)\Pi_\rho^{eq}\diamondsuit\left\{\begin{bmatrix}1-f\\-f\end{bmatrix}\right\}\{G_A^{eq}+\delta G_A\}\\
&-i\diamondsuit\left(\{\delta\Pi_{\lessgtr}\}+i\begin{bmatrix}1-f\\-f\end{bmatrix}\{\delta\Pi_\rho\}\right)\{G_A^{eq}\}\ . \qquad (5.10)
\end{aligned}
$$

The deviation from $G_{\lessgtr}^{eq}$ occurs due to the expansion of the universe, and hence $\delta G_{\lessgtr}$ is proportional to the Hubble parameter H. Since the derivative expansion of $\diamondsuit$ is an expansion of H, we can drop terms containing more than one δ or $\diamondsuit$ when $H \ll \Gamma_N, \Gamma_{\ell,\phi}$. Then (5.10) is simplified as

$$
\begin{aligned}
&-i\diamondsuit\left\{\gamma^0 q_0-\frac{\mathbf{q}\cdot\boldsymbol{\gamma}}{a}-\hat{M}-\Pi_R^{eq}\right\}\{if\}\,G_\rho^{eq}+i\Pi_\rho^{eq}\diamondsuit\{if\}\{G_A^{eq}\}\\
&=\widetilde{\delta\Pi_{\lessgtr}}G_A^{eq}-\left(\gamma^0 q_0-\frac{\mathbf{q}\cdot\boldsymbol{\gamma}}{a}-\hat{M}-\Pi_R^{eq}\right)\widetilde{\delta G_{\lessgtr}} \qquad (5.11)
\end{aligned}
$$

Instead of (5.2), we can start from

$$
iG_{\lessgtr}G_0^{-1}-G_{\lessgtr}\Pi_A=G_R\Pi_{\lessgtr} \qquad (5.12)
$$

and obtain a similar equation to (5.11),

$$
\begin{aligned}
&-iG_\rho^{eq}\diamondsuit\{if\}\left\{\gamma^0 q_0-\frac{\mathbf{q}\cdot\boldsymbol{\gamma}}{a}-\hat{M}-\Pi_A^{eq}\right\}+i\diamondsuit\{G_R^{eq}\}\{if\}\,\Pi_\rho^{eq}\\
&=G_R^{eq}\widetilde{\delta\Pi_{\lessgtr}}-\widetilde{\delta G_{\lessgtr}}\left(\gamma^0 q_0-\frac{\mathbf{q}\cdot\boldsymbol{\gamma}}{a}-\hat{M}-\Pi_A^{eq}\right). \qquad (5.13)
\end{aligned}
$$

By multiplying a helicity projection operator with $h=\pm1$

$$
P_h \equiv \frac{1+h\mathbf{n}\cdot\boldsymbol{\sigma}}{2}, \quad \mathbf{n}=\frac{\mathbf{q}}{q}, \quad \sigma^i=\gamma^0\gamma^i\gamma_5 \qquad (5.14)
$$

on [(5.11) – (5.13)], and taking trace of spinors, we get

$$
\begin{aligned}
&-i\mathrm{tr}\Big[P_h\Big(\diamondsuit\Big\{\gamma^0 q_0-\frac{\mathbf{q}\cdot\boldsymbol{\gamma}}{a}-\hat{M}-\Pi_R^{eq}\Big\}\{if\}\,G_\rho^{eq}-\Pi_\rho^{eq}\diamondsuit\{if\}\{G_A^{eq}\}\\
&-G_\rho^{eq}\diamondsuit\{if\}\Big\{\gamma^0 q_0-\frac{\mathbf{q}\cdot\boldsymbol{\gamma}}{a}-\hat{M}-\Pi_A^{eq}\Big\}+\diamondsuit\{G_R^{eq}\}\{if\}\,\Pi_\rho^{eq}\Big)\Big]\\
=&\mathrm{tr}\Big[P_h\Big(\Big(\hat{M}+\Pi_H^{eq}\Big)\widetilde{\delta G_{\lessgtr}}-\widetilde{\delta G_{\lessgtr}}\Big(\hat{M}+\Pi_H^{eq}\Big)\Big)\Big]\\
&+\mathrm{tr}\Big[P_h\Big(\widetilde{\delta\Pi_{\lessgtr}}G_A^{eq}+\frac{1}{2}\Pi_\rho^{eq}\widetilde{\delta G_{\lessgtr}}-G_R^{eq}\widetilde{\delta\Pi_{\lessgtr}}+\frac{1}{2}\widetilde{\delta G_{\lessgtr}}\Pi_\rho^{eq}\Big)\Big].
\end{aligned}
\tag{5.15}
$$

where $\Pi_H = (\Pi_R + \Pi_A)/2$.

We make the following quasi-particle ansatz for $\widetilde{\delta G_{\lessgtr}}$. In this thesis, we consider a situation that two RH neutrinos have almost degenerate masses. Hence their poles in the Green function can be approximated by a single pole of Breit-Wigner type [70]:

$$
\begin{aligned}
\widetilde{\delta G_{\lessgtr}} \simeq& \sum_{h=\pm} i\delta f_{N,h}(q_0,X)G_\rho^{eq}P_h\\
\simeq& \sum_{h=\pm}(-\delta f_{N,h,q})\frac{\Gamma_q}{(q_0-\omega_q)^2+\Gamma_q^2/4}\frac{\not{q}_+ + M}{2\omega_q}P_h\\
&+\sum_{h=\pm}(-\delta f^*_{N,h,q})\frac{\Gamma_q}{(q_0+\omega_q)^2+\Gamma_q^2/4}\frac{\not{q}_- + M}{2\omega_q}P_h.
\end{aligned}
\tag{5.16}
$$

where we set the momentum at on-shell $q_{\pm\mu} = (\pm\omega_q, -\mathbf{q})_\mu$ and

$$
G_\rho^{eq} = \simeq \sum_{h=\pm}\frac{i2q_0\Gamma_q(\not{q}+M)}{({q_0}^2-\omega_q^2)+\omega_q^2\Gamma_q^2}P_h \tag{5.17}
$$

is the spectral density of RH neutrino. By putting these ansatz, we assumed that deviation from local equilibrium appears only in the matrix-valued distribution function $f_{N,h}$. As a result, we obtain the same equation as derived in [70]. Two mass eigenstates are summed in the distribution function δf_N. As seen in Section 4, flavor off-diagonal components of the distribution function is suppressed by a cancellation of two mass eigenstates. But when the system is out-of-equilibrium, off-diagonal component of δf_N becomes comparable to its diagonal one.

Also note that hermiticity of Wightman function

$$
[G_<(q_0,\mathbf{q})]^\dagger = \gamma^0 G_<(q_0,\mathbf{q})\gamma^0 \tag{5.18}
$$

together with spatial homogeneity and isotropy require the relation $\delta f^\dagger_{N,h,q} = \delta f_{N,h,q}$. Majorana condition

$$
[G_<(q_0,\mathbf{q})]^C = C[G_>(-q_0,-\mathbf{q})]^{\mathrm{t}}C^{-1} = G_<(q_0,\mathbf{q}) \tag{5.19}
$$

relates the positive and negative frequency parts as in (5.16).

We then insert the ansatz of $\widetilde{\delta G_{\lessgtr}}$ of (5.16) into (5.15) and perform q_0 integration: $\int_0^\infty dq_0/2\pi$. It is dominated near the region $q_0 \sim \omega_{\mathbf{q}} = \sqrt{M^2 + |\mathbf{q}|^2}$ (see Appendix D), and we get an evolution equation for the density matrix:

$$-id_t f_{N,h,q} = -[\omega_{qh}^{\text{eff}}, \delta f_{N,h,q}] + \frac{\mathcal{S}}{2} \tag{5.20}$$

where $f_{N,h,q}$ is the density matrix defined as

$$f_{N,h,q} = f_{N,h,q}^{eq} + \delta f_{N,h,q} \ , \quad f_{N,h,q}^{eq} = f_N^{eq}(\omega_q)\mathbf{1}_{2\times 2} \ . \tag{5.21}$$

This quantity is identified as the expectation value of the off-diagonal number operator $a_{N_i,h,\mathbf{q}}^\dagger a_{N_j,h,\mathbf{q}}$:

$$f_{N,h,q} = \begin{pmatrix} \langle a_{N_1,h,\mathbf{q}}^\dagger a_{N_1,h,\mathbf{q}} \rangle & \langle a_{N_1,h,\mathbf{q}}^\dagger a_{N_2,h,\mathbf{q}} \rangle \\ \langle a_{N_2,h,\mathbf{q}}^\dagger a_{N_1,h,\mathbf{q}} \rangle & \langle a_{N_2,h,\mathbf{q}}^\dagger a_{N_2,h,\mathbf{q}} \rangle \end{pmatrix} \ . \tag{5.22}$$

$\langle \cdots \rangle$ is defined by (3.3) using the initial (full) density matrix $\rho(t_i)$. The matrix-valued quantity $f_{N,h,q}$, which is called density matrix here, should be understood as the gaussian part of the full density matrix $\rho(t)$. Remember the underlying assumption on which analyses using the ordinary Boltzmann equation are based is that the system can be well described by the time evolution of *classical* one-particle distribution functions which parametrize the gaussian part of the full density matrix. In a similar way, the quantity $f_{N,h,q}$ is assumed to be enough to describe the system, but the matrix $f_{N,h,q}$ is regarded as the *quantum* extension of one-particle distribution function involving quantum superposition states of the different mass eigenstates.

The derivation of the l.h.s. of (5.20) is given in Appendix D. The first line of the r.h.s. of (5.15) gives an effective Hamiltonian in the r.h.s. of (5.20),

$$\omega_{qh}^{\text{eff}} = \text{tr}\left\{ \left(\hat{M} + \Pi_H^{eq}(q) \right) \frac{\not{q} + M}{2\omega_q} P_h \right\} , \tag{5.23}$$

while the second line of the r.h.s. of (5.15) gives the collision term,

$$\begin{aligned} \mathcal{S} &= -\text{tr}\Big[P_h \Big(\widetilde{\delta\Pi_{\lessgtr}} G_\rho^{eq} - \Pi_\rho^{eq} \widetilde{\delta G_{\lessgtr}} + G_\rho^{eq} \widetilde{\delta \Pi_{\lessgtr}} - \widetilde{\delta G_{\lessgtr}} \Pi_\rho^{eq} \Big) \Big] \\ &= +i\text{tr}\Big[P_h \Big(\{ \widetilde{\delta\Pi_>}, G_<^{eq} \} + \{ \Pi_>^{eq}, \widetilde{\delta G_<} \} - \{ \widetilde{\delta\Pi_<}, G_>^{eq} \} - \{ \Pi_<^{eq}, \widetilde{\delta G_>} \} \Big) \Big] \\ &= +i \left\{ \text{tr}\left[P_h \frac{\not{q} + M}{2\omega_q} \delta\Pi_>(q) \right], -f_{N,h,q}^{eq} \right\} + i \left\{ \text{tr}\left[P_h \frac{\not{q} + M}{2\omega_q} \Pi_>^{eq}(q) \right], -\delta f_{N,h,q} \right\} \\ &\quad - i \left\{ \text{tr}\left[P_h \frac{\not{q} + M}{2\omega_q} \delta\Pi_<(q) \right], 1 - f_{N,h,q}^{eq} \right\} - i \left\{ \text{tr}\left[P_h \frac{\not{q} + M}{2\omega_q} \Pi_<^{eq}(q) \right], -\delta f_{N,h,q} \right\} \end{aligned} \tag{5.24}$$

Here we have used smallness of the flavor off-diagonal components $G_{i\neq j}^{eq}$ in the q_0 integration corresponding to taking coincidence in time (see discussion after (5.6)), and smallness of flavor dependent thermal corrections to G_R.

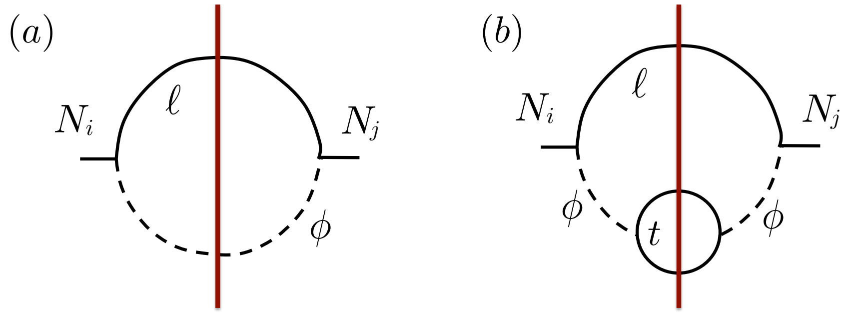

Figure 14: Two dominant contributions to the self-energy diagrams of Figure 13 (a). Propagators that cross with the cut-line in the middle are put on mass-shell. Internal lines are no longer full propagators. The left figure (a) gives a decay and an inverse-decay term of RH neutrinos in the KB equation. In the right figure (b), we consider a loop correction of the Higgs propagator by top quarks. It gives scattering terms such as $N + \bar{\ell} \leftrightarrow t + \bar{Q}$ or $N + Q \leftrightarrow \ell + t$ in the KB equation[74].

5.2 Kinetic equation for density matrix

In deriving kinetic equations for the density matrix, we need to make quasi-particle ansatz in (5.16). Similar ansatz must be imposed on the internal lines in the self-energy diagrams Π because distribution functions (even when they are matrix-valued) are defined only on mass-shell. This is the most subtle point in the KB approach. In order to take various diagrams contained in each self-energy diagram in Fig.13, an often-adopted method is to expand the full propagators and cut the self-energy diagram into two. Examples are shown in Fig.14. On the cut-line, on-shell propagators are used.

5.2.1 Kinetic equation for RH neutrinos

The collision term (5.24) is proportional to

$$\mathrm{tr}\left[P_h\left(\{\Pi_>, G_<\} - \{\Pi_<, G_>\}\right)\right]. \tag{5.25}$$

The first term with $G_<$ describes decay (or scattering) of RH neutrino (plus other particles) into others while the second term with $G_>$ is an inverse-decay (or inverse scattering). By expanding the full propagators in the self-energy Π and cutting the diagram into two, we have various diagrams with on-shell external lines. External lines are assigned to either incoming or outgoing particles. If a cut diagram with $G_<$ represents a scattering process of $N+i+j\cdots \to a+b+\cdots$, it can be expressed as

$$-\operatorname{tr}\{\Pi_>(q)(\not{q}+M)P_h\} = \sum_{i,..,a,..}\int d\Pi_{i,..,a,..}\gamma^{ab..}_{hij..} f_i f_j ...(1-\eta_a f_a)(1-\eta_b f_b)... \tag{5.26}$$

$\eta_{a,i} = \pm 1$ corresponding to boson or fermion. Here the integral measure is defined as

$$d\Pi_{i,..,a,..} = \prod_{i,..,a,..} \frac{d^3 q_i}{(2\pi)^3 2\omega_i} \cdots \frac{d^3 p_a}{(2\pi)^3 2\omega_a} \cdots \times (2\pi)^4 \delta^{(4)}(q + \textstyle\sum_i q_i - \sum_a p_a) \tag{5.27}$$

where q is a momentum of incoming RH neutrino, q_i and p_a are momenta of other incoming and outgoing particles. On the other hand, if a diagram with $G_>$ represents an inverse scattering process of $a + b + \cdots \to N + i + j + \cdots$, it can be expressed as

$$\operatorname{tr}\{\Pi_<(q)(\not{q}+M)P_h\} = \sum_{i,..,a,..}\int d\Pi_{i,..,a,..}\gamma^{ab..}_{hij..}(1-\eta_i f_i)(1-\eta_j f_j)...f_a f_b... \tag{5.28}$$

Combining these two contributions, the evolution equation for the density matrix $f_{N,h,q}$ (5.20) is written as

$$\begin{aligned} d_t f_{N,h,q} =& -i\left[\omega^{\rm eff}_{qs}, f_{N,h,q}\right] \\ & -\frac{1}{2}\frac{1}{2\omega_q}\sum_{i,..,a,..}\int d\Pi_{i,..,a,..}\{\gamma^{ab..}_{hij..}, f_{N,h,q}\} f_i f_j ...(1-\eta_a f_a)(1-\eta_b f_b)... \\ & +\frac{1}{2}\frac{1}{2\omega_q}\sum_{i,..,a,..}\int d\Pi_{i,..,a,..}\{\gamma^{ab..}_{hij..}, (1-f_{N,h,q})\}(1-\eta_i f_i)(1-\eta_j f_j)...f_a f_b... \end{aligned} \tag{5.29}$$

In this expression, we combined variations as $\Pi = \Pi^{(eq)} + \delta\Pi$ and $f_N = f_N^{(eq)} + \delta f_N$ for notational simplicity. zeroth order term of the variation δ automatically cancels due to the detailed balance condition in the equilibrium.

Let us now consider a specific diagram of Figure 14 (a). This diagram is reduced to the cut diagram of Fig. 15 (a). Fig. 15 (b) is its conjugate and N decays into $(\bar{\ell}, \phi^*)$. Other diagrams like Fig. 14 (b) are of higher orders in the Yukawa couplings, and we omit them in the following. From Fig. 14(a) and its conjugate, we have

$$\sum_\alpha \int d\Pi_{pk}\big(\gamma_h^{\ell^\alpha\phi}(1-f_{\ell^\alpha p})(1+f_{\phi k}) + (\gamma_{-h}^{\ell^\alpha\phi})^*(1-f_{\overline{\ell}^\alpha p})(1+f_{\overline{\phi}k})\big) \tag{5.30}$$

for (5.26), and

$$\sum_\alpha \int d\Pi_{pk}\big(\gamma_h^{\ell^\alpha\phi} f_{\ell^\alpha p} f_{\phi k} + (\gamma_{-h}^{\ell^\alpha\phi})^* f_{\overline{\ell}^\alpha p} f_{\overline{\phi}k}\big) \tag{5.31}$$

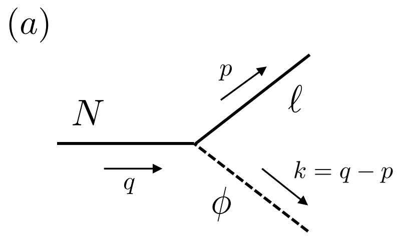

Figure 15: Decay of RH neutrino into (ℓ, ϕ) and $(\bar{\ell}, \phi^*)$.

for (5.28) where the following relation

$$\gamma_h^{\overline{\ell}^\alpha \overline{\phi}} = (\gamma_{-h}^{\ell^\alpha \phi})^* \tag{5.32}$$

is used. The decay matrix $\gamma_h^{\ell^\alpha \phi}$ is given by

$$\left(\gamma_h^{\ell^\alpha \phi}\right)_{ij} \equiv (h^\dagger_{i\alpha} h_{\alpha j}) g_w \left(q \cdot p - h(\omega_q \frac{\mathbf{q} \cdot \mathbf{p}}{|\mathbf{q}|} - \omega_p |\mathbf{q}|) \right), \tag{5.33}$$

where we have used the relation

$$\mathrm{tr}\left((\not{q} + M) \frac{1 + h\mathbf{n} \cdot \boldsymbol{\sigma}}{2} \frac{1 - \gamma^5}{2} \not{p} \right) = \left(q \cdot p - h(\omega_q \frac{\mathbf{q} \cdot \mathbf{p}}{|\mathbf{q}|} - \omega_p |\mathbf{q}|) \right). \tag{5.34}$$

The first term $q \cdot p$ is even under the helicity flip $h \to -h$, while the second term is odd. The integral

$$\int \frac{d^3 p d^3 k}{2\omega_p 2\omega_k} \delta^4(q - p - k)(\omega_q \frac{\mathbf{q} \cdot \mathbf{p}}{|\mathbf{q}|} - \omega_p |\mathbf{q}|) \tag{5.35}$$

vanishes when thermal effects of the SM particles, namely the thermal mass ($\sim gT$) and the statistical factor (Pauli blocking) of leptons, are neglected.

The kinetic equaction (5.29) describes an evolution of the density matrix f_N of the RH neutrinos. Since the equilibrium distribution satisfies the detailed balance condition, the r.h.s. is non-vanishing only when various quantities are out-of-equilibrium. We take a variation of (5.29) around the equilibrium. Here note that the relations $\delta f_\ell = -\delta f_{\overline{\ell}}$, $\delta f_\phi = -\delta f_{\overline{\phi}}$ hold since the SM gauge particles are in thermal equilibrium and their chemical potentials are vanishing.

In order to solve the kinetic equations, it is convenient to define helicity even and odd combinations $\delta f_{N,q}^{even,odd}$ by

$$\delta f_{N,q}^{even} \equiv \delta f_{N,+,q} + \delta f_{N,-,q}\ , \quad \delta f_{N,q}^{odd} \equiv \delta f_{N,+,q} - \delta f_{N,-,q}. \tag{5.36}$$

Since helicity operator $\mathbf{n} \cdot \boldsymbol{\sigma}$ is parity-odd and RH neutrino is invariant under the charge conjugation, $\delta f_{N,q}^{even,odd}$ are CP-even and odd components respectively;

In terms of these components, eq. (5.29) with the cut-diagram in Figure 14(a) can be rewritten as a set of equations

$$\begin{aligned}
&d_t(2f^{eq}_{N,q}+\delta f^{even}_{N,q})\\
=&-i\left[\frac{\omega^{\rm eff}_{q+}+\omega^{\rm eff}_{q-}}{2},\delta f^{even}_{N,q}\right]-i\left[\frac{\omega^{\rm eff}_{q+}-\omega^{\rm eff}_{q-}}{2},\delta f^{odd}_{N,q}\right]\\
&-\frac{1}{2}\frac{1}{2\omega_q}\int d\Pi_{pk}\sum_\alpha\{\Re(\gamma^{\ell^\alpha\phi}_{+}+\gamma^{\ell^\alpha\phi}_{-}),\delta f^{even}_{N,q}\}(1-f^{eq}_{\ell^\alpha p}+f^{eq}_{\phi k})\\
&-\frac{1}{2}\frac{1}{2\omega_q}\int d\Pi_{pk}\sum_\alpha\{i\Im(\gamma^{\ell^\alpha\phi}_{+}-\gamma^{\ell^\alpha\phi}_{-}),\delta f^{odd}_{Nq}\}(1-f^{eq}_{\ell^\alpha p}+f^{eq}_{\phi k})\\
&+\frac{1}{2\omega_q}\int d\Pi_{pk}\sum_\alpha\ i\Im(\gamma^{\ell^\alpha\phi}_{+}+\gamma^{\ell^\alpha\phi}_{-})\big(\delta f_{\ell^\alpha p}(f_{\phi k}+f^{eq}_{N,q})+\delta f_{\phi k}(f_{\ell^\alpha p}-f^{eq}_{N,q})\big),
\end{aligned}\tag{5.37}$$

$$\begin{aligned}
&d_t(\delta f^{odd}_{Nq})=-i\left[\frac{\omega^{\rm eff}_{q+}+\omega^{\rm eff}_{q-}}{2},\delta f^{odd}_{Nq}\right]-i\left[\frac{\omega^{\rm eff}_{q+}-\omega^{\rm eff}_{q-}}{2},\delta f^{even}_{Nq}\right]\\
&-\frac{1}{2}\frac{1}{2\omega_q}\int d\Pi_{pk}\sum_\alpha\{\Re(\gamma^{\ell^\alpha\phi}_{+}+\gamma^{\ell^\alpha\phi}_{-}),\delta f^{odd}_{Nq}\}(1-f^{eq}_{\ell^\alpha p}+f^{eq}_{\phi k})\\
&-\frac{1}{2}\frac{1}{2\omega_q}\int d\Pi_{pk}\sum_\alpha\{i\Im(\gamma^{\ell^\alpha\phi}_{+}-\gamma^{\ell^\alpha\phi}_{-}),\delta f^{even}_{Nq}\}(1-f^{eq}_{\ell^\alpha p}+f^{eq}_{\phi k})\\
&+\frac{1}{2\omega_q}\int d\Pi_{pk}\sum_\alpha\Re(\gamma^{\ell^\alpha\phi}_{+}-\gamma^{\ell^\alpha\phi}_{-})\big(\delta f_{\ell^\alpha p}(f_{\phi k}+f^{eq}_{N,q})+\delta f_{\phi k}(f_{\ell^\alpha p}-f^{eq}_{N,q})\big).
\end{aligned}\tag{5.38}$$

If we can neglect the helicity odd part of the decay width $\gamma^{\ell\phi}_h$ as discussed in (5.35) and the backreaction from lepton asymmetry (the last terms) is dropped, these equations for δf^{even} and δf^{odd} are almost decoupled. Note that the helicity dependent mass term $(\omega_+-\omega_-)$ is also negligible if thermal corrections are small.

The dominant source to generate deviations is the time variation of the local equilibrium distribution $d_t f^{eq}$, which is absent in the equation of δf^{odd}. Hence in the decoupling limit, it is sufficient to consider only the equation for for δf^{even}. In section 5.3.4, we obtain the CP-violating parameter under such a condition.

5.2.2 Kinetic equation for lepton number

The evolution equation for the lepton number is similarly obtained from the KB equation. Details of the derivation is given in Section 3.3 and 3.4. α-th flavor

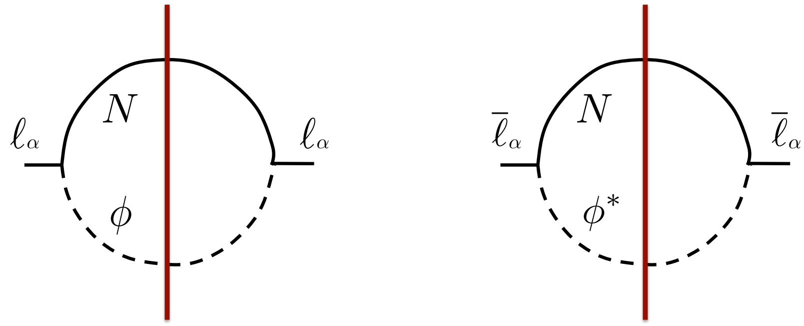

Figure 16: Cutting the self-energy diagram Σ of leptons ℓ. The cut diagram is the same as Figure 15 (a).

lepton number current is defined by

$$\begin{aligned}\sum_a \langle \bar{\hat{\ell}}^\alpha_a(x)\gamma^\mu(x)\ell^\alpha_a\rangle &= -\sum_a \operatorname{tr}\{\gamma(x)S^{\alpha\alpha}_{aa\lessgtr}(x,y)\}\Big|_{y=x} \\ &= -g_w \operatorname{tr}\{\gamma(x)S^{\alpha\alpha}_{\lessgtr}(x,y)\}\Big|_{y=x} \end{aligned} \tag{5.39}$$

where a is an $SU(2)$ isospin index. Around TeV scale, the charged Yukawa couplings distinguishing the lepton flavors are in equilibrium and the off-diagonal components of lepton flavor density matrix are negligible compared to diagonal ones. In the second equality, we have assumed that $SU(2)$ isospin symmetry is restored.

Since the derivative expansion is an expansion of $H/\Gamma_{\ell,\phi}$, higher order terms are highly suppressed and we have

$$\begin{aligned} & d_t n_{L^\alpha} + 3Hn_{L^\alpha} \\ &\quad = g_w \int d\Pi_p \Big[\operatorname{tr}\left[P_L \not{p}\Sigma^{\alpha\alpha}_<(p)\right](1-f_{\ell^\alpha p}) + \operatorname{tr}\left[P_L\not{p}\Sigma^{\alpha\alpha}_>(p)\right] f_{\ell^\alpha p} \\ &\qquad\qquad -\operatorname{tr}\left[P_L\not{p}\overline{\Sigma}^{\alpha\alpha}_<(p)\right](1-f_{\overline{\ell}^\alpha p}) - \operatorname{tr}\left[P_L\not{p}\overline{\Sigma}^{\alpha\alpha}_>(p)\right] f_{\overline{\ell}^\alpha p}\Big]. \end{aligned} \tag{5.40}$$

Σ is the self-energy of the SM lepton ℓ. If we consider, as an example, the Yukawa interaction of (ℓ, ϕ, N), the self-energy function for leptons in Figure 16 gives the same cut diagram Fig. 15 (a). By using the same $\gamma_h^{\ell^\alpha\phi}$ in (5.33), the kinetic equation is reduced to the following Boltzmann equation;

$$\begin{aligned} & d_t n_{L^\alpha} + 3Hn_{L^\alpha} \\ &= \sum_h \int d\Pi_{qpk}\Big[\operatorname{Tr}\left[\gamma_h^{\ell^\alpha\phi}\{f_{N,h,q}(1-f_{\ell^\alpha p})(1+f_{\phi k}) - (1-f_{N,h,q})f_{\ell^\alpha p}f_{\phi k}\}\right] \\ &\qquad -\operatorname{Tr}\left[(\gamma_{-h}^{\ell^\alpha\phi})^*\left\{f_{N,h,q}(1-f_{\overline{\ell}^\alpha p})(1+f_{\overline{\phi}k}) - (1-f_{N,h,q})f_{\overline{\ell}^\alpha p}f_{\overline{\phi}k}\right\}\right]\Big]. \end{aligned} \tag{5.41}$$

Here Tr is trace of the RH neutrino flavour.

5.2.3 Kinetic equations in terms of Yield variables

We rewrite the kinetic equations, (5.37), (5.38) and (5.41), in terms of the Yield variables Y defined by

$$Y_N^{eq} = \frac{2}{s}\int \frac{d^3q}{(2\pi)^3} f_{N,q}^{eq} \ , \ Y_{\ell^\alpha}^{eq} = \frac{g_w}{s}\int \frac{d^3p}{(2\pi)^3} f_{\ell^\alpha p}^{eq} \ , \ Y_{L^\alpha} = \frac{g_w}{s}\int \frac{d^3p}{(2\pi)^3}\left(\delta f_{\ell^\alpha} - \delta f_{\bar{\ell}^\alpha}\right). \tag{5.42}$$

Here s is the entropy of the universe. Note that Y_N is a flavour matrix while Y_ℓ^α is a c-number (or α-th eigenvalue of a diagonal flavour matrix). In the following, we consider deviations of distribution functions of RH neutrinos N_i and charged leptons ℓ_α, and other SM particles are assumed to be in the equilibrium distributions. We assume N_i and ℓ are in the kinematical equilibrium. Then we can set

$$\frac{\delta f_{N,q}^{even}}{f_{N,q}^{eq}} = 2\frac{\delta Y_N^{even}}{Y_N^{eq}} \ , \ \frac{\delta f_{N,q}^{odd}}{f_{N,q}^{eq}} = 2\frac{\delta Y_N^{odd}}{Y_N^{eq}} \ , \ \frac{\delta f_{\ell^\alpha}}{f_{\ell^\alpha}^{eq}} = \frac{Y_{L^\alpha}}{2Y_{\ell^\alpha}^{eq}}. \tag{5.43}$$

Since the equations for δY are approximated by coupled linear differential equations, equations (5.37), (5.38) can be written in a generic form with matrices $\mathrm{H}, \widetilde{\mathrm{H}}, \Gamma_N, \widetilde{\Gamma_N}, \Gamma_L, \widetilde{\Gamma_L}$;

$$\begin{aligned} d_t(Y_N^{eq} + \delta Y_N^{even}) &= -i[\mathrm{H}, \delta Y_N^{even}] - i[\widetilde{\mathrm{H}}, \delta Y_N^{odd}] \\ &\quad - \frac{1}{2}\{\Gamma_N, \delta Y_N^{even}\} - \frac{1}{2}\{\widetilde{\Gamma}_N, \delta Y_N^{odd}\} + \sum_\alpha \Gamma_{L^\alpha} Y_{L^\alpha} \ , \end{aligned} \tag{5.44}$$

$$\begin{aligned} d_t(\delta Y_N^{odd}) &= -i[\mathrm{H}, \delta Y_N^{odd}] - i[\widetilde{\mathrm{H}}, \delta Y_N^{even}] \\ &\quad - \frac{1}{2}\{\Gamma_N, \delta Y_N^{odd}\} - \frac{1}{2}\{\widetilde{\Gamma}_N, \delta Y_N^{even}\} + \sum_\alpha \widetilde{\Gamma}_{L^\alpha} Y_{L^\alpha}. \end{aligned} \tag{5.45}$$

In the model with only Yukawa interactions, these matrices are given as follows:

$$\begin{aligned} \mathrm{H} &\equiv \frac{2}{sY_N^{eq}}\int \frac{d^3q}{(2\pi)^3} f_{N,q}^{eq} \frac{\omega_{+,q}^{\mathrm{eff}} + \omega_{-,q}^{\mathrm{eff}}}{2} \ , \\ \widetilde{\mathrm{H}} &\equiv \frac{2}{sY_N^{eq}}\int \frac{d^3q}{(2\pi)^3} f_{N,q}^{eq} \frac{\omega_{+,q}^{\mathrm{eff}} - \omega_{-,q}^{\mathrm{eff}}}{2} \ , \end{aligned} \tag{5.46}$$

$$\Gamma_N = \Re\left(\sum_\alpha \Gamma_\alpha\right) \ , \ \widetilde{\Gamma}_N = i\Im\left(\sum_\alpha \widetilde{\Gamma}_\alpha\right) \ , \ \Gamma_{L^\alpha} = i\Im[\Gamma_\alpha^W] \tag{5.47}$$

$$\widetilde{\Gamma}_{L^\alpha} \equiv \tfrac{1/s}{2Y_{\ell^\alpha}^{eq}} \int d\Pi_{qpk} \Re(\gamma_+^{\ell^\alpha\phi} - \gamma_-^{\ell^\alpha\phi}) f_{\ell^\alpha p}^{eq}(f_{\phi k} + f_{N,q}^{eq}), \tag{5.48}$$

where[26]

$$\Gamma_\alpha \equiv \frac{2}{sY_N^{eq}} \int d\Pi_{qpk} (\gamma_+^{\ell^\alpha\phi} + \gamma_-^{\ell^\alpha\phi}) f_{N,q}^{eq} (1 - f_{\ell^\alpha p}^{eq} + f_{\phi k}^{eq}) \tag{5.49}$$

$$\widetilde{\Gamma}_\alpha \equiv \frac{2}{sY_N^{eq}} \int d\Pi_{qpk} (\gamma_+^{\ell^\alpha\phi} - \gamma_-^{\ell^\alpha\phi}) f_{N,q}^{eq} (1 - f_{\ell^\alpha p}^{eq} + f_{\phi k}^{eq}) \tag{5.50}$$

$$\Gamma_\alpha^W \equiv \frac{1/s}{2Y_{\ell^\alpha}^{eq}} \int d\Pi_{qpk} (\gamma_+^{\ell^\alpha\phi} + \gamma_-^{\ell^\alpha\phi}) f_{\ell^\alpha p}^{eq} (f_{\phi k} + f_{Nq}^{eq}) \ . \tag{5.51}$$

Similarly the kinetic equation for lepton number (5.41) is also rewritten as

$$\begin{aligned}
d_t Y_{L^\alpha} =& \mathrm{Tr}\left[2 \int d\Pi_{qpk}\ i\Im(\gamma_+^{\ell^\alpha\phi} + \gamma_-^{\ell^\alpha\phi}) f_{N,q}^{eq} (1 - f_{\ell^\alpha p}^{eq} + f_{\phi k}^{eq}) \frac{\delta Y_N^{even}}{sY_N^{eq}} \right] \\
&+ \mathrm{Tr}\left[2 \int d\Pi_{qpk}\ \Re(\gamma_+^{\ell^\alpha\phi} - \gamma_-^{\ell^\alpha\phi}) f_{N,q}^{eq} (1 - f_{\ell^\alpha p}^{eq} + f_{\phi k}^{eq}) \frac{\delta Y_N^{odd}}{sY_N^{eq}} \right] \\
&- \left[\int d\Pi_{qpk}\ \mathrm{Tr}[\Re(\gamma_+^{\ell^\alpha\phi} + \gamma_-^{\ell^\alpha\phi})] f_{N,q}^{eq} (1 + f_{\phi k}^{eq}) \frac{Y_{L^\alpha}}{s2Y_{\ell^\alpha}^{eq}} \right] \qquad (5.52) \\
=& \mathrm{Tr}\Big[i\Im[\Gamma_\alpha] \delta Y_N^{even} \Big] + \mathrm{Tr}\Big[\Re[\widetilde{\Gamma}_\alpha] \delta Y_N^{odd} \Big] - \mathrm{Tr}\Big[\Re[\Gamma_\alpha^W] \Big] Y_{L^\alpha} \ . \qquad (5.53)
\end{aligned}$$

Hence the lepton asymmetry is generated if the r.h.s. is nonvanishing. CP-violating parameter ε can be read from the equation by inserting solutions of the kinetic equations for δY_N^{even} (5.44) and δY_N^{odd} (5.45).

5.3 Solution of the kinetic equations

In order to obtain the CP-violating parameter, we solve the kinetic equations for δY_N. In the derivation of the kinetic equation from the KB equation, we assumed that the system is not far from the local equilibrium at each time of the expanding universe. But smallness of the off-diagonal Yukawa coupling is *not* assumed, and the coherent flavor oscillation is fully taken into account. Since the deviation from local equilibrium is caused by the Hubble expansion, both of δ and ∂_t are proportional to the Hubble parameter H. Hence we can set

$$d_t(\delta Y_N^{even}) \simeq 0 \ , \quad d_t(\delta Y_N^{odd}) \simeq 0 \tag{5.54}$$

in the l.h.s. of Eq. (5.44), (5.45) under the condition $H \ll \Gamma_i \ll \Gamma_{\ell,\phi}$. This is the approximation employed in section 2.2 to obtain the analytic formula of the efficiency factor (2.91) in the strong washout regime where $\mathcal{K}_1 = \Gamma_1/H(T = M_1) \gg 1$.

[26] The real and imaginary properties of Γ_N and $\widetilde{\Gamma_N}$ are valid when we neglect the direct CP-violation, an interference between the tree and one-loop vertex corrections. In the resonant leptogenesis, this approximation is justified.

5.3.1 Formal solution of δY_N

In the two-flavor case, Y_N, H, Γ_N etc. are 2×2 matrices. We here express a 2×2 matrix A as $A=\sum_{a=0}^{3}[A]^a\sigma^a$ where $\sigma^0=1_{2\times 2}$ and σ^i $(i=1,2,3)$ is the Pauli matrix. Then Eqs. (5.44), (5.45) are rewritten as

$$\begin{aligned}[d_tY_N^{eq}]^a &= C^{ab}[\delta Y_N^{even}]^b+\widetilde{C}^{ab}[\delta Y_N^{odd}]^b+[\mu]^a\\ 0 &= C^{ab}[\delta Y_N^{odd}]^b+\widetilde{C}^{ab}[\delta Y_N^{even}]^b+[\widetilde{\mu}]^a\end{aligned} \tag{5.55}$$

where

$$\begin{aligned}C^{ab} &\equiv -\left(\delta^{ab}[\Gamma_N]^0+\delta_0^a\delta_i^b[\Gamma_N]^i+\delta_i^a\delta_0^b[\Gamma_N]^i+2\delta_i^a\delta_j^b\epsilon^{ijk}[\mathrm{H}]^k\right),\\ \widetilde{C}^{ab} &\equiv -\left(\delta^{ab}[\widetilde{\Gamma}_N]^0+\delta_0^a\delta_i^b[\widetilde{\Gamma}_N]^i+\delta_i^a\delta_0^b[\widetilde{\Gamma}_N]^i+2\delta_i^a\delta_j^b\epsilon^{ijk}[\widetilde{\mathrm{H}}]^k\right),\\ [\mu]^a &\equiv \sum_\alpha[\Gamma_{L^\alpha}]^aY_{L^\alpha}\ ,\quad [\widetilde{\mu}]^a\equiv\sum_\alpha[\widetilde{\Gamma}_{L^\alpha}]^aY_{L^\alpha}\ .\end{aligned} \tag{5.56}$$

The Yield density matrix $Y_N^{(eq)}$ in equilibrium has only $a=0$ component

$$[d_tY_N^{eq}]^a=\delta_0^a(d_tY_N^{eq})\ . \tag{5.57}$$

From (5.46), (5.47) and (5.48), H,Γ_N and $\widetilde{\Gamma}_L$ (hence $\widetilde{\mu}$) are real matrices. Hence $[\Gamma_N]^a,[\mathrm{H}]^a,[\widetilde{\mu}]^a$ do not have an $a=2$ component. On the other hand, $[\widetilde{\Gamma}_N]^a,[\widetilde{\mathrm{H}}]^a,[\mu]^a$ have only an $a=2$ component since they are imaginary matrices[27].

The equations (5.55) are linear equations with respect to δY_N and can be solved in terms of the time-variation of the local equilibrium distribution $d_tY_N^{(eq)}$ and the lepton asymmetry $\mu,\widetilde{\mu}$ as

$$\begin{pmatrix}[\delta Y_N^{even}]\\ [\delta Y_N^{odd}]\end{pmatrix}=\mathbf{C}^{-1}\begin{pmatrix}[d_tY_N^{eq}]-[\mu]\\ -[\widetilde{\mu}]\end{pmatrix},\quad \mathbf{C}\equiv\begin{pmatrix}C & \widetilde{C}\\ \widetilde{C} & C\end{pmatrix}\ . \tag{5.58}$$

In the expanding universe, the deviation of RH neutrino number densities from equilibrium δY_N is first generated and then lepton asymmetry Y_L is generated by the flavor oscillation and decay. For the moment, we neglect backreaction from Y_L and evaluate the deviation of RH neutrino density directly caused by the expansion of universe. Setting $\widetilde{\mu}=0$, δY_N is solved as

$$\begin{aligned}[\delta Y_N^{even}]^a &= (\mathcal{C}^{-1})^{ab}[d_tY_N^{eq}]^b=(\mathcal{C}^{-1})^{a0}\times d_tY_N^{eq}\ ,\\ [\delta Y_N^{odd}]^a &= (\widetilde{\mathcal{C}}^{-1})^{ab}[d_tY_N^{eq}]^b=(\widetilde{\mathcal{C}}^{-1})^{a0}\times d_tY_N^{eq}\end{aligned} \tag{5.59}$$

where

$$\mathbf{C}^{-1}\equiv\begin{pmatrix}\mathcal{C}^{-1} & \widetilde{\mathcal{C}}^{-1}\\ \widetilde{\mathcal{C}}^{-1} & \mathcal{C}^{-1}\end{pmatrix}\ . \tag{5.60}$$

[27] Flavor covariance is explicitly broken by setting the Majorana mass matrix of the RH neutrinos diagonal with eigenvalues M_1,M_2.

Components in the 0-th column of $\mathbf{C}^{-1}$ are given by

$$
\begin{aligned}
(\mathcal{C}^{-1})^{00} =&\frac{-1}{D}[\Gamma_N]^0 \left\{([\Gamma_N]^0)^2 + 4([\mathrm{H}\cdot\mathrm{H}] + [\widetilde{\mathrm{H}}\cdot\widetilde{\mathrm{H}}])\right\},\\
(\mathcal{C}^{-1})^{i0} =&\frac{1}{D}\big\{([\Gamma_N]^0)^2[\Gamma_N]^i + 4([\Gamma_N\cdot\mathrm{H}] - [\widetilde{\Gamma}_N\cdot\widetilde{\mathrm{H}}])[\mathrm{H}]^i - 2[\Gamma_N]^0\epsilon^{ijk}[\Gamma_N]^j[\mathrm{H}]^k\big\},\\
(\widetilde{\mathcal{C}}^{-1})^{00} =&0,\\
(\widetilde{\mathcal{C}}^{-1})^{i0} =&\frac{1}{D}\big\{([\Gamma_N]^0)^2[\widetilde{\Gamma}_N]^i + 4([\Gamma_N\cdot\mathrm{H}] - [\widetilde{\Gamma}_N\cdot\widetilde{\mathrm{H}}])[\widetilde{\mathrm{H}}]^i\\
&\qquad - 2[\Gamma_N]^0\epsilon^{ijk}[\Gamma_N]^j[\widetilde{\mathrm{H}}]^k - 2[\Gamma_N]^0\epsilon^{ijk}[\widetilde{\Gamma}_N]^j[\mathrm{H}]^k\big\},
\end{aligned}
\tag{5.61}
$$

where D is the determinant,

$$
\begin{aligned}
D \equiv&([\Gamma_N]^0)^2\big\{([\Gamma_N]^0)^2 - [\Gamma_N\cdot\Gamma_N] + [\widetilde{\Gamma}_N\cdot\widetilde{\Gamma}_N] + 4([\mathrm{H}\cdot\mathrm{H}] + [\widetilde{\mathrm{H}}\cdot\widetilde{\mathrm{H}}])\big\}\\
&- 4\big\{[\Gamma_N\cdot\mathrm{H}] + [\widetilde{\Gamma}_N\cdot\widetilde{\mathrm{H}}]\big\}^2 .
\end{aligned}
\tag{5.62}
$$

$[\,\cdot\,]$ denotes a summation over $i = 1, 2, 3$.

5.3.2 CP-violation parameter ε

In order to read the effective CP-violating parameter ε, we set $Y_L = 0$ and insert (5.59) into the kinetic equation of the lepton numbers (5.53),

$$
\begin{aligned}
d_t Y_{L^\alpha} =&\mathrm{Tr}\left[i\Im(\Gamma_\alpha)\delta Y_N^{even}\right] + \mathrm{Tr}\left[\Re(\widetilde{\Gamma}_\alpha)\delta Y_N^{odd}\right]\\
=&2[\Gamma_\alpha]^2[\delta Y_N^{even}]^2 + 2\sum_{a=0,1,3}[\widetilde{\Gamma}_\alpha]^a[\delta Y_N^{odd}]^a\\
=&2\left\{[\Gamma_\alpha]^2(\mathcal{C}^{-1})^{20} + [\widetilde{\Gamma}_\alpha]^1(\widetilde{\mathcal{C}}^{-1})^{10} + [\widetilde{\Gamma}_\alpha]^3(\widetilde{\mathcal{C}}^{-1})^{30}\right\} \times d_t Y_N^{eq}\\
=&\frac{4[\Gamma_N]^0}{D}\epsilon^{ijk}\left\{[\Gamma_\alpha]^i[\Gamma_N]^j[\mathrm{H}]^k + [\widetilde{\Gamma}_\alpha]^i[\Gamma_N]^j[\widetilde{\mathrm{H}}]^k + [\widetilde{\Gamma}_\alpha]^i[\widetilde{\Gamma}_N]^j[\mathrm{H}]^k\right\} \times (-d_t Y_N^{eq})
\end{aligned}
\tag{5.63}
$$

The r.h.s. can be rewritten in terms of $2[\delta Y_N]^0 = \mathrm{Tr}(\delta Y_N)$, which is the total RH neutrino number deviated from the local equilibrium. Especially, neglecting the difference of helicity, we can write the r.h.s. of (5.63) in terms of $[\delta Y_N^{even}]^0$ in (5.59) as

$$
d_t Y_{L^\alpha} =2\varepsilon^\alpha[\Gamma_N]^0[\delta Y_N^{even}]^0 . \tag{5.64}
$$

Here $[\Gamma_N]^0$ is an averaged decay rate of RH neutrinos into charged lepton ℓ^α. The CP-violating parameter ε^α defined by the coefficient[28] is read as

$$\begin{aligned}\varepsilon^\alpha &= \frac{2\epsilon^{ijk}\left\{[\Gamma_\alpha]^i[\Gamma_N]^j[\mathrm{H}]^k + [\widetilde{\Gamma}_\alpha]^i[\Gamma_N]^j[\widetilde{\mathrm{H}}]^k + [\widetilde{\Gamma}_\alpha]^i[\widetilde{\Gamma}_N]^j[\mathrm{H}]^k\right\}}{\left(([\Gamma_N]^0)^2 + 4([\mathrm{H}\cdot\mathrm{H}] + [\widetilde{\mathrm{H}}\cdot\widetilde{\mathrm{H}}])\right)[\Gamma_N]^0} \\ &= -i\frac{\mathrm{tr}\left(\Gamma_\alpha\Gamma_N\mathrm{H} + \widetilde{\Gamma}_\alpha\Gamma_N\widetilde{\mathrm{H}} + \widetilde{\Gamma}_\alpha\widetilde{\Gamma}_N\mathrm{H}\right)}{\left(([\Gamma_N]^0)^2 + 4([\mathrm{H}\cdot\mathrm{H}] + [\widetilde{\mathrm{H}}\cdot\widetilde{\mathrm{H}}])\right)[\Gamma_N]^0}. \end{aligned} \tag{5.65}$$

The result is valid when it is justified to replace d_tY_N by its equilibrium value $d_tY_N^{eq}$. Though our calculation fixes the flavor basis in which the Majorana masses are diagonal, the final form is written in a flavor covariant way. The above definition of ε is appropriate since the numerator of the ordinary definition

$$\varepsilon \equiv \frac{\Gamma_{N\to\ell\phi} - \Gamma_{N\to\overline{\ell\phi}}}{\Gamma_{N\to\ell\phi} + \Gamma_{N\to\overline{\ell\phi}}} \tag{5.66}$$

is replaced by $d_tY_L/2[\delta Y_N]^0$ while the denominator is approximated by Γ_N.

5.3.3 Explicit forms of δY_N

In this section, we use explicit forms of various quantities to rewrite the formal expression (5.65) in a more familiar form.

H ($\widetilde{\mathrm{H}}$) is the helicity even (odd) part of the mass (with thermal corrections included) and given in (5.46). $\widetilde{\mathrm{H}}$ has an $a = 2$ component only. For H, $a = 0$ component is the total mass and decouples from the equation. $a = 3$ component of H gives the mass difference

$$2[\mathrm{H}]^3 = \frac{\xi_0}{sY_N^{eq}}(M_1 - M_2) + \cdots \tag{5.67}$$

where

$$\xi_0 \equiv 2M\int\frac{dq^3}{(2\pi)^3}\frac{1}{\omega_q}f_{Nq}^{eq}. \tag{5.68}$$

The $\cdots$ in $[\mathrm{H}]^3$ represents finite temperature (and density) corrections to the RH neutrino potential. Off-diagonal components $[\mathrm{H}]^1$ and $[\widetilde{\mathrm{H}}]^2$ represent kinetic mixing induced by the thermal effects, and can be removed by flavor rotation at

[28]Such a definition of ε was also adopted in [59][60]. They concluded that, since the quantity corresponding to $[\delta Y_N^{even}]^2$ oscillates with time as can be seen from eq.(5.44), the lepton asymmetry also behaves similar oscillatory behavior. Such behavior is interpreted in their analysis as an oscillating CP-violating parameter by expressing $[\delta Y_N]^2$ in terms of the non-oscillatory quantity $[\delta Y_N]^0$. In the strong washout regime, the effect of the oscillation is averaged out. The averaged CP-violating parameter in the papers [59][60] is inconsistent with ours. The discrepancy seems to be caused by neglecting one of the decay widths in their analysis, corresponding a partial truncation of the self-energy diagrams.

each time. Unitary matrix diagonalizing the mass matrix is time dependent, but in the following analysis, we neglect time-dependence of the thermal mass and mixing. If we neglect the statistical effects, the coefficient in $[H]^3$ is given by $(\xi_0/sY_N^{eq}) = K_1(M/T)/K_2(M/T)$. At low temperature $T \ll M$ it approaches $(\xi_0/sY_N^{eq}) \to 1$ while at high temperature $T \gg M$, it behaves as $(\xi_0/sY_N^{eq}) \sim M/(2T)$.

Γ_N comes from the self-energy diagrams of RH neutrinos, and contains information of (inverse) decay or scattering of RH neutrinos. We decompose Γ_N into Γ_α by fixing the flavor α of lepton ℓ^α in the final state. Only the real part appears in the KB equation. From (5.47), we can decompose Γ_N in the model (2.2) as

$$\Gamma_N = \frac{\xi}{sY_N^{eq}} \frac{\Re(h^\dagger h)M}{8\pi} + \Gamma_N^{\text{scatt}} + \Gamma_N^{\text{vertex}} , \tag{5.69}$$

where

$$\xi \equiv 32\pi \left(M - \frac{m_\phi^2 - m_\ell^2}{M} \right) \int d\Pi_{N\ell^\alpha\phi} f_{Nq}^{eq}(1 - f_{\ell p}^{eq} + f_{\phi k}^{eq}) . \tag{5.70}$$

Γ_α is a partial decay width that RH neutrino decays into ℓ^α. At the leading order, it is given by replacing $(h^\dagger h)_{ij}$ in (5.69) by $(h^\dagger_{i\alpha} h_{\alpha j})$ (no summation over α).

The first term of Γ_N is the decay amplitude at the tree level and if we neglect the statistical effects and the thermal mass of the Higgs and lepton, ξ coincides with ξ_0, and approaches

$$(\xi/sY_N^{eq}) = (\xi_0/sY_N^{eq}) \to M/(2T) \tag{5.71}$$

at high temperature. Γ_N^{scatt} are corrections to the decay rate from scattering with the top quarks or gauge particles in the thermal media. Γ_N^{vertex} are corrections to the vertex diagram. It is negligible compared to the first term. In the resonant leptogenesis, the direct CP-violating parameter associated with an interference between the tree and the vertex correction can be neglected compared to the indirect CP-violation through the flavor oscillation. Then the relations $[\Gamma_N]^2 = [\widetilde{\Gamma}_N]^{0,1,3} = 0$ hold. (See footnote 26.)

In order to simplify the notation, we write

$$(\Gamma_N)_{ij} = \frac{\xi_0}{sY_N^{eq}} \Gamma_{ij}^{\text{eff}} , \quad (\widetilde{\Gamma}_N)_{ij} = \frac{\xi_0}{sY_N^{eq}} \widetilde{\Gamma}_{ij}^{\text{eff}} \tag{5.72}$$

where Γ_{ij}^{eff} and $\widetilde{\Gamma}_{ij}^{\text{eff}}$ are effective decay rates including not only thermal effects but also scattering contributions. If interactions do not change the flavor structure, the effective decay matrix is written as

$$\Gamma_{ij}^{\text{eff}} = (1+\alpha)M \frac{\Re(h^\dagger h)_{ij}}{8\pi} , \quad \widetilde{\Gamma}_{ij}^{\text{eff}} = \tilde{\alpha} M \frac{i\Im(h^\dagger h)_{ij}}{8\pi} . \tag{5.73}$$

for $a = 1, 2, 3$ component. Furthermore, if we consider flavor independent interactions such as $B-L$ gauge interaction of RH neutrinos, an additional contribution is added to $a = 0$ component $[\Gamma_N]^0$. In the following, we neglect this contribution for simplicity. When we neglect thermal effects and scattering contributions, α and $\widetilde{\alpha}$ vanish and diagonal components of $\Gamma_{ii}^{\rm eff}$ are reduced to the tree-level vacuum decay rate $\Gamma_i^{\rm vac} \equiv (h^\dagger h)_{ii} M/(8\pi)$. In the following we write $\Gamma_i = \Gamma_{ii}^{\rm eff}$ as a decay rate including the above corrections.

Using these quantities of H and Γ_N, we can express each component of the inverse matrix $\mathcal{C}^{-1}$ in terms of masses M_i and decay rates Γ_i. The explicit forms are written in Appendix E.

By using the explicit forms of $\mathcal{C}^{-1}$ in Appendix E, we can write down each component of δY as follows. First, the diagonal components of δY_N^{even} $(a = 0, 3)$ are given by

$$[\delta Y_N^{even}]^0 = -\frac{d_t Y_N^{eq}}{\xi_0/(sY_N^{eq})}\frac{\Gamma_1 + \Gamma_2}{2\Gamma_1\Gamma_2}U, \tag{5.74}$$

$$[\delta Y_N^{even}]^3 = -\frac{d_t Y_N^{eq}}{\xi_0/(sY_N^{eq})}\frac{-\Gamma_1 + \Gamma_2}{2\Gamma_1\Gamma_2}U, \tag{5.75}$$

where

$$U \equiv \frac{(M_1^2 - M_2^2)^2 + M^2(\Gamma_1 + \Gamma_2)^2}{(M_1^2 - M_2^2)^2 + M^2(\Gamma_1 + \Gamma_2)^2 X} \ . \tag{5.76}$$

and

$$X = \frac{\det[\Re(h^\dagger h)](1+\alpha)^2 - (\widetilde{\alpha}\Im(h^\dagger h))^2}{(h^\dagger h)_{11}(h^\dagger h)_{22}(1+\alpha)^2} \ . \tag{5.77}$$

$[\delta Y_N^{even}]^0$ gives an averaged number of the RH neutrinos deviated from the local equilibrium. Equivalently, ii-component of the matrix δY_N^{even} is given by

$$(\delta Y_N^{even})_{ii} = [\delta Y_N^{even}]^0 \pm [\delta Y_N^{even}]^3 = -\frac{d_t Y_N^{eq}}{\xi_0/(sY_N^{eq})}\frac{U}{\Gamma_i} \tag{5.78}$$

where $\pm$ represents $i = 1, 2$ respectively.

Off-diagonal components can be similarly obtained. The real part $a = 1$ and the imaginary part $a = 2$ of δY_N^{even} are given by

$$[\delta Y_N^{even}]^1 = \Re\delta Y_{N12}^{even} = -2(1+\alpha)\Re[h^\dagger h]_{12}(\Gamma_1 + \Gamma_2) M V [\delta Y_N^{even}]^0, \tag{5.79}$$

$$[\delta Y_N^{even}]^2 = -\Im\delta Y_{N12}^{even} = -2(1+\alpha)\Re[h^\dagger h]_{12}(M_1^2 - M_2^2) V [\delta Y_N^{even}]^0. \tag{5.80}$$

For δY_N^{odd}, we have

$$[\delta Y_N^{odd}]^1 = \Re\delta Y_{N12}^{even} = 2\widetilde{\alpha}\Im[h^\dagger h]_{12}(\Gamma_1 + \Gamma_2) M V [\delta Y_N^{even}]^0, \tag{5.81}$$

$$[\delta Y_N^{odd}]^2 = -\Im\delta Y_{N12}^{even} = -2\widetilde{\alpha}\Im[h^\dagger h]_{12}(M_1^2 - M_2^2) V [\delta Y_N^{even}]^0. \tag{5.82}$$

Here we defined

$$V \equiv \frac{M^2/(8\pi)}{(M_1^2 - M_2^2)^2 + M^2(\Gamma_1 + \Gamma_2)^2}. \tag{5.83}$$

$[\delta Y_N^{even}]^2$ and $[\delta Y_N^{odd}]^1$ give the CP-violating parameter ε. It is given in a simplified case in the next section.

We comment on a situation when $\det[\Re(h^\dagger h)]$ becomes small. (For simplicity we set $\tilde{\alpha}$=0.) Then X and accordingly $[\delta Y_N^{even}]^0$ is largely enhanced. The situation corresponds to a case that an effective decay rate (cf.(5.44)) is small. Especially when the mass difference vanishes $M_1 = M_2$, it diverges at $\det[\Re(h^\dagger h)] = 0$, namely when $\det C = 0$. In such a situation, the deviation of RH neutrino number density becomes large and the assumption of our investigation, smallness of the deviation from local equilibrium, becomes invalid.

5.3.4 CP-violating parameter ε when $\widetilde{\mathcal{C}} = 0$

We write the formal expression of (5.65) in a more familiar form by introducing further simplifications. We neglect the thermal mass of leptons and drop the Pauli blocking terms. Then the helicity odd part of $\gamma_h^{\ell\phi}$ disappears as explained in (5.35) and the off-diagonal components $\widetilde{\mathcal{C}}$ connecting the CP-even and odd parts in δY vanish. Furthermore we use the vacuum value of Γ_N ($\alpha = \widetilde{\alpha} = 0$). Then, by using explicit forms of H in (5.67) and Γ_N in (5.72) with $\Gamma_{ij}^{\rm eff} = \Gamma_{ij}^{\rm vac}$, the CP-violating parameter ε^α is given by

$$\begin{aligned}
\varepsilon^\alpha &= \frac{2\epsilon^{ijk}[\Gamma_\alpha]^i[\Gamma_N]^j[\mathrm{H}]^k}{([\Gamma_N]^0)^2 + 4[\mathrm{H}\cdot\mathrm{H}]} \\
&= \frac{2\Re(h^\dagger h)_{12}\Im(h_{1\alpha}^\dagger h_{\alpha 2})}{((h^\dagger h)_{11} + (h^\dagger h)_{22})^2/4} \frac{(M_1^2 - M_2^2)M(\Gamma_1+\Gamma_2)/2}{(M_1^2 - M_2^2)^2 + M^2(\Gamma_1+\Gamma_2)^2} \ .
\end{aligned} \tag{5.84}$$

This CP-violating parameter has the regulator $M^2(\Gamma_1 + \Gamma_2)^2$ which is consistent with our previous result (4.90). In the previous analysis we obtained the same result under an assumption that the off-diagonal Yukawa couplings are smaller than the diagonal ones. In the present analysis, we do not use such a condition, and take effects of coherent flavor oscillation fully into account. The decay widths $\Gamma_i^{\rm eff}$ are determined by the effective decay width (5.72), which are obtained from the 1PI self-energy diagrams Π by cutting the diagrams and putting external lines on mass-shell.

Finally we note that we can decompose the r.h.s. of (5.64) into N_i ($i = 1, 2$) as

$$d_t Y_{L^\alpha} = \sum_{i=1,2} \varepsilon_i^\alpha \, (\Gamma_N)_{ii} \, (\delta Y_N^{even})_{ii} \tag{5.85}$$

where we define the CP-violating parameter of each N_i as

$$\varepsilon_i^\alpha = \frac{2\Re(h^\dagger h)_{12}\Im(h_{1\alpha}^\dagger h_{\alpha 2})}{(h^\dagger h)_{11}(h^\dagger h)_{22}} \frac{(M_1^2 - M_2^2)M\Gamma_{j(\neq i)}}{(M_1^2 - M_2^2)^2 + M^2(\Gamma_1+\Gamma_2)^2} \ . \tag{5.86}$$

When $i=1$, j takes 2, and vice-versa. Such a separation into a different flavor of RH neutrinos is, of course, valid only when the off-diagonal component $(h^\dagger h)$ is smaller than the diagonal one. The numerator of the first factor can be rewritten as

$$2\Re(h^\dagger h)_{12}\Im(h^\dagger_{1\alpha}h_{\alpha 2}) = \Im[(h^\dagger h)_{12}(h^\dagger_{1\alpha}h_{\alpha 2})] + \Im[(h^\dagger h)_{21}(h^\dagger_{1\alpha}h_{\alpha 2})] \tag{5.87}$$

which gives a consistent result with (2.28).

5.4 Final lepton asymmetry

Finally, we calculate the final yield value of the lepton number. In the resonant case, because of $\varepsilon \sim 1$, the generated lepton asymmetry can become of the same order as the deviation of Y_N from equilibrium value. Therefore, in order to obtain the final lepton abundance, $[\mu]$ should not be neglected in (5.58):

$$[\delta Y_N^{even}] = C^{-1}\left([d_t Y_N^{eq}] - [\mu]\right) \ . \tag{5.88}$$

Again $\widetilde{C}$ is neglected here. The component which affects to the evolution equation of Y_L is

$$[\delta Y_N^{even}]^2 = (C^{-1})^{20} d_t Y_N^{eq} - (C^{-1})^{22}\sum_\alpha (-\Im[\Gamma^W_\alpha]_{12}) Y_{L^\alpha} \ . \tag{5.89}$$

Plugging this into (5.53), we get the effective equation of the lepton yield value:

$$d_t Y_{L^\alpha} = 2\varepsilon'_\alpha [\Gamma_N]^0 \frac{-d_t Y_N^{eq}}{[\Gamma_N]^0} - \mathrm{Tr}\{\Re[\Gamma^W_\alpha]\} Y_{L^\alpha} + 2\sum_\beta \Gamma^{\mathrm{BR}}_{\alpha\beta} Y_{L^\beta} \tag{5.90}$$

where

$$\begin{aligned} \varepsilon'_\alpha &\equiv -[\Gamma_\alpha]^2 (C^{-1})^{20} \\ &= \frac{2\Re(h^\dagger h)_{12}\Im(h^\dagger_{1\alpha}h_{\alpha 2})}{(h^\dagger h)_{11}(h^\dagger h)_{22}} \frac{(M_1^2 - M_2^2) M (\Gamma_1+\Gamma_2)/2}{(M_1^2-M_2^2)^2 + \frac{\det[\Re[h^\dagger h]]}{(h^\dagger h)_{11}(h^\dagger h)_{22}} M^2 (\Gamma_1+\Gamma_2)^2} \end{aligned} \tag{5.91}$$

is practically convenient definition [89] of the CP-violating parameter, differ from (5.64). And the backreaction contribution due to the generated lepton asymmetry is included as

$$\begin{aligned} \Gamma^{\mathrm{BR}}_{\alpha\beta} &\equiv -[\Gamma_\alpha]^2 (C^{-1})^{22} (-\Im[\Gamma^W_\beta]_{12}) \\ &= \frac{\mathrm{Tr}\{\Re[\Gamma^W_\alpha]\}}{(h^\dagger_{1\alpha}h_{\alpha 1} + h^\dagger_{2\alpha}h_{\alpha 2})} \times \frac{(h^\dagger h)_{11} + (h^\dagger h)_{22}}{2} \frac{\det[\Re[h^\dagger h]]}{(h^\dagger h)_{11}(h^\dagger h)_{22}} \\ &\quad \times \frac{4M^2 \Im[h^\dagger_{1\alpha}h_{\alpha 2}]\Im[h^\dagger_{1\beta}h_{\beta 2}](M/8\pi)^2}{(M_1^2-M_2^2)^2 + \frac{\det[\Re[h^\dagger h]]}{(h^\dagger h)_{11}(h^\dagger h)_{22}} M^2(\Gamma_1+\Gamma_2)^2} \end{aligned} \tag{5.92}$$

For the later convenience, let us rewrite (5.90) using $z \equiv M/T$ as

$$d_z Y_{L^\alpha} = 2\varepsilon'_\alpha(-d_z Y_N^{eq}) - \frac{z}{H(M)} \sum_\gamma \mathrm{Tr}\{\Re[\Gamma_\gamma^W]\} \times \sum_\beta \kappa_{\alpha\beta} Y_{L^\beta} \ . \tag{5.93}$$

In this expression, the effective wash-out term with its coefficient

$$\kappa_{\alpha\beta} \equiv \Bigg[\frac{h^\dagger_{1\alpha} h_{\alpha 1} + h^\dagger_{2\alpha} h_{\alpha 2}}{(h^\dagger h)_{11} + (h^\dagger h)_{22}} \delta_{\alpha\beta} \left((M_1^2 - M_2^2)^2 + \frac{\det[\Re[h^\dagger h]]}{(h^\dagger h)_{11}(h^\dagger h)_{22}} M^2 (\Gamma_1 + \Gamma_2)^2 \right) \tag{5.94}$$

$$- \frac{\det[\Re[h^\dagger h]]}{(h^\dagger h)_{11}(h^\dagger h)_{22}} \frac{M^2 \Im[h^\dagger_{1\alpha} h_{\alpha 2}]}{4\pi} \frac{M^2 \Im[h^\dagger_{1\beta} h_{\beta 2}]}{4\pi} \Bigg]$$

$$\times \left[(M_1^2 - M_2^2)^2 + \frac{\det[\Re[h^\dagger h]]}{(h^\dagger h)_{11}(h^\dagger h)_{22}} M^2 (\Gamma_1 + \Gamma_2)^2 \right]^{-1} \ . \tag{5.95}$$

In the unflavored approximation, (5.93) becomes

$$d_z Y_L = 2\varepsilon'(-d_z Y_N^{eq}) - \kappa \frac{\Gamma_1 + \Gamma_2}{H(M)} \frac{z^3 K_1(z)}{4} Y_L \tag{5.96}$$

where $\varepsilon' \equiv \sum_\alpha \varepsilon'_\alpha$ and

$$\kappa \equiv \sum_{\alpha,\beta} \kappa_{\alpha\beta} = \left[(M_1^2 - M_2^2)^2 + \frac{\det[\Re[h^\dagger h]]}{(h^\dagger h)_{11}(h^\dagger h)_{22}} M^2 \left((\Gamma_1 + \Gamma_2)^2 - \left(\frac{M \Im[(h^\dagger h)_{12}]}{4\pi} \right)^2 \right) \right]$$

$$\times \left[(M_1^2 - M_2^2)^2 + \frac{\det[\Re[h^\dagger h]]}{(h^\dagger h)_{11}(h^\dagger h)_{22}} M^2 (\Gamma_1 + \Gamma_2)^2 \right]^{-1} \tag{5.97}$$

which is always smaller than unity: $\kappa < 1$, and reduce the wash-out effect. (5.97) is consistent with the back reaction term (4.89) obtained with the assumption of the small off-diagonal component $(h^\dagger h)'/(h^\dagger h)^d < 1$. Such an suppression is also found in the conventional approach, where the RIS-subtracted scattering term brings this effect and but gives a different form of κ [40, 41, 46]. Using the method reviewed in Section 2.2 and the approximated analytic form of the efficiency factor (2.91), the final yield value of the lepton number is obtained as[29]

$$Y_L \approx 2\varepsilon' Y_N^{eq}(0) \frac{2}{\mathcal{K}_{\mathrm{eff}} z_B(\mathcal{K}_{\mathrm{eff}})} \left(1 - e^{-\mathcal{K}_{\mathrm{eff}} z_B(\mathcal{K}_{\mathrm{eff}})/2} \right) \tag{5.98}$$

where

$$\mathcal{K}_{\mathrm{eff}} \equiv 2\kappa \times \frac{(\Gamma_1 + \Gamma_2)/2}{H(M)} \ . \tag{5.99}$$

The pre-factors of 2 come from the fact that two RH neutrinos contribute to the generation and wash-out of the lepton asymmetry.

[29]Note that the factor κ, defined by (5.97) can vanish. In such an extreme situation, the evolution of lepton number cannot be simply estimated by the formula (2.91) obtained with large washout effect. In addition, the various terms we omitted above may get to be significant and then more careful analysis would be needed.

5.5 Summary and comments

In the section, we solved the KB equation without assuming that the off-diagonal component of the Yukawa couplings are small compared to the diagonal ones. In order to solve it, we first derive the kinetic equation for the density matrix. The differential equation can be reduced to a linear equation if the background is slowly changing and the deviation of the distribution function from local equilibrium is small. Then the density matrix of RH neutrino can be solved in terms of the time variation of the equilibrium distribution function and the generated lepton asymmetry. Its off-diagonal component determines the CP-violating parameter ε defined by (5.64) and a practically convenient definition (5.91). they are resonantly enhanced due to the almost degenerate Majorana masses and the regulator in the CP-violating parameter (5.64) is given by $R_{ij} = M_i\Gamma_i + M_j\Gamma_j$. In the 2PI formalism, the decay width Γ_i is given by the imaginary part of the self-energy function of the RH neutrinos. In addition to the loop corrections of the vertex functions, scattering effects with particles in medium are contained. The back reaction effect of the generated lepton asymmetry is also obtained. It reduces the wash-out effect. The density matrix formalism gives the consistent results with the method directly solving the KB equation.

Because we have not directly solved the KB equation of the RH neutrino, the non-Markovian expression have not appeared. However, note that the multi-flavor generalization of the quasi-particle approximation makes it possible to reduce the KB equation into the Markovian density matrix equation. If we didn't take such an ansatz, then the non-Markovian expression with the formal solution of the KB equation (3.48) would be necessary. By adopting the multi-flavor generalization of the quasi-particle approximation, we have been able to obtain the evolution equation of the "distribution function of the quantum state" involving the superposition of the different mass eigenstates.

The authors of [87, 88] derived the kinetic equation of density matrix based on the Hamiltonian approach, and solve the equation to obtain δY_N^{even} in the flavor covariant way. The result is consistent with ours but the interpretation of the CP-violating parameter seems to be different. In their paper, the one-loop resummed effective Yukawa coupling is used to define decay and inverse-decay amplitudes (Γ_N in our notation), in which the effect of coherent oscillation is included in their analysis. In our approach based on the 2PI formalism, Γ_N comes from 1PI self-energies and the effect of coherent oscillation is not contained. The indirect CP-violating parameter ε generated by resummation of RH neutrino propagators is encoded by the non-diagonal density matrix.

6 Conclusion

In this thesis, we have investigated the formal aspects of the thermal resonant leptogenesis in the expanding universe using the non-equilibrium quantum field theory. The lepton asymmetry is generated in the CP asymmetric decay of the RH neutrinos whose distribution functions are out of thermal equilibrium. If the RH neutrinos have almost degenerate masses, the time scale of the quantum flavor oscillation $\sim 1/\Delta M$ becomes of the same order of the time scale of the decay processes $\sim 1/\Gamma$. In such a situation, the classical Boltzmann equation is not valid because the propagating process of RH neutrino is no longer a classical process, and the full quantum mechanical approach is necessary.

In Section 2, we reviewed the conventional calculations of the leptogenesis scenarios, and mentioned the insufficiency of them to describe the resonant leptogenesis. Because of the artificial separation between the (inverse) decay and scattering process, the RIS subtraction is needed to get a physically acceptable evolution equation of the lepton number. In addition, when deriving the CP-violating parameter (2.151) with almost degenerate Majorana mass, one has to pick up only the on-shell part because of the cancelation between on- and off-shell contribution with *equilibrium* propagator of RH neutrino mediating the $2 \to 2$ scattering process. These are mentioned, in section 2.4, as the motivation to employ the Kadanoff-Baym (KB) equation which is obtained from the first principle of the QFT.

In Section 3, we summarized the derivation of the evolution equation of the SM lepton number from the KB equation of the SM lepton propagator. Plugging the quasi-particle approximation of the SM particle's propagators with thermal damping much faster than the Hubble expansion into (3.43), we get the Boltzmann equation-like form (3.47). However, the contribution from RH neutrinos are kept in the original form of the propagator. Then, as seen in section 3.5, the question is reduced to how the Wightman propagator of RH neutrino should be treated so as not to lose the important quantum effects.

In section 4, By extending the method developed in [71] to the expanding universe, we have derived the evolution equation of lepton number and explicitly obtained the CP-violating parameter in the decay process of RH neutrino. It has been clarified where the difference between the regulator $R_{ij} = |M_i\Gamma_i + M_j\Gamma_j|$ and $R_{ij} = |M_i\Gamma_i - M_j\Gamma_j|$ comes from. Because of resonant mixing of different flavors, the state of RH neutrino in propagating process before (after) a decay (inverse decay) process consists of a quantum superposition of different mass eigenstates. Such a quantum process is involved as the non-local $2 \to 2$ scattering (Fig.8 (b)). It has been shown that if we erroneously neglect the off-shell contribution comes from interference between different mass eigenstates, then the wrong regulator $R_{ij} = |M_i\Gamma_i - M_j\Gamma_j|$ is reproduced.

As mentioned in Section 4.5, the difference from the conventional calculation reviewed in Section 2.3 is twofold: The first is to take into account the interferences between the different mass eigenstates. We called them off-shell contribution. And the second is to consider the *non-equilibrium* propagators of the RH neutrinos. For the equilibrium propagators, the on-shell and off-shell contributions are canceled each other. Then, what contribute to the CP-violating parameter is only the non-equilibrium part of the Wightman propagator containing the regulator $R_{ij} = |M_i\Gamma_i + M_j\Gamma_j|$. In other words, in the conventional calculation based on the equilibrium QFT, the interferences have to be neglected to get the non-zero result of CP-violating parameter. And then, it necessarily leads to the wrong form of the regulator as well as the problem to be solved by the RIS subtraction.

In Section 5, based on the observation obtained in Section 4, we have adopted the multi-flavor generalization of the KB ansatz for the RH neutrino propagators, so-called density matrix. By taking the lowest order in the Kramers-Moyal expansion, the KB equation has been reduced to the evolution of the density matrix. In this case, the equation becomes Markovian with incoming and outgoing quantum superposition state of RH neutrino (Fig.8 (a)). When we consider the situation where the deviation from thermal equilibrium is small, we can obtain the analytic solution of the evolution equation without assuming the smallness of the off-diagonal component of Yukawa coupling. By plugging the solution into the evolution equation of lepton number, we have read off the CP-violating parameter with the modified regulator consistent with the one obtained in Section 4.

The advantage of the reduction to the density matrix formalism is that additional interactions coming from some extension of the model can be easily taken into account. For example, if we consider the B-L gauged model of the SM, the new contributions to the collision term appear, such as the annihilation (production) process of RH neutrinos mediated by B-L gauge boson. From the derivation of the CP-violating parameter, it's clear that the regulator in the CP-violating parameter also includes the various annihilation rates as well as the decay rate of RH neutrino. This reflects the fact that the important quantum coherence is spoiled by them. Then if the B-L gauge coupling is so large that the annihilation rates coming from gauge interaction become larger than the usual decay rate, the CP-violating parameter becomes smaller.

The KB equation as an approximation of the SD equation derived from the non-equilibrium QFT is the self-consistent equation of full two point functions with the memory integral. Two point functions are defined without distinguishing on-shell and off-shell states. The KB equation of Wightman function can be formally solved as (3.48) with the double time integration. In the formal solution, it's clear how the state in past affects the physical quantities encoded on the two point function at the present time.

In the resonant leptogenesis in strong washout regime, by investigating the formal solution of the Wightman propagator, significant part of quantum effects can be taken into account. Also, the multi-flavor generalization of the quasi-particle approximation is applicable and the Markovian density matrix equation can be derived from the KB equation because the relaxation rate of the RH neutrino is faster than the other time scales related to the Hubble expansion of the universe and then the Kramers-Moyal derivative expansion can be justified. Even though the density matrix equation is already well known, the first principle derivation from the non-equilibrium QFT is useful in order to consider finite temperature and density effects and to be careful for its validity. Especially, the derivation from the KB equation manifests the fact that the explicit form of the CP-violation coming from the resonant oscillation of the degenerate RH neutrinos doesn't appear in the density matrix equation even as the effective coupling constant at the vertices, and it appears only after solving the equation for the density matrix of the RH neutrinos. Besides the form of the regulator R_{ij}, that is one of the consequences of the derivation from the KB equation which describes both of the collision and propagating processes together.

For more complicated non-equilibrium systems in which various time scales exist, we always need to start with the first principle of QFT and find the reasonable approximation not to miss important effects.

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A Non-equilibrium QFT

A.1 CTP formalism

In equilibrium field theories, we implicitly assume that the initial and final states asymptotically approach the ground state of the free Hamiltonian. But this does not hold in general, especially in time-dependent backgrounds such as the evolving universe. The final state is generally different from the initial state. The closed time path (CTP) formalism, or the Schwinger-Keldish formalism, is the general formalism to calculate physical quantities for time-dependent wave functions.

Suppose that a system is described by a Hamiltonian $\hat{H}_0 + \hat{H}_1$, where $\hat{H}_0$ and $\hat{H}_1$ are free and interaction Hamiltonians, and that the system is in the initial state $|\psi_i\rangle$ at time $t = t_i$. In the interaction picture, the expectation value of an observable $\hat{\mathcal{O}}$ at time t is given by

$$\mathcal{O}(t) = \langle \psi_i^I(t) | \hat{\mathcal{O}}^I(t) | \psi_i^I(t) \rangle = \langle \psi_i | U^I(t_i, t) \hat{\mathcal{O}}^I(t) U^I(t, t_i) | \psi_i \rangle \ . \tag{A.1}$$

Here the operator in the interaction picture $\hat{\mathcal{O}}^I(t)$ is related to the operator in the Heisenberg picture as

$$\begin{aligned} \hat{\mathcal{O}}^H(t) &= U^I(t, t_i) \hat{\mathcal{O}}^I(t) U^{I\dagger}(t, t_i) \ , \\ U^I(t, t') &= T \exp\left(-i \int_{t'}^{t} dt'' \hat{H}_1{}^I(t'') \right) \ . \end{aligned} \tag{A.2}$$

In equilibrium cases, the final state at time $t = t_f$ is assumed to be proportional to the initial state $U^I(t_f, t_i)|\psi_i\rangle = e^{i\theta}|\psi_i\rangle$ where $\theta(t_f, t_i)$ is a c-number phase. Then we can factorize $\mathcal{O}(t)$ of (A.1) as

$$\begin{aligned} \mathcal{O}(t) &= \langle \psi_i | U^I(t_i, t_f) | \psi_i \rangle \langle \psi_i | U^I(t_f, t) \hat{\mathcal{O}}^I(t) U^I(t, t_i) | \psi_i \rangle \\ &= e^{-i\theta} \langle \psi_i | U^I(t_f, t) \hat{\mathcal{O}}^I(t) U^I(t, t_i) | \psi_i \rangle \ . \end{aligned} \tag{A.3}$$

Similarly, an expectation value of the time-ordering product of two operators $\hat{\mathcal{O}}_1^I(t_1)$ and $\hat{\mathcal{O}}_2^I(t_2)$ is given by

$$\mathcal{O}(t_1, t_2) = e^{-i\theta} \langle \psi_i | T \left(\hat{\mathcal{O}}_1^I(t_1) \hat{\mathcal{O}}_2^I(t_2) U^I(t_f, t_i) \right) | \psi_i \rangle \ . \tag{A.4}$$

This formula gives an ordinary perturbative expansion of correlation functions in equilibrium field theories. Namely, if we take $t_i \to -\infty$ and $t_f \to \infty$, the interaction vertices $\hat{H}^I(t)$ are inserted in $-\infty < t < \infty$.

In non-equilibrium cases where the final state is no longer proportional to the initial state, the factorization property does not hold and we have

$$\mathcal{O}(t_1, t_2) = \langle \psi_i | U^I(t_i, t_f) T \left(\hat{\mathcal{O}}_1^I(t_1) \hat{\mathcal{O}}_2^I(t_2) U^I(t_f, t_i) \right) | \psi_i \rangle \ . \tag{A.5}$$

In perturbative expansions, the interaction vertices are inserted not only on the path $\mathcal{C}_+$ from t_i to t_f, but also on the backward path $\mathcal{C}_-$ from t_f to t_i.

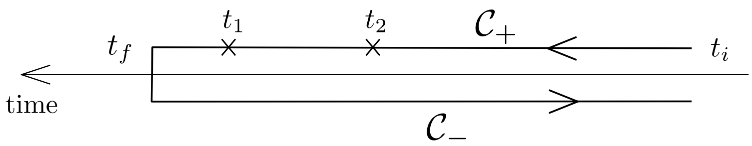

Figure 17: A closed time path $\mathcal{C}$ from t_i to t_f and then back to t_i. (i.e., $\mathcal{C} = \mathcal{C}_+ + \mathcal{C}_-$.) Operators are inserted at $t = t_1$ and t_2 for the time ordered product $\mathcal{O}(t_1, t_2)$. Interaction vertices are inserted everywhere on the CTP.

Figure 17 shows the closed time path (CTP), $\mathcal{C} = \mathcal{C}_+ + \mathcal{C}_-$. In this formalism, the final state is not specified at all and we can calculate time-dependence of various quantities as in (A.5). The time-ordering T_C is defined on the CTP as a path-ordering along $\mathcal{C} = \mathcal{C}_+ + \mathcal{C}_-$.

A.2 Evolution equations of various propagators

We define various propagators and give a brief derivation of their evolution equations. In the following we consider a real (Majorana) fermion field $\hat{\psi}$ and write its conjugate by $\overline{\hat{\psi}} = \hat{\psi}^t C$ where $C = i\gamma^2\gamma^0$. For the simplest case of a real scalar field, see e.g. [86]. In the CTP formalism, a generating function of time-ordered products of operators is given by

$$\begin{aligned} Z[\overline{J}] =& e^{iW[\overline{J}]} = \left\langle \mathrm{T}_{\mathcal{C}}\ e^{i\int_{\mathcal{C}} d^4x\sqrt{-g}\ \overline{J}(x)\hat{\psi}(x)} \right\rangle \\ =& \int d\Psi_+ d\Psi_- \langle \Psi_+ | \hat{\rho}_{(t_i)} | \Psi_- \rangle \int \mathcal{D}'\psi\ e^{iS[\psi]+i\int_{\mathcal{C}} d^4x\sqrt{-g}\ \overline{J}(x)\psi(x)} \end{aligned} \tag{A.6}$$

where $\hat{\rho}(t_i) = |\psi_i\rangle\langle\psi_i|$. The path integral $\int \mathcal{D}'\psi$ denotes an integration of the Grassmann variables $\psi_\pm$ on $\mathcal{C}_\pm$ with the fixed boundary conditions $\psi_\pm(t_i) = \Psi_\pm$. The integrations of $\Psi_\pm$ represent an weighting by the initial wave function $|\psi_i\rangle$. The source $\overline{J}(x)$ is defined on $\mathcal{C} = \mathcal{C}_+ + \mathcal{C}_-$. The 1PI effective action is obtained from the generating function $W[\overline{J}]$ of the connected Green function by the Legendre transformation. Defining the classical field by the left-derivative of $W[\overline{J}]$ with respect to the Grassmannian source $\overline{J}$:

$$\Psi(x) = +\frac{\delta W[\overline{J}]}{\delta \overline{J}(x)}\ , \tag{A.7}$$

we have

$$\Gamma[\Psi] = W[\overline{J}] - \int_{\mathcal{C}} d^4x_g \overline{J}(x)\Psi(x)\ . \tag{A.8}$$

For notational simplicity, we use the following abbreviation

$$d^4x_g \equiv d^4x\sqrt{-g(x)} \tag{A.9}$$

unless the explicit dependence of the measure on x is necessary. The stationary condition $(\delta/\delta\Psi(x))\Gamma[\Psi] = 0$ at $\overline{J} = 0$ gives the equation of motion of Ψ.

By taking the second derivative of the effective action Γ with respect to Ψ, we obtain the Schwinger-Dyson (SD) equation

$$iG^{-1}(x,y) = iG_0^{-1}(x,y) - i\Pi(x,y) \ . \tag{A.10}$$

Π is the self-energy and only 1PI diagrams contribute to it. The connected Green function G on $\mathcal{C}$ is defined by

$$\begin{aligned} G(x,y) &= \langle \mathrm{T}_{\mathcal{C}} \hat{\psi}(x)\overline{\hat{\psi}}(y)\rangle \\ &= \Theta_{\mathcal{C}}(x^0 - y^0) G_>(x,y) + \Theta_{\mathcal{C}}(y^0 - x^0) G_<(x,y) \ , \end{aligned} \tag{A.11}$$

where $\Theta_C(x^0 - y^0)$ is the step function on $\mathcal{C}$ and

$$G_<(x,y) \equiv -\langle \overline{\hat{\psi}}(y)\hat{\psi}(x)\rangle \ , \quad G_>(x,y) \equiv \langle \hat{\psi}(x)\overline{\hat{\psi}}(y)\rangle \tag{A.12}$$

are the Wightman Green functions. $iG_0^{-1}(x,y) = iG_{0(x)}^{-1}\delta_{\mathcal{C}}^g(x-y)$ is an inverse of the free propagator and $\Pi(x,y)$ is the self-energy of the fermion field ψ.

The statistical propagator $G_F(x,y)$ and the spectral function $G_\rho(x,y)$ are defined by

$$G_F(x,y) = \frac{1}{2}\left(G_>(x,y) + G_<(x,y)\right) = \frac{1}{2}\langle [\hat{\psi}(x), \overline{\hat{\psi}}(y)]\rangle \ , \tag{A.13}$$

$$G_\rho(x,y) = i\left(G_>(x,y) - G_<(x,y)\right) = i\langle \{\hat{\psi}(x), \overline{\hat{\psi}}(y)\}\rangle \ . \tag{A.14}$$

G_F contains information of the distribution function of the specified state while G_ρ depends only on the spectrum of the system. In this sense, G_F is dynamical while G_ρ is kinematical. Especially, $G_\rho(x,y)$ becomes proportional to the spatial delta-function $\delta^{(3)}(\mathbf{x}-\mathbf{y})$ in the equal-time limit. We further define the retarded and advanced Green functions by

$$G_{R/A}(x,y) = \pm\Theta(\pm(x^0 - y^0))G_\rho(x,y). \tag{A.15}$$

They are related to G_ρ as

$$\begin{aligned} G_R(x,y) - G_A(x,y) &= G_\rho(x,y) \ , \\ G_R(x,y) + G_A(x,y) &= \mathrm{sign}(x^0 - y^0) G_\rho(x,y) \ . \end{aligned} \tag{A.16}$$

In terms of G_F and G_ρ, the Green function (A.11) can be written as

$$G(x,y) = G_F(x,y) - \frac{i}{2}\mathrm{sign}_{\mathcal{C}}(x^0 - y^0) G_\rho(x,y) \ , \tag{A.17}$$

where the sign-function on $\mathcal{C}$ is defined by

$$\mathrm{sign}_{\mathcal{C}}(x^0 - y^0) = \Theta_{\mathcal{C}}(x^0 - y^0) - \Theta_{\mathcal{C}}(y^0 - x^0) \ . \tag{A.18}$$

By convoluting (A.10) with the full propagator G, we have

$$iG_{0(x)}^{-1}G(x,y) - i\int_{\mathcal{C}} d^4z_g\ \Pi(x,z)G(z,y) = i\delta_{\mathcal{C}}^g(x-y)\ . \tag{A.19}$$

Here $\delta_{\mathcal{C}}^g(x-y)$ is the delta-function on $\mathcal{C}$ with the space-time metric g, and satisfies $\int_C d^4z_g\delta_{\mathcal{C}}^g(x-y) = 1$. By denoting x on $\mathcal{C}_\pm$ as $x_\pm$ respectively, the delta-function on $\mathcal{C}$ can be expressed by a 2×2 matrix:

$$\delta_{\mathcal{C}}^g(x_a - y_b) = c_{ab}\delta^g(x-y)\ , \quad c_{ab} = \mathrm{diag}(1,-1) \tag{A.20}$$

where a, b takes $+$ or $-$. The minus sign on $\mathcal{C}_-$ comes from the backward integral of the time variable and corresponds to the anti-time-ordering of the Green function G in (A.11). $\delta^g(x-y)$ is an ordinary delta-function for $(x-y)$.

The 2-point function $G(x,y)$ of (A.11) with $x, y \in \mathcal{C}$ can be similarly decomposed (depending on whether x, y are on $\mathcal{C}_+$ or $\mathcal{C}_-$) into a 2×2 matrix form as

$$G_{ab}(x,y) = \begin{pmatrix} G_{++}(x,y) & G_{+-}(x,y) \\ G_{-+}(x,y) & G_{--}(x,y) \end{pmatrix} = \begin{pmatrix} G_{\mathrm{T}}(x,y) & G_{<}(x,y) \\ G_{>}(x,y) & G_{\widetilde{\mathrm{T}}}(x,y) \end{pmatrix} \tag{A.21}$$

where $\mathrm{T}, \widetilde{\mathrm{T}}$ denote time and anti-time orderings respectively, and

$$\begin{aligned} G_{\mathrm{T}}(x,y) &= \Theta(x^0-y^0)G_>(x,y) + \Theta(y^0-x^0)G_<(x,y)\ , \\ G_{\widetilde{\mathrm{T}}}(x,y) &= \Theta(x^0-y^0)G_<(x,y) + \Theta(y^0-x^0)G_>(x,y)\ . \end{aligned} \tag{A.22}$$

$\Theta(x^0-y^0)$ is the ordinary step-function. By using (A.17) and (A.16), we have

$$\begin{aligned} G_{ab}(x,y) &= \begin{pmatrix} G_F - \frac{i}{2}\mathrm{sign}(x^0-y^0)G_\rho & G_F + \frac{i}{2}G_\rho \\ G_F - \frac{i}{2}G_\rho & G_F + \frac{i}{2}\mathrm{sign}(x^0-y^0)G_\rho \end{pmatrix} \\ &= \begin{pmatrix} G_F - \frac{i}{2}(G_R+G_A) & G_F + \frac{i}{2}(G_R-G_A) \\ G_F - \frac{i}{2}(G_R-G_A) & G_F + \frac{i}{2}(G_R+G_A) \end{pmatrix} \\ &= U^t \begin{pmatrix} 0 & G_A \\ G_R & G_F \end{pmatrix} U \end{aligned} \tag{A.23}$$

where

$$U \equiv \begin{pmatrix} -i/2 & i/2 \\ 1 & 1 \end{pmatrix}\ . \tag{A.24}$$

We also decompose the self-energy $\Pi(x,y)$ as

$$\begin{aligned} \Pi(x,y) &= \Theta_{\mathcal{C}}(x^0-y^0)\Pi_>(x,y) + \Theta_{\mathcal{C}}(y^0-x^0)\Pi_<(x,y) \\ &= \Pi_F(x,y) - \frac{i}{2}\mathrm{sign}_{\mathcal{C}}(x^0-y^0)\Pi_\rho(x,y)\ . \end{aligned} \tag{A.25}$$

Defining $\Pi_{R/A}$ by

$$\Pi_{R/A}(x,y) = \pm\Theta(\pm(x^0-y^0))\Pi_\rho(x,y)\ , \tag{A.26}$$

the matrix form of the self-energy is obtained as

$$\Pi_{ab} = \begin{pmatrix} \Pi_{\mathrm{T}} & \Pi_{<} \\ \Pi_{>} & \Pi_{\widetilde{\mathrm{T}}} \end{pmatrix} = U^t \begin{pmatrix} 0 & \Pi_A \\ \Pi_R & \Pi_F \end{pmatrix} U \ . \tag{A.27}$$

Using these matrix forms of G_{ab} and Π_{ab}, the equation (A.19) becomes

$$iG^{-1}_{0(x)} G_{ab}(x,y) - i \int_{t_i}^{t_f} d^4 z_g \ \Pi_{ac}(x,z) c_{cd} G_{db}(z,y) = i\delta^g(x-y) c_{ab} \ . \tag{A.28}$$

The matrix c_{cd} between Π and G comes from the backward integration of the time variable in the original integral in (A.19). By multiplying $(U^t)^{-1}$ on the left and U^{-1} on the right, using (A.23) and (A.27) and noting

$$Uc\, U^t = \begin{pmatrix} 0 & -i \\ -i & 0 \end{pmatrix} \ , \tag{A.29}$$

we obtain the following set of the evolution equations:

$$iG^{-1}_{0(x)} G_F(x,y) - \int_{t_{int}}^{\infty} d^4 z_g \ \Pi_R(x,z) G_F(z,y) - \int_{t_{int}}^{\infty} d^4 z_g \ \Pi_F(x,z) G_A(z,y) = 0 \ , \tag{A.30}$$

$$iG^{-1}_{0(x)} G_{R/A}(x,y) - \int_{t_{int}}^{\infty} d^4 z_g \ \Pi_{R/A}(x,z) G_{R/A}(z,y) \quad = -\delta^g(x-y) \ . \tag{A.31}$$

From the equations (A.31), we obtain the evolution equation for the spectral density $G_\rho = G_R - G_A$:

$$iG^{-1}_{0(x)} G_\rho(x,y) - \int_{t_{int}}^{\infty} d^4 z_g \ \Pi_R(x,z) G_\rho(z,y) - \int_{t_{int}}^{\infty} d^4 z_g \ \Pi_\rho(x,z) G_A(z,y) = 0 \ . \tag{A.32}$$

The Wightman Green function $G_{\gtrless} = G_F \mp (i/2) G_\rho$ satisfies

$$iG^{-1}_{0(x)} G_{\gtrless}(x,y) - \int_{t_{int}}^{\infty} d^4 z_g \ \Pi_R(x,z) G_{\gtrless}(z,y) - \int_{t_{int}}^{\infty} d^4 z_g \ \Pi_{\gtrless}(x,z) G_A(z,y) = 0 \ . \tag{A.33}$$

A.3 2PI formalism

In this appendix, we give a brief review of a systematic approach to evaluate the self-energy based on the 2PI formalism (see, e.g., [86] for more details). The generating functional $Z[\overline{J}, R]$ in the presence of sources $J(x)$ and $R(x,y)$[30] is given by

$$\begin{aligned} Z[\overline{J}, R] =& e^{iW[\overline{J},R]} \\ =& \left\langle \mathrm{T}_{\mathcal{C}} \ e^{i\int_{\mathcal{C}} d^4 x_g \ \overline{J}(x)\hat{\psi}(x) + \frac{i}{2}\int_{\mathcal{C}} d^4 x_g d^4 y_g \ \overline{\hat{\psi}}(x) R(x,y) \hat{\psi}(y)} \right\rangle \ . \end{aligned} \tag{A.34}$$

[30]Note that the Majorana condition $R(x,y) = CR^t(y,x)C^{-1}$ is not imposed on the source field $R(x,y)$.

By taking a variation with respect to the source fields $J(x)$ and $R(y,x)$, we have

$$\begin{aligned}\frac{\delta W[\overline{J},R]}{\delta \overline{J}_\zeta(x)} &= +\Psi_\zeta(x) \ ,\\ \frac{\delta W[\overline{J},R]}{\delta R_{\eta\zeta}(y,x)} &= -\frac{1}{2}\left(\Psi_\zeta(x)\overline{\Psi}_\eta(y) + G_{\zeta\eta}(x,y)\right) \ . \end{aligned} \tag{A.35}$$

Here ζ, η represent Spinor indices.

$\overline{\Psi}$ is defined as $\overline{\Psi}(x) \equiv \Psi^t(x)C$ and the connected Green function G is given by

$$G_{\zeta\eta}(x,y) = i\frac{\delta^2 W[\overline{J},R]}{\delta \overline{J}_\zeta(x)\delta J_\eta(y)} \ . \tag{A.36}$$

By taking the Legendre transform of $W[\overline{J},R]$ with respect to the sources J, R, we obtain the effective action in the presence of source fields

$$\begin{aligned}\Gamma[\Psi,G] &\equiv W[\overline{J},R] - \int_{\mathcal{C}} d^4x_g \ \overline{J}_\zeta(x)\frac{\delta W[\overline{J},R]}{\delta \overline{J}_\zeta(x)} - \int_{\mathcal{C}} d^4x_g d^4y_g R_{\zeta\eta}(x,y)\frac{\delta W[\overline{J},R]}{\delta R_{\zeta\eta}(x,y)}\\ &= W[\overline{J},R] - \int_{\mathcal{C}} d^4x_g \overline{J}\Psi - \frac{1}{2}\int_{\mathcal{C}} d^4x_g d^4y_g \overline{\Psi}(x)R(x,y)\Psi(y) + \frac{1}{2}\mathrm{Tr}GR \ . \end{aligned} \tag{A.37}$$

Tr in the last term represents a trace in the Spinor indices and an integration over the closed time path $\mathcal{C}$.

Now we decompose the effective action into

$$\Gamma[\Psi,G] = S(\Psi) - \frac{i}{2}\mathrm{Tr}\ln G^{-1} - \frac{i}{2}\mathrm{Tr}G_0^{-1}G + \Gamma_2[\Psi,G] \ . \tag{A.38}$$

The first term is the classical action. The second and the third term are '1-loop' type contributions to the effective action. The meaning of the decomposition can be understood by taking a functional derivative[31] with respect to G:

$$\left.\frac{\delta\Gamma[\Psi,G]}{\delta G_{\eta\zeta}(y,x)}\right|_{R=0} = \frac{i}{2}G^{-1}_{\zeta\eta}(x,y) - \frac{i}{2}G^{-1}_{0,\zeta\eta}(x,y) + \frac{\delta\Gamma_2(\Psi,G)}{\delta G_{\eta\zeta}(y,x)} = 0 \ . \tag{A.39}$$

Compared with the SD equation (A.10), the last term can be identified as the self-energy Π:

$$\Pi_{\zeta\eta}(x,y;\Psi,G) = -2i\frac{\delta\Gamma_2(\Psi,G)}{\delta G_{\eta\zeta}(y,x)} \ . \tag{A.40}$$

In this way, the proper self-energy Π is obtained by differentiating Γ_2 with respect to the full propagator G. Since the proper self-energy Π is calculated as a

[31] In taking the functional derivative with respect to G, the Majorana condition $G(x,y) = CG^t(y,x)C^{-1}$ should be used after setting the source field R zero.

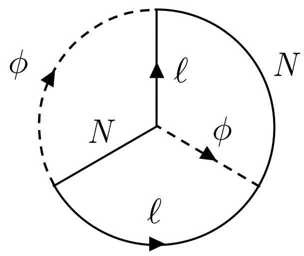

Figure 18: Another example of 2PI diagrams.

sum of contributions from 1PI diagrams, Γ_2 becomes a sum of contributions from 2PI diagrams with respect to the full propagator. In another word, the proper self-energy can be systematically obtained by taking a functional derivative of 2PI diagrams (in which all internal lines are full propagators) with respect to the full propagator.

The SD equation (A.39) can be interpreted as a self-consistent equation for the full propagators. By rewriting this equation on the forward time line $\mathcal{C}_+$, we obtain the set of KB equations (A.30), (A.31), (A.32), (A.33) which can be interpreted as equations for the full propagators.

For Dirac or Weyl fermions, we introduce additional source terms

$$+i \int_{\mathcal{C}} d^4x_g \ \overline{\hat{\psi}}(x)J(x) + i \int_{\mathcal{C}} d^4x d^4y \ \overline{\hat{\psi}}(x)R(x,y)\hat{\psi}(y) \ .$$

The self-energy is similarly obtained as a functional of the full propagators:

$$\Pi_{\zeta\eta}(x,y;\Psi,\overline{\Psi},G) = -i\frac{\delta\Gamma_2(\Psi,\overline{\Psi},G)}{\delta G_{\eta\zeta}(y,x)} \ . \tag{A.41}$$

B Self-energies Σ, Π

In this appendix, using the 2PI formalism, we give an expression of the self-energy function for the RH neutrino $\Pi(x,y)$ in terms of the lepton and Higgs propagators. The simplest and the most important contribution to the 2PI effective action Γ_2 in the model (2.2) is given by the 2-loop diagram of Figure 9. The second simplest 2PI diagram is given by Figure 18. Note that each internal line represents a *full* propagator of the SM lepton, Higgs and the RH neutrino. If the RH neutrinos have almost degenerate mass, we need to use the resummed propagators for the RH neutrinos. Once resummed, we can use an ordinary perturbative expansion with respect to the Yukawa coupling $h_{i\alpha}$. Hence it will not be a bad approximation to use the simplest 2PI diagram to evaluate the self-energy.

In terms of the full propagators, the contribution from the diagram Figure

9 becomes

$$\Gamma_2^{(2\text{loop})}[G,S,\Delta] = \frac{i}{2} h^\dagger_{i\alpha} h_{\beta j} \int_{\mathcal{C}} d^4w_g d^4z_g \ \epsilon_{a'a}\epsilon_{bb'}\Delta_{a'b'}(w,z) \\ \times \mathrm{tr}\left[P_{\mathrm{R}}\left(G^{ji}(z,w) + CG^{t,ij}(w,z)C^{-1}\right)P_{\mathrm{L}}S^{\alpha\beta}_{ab}(w,z)\right] \ . \tag{B.1}$$

Here G, S, Δ are full propagators of the RH neutrino, the SM lepton doublet and the Higgs doublet respectively. $(i,j),(\alpha,\beta),(a,b,a',b')$ represent the flavor indices of the RH neutrino, the flavor indices of the leptons and the $SU(2)_L$ indices of the SM doublets respectively.

By using the formula (A.41), the self-energy of the SM lepton doublet is given by taking a functional derivative of Γ_2 with respect to the lepton propagator S:

$$\begin{aligned}\Sigma^{\alpha\beta}_{ab}(x,y) &= h_{\alpha i}h^\dagger_{j\beta}P_{\mathrm{R}}G^{ij}(x,y)P_{\mathrm{L}}\epsilon_{aa'}\epsilon_{b'b}\Delta_{b'a'}(y,x) \\ &= -\,\delta_{ab}h_{\alpha i}h^\dagger_{j\beta}P_{\mathrm{R}}G^{ij}(x,y)P_{\mathrm{L}}\Delta(y,x) \ .\end{aligned} \tag{B.2}$$

Here we have used the Majorana property $G^{ij}(x,y) = CG^{tji}(y,x)C^{-1}$ of the RH neutrinos. In the second equality, we have used the fact that the lepton and the Higgs propagators are $SU(2)_L$ symmetric and proportional to δ_{ab}, $S_{ab} = S\delta_{ab}, \Delta_{ab} = \Delta\delta_{ab}$, in the early universe where the $SU(2)_L$ symmetry is restored. This is indeed the case in the era of the lepton asymmetry generation through the decay of the RH neutrino. Similarly the self-energy of the RH neutrino is obtained by taking a functional derivative of Γ_2 with respect to G:

$$\begin{aligned}\Pi^{ij}(x,y) =& \{h^\dagger_{i\alpha}h_{\beta j}P_{\mathrm{L}}S^{\alpha\beta}_{ab}(x,y)P_{\mathrm{R}}\Delta_{a'b'}(x,y) \\ &+ h^\dagger_{j\alpha}h_{\beta i}P_{\mathrm{R}}P\overline{S}^{\beta\alpha}_{ba}(\overline{x},\overline{y})PP_{\mathrm{L}}\overline{\Delta}_{b'a'}(\overline{x},\overline{y})\}\epsilon_{a'a}\epsilon_{bb'} \\ =& -g_w h^\dagger_{i\alpha}h_{\beta j}P_{\mathrm{L}}S^{\alpha\beta}(x,y)P_{\mathrm{R}}\Delta(x,y) \\ &- g_w(h^\dagger_{i\alpha}h_{\beta j})^* P_{\mathrm{R}}P\overline{S}^{\alpha\beta}(\overline{x},\overline{y})PP_{\mathrm{L}}\overline{\Delta}(\overline{x},\overline{y})\end{aligned} \tag{B.3}$$

where $P = \gamma^0$. In the first equality, we have used

$$\overline{S}^{\beta\alpha}_{ba}(\overline{x},\overline{y}) = CPS^{t\alpha\beta}_{\ \ ab}(y,x)(CP)^{-1} \ , \quad \overline{\Delta}_{ba}(\overline{x},\overline{y}) = \Delta_{ab}(y,x) \ . \tag{B.4}$$

In the second equality, $SU(2)_L$ symmetry of S and Δ is used. Decomposing these self-energies into the Wightman functions as in (A.25), we have

$$\Sigma^{\alpha\beta}_{ab\ \gtrless}(x,y) = -\,\delta_{ab}h_{\alpha i}h^\dagger_{j\beta}P_{\mathrm{R}}G^{ij}_{\gtrless}(x,y)P_{\mathrm{L}}\Delta_{\lessgtr}(y,x) \equiv \ \delta_{ab}\Sigma^{\alpha\beta}_{\gtrless}(x,y) \ , \tag{B.5}$$

$$\begin{aligned}\Pi^{ij}_{\gtrless}(x,y) =& -g_w h^\dagger_{i\alpha}h_{\beta j}P_{\mathrm{L}}S^{\alpha\beta}_{\gtrless}(x,y)P_{\mathrm{R}}\Delta_{\gtrless}(x,y) \\ &- g_w(h^\dagger_{i\alpha}h_{\beta j})^* P_{\mathrm{R}}P\overline{S}^{\alpha\beta}_{\gtrless}(\overline{x},\overline{y})PP_{\mathrm{L}}\overline{\Delta}_{\gtrless}(\overline{x},\overline{y}) \ .\end{aligned} \tag{B.6}$$

In the following, we derive the self-energy of the RH neutrino $\Pi^{(eq)}$ under an assumption that the lepton and the Higgs are in the thermal equilibrium. The approximation is justified in the leading order calculation since the SM leptons and the Higgs particles interact faster than the Hubble expansion rate in the era of the leptogenesis. See (3.42). Hence the deviation from the equilibrium can be neglected in the calculation of Π. In the equilibrium, the lepton and the Higgs propagators become CP-symmetric and satisfy

$$\overline{S}(x,y) = S(x,y)\ , \quad \overline{\Delta}(x,y) = \Delta(x,y)\ . \tag{B.7}$$

By using the quasi-particle approximation for the propagators (3.36) and (3.37), the Fourier transform of the self-energy $\Pi_\rho = i(\Pi_> - \Pi_<) = \Pi_R - \Pi_A$ of the RH neutrino becomes

$$\Pi_\rho^{(eq)ij}(q) = \left(\Re(h^\dagger h)_{ij} - i\Im(h^\dagger h)_{ij}\gamma_5\right)\pi_\rho^{(eq)}(q)\ , \tag{B.8}$$

where

$$\begin{aligned}\pi_\rho^{(eq)}(q) =&(-g_w)\sum_{\epsilon_\ell,\epsilon_\phi}\int\frac{d^3p}{(2\pi)^3 2\omega_p}\frac{d^3k}{2\omega_k}\ \delta^3(q-p-k)\\ &\times\frac{i\Gamma_{\ell\phi}}{(q_0-\epsilon_\ell\omega_p-\epsilon_\phi\omega_k)^2+\Gamma_{\ell\phi}^2/4}\ \not{p}_{\epsilon_\ell}\mathcal{D}_{\rho(p,k)}^{\epsilon_\ell\cdot\epsilon_\phi(eq)}\ .\end{aligned} \tag{B.9}$$

$\mathcal{D}_{\rho(p,k)}$ is given by

$$\mathcal{D}_{\rho(p,k)}^{\epsilon_\ell\epsilon_\phi} \equiv \mathcal{D}_{>(p,k)}^{\epsilon_\ell\epsilon_\phi} - \mathcal{D}_{<(p,k)}^{\epsilon_\ell\epsilon_\phi} = (-1)^{\epsilon_\ell}(-1)^{\epsilon_\phi}\left(1 - f_{\ell p}^{\epsilon_\ell} + f_{\phi k}^{\epsilon_\phi}\right) \tag{B.10}$$

with the definition of $\mathcal{D}_{\gtrless(p,k)}$ in (3.46), and satisfy the relation $\mathcal{D}_{\rho(p,k)}^{-\epsilon_\ell-\epsilon_\phi(eq)} = -\mathcal{D}_{\rho(p,k)}^{+\epsilon_\ell+\epsilon_\phi(eq)}$ and is followed by the relation of

$$\pi_\rho^{(eq)}(-q_0,\mathbf{q}) = +\gamma^0\pi_\rho^{(eq)}(+q_0,\mathbf{q})\gamma^0\ . \tag{B.11}$$

In the calculation we have used the integral (in the limit $t_{int}\to-\infty$)

$$2\int_{-\infty}^{t} d\tau e^{i(-q^0+\epsilon_\ell\omega_p+\epsilon_\phi\omega_k+i\Gamma_{\ell\phi}/2)(t-\tau)} = \frac{\Gamma_{\ell\phi}-2i(q^0-\epsilon_\ell\omega_p-\epsilon_\phi\omega_k)}{(q^0-\epsilon_\ell\omega_p-\epsilon_\phi\omega_k)^2+\Gamma_{\ell\phi}^2/4}\ . \tag{B.12}$$

The contribution from the boundary at $\tau=-\infty$ vanishes because of the damping factor $\sim e^{-\Gamma_{\ell\phi}(t-\tau)/2}$.

In the weak coupling limit of the SM gauge couplings, $\Gamma_{\ell\phi}$ becomes much less than the typical energy transfer $(q_0-\epsilon_\ell\omega_p-\epsilon_\phi\omega_k)\sim T$ where T is the temperature at which the leptogenesis occurs. In such a limit, the above integral becomes proportional to $\delta(q_0-\epsilon_\ell\omega_p-\epsilon_\phi\omega_k)$, and the exact energy conservation is satisfied instead of the Lorentz type in (B.9). Furthermore, in order to simplify the form of the self-energy (B.9), we neglect the medium effects (e.g., the Pauli exclusion of the SM lepton and the induced emission of the Higgs) encoded in $\mathcal{D}_\rho$ in (B.9) and drop the distribution function f.

Adopting these two simplifications of the weak coupling limit and neglecting the medium effects, the self-energy (B.9) reduces to the vacuum one:

$$\pi_\rho(q) \to \frac{-ig_w}{16\pi}\Theta(q^2)\mathrm{sign}(q_0)\not{q} \ . \tag{B.13}$$

Since the main purpose of the thesis is to obtain the effect of quantum oscillations of almost degenerate RH neutrinos, we use this simplified form of the self-energy. The full treatment is investigated by using the integral form (B.9) of the self-energy instead of (B.13).

Similarly, for $2\Pi_h = \Pi_R + \Pi_A$, we have

$$\Pi_h^{(eq)ij}(q) = \left(\Re(h^\dagger h)_{ij} - i\Im(h^\dagger h)_{ij}\gamma_5\right)\pi_h^{(eq)}(q) \ , \tag{B.14}$$

where

$$\begin{aligned}\pi_h^{(eq)}(q) =& (-g_w)\sum_{\epsilon_\ell,\epsilon_\phi}\int \frac{d^3p}{(2\pi)^3 2\omega_p}\frac{d^3k}{2\omega_k}\ \delta^3(q-p-k) \\ &\times \frac{-(q_0-\epsilon_\ell\omega_p-\epsilon_\phi\omega_k)}{(q_0-\epsilon_\ell\omega_p-\epsilon_\phi\omega_k)^2+\Gamma_{\ell\phi}^2/4}\ \not{p}_{\epsilon_\ell}\mathcal{D}_{\rho(p,k)}^{\epsilon_\ell\epsilon_\phi(eq)} \ .\end{aligned} \tag{B.15}$$

It satisfies the relation

$$\pi_h^{(eq)}(-q_0,\mathbf{q}) = -\gamma^0\pi_h^{(eq)}(+q_0,\mathbf{q})\gamma^0 \ . \tag{B.16}$$

Note that $\pi_\rho^{(eq)}(q)$ is pure imaginary while $\pi_h^{(eq)}(q)$ is real. The real part $\pi_h(q)$ contains a diverging integral which is subtracted by the mass renormalisation. In the body of the thesis, we have implicitly assumed that the self-energy $\pi_h^{(eq)}(q)$ is already regularized. The imaginary part $\pi_\rho^{(eq)}$ gives a decay width of the RH neutrino.

C Another derivation of $\Delta G'_{\gtrless}$

In this appendix we give another, quick and heuristic, derivation of $\Delta G'_{\gtrless}$. The derivation use some of the results justified in the systematic derivation adopted in this thesis. First we assume that the deviations of the Wightman functions from the thermal value at time t is given by the following form:

$$\begin{aligned}\Delta G_{\gtrless}^{ij}(X_{xy}, s_{xy}=0;\mathbf{q}) =& \sum_\epsilon \frac{\epsilon}{2\omega_q}(\not{q}_\epsilon + M)\big\{\Delta\mathcal{A}_{\gtrless}^{ij} + \Delta\dot{\mathcal{A}}_{\gtrless}^{ij}(X_{xy}-t) \\ &+ \epsilon\Delta\mathcal{B}_{\gtrless}^{ij} + \epsilon\Delta\dot{\mathcal{B}}_{\gtrless}^{ij}(X_{xy}-t) + \cdots\big\} \ .\end{aligned} \tag{C.17}$$

$\Delta\mathcal{A}, \Delta\mathcal{B}$ are terms which remain at $X_{xy} = t$, and $\mathcal{A}$ and $\mathcal{B}$ are introduced to represent ϵ dependence of the sum. Here we take the leading order with respect to $(X_{xy}-t)H \sim H/\Gamma$. Both of $\Delta\mathcal{A}$ and $\Delta\mathcal{B}$ have no spinor indices.

C.1 Solving KB equation for $G_{\gtrless}^{(eq)ij}$

For the diagonal component, (4.60) shows that

$$\Delta\mathcal{A}_{\gtrless}^{d} = 0 \ , \quad \Delta\mathcal{B}_{\gtrless}^{d} = \frac{d_t f_q^{(eq)}}{\Gamma_q} = \Delta\begin{Bmatrix} 1 - f_q \\ -f_q \end{Bmatrix} \ , \tag{C.18}$$

$$\Delta\dot{\mathcal{A}}_{\gtrless}^{d} = 0 \ , \quad \Delta\dot{\mathcal{B}}_{\gtrless}^{d} = -d_t f_q^{(eq)} = -\Gamma_q \Delta\mathcal{B}_{\gtrless}^{d} \ . \tag{C.19}$$

The Wightman functions in the thermal equilibrium at t are given in (4.40) for the diagonal component and (4.44) for the off-diagonal component. Hence they are similarly written in terms of $\mathcal{A}$ and $\mathcal{B}$ as

$$G_{\gtrless}^{(eq)ij}(X_{xy}, s_{xy} = 0; \mathbf{q}) = \sum_{\epsilon} \frac{\epsilon}{2\omega_q}(\not{q}_\epsilon + M)\left\{\mathcal{A}_{\gtrless}^{(eq)ij} + \epsilon\mathcal{B}_{\gtrless}^{(eq)ij}\right\}$$

where

$$\mathcal{A}_{>}^{d(eq)} = -\mathcal{A}_{<}^{d(eq)} = \frac{1}{2} \ , \quad \mathcal{B}_{>}^{d(eq)} = \mathcal{B}_{<}^{d(eq)} = \frac{1}{2}(1 - 2f_q^{(eq)}) \tag{C.20}$$

for the diagonal component and

$$\mathcal{A}_{\gtrless}^{'(eq)} = \mathcal{B}_{\gtrless}^{'(eq)} = 0 \tag{C.21}$$

for the off-diagonal component.

C.2 KB equation for $\Delta G_{\gtrless}^{ij}$

In the following we obtain the deviation of the off-diagonal component of the Wightman functions $\Delta\mathcal{A}_{\gtrless}^{'}$ directly by solving the KB equations using the above information. The KB equations for the off-diagonal Wightman functions are given by

$$\begin{aligned} &i\gamma^0\partial_{x^0} G_{\gtrless}^{ij}(x^0, y^0; \mathbf{q}) - \left\{\frac{\boldsymbol{\gamma}\cdot\mathbf{q}}{a(x^0)} + M_i\right\} G_{\gtrless}^{ij}(x^0, y^0; \mathbf{q}) \\ &\quad = \int dz^0\ \Pi_R^{ik}(x^0, z^0; \mathbf{q}) G_{\gtrless}^{kj}(z^0, y^0; \mathbf{q}) + \int dz^0\ \Pi_{\gtrless}^{ik}(x^0, z^0; \mathbf{q}) G_A^{kj}(z^0, y^0; \mathbf{q}) \end{aligned} \tag{C.22}$$

or

$$\begin{aligned} &-i\gamma^0\partial_{y^0} G_{\gtrless}^{ij}(x^0, y^0; \mathbf{q}) - G_{\gtrless}^{ij}(x^0, y^0; \mathbf{q})\left\{\frac{\boldsymbol{\gamma}\cdot\mathbf{q}}{a(y^0)} + M_j\right\} \\ &\quad = \int dz^0\ G_{\gtrless}^{ik}(x^0, z^0; \mathbf{q}) \Pi_A^{kj}(z^0, y^0; \mathbf{q}) + \int dz^0\ G_R^{ik}(x^0, z^0; \mathbf{q}) \Pi_{\gtrless}^{kj}(z^0, y^0; \mathbf{q}) \end{aligned} \tag{C.23}$$

Setting $x^0 = y^0 = t$ and take a difference of these two equations. Summing over the spinor indices, we have

$$
\begin{aligned}
& i\partial_X G^{ij}_{\gtrless V^0}(X;\mathbf{q})\Big|_{X=t} - (M_i - M_j) G^{ij}_{\gtrless S}(X=t;\mathbf{q}) \\
&= \int dz^0 \ \frac{1}{4}\mathrm{tr}\left\{\Pi^{ik}_R(t,z^0;\mathbf{q}) G^{kj}_{\gtrless}(z^0,t;\mathbf{q}) - G^{ik}_{\gtrless}(t,z^0;\mathbf{q})\Pi^{kj}_A(z^0,t;\mathbf{q})\right\} \\
&+ \int dz^0 \ \frac{1}{4}\mathrm{tr}\left\{\Pi^{ik}_{\gtrless}(t,z^0;\mathbf{q}) G^{kj}_A(z^0,t;\mathbf{q}) - G^{ik}_R(t,z^0;\mathbf{q})\Pi^{kj}_{\gtrless}(z^0,t;\mathbf{q})\right\} \ . \quad \text{(C.24)}
\end{aligned}
$$

On the other hand, multiplying γ_0 and then summing over the spinor indices, we have

$$
\begin{aligned}
& i\partial_X G^{ij}_{\gtrless S}(X;\mathbf{q})\Big|_{X=t} - \frac{2}{4}\mathrm{tr}\left\{\gamma^0 \frac{\boldsymbol{\gamma}\cdot\mathbf{q}}{a(t)} G^{ij}_{\gtrless}(X=t;\mathbf{q})\right\} - (M_i - M_j) G^{ij}_{\gtrless V^0}(X=t;\mathbf{q}) \\
&= \int dz^0 \ \frac{1}{4}\mathrm{tr}\left\{\gamma^0\Pi^{ik}_R(t,z^0;\mathbf{q}) G^{kj}_{\gtrless}(z^0,t;\mathbf{q}) - \gamma^0 G^{ik}_{\gtrless}(t,z^0;\mathbf{q})\Pi^{kj}_A(z^0,t;\mathbf{q})\right\} \\
&+ \int dz^0 \ \frac{1}{4}\mathrm{tr}\left\{\gamma^0\Pi^{ik}_{\gtrless}(t,z^0;\mathbf{q}) G^{kj}_A(z^0,t;\mathbf{q}) - \gamma^0 G^{ik}_R(t,z^0;\mathbf{q})\Pi^{kj}_{\gtrless}(z^0,t;\mathbf{q})\right\}
\end{aligned}
\quad \text{(C.25)}
$$

where

$$
G^{ij}_{\gtrless S}(X;\mathbf{q}) \equiv \frac{1}{4}\mathrm{tr}\{G^{ij}_{\gtrless}(X,s=0;\mathbf{q})\} \ , \quad G^{ij}_{\gtrless V^\mu}(X;\mathbf{q}) \equiv \frac{1}{4}\mathrm{tr}\{\gamma^\mu G^{ij}_{\gtrless}(X,s=0;\mathbf{q})\} \ . \quad \text{(C.26)}
$$

We are now interested in the deviation from the thermal values at time t. The equations (C.24) and (C.25) are rewritten as

$$
\begin{aligned}
& i\Delta\dot{\mathcal{A}}^{ij}_{\gtrless} - (M_i - M_j)\frac{M}{\omega_q}\Delta\mathcal{B}^{ij}_{\gtrless} \\
&= \int dz^0 \ \frac{1}{4}\mathrm{tr}\left\{\Pi^{(eq)ik}_R(t,z^0;\mathbf{q})\Delta\mathcal{G}^{kj}_{\gtrless}(z^0,t;\mathbf{q}) - \Delta\mathcal{G}^{ik}_{\gtrless}(t,z^0;\mathbf{q})\Pi^{(eq)kj}_A(z^0,t;\mathbf{q})\right\} \\
&= \sum_\epsilon \frac{\epsilon}{2\omega_q}\eta^{\mu\nu} q_{\epsilon\nu}\left\{\Pi^{(eq)ik}_{RV^\mu}(\epsilon\omega_q,\mathbf{q})(\Delta\mathcal{A}^{kj}_{\gtrless} + \epsilon\Delta\mathcal{B}^{kj}_{\gtrless}) - \Pi^{(eq)kj}_{AV^\mu}(\epsilon\omega_q,\mathbf{q})(\Delta\mathcal{A}^{ik}_{\gtrless} + \epsilon\Delta\mathcal{B}^{ik}_{\gtrless})\right\} \\
&= \frac{1}{2\omega_q}\left\{q\cdot\Pi^{(eq)ik}_\rho(\omega_q,\mathbf{q})\Delta\mathcal{A}^{kj}_{\gtrless} + q\cdot\Pi^{(eq)kj}_\rho(\omega_q,\mathbf{q})\Delta\mathcal{A}^{ik}_{\gtrless}\right\} \\
&\qquad + \frac{1}{\omega_q}\left\{q\cdot\Pi^{(eq)ik}_h(\omega_q,\mathbf{q})\Delta\mathcal{B}^{kj}_{\gtrless} - q\cdot\Pi^{(eq)kj}_h(\omega_q,\mathbf{q})\Delta\mathcal{B}^{ik}_{\gtrless}\right\} \quad \text{(C.27)}
\end{aligned}
$$

and

$$
\begin{aligned}
& i\frac{M}{\omega_q}\Delta\dot{\mathcal{B}}^{ij}_{\gtrless} - \frac{2}{4}\mathrm{tr}\left\{\gamma^0\frac{\boldsymbol{\gamma}\cdot\mathbf{q}}{a(t)}\Delta G^{ij}_{\gtrless}(X=t;\mathbf{q})\right\} - (M_i - M_j)\Delta\mathcal{A}^{ij}_{\gtrless} \\
&= \int dz^0\ \frac{1}{4}\mathrm{tr}\left\{\gamma^0\Pi^{(eq)ik}_R(t,z^0;\mathbf{q})\Delta\mathcal{G}^{kj}_{\gtrless}(z^0,t;\mathbf{q}) - \gamma^0\Delta\mathcal{G}^{ik}_{\gtrless}(t,z^0;\mathbf{q})\Pi^{(eq)kj}_A(z^0,t;\mathbf{q})\right\} \\
&= \sum_\epsilon \frac{\epsilon M}{2\omega_q}\left\{\Pi^{(eq)ik}_{RV^0}(\epsilon\omega_q,\mathbf{q})(\Delta\mathcal{A}^{kj}_{\gtrless} + \epsilon\Delta\mathcal{B}^{kj}_{\gtrless}) - \Pi^{(eq)kj}_{AV^0}(\epsilon\omega_q,\mathbf{q})(\Delta\mathcal{A}^{ik}_{\gtrless} + \epsilon\Delta\mathcal{B}^{ik}_{\gtrless})\right\} \\
&= \frac{M}{2\omega_q}\left\{\Pi^{(eq)ik}_{\rho V^0}(\omega_q,\mathbf{q})\Delta\mathcal{B}^{kj}_{\gtrless} + \Pi^{(eq)kj}_{\rho V^0}(\omega_q,\mathbf{q})\Delta\mathcal{B}^{ik}_{\gtrless}\right\} \\
&\qquad + \frac{M}{\omega_q}\left\{\Pi^{(eq)ik}_{hV^0}(\omega_q,\mathbf{q})\Delta\mathcal{A}^{kj}_{\gtrless} - \Pi^{(eq)kj}_{hV^0}(\omega_q,\mathbf{q})\Delta\mathcal{A}^{ik}_{\gtrless}\right\}\ . \qquad \text{(C.28)}
\end{aligned}
$$

The first line of each equation is nothing but the l.h.s. of (C.24) and (C.25) written in term of the definitions in (C.17). The second lines of them are the r.h.s. of (C.24) and (C.25) in which small deviations from the thermal values are considered. The terms represent the dominant contributions and terms like $\Delta\Pi_{R/A}$, $\Delta\Pi_{\gtrless}$ and $\Delta G_{R/A}$ are dropped. This is justified because of the large damping factor $\Pi(t,z^0) \sim e^{-|t-z^0|\Gamma_{\ell\phi}/2}$ of the self-energies. In the second equalities, we performed time integrations and taking the trace with respect to indices of spinor. In the third equalities, we used (B.11) and (B.16).

C.3 Diagonal component $\Delta G^{dii}_{\gtrless}$

Let us first look at the diagonal component. We use the simple expression of the self-energy (B.13) by neglecting the medium effects and in the weak coupling limit. Then we have

$$
\begin{aligned}
\Pi^{d(eq)}_{\rho V^0}(\omega_q,\mathbf{q}) &= (\omega_q/M^2)q\cdot\Pi^{(eq)ik}_{\rho}(\omega_q,\mathbf{q}) \\
&= -i(\omega_q/M)\Gamma\ =\ -i(\omega_q^2/M^2)\Gamma_q\ . \qquad \text{(C.29)}
\end{aligned}
$$

With this relation, (C.27) and (C.28) are simplified to be

$$
i\Delta\dot{\mathcal{A}}^d_{\gtrless} = -i\Gamma_q\Delta\mathcal{A}^d_{\gtrless} \qquad \text{(C.30)}
$$

and

$$
i\frac{M}{\omega_q}\Delta\dot{\mathcal{B}}^d_{\gtrless} - \frac{2}{4}\mathrm{tr}\left\{\gamma^0\frac{\boldsymbol{\gamma}\cdot\mathbf{q}}{a(t)}\Delta G^d_{\gtrless}(X=t;\mathbf{q})\right\} = -i\frac{\omega_q}{M}\Gamma_q\Delta\mathcal{B}^d_{\gtrless}\ . \qquad \text{(C.31)}
$$

(C.19) indeed satisfies (C.30). The second term of the l.h.s. of (C.31) vanishes in the leading order approximation (C.17), but using the next to leading order approximation of ΔG^d, the second term becomes $-i\Gamma_q(|\mathbf{q}|^2/a^2)/(M\omega_q)$. Then eq. (C.31) is satisfied.

C.4 Off-diagonal component $\Delta G^{'ij}_{\gtrless}$

Then we study the off-diagonal component. Using $\Pi(\omega_q, \mathbf{q}) \propto \not{q}$, (C.27) and (C.28) become

$$
\begin{aligned}
& i\Delta\dot{\mathcal{A}}^{'ij}_{\gtrless} - (M_i - M_j)\frac{M}{\omega_q}\Delta\mathcal{B}^{'ij}_{\gtrless} \\
& = -i\frac{M}{2\omega_q}\left\{\Gamma_i + \Gamma_j\right\}\Delta\mathcal{A}^{'ij}_{\gtrless} + \frac{1}{2\omega_q}(q\cdot\Pi^{'(eq)ij}_{\rho}(\omega_q, \mathbf{q}))\left\{\Delta\mathcal{A}^{djj}_{\gtrless} + \Delta\mathcal{A}^{dii}_{\gtrless}\right\} \\
& + \frac{1}{\omega_q}(q\cdot\Pi^{'(eq)ij}_{h}(\omega_q, \mathbf{q}))\left\{\Delta\mathcal{B}^{djj}_{\gtrless} - \Delta\mathcal{B}^{dii}_{\gtrless}\right\}
\end{aligned}
\tag{C.32}
$$

and

$$
\begin{aligned}
& i\frac{M}{\omega_q}\Delta\dot{\mathcal{B}}^{'ij}_{\gtrless} - \frac{2}{4}\mathrm{tr}\left\{\gamma^0\frac{\boldsymbol{\gamma}\cdot\mathbf{q}}{a(t)}\Delta G^{'ij}_{\gtrless}(X=t;\mathbf{q})\right\} - (M_i - M_j)\Delta\mathcal{A}^{'ij}_{\gtrless} \\
& = -i\frac{1}{2}\left\{\Gamma_i + \Gamma_j\right\}\Delta\mathcal{B}^{'ij}_{\gtrless} + \frac{1}{2M}(q\cdot\Pi^{'(eq)ij}_{\rho}(\omega_q, \mathbf{q}))\left\{\Delta\mathcal{B}^{djj}_{\gtrless} + \Delta\mathcal{B}^{dii}_{\gtrless}\right\} \\
& + \frac{1}{M}(q\cdot\Pi^{'(eq)ij}_{h}(\omega_q, \mathbf{q}))\left\{\Delta\mathcal{A}^{djj}_{\gtrless} - \Delta\mathcal{A}^{dii}_{\gtrless}\right\} \ .
\end{aligned}
\tag{C.33}
$$

Here we absorbed the real part of the self-energy Π_h into the mass term in the l.h.s. by the mass renormalisation.

For the off-diagonal component, we can expect that

$$
\Delta\dot{\mathcal{A}}^{'}_{\gtrless} = \Delta\dot{\mathcal{B}}^{'}_{\gtrless} = 0 \ . \tag{C.34}
$$

This comes from the relation (4.45) or equivalently (C.21). Since it vanishes in the thermal equilibrium, its variation due to the change of the local temperature is also expected to vanish in the leading order approximation. On the contrary, since the equilibrium diagonal Wightman function survives in the same limit, its variation (or $\Delta\dot{\mathcal{B}}^{d} \neq 0$) does not vanish either. Furthermore, the second term in the l.h.s. of (C.33) is also expected to give no leading contribution like the first term $\Delta\dot{\mathcal{B}}^{'}_{\gtrless}$.

Using the above arguments, the equations (C.32) and (C.33) are simplified as equations to determine $\Delta\mathcal{A}^{'}_{\gtrless}$ and $\Delta\mathcal{B}^{'}_{\gtrless}$ in terms of $\Delta\mathcal{A}^{d}_{\gtrless}$ and $\Delta\mathcal{B}^{d}_{\gtrless}$, and they are solved as

$$
\begin{aligned}
\begin{pmatrix}\Delta\mathcal{A}^{'ij}_{\gtrless} \\ \Delta\mathcal{B}^{'ij}_{\gtrless}\end{pmatrix} = & \frac{-1}{(\Delta M^2_{ij})^2 + (M\Gamma_i + M\Gamma_j)^2}\begin{pmatrix} i(M\Gamma_i + M\Gamma_j) & \Delta M^2_{ij} \\ \Delta M^2_{ij} & i(M\Gamma_i + M\Gamma_j)\end{pmatrix} \\
& \times \begin{pmatrix} 2(q\cdot\Pi^{'(eq)ij}_{h}(\omega_q, \mathbf{q}))\left\{\Delta\mathcal{B}^{djj}_{\gtrless} - \Delta\mathcal{B}^{dii}_{\gtrless}\right\} \\ (q\cdot\Pi^{'(eq)ij}_{\rho}(\omega_q, \mathbf{q}))\left\{\Delta\mathcal{B}^{djj}_{\gtrless} + \Delta\mathcal{B}^{dii}_{\gtrless}\right\}\end{pmatrix} \ .
\end{aligned}
\tag{C.35}
$$

The regulator $M_i\Gamma_i + M_j\Gamma_j$ controls the enhancement of the solutions for $\Delta G^{'}_{\gtrless}$. This expression corresponds to (4.69).

C.5 $\Delta G'_{\gtrless}$ based on a wrong assumption $G'_{\gtrless} \neq 0$

Finally in this appendix, we discuss how we will obtain an erroneous answer with the regulator of the type $M_i\Gamma_i - M_j\Gamma_j$. Let us assume (which turns out to be wrong) that the off-diagonal component did not vanish and is given by

$$i\Delta\dot{\mathcal{A}}''_{\gtrless} = -i\overline{\Gamma}_q \Delta\mathcal{A}''_{\gtrless} \tag{C.36}$$

and

$$i\frac{M}{\omega_q}\Delta\dot{\mathcal{B}}''_{\gtrless} - \frac{2}{4}\mathrm{tr}\left\{\gamma^0 \frac{\boldsymbol{\gamma}\cdot\mathbf{q}}{a(t)}\Delta G''_{\gtrless}(X=t;\mathbf{q})\right\} = -i\frac{\omega_q}{M}\overline{\Gamma}_q\Delta\mathcal{B}''_{\gtrless}\ . \tag{C.37}$$

Here $\overline{\Gamma}_q = \overline{\Gamma}M/\omega_q$ is of the same order as $\Gamma_{iq} = \Gamma_i M/\omega_q$ $(i = 1, 2)$. These are similar to the correct relations for the diagonal components, (C.30) and (C.31).

The above equations (C.36) and (C.37) are based on a correct-looking assumption that the deviations of the off-diagonal Wightman functions out of equilibrium are obtained by taking a variation of the the equilibrium value with respect to the local temperature. In other words, it is assumed that there exists an "off-diagonal distribution function $f_q^{'(eq)}$" which does not vanish at $s_{xy} = x - y = 0$ and its deviation from the equilibrium value satisfies the relation $\Delta f'_q = -d_t f_q^{'(eq)}/\overline{\Gamma}_q$. (As a matter of fact, such a function does not exist.)

Under such incorrect assumptions, additional terms change the l.h.s of (C.32) and (C.33), and the regulator is modified to be

$$\Gamma_i + \Gamma_j \to \Gamma_i + \Gamma_j - 2\overline{\Gamma} \sim \Gamma_i - \Gamma_j\ . \tag{C.38}$$

This is the way we could obtain an erroneous enhancement factor.

D Derivation of the kinetic term $d_t f_N$

In this appendix, we show how the the kinetic term in (5.20) $-id_t f_{N,h,q}$ is derived from the l.h.s. in (5.15):

$$\begin{aligned}-i\mathrm{tr}\Big[P_h\Big(&\Diamond\left\{\gamma^0 q_0 - \tfrac{\mathbf{q}\cdot\boldsymbol{\gamma}}{a} - \hat{M} - \Pi_R^{eq}\right\}\{if\}\,G_\rho^{eq} - \Pi_\rho^{eq}\Diamond\{if\}\{G_A^{eq}\}\\ &-G_\rho^{eq}\Diamond\{if\}\left\{\gamma^0 q_0 - \tfrac{\mathbf{q}\cdot\boldsymbol{\gamma}}{a} - \hat{M} - \Pi_A^{eq}\right\} + \Diamond\{G_R^{eq}\}\{if\}\,\Pi_\rho^{eq}\Big)\Big]\ .\end{aligned} \tag{D.1}$$

First we look at the leading term. For simplicity, we drop the self-energy correction Π_R^{eq}. Then we have

$$\begin{aligned}&i\mathrm{tr}\left[P_h\sum_{h'}\Diamond\left\{\gamma^0 q_0 - \frac{\mathbf{q}\cdot\boldsymbol{\gamma}}{a} - \hat{M}\right\}\{if_{h'}^{eq}\}(\not{q}+M)P_{h'}\right]\frac{\Gamma_a}{((q_0-\omega_q)^2+\Gamma_q^2/4)}\\ &= q_0\left(\partial_X f_h^{eq}(q_0,X) - \frac{H|\mathbf{q}|^2}{q_0 a^2}\partial_{q_0} f_h^{eq}(q_0,X)\right)\frac{\Gamma_a}{((q_0-\omega_q)^2+\Gamma_q^2/4)}\end{aligned} \tag{D.2}$$

If we set $q_0 = \omega_q$, two terms in the bracket give a total derivative

$$d_t = (\partial_t T)\partial_T + (\partial_t \omega_q)\partial_{\omega_q} \tag{D.3}$$

of the on-shell Fermi distribution function $f_{hq}^{eq} \equiv f^{eq}(t, \omega_q(t))$ in equilibrium. But the propagator has a Lorentz type structure and q_0 is extended around the position of the pole $q_0 = \omega_q$.

We then take an effect of the remaining terms in (D.1). These terms can be rewritten as

$$\begin{aligned} & i\mathrm{tr}\left\{P_h \Pi_\rho \diamondsuit \left\{if^{eq}\right\}\left\{G_A\right\} - P_h \diamondsuit \left\{G_R\right\}\left\{if^{eq}\right\}\Pi_\rho\right\} \\ & = i\mathrm{tr}\left\{P_h \Pi_\rho G_A \diamondsuit \left\{if^{eq}\right\}\left\{G_A^{-1}\right\} G_A - P_h G_R \diamondsuit \left\{G_R^{-1}\right\}\left\{if^{eq}\right\} G_R \Pi_\rho\right\} \\ & \simeq -\mathrm{tr}\left\{P_h \diamondsuit \left\{\gamma^0 q_0 - \frac{\mathbf{q}\cdot\boldsymbol{\gamma}}{a} - \hat{M}\right\}\left\{f^{eq}\right\}\left(G_R \Pi_\rho G_R + G_A \Pi_\rho G_A\right)\right\} \ . \end{aligned} \tag{D.4}$$

In the first equality, we have used the relation $\diamondsuit\{f\}\{A\} = -\diamondsuit\{A\}\{f\}$ and $\diamondsuit\{f\}\{A\} = A\diamondsuit\{f\}\{A^{-1}\}A$ for a given matrix A. In the second line, we have used $G_{R/A}^{-1} = -(\not{q} - \hat{M} - \Pi_{R/A})$ and dropped next-to-leading order contributions $\Pi_{R,A}$.

Using (D.4), four terms in (D.1) are combined to become

$$\begin{aligned} & 2\mathrm{tr}\left\{P_h \diamondsuit \left\{\gamma^0 q_0 - \frac{\mathbf{q}\cdot\boldsymbol{\gamma}}{a} - \hat{M}\right\}\left\{f\right\} G_\rho\right\} \\ & - \mathrm{tr}\left\{P_h \diamondsuit \left\{\gamma^0 q_0 - \frac{\mathbf{q}\cdot\boldsymbol{\gamma}}{a} - \hat{M}\right\}\left\{f^{eq}\right\}\left(G_R \Pi_\rho G_R + G_A \Pi_\rho G_A\right)\right\} \\ & \simeq \left(\partial_X f_h(q_0, X) - \frac{H|\mathbf{q}|^2}{q_0 a^2}\partial_{q_0} f_h(q_0, X)\right) \times (-i) \\ & \quad \times \left(\frac{\Gamma_q}{(q_0-\omega_q)^2 + \Gamma_q^2/4} - \frac{\Gamma_q\,(q_0-\omega_q - i\Gamma_q/2)^2}{2((q_0-\omega_q)^2+\Gamma_q^2/4)^2} - \frac{\Gamma_q\,(q_0-\omega_q + i\Gamma_q/2)^2}{2((q_0-\omega_q)^2+\Gamma_q^2/4)^2}\right) \\ & = -i\left(\partial_X f_h(q_0, X) - \frac{H|\mathbf{q}|^2}{q_0 a^2}\partial_{q_0} f_h(q_0, X)\right) \times \frac{\Gamma_q^3/2}{((q_0-\omega_q)^2+\Gamma_q^2/4)^2} \end{aligned} \tag{D.5}$$

around the position of the pole $q_0 = \omega_q$. Here, we used the approximate form $\Pi_\rho \sim \not{q} \times (-i\omega_{q_0}\Gamma_q/M^2)$ and dropped higher order terms with respect to $(q_0 - \omega_q)$. Hence, the original Lorentz type distribution becomes to have a sharper spectrum after adding the higher order terms in the KB equation. Namely, the term $\Gamma_q^3/2/((q_0-\omega_q)^2+\Gamma_q^2/4)^2$ approaches Dirac delta function $2\pi\delta(q_0-\omega_q)$ much faster than the usual Lorentz type form $\Gamma_a/((q_0-\omega_q)^2+\Gamma_q^2/4)$ in the limit $\Gamma_q \to 0$ [85].

E Explicit forms of $\mathcal{C}^{-1}$ and $\widetilde{\mathcal{C}}^{-1}$

In this appendix, we show explicit forms of $\mathcal{C}^{-1}$ and $\widetilde{\mathcal{C}}^{-1}$ used in the section 5.3.3. For brevity, we write each coefficient of the 2×2 matrices Γ^{eff} and $\widetilde{\Gamma}^{\mathrm{eff}}$

expanded in terms of $(1_{2\times 2}, \sigma^a)$ as $[\Gamma]^a$ and $[\widetilde{\Gamma}]^a$ without the supersctipt "eff".

$$(\mathcal{C}^{-1})^{a0} = \frac{-1}{D}\begin{pmatrix} [\Gamma_N]^0 \left\{([\Gamma_N]^0)^2 + (2[\mathrm{H}]^3)^2\right\} \\ -([\Gamma_N]^0)^2[\Gamma_N]^1 \\ -2[\mathrm{H}]^3[\Gamma_N]^0[\Gamma_N]^1 \\ -[\Gamma_N]^3 \left\{([\Gamma_N]^0)^2 + (2[\mathrm{H}]^3)^2\right\} \end{pmatrix}^a$$
$$= \frac{-\xi_0^3}{D(sY_N^{eq})^3}\begin{pmatrix} [\Gamma]^0 \left\{([\Gamma]^0)^2 + (M_1 - M_2)^2\right\} \\ -([\Gamma]^0)^2[\Gamma]^1 \\ -(M_1 - M_2)[\Gamma]^0[\Gamma]^1 \\ -[\Gamma]^3 \left\{([\Gamma]^0)^2 + (M_1 - M_2)^2\right\} \end{pmatrix}^a ,$$
$$(\widetilde{\mathcal{C}}^{-1})^{a0} = \frac{-1}{D}\begin{pmatrix} 0 \\ +2[\mathrm{H}]^3[\Gamma_N]^0[\widetilde{\Gamma}_N]^2 \\ -([\Gamma_N]^0)^2[\widetilde{\Gamma}_N]^2 \\ 0 \end{pmatrix}^a = \frac{-\xi_0^3}{D(sY_N^{eq})^3}\begin{pmatrix} 0 \\ +(M_1 - M_2)[\Gamma]^0[\widetilde{\Gamma}]^2 \\ -([\Gamma]^0)^2[\widetilde{\Gamma}]^2 \\ 0 \end{pmatrix}^a , \tag{E.1}$$

$$(C^{-1})^{a2} = \frac{-1}{D}\begin{pmatrix} +2[\mathrm{H}]^3[\Gamma_N]^0[\Gamma_N]^1 \\ -2[\mathrm{H}]^3 \left\{([\Gamma_N]^0)^2 - ([\Gamma_N]^3)^2\right\} \\ [\Gamma_N]^0 \left\{([\Gamma_N]^0)^2 - ([\Gamma_N]^1)^2 - ([\Gamma_N]^3)^2\right\} \\ -2[\mathrm{H}]^3[\Gamma_N]^1[\Gamma_N]^3 \end{pmatrix}^a$$
$$= \frac{-\xi_0^3}{D(sY_N^{eq})^3}\begin{pmatrix} +(M_1 - M_2)[\Gamma]^0[\Gamma]^1 \\ -(M_1 - M_2) \left\{([\Gamma]^0)^2 - ([\Gamma]^3)^2\right\} \\ [\Gamma]^0 \left\{([\Gamma]^0)^2 - ([\Gamma]^1)^2 - ([\Gamma]^3)^2\right\} \\ -(M_1 - M_2)[\Gamma]^1[\Gamma]^3 \end{pmatrix}^a . \tag{E.2}$$

where determinant D is given by

$$D = \left\{([\Gamma_N]^0)^2 - ([\Gamma_N]^3)^2\right\}\left[(2[\mathrm{H}]^3)^2 + ([\Gamma_N]^0)^2 \frac{([\Gamma_N]^0)^2 - [\Gamma_N \cdot \Gamma_N] - [\widetilde{\Gamma}_N \cdot \widetilde{\Gamma}_N]}{([\Gamma_N]^0)^2 - ([\Gamma_N]^3)^2}\right]$$
$$= \frac{\xi_0^4}{(sY_N^{eq})^4}\Gamma_1\Gamma_2\left[(M_1 - M_2)^2 + ([\Gamma]^0)^2 \frac{\det\{\Gamma\} - ([\widetilde{\Gamma}]^2)^2}{\Gamma_1\Gamma_2}\right]. \tag{E.3}$$

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