

SPONTANEOUS SYMMETRY BREAKING AND MASS GENERATION AS BUILT-IN PHENOMENA IN LOGARITHMIC NONLINEAR QUANTUM THEORY

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Our primary task is to demonstrate that the logarithmic nonlinearity in the quantum wave equation can cause the spontaneous symmetry breaking and mass generation phenomena on its own, at least in principle. To achieve this goal, we view the physical vacuum as a kind of the fundamental Bose–Einstein condensate embedded into the fictitious Euclidean space. The relation of such description to that of the physical (relativistic) observer is established via the fluid/gravity correspondence map, the related issues, such as the induced gravity and scalar field, relativistic postulates, Mach’s principle and cosmology, are discussed. For estimate the values of the generated masses of the otherwise massless particles such as the photon, we propose few simple models which take into account small vacuum fluctuations. It turns out that the photon’s mass can be naturally expressed in terms of the elementary electrical charge and the extensive length parameter of the nonlinearity. Finally, we outline the topological properties of the logarithmic theory and corresponding solitonic solutions.

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1. Introduction

Current observational data in astrophysics are probing a regime of departures from classical relativity with sensitivities that are relevant for the study of the quantum-gravity problem [1,2]. On the other hand, the quantum theory of gravity which would be both widely agreed upon and capable of making unique testable predictions is still pending. In this connection, the effective non-axiomatic theories and semi-phenomenological approaches guided by the physical intuition can be very helpful as they may provide

new ideas and insights [3]. We already proposed elsewhere [4] that the non-trivial vacuum causes the deformation of the quantum wave equations of the universal form:

$$\left[\hat{\mathcal{H}} - \beta^{-1} \ln(\Omega |\Psi|^2) \right] \Psi = 0, \quad (1)$$

where Ψ refers in general to the complex-valued wave functional and $\hat{\mathcal{H}}$ is the operator which form is determined by a physical setup. The physical motivation behind this equation as well as its unique properties are listed in the Appendix. Here β and Ω are constant parameters. If we impose that Ω has the dimensionality of a spatial volume then the logarithmic term (1) introduces the primary (extensive) length scale,

$$\ell_\Omega = \Omega^{1/(D-1)}, \quad (2)$$

which role and possible physical meaning will be discussed below; here $D - 1$ refers to the number of spatial dimensions, throughout the paper it is assumed $D = 4$.

It was shown that some phenomenological consequences of such theory are actually model-independent and can be derived even at the kinematical level, *i.e.*, prior to specifying the dynamical details of a quantum-gravitational model. One of the primary phenomenological implications of this theory is that for any two freely-moving particles the following relation is valid

$$\frac{d\tau_2}{d\tau_1} = \frac{E_2 - E_0}{E_1 - E_0} = 1 - \frac{\Delta E}{E_0} + \mathcal{O}\frac{E^2}{E_0^2}, \quad (3)$$

where τ_i and E_i are the proper time and energy of the i th particle, E_0 is the energy of the vacuum of a theory; for the vacuum not affected by external fields that would be $E_0 = \pm E_{\text{QG}}$, $E_{\text{QG}} \lesssim 10^{19}$ GeV. The value E_0 defines another length scale, the Compton-type one:

$$\ell_0 = \frac{hc}{|E_0|}, \quad (4)$$

which value thus can be as small as the Planck one. We expect that the properties of any dynamical systems immersed in such vacuum can change drastically when their characteristic length scales approach either of the critical values ℓ_0 and ℓ_Ω , or, in terms of energy, E_0 and $E_\Omega = hc/\ell_\Omega$.

The effective refractive index can be directly computed from corresponding dispersion relations (taking into account that both the Planck relation and energy additivity of uncorrelated systems survive in the logarithmic theory [5], in contrast to other nonlinear extensions of quantum mechanics, see also the Appendix). In the Cauchy form the index can be written as

$$n^2 = 1 + \mu_\gamma \left[1 + \mathcal{M}(\omega) \left(\frac{\omega}{2\pi c} \right)^2 \right], \quad (5)$$

where $\mu_\gamma = \chi_\gamma^2 - 1$ and $\mathcal{M}(\omega) = (2\pi c/\omega_0)^2 (1 \pm 2\omega_0/\omega)$ are, respectively, the constant of refraction and dispersion coefficient of the vacuum, ω is the angular frequency of the electromagnetic wave, $\omega_0 = |E_0|/\hbar$ is the proper frequency of the vacuum, $\pm = -\text{sign}(E_0)$.

All this suggests that the vacuum is the medium with non-trivial properties which affects photons and other particles propagating through it, and the effects grow along with particles' energies. The predicted phenomena which can be derived from Eq. (3) can be cast into three groups:

- (i) *subluminal phenomena*: the estimates imply that the particles with higher energy propagate slower than those with lower one, therefore, for a high-energy particle the mean free path, lifetime in a high-energy state and, therefore, travel distance from the source can be significantly larger than one would expect from the conventional relativity theory. There already exists tentative evidence of this effect, often referred as the “high-energy tail” [6];
- (ii) *transluminal phenomena*: according to the theory, particles can reach the speed of light in vacuum at finite energy. This may cause the “luminal boom” in vacuum and appearance of a conical front of the Cherenkov-type shock wave. These effects can be detected at the Earth's particle accelerators — the special feature of the latter is the particles get accelerated to ultrarelativistic speeds in a controlled way whereas the cosmic-ray particles have been accelerated somewhere else, usually very far from our detectors. Of course, the outcomes of the accelerator studies will totally depend on the value of E_0 which is not that simple to compute because the vacuum inside the accelerator pipe is distorted by external fields;
- (iii) *superluminal phenomena*: unlike the tachyons in the classical relativity, in the logarithmic theory the energies of the superluminal particles are real-valued and stay finite when their propagation speed approaches c . The electromagnetic component of their Cherenkov radiation may exhibit the anomalous Doppler effect — similar to the one for the superluminal (non-point) sources in vacuum which was predicted even at the classical relativistic level by Bolotovskii and Ginzburg [7]. Also there may exist the phenomenon of mimicking the double-lobed radio sources in astrophysics. In general, the current understanding of physical phenomena happening in supernovae, active galactic nuclei and gamma-ray bursts may need a serious revision.

As mentioned earlier, these phenomena are determined mainly by the kinematics of the theory — in a sense, they are analogues of the kinematic effects of special relativity. What about the dynamical ones, is it possible to

find any without specifying an underlying microscopical model? In general the answer is naturally “no” but there exists (at least) one exception: the mechanism of the spontaneous symmetry breaking is actually hidden in the logarithmic term itself. Of course, this does not exclude the existence of other symmetry-breaking mechanisms caused by the dynamics of a concrete model.

Spontaneous symmetry breaking occurs when the ground state of a system does not possess the full symmetry of the theory. The most famous its realization in physics is known as the (Englert–Brout–)Higgs(–Guralnik–Hagen–Kibble–Nambu–Anderson) mechanism [8]. The closely related phenomenon is the mass generation which has been employed in the Glashow–Weinberg–Salam electroweak theory as to explain the nonzero masses of the intermediate vector bosons by breaking the electroweak symmetry group $SU(2) \times U(1)$ down to the electromagnetic $U(1)$ [9]. This mechanism is mediated by the yet undiscovered particle, Higgs boson, which mass is currently narrowed to be between 114 and 158 GeV — provided that the Standard Model (SM) remains valid at that energy range.

Despite the overall success of the electroweak theory, few questions about its Higgs mechanism remain open. The one of them is the following. Intuitively one would expect that anything related to the mass creation must be governed by gravity, be it classical or quantum — as the Mach’s principle suggests, for instance. But SM, in its current formulation, does not have the gravitational sector. Instead, the role of the “mass generator” is transferred to the Higgs particle from the electroweak sector. The gravity seems to be totally excluded from this process. From the mathematical point of view, no mass generation mechanism which would naturally appear as a solely (quantum-)gravitational effect, *i.e.*, without involving other matter fields, has been proposed so far, to our best knowledge. On the other hand, in quantum field theory it has been already known that the radiation corrections themselves can cause the spontaneous symmetry breaking [10].

This issue is closely related to the second question — what *is* the physical vacuum: what are its properties, how do they change at higher energies and shorter scales of length, *etc.* Regrettably, up to now no reliable theory of the physical vacuum actually exists. The two most popular nowadays theories, SM and string theory, are practically useless in this regard. The former is the operational Lorentz-invariant renormalizable theory which means that it does not take into account that the physical vacuum can break the Lorentz invariance at high energies (of order TeV and above) and shorter length scales, also the theory replaces important parameters, such as masses and charges of elementary particles, by their experimentally measured values thus giving no theoretical explanations for why their values are the way they are. In particular, the value of zero-point energy when computed in the

electroweak or QCD sectors disagrees with the one restricted by astrophysical observations by more than a hundred orders of magnitude — one of the most striking manifestations of the so-called “vacuum catastrophe” or “cosmological constant problem” noticed by Nernst almost a century ago [11]. The superstring theory, apart from being based on the Lorentz symmetry too, suffers from the so-called “landscape problem”: it gives almost infinitely many mutually exclusive predictions about the structure of the physical vacuum. It may turn out that this problem is not just a temporary difficulty of the theory but the indication of the Lorentz symmetry’s breakdown in Nature at some energy and length scale. As a result, certain mathematical constructions heavily relying upon (or motivated by) this symmetry, such as supersymmetry or tensor representations of the Poincaré group, should be attributed to the real world with utmost care — as their characteristic energy scales can lie outside the validity range of the Lorentz-symmetric approach.

The third issue is the mass of the photon. In the current Standard Model the photon is assumed to be strangely exceptional — its mass remains zero even after the electroweak symmetry breaking. On the other hand, recent observational data bring certain evidence that the photon propagates with the subluminal speed and thus can be assigned a mass, at least effectively, but of an extremely small value, as compared to that of the intermediate vector bosons. This suggests that the mass generation mechanism for the photon must be in something drastically different from the electroweak one.

The fourth, last for the moment, issue is almost obvious to guess: if the electroweak Higgs boson does exist what is the mechanism which generates its mass?

Thus, regardless of whether the electroweak Higgs particle exists or not, there should be at least one mass generation mechanism which lies outside the scope of the Glashow–Weinberg–Salam theory. What about the logarithmic nonlinearity, can it help in understanding these problems? Also, once we have established that the particles freely propagating in the logarithmic theory can be effectively viewed as propagating in some non-trivial background medium, what is the physical nature of this medium?

2. Spontaneous symmetry breaking

The first thing to notice is if in some representation the operator $\hat{\mathcal{H}}$ can be written as a second-order differential operator with respect to some variable X , i.e., $\hat{\mathcal{H}} \sim f_1 \frac{\partial^2}{\partial X^2} + f_2 \frac{\partial}{\partial X}$ (we assume $f_1 > 0$ otherwise one must invert the sign of β or perform the Wick rotation of X) then the wave equation (1) can be viewed as the equation of motion of the fictitious particle moving on

a plane $\{\Re(\Psi), \Im(\Psi)\}$ in the rotationally-invariant external potential

$$V(\Psi) = \beta^{-1} \{ \Omega |\Psi|^2 [\ln (\Omega |\Psi|^2) - 1] + 1 \} + V_0, \quad (6)$$

where $V_0 \equiv V(|\Psi| = 1/\sqrt{\Omega})$, with the role of time coordinate being assigned to X or to iX , as in the semi-classical approach. It is not difficult to check that for positive β and Ω this potential has the Mexican-hat shape: its local maximum is located at $|\Psi| = 0$ whereas the degenerate minima lie on the circle $|\Psi| = 1/\sqrt{\Omega}$ where the energy of the “particle” reaches its minimum.

To present things in a more rigorous way we use the ideology of the Bogoliubov–Ginzburg–Landau(–Gross–Pitaevskii) mean-field approach [12] which is a special case of the Schrödinger field method and originates from the following idea. Suppose Ψ is originally the functional on a space of field operators $\hat{\psi}_{(i)}$ which maps this space onto the field of c -numbers. As long as those fields themselves depend on space and time variables x then in certain cases, for instance, when they describe identical particles in the same state, the functional $\Psi[\hat{\psi}_{(i)}(x)]$ can be replaced by the function $\Psi(x)$. The latter is nothing but the probability amplitude which complex square is a measurable quantity but now the wave equation it satisfies is not necessarily linear. This $\Psi(x)$ is traditionally called the wave function of the Bose–Einstein condensate (BEC). The type of the nonlinearity is determined by the way the condensate particles interact with each other. For most dilute Bose systems it suffices to consider only the Gross–Pitaevskii (GP) quartic non-linearity which leads to the cubic Schrödinger equation (although, even for such systems the beyond-GP approximations are unavoidable in some cases [13]). In general case, however, higher-order terms (which can account, for instance, for multi-body interactions, self-energy effects, *etc.*) can result in entirely new physics as their infinite sum is an essentially non-perturbative object with the features drastically different from what one might expect from a perturbation theory [14], an example to be given shortly after Eq. (12).

Thus, here we are going to view our Ψ as a wave function of the effective BEC described by the field operator $\hat{\Psi}$. Then Ψ can be considered as the expectation value of the latter, $\langle \hat{\Psi} \rangle = \Psi$. We assume that the full classical action can be decomposed into two parts (unless stated otherwise, in this section we work in the high-energy units $c = \hbar = 1$):

$$S = \tilde{S}(\phi_i, \Psi) - \int \mathcal{V}(\Psi), \quad (7)$$

where the action $\tilde{S}(\phi_i, \Psi) = \int \tilde{\mathcal{L}}$ and integration measure are defined on some suitably chosen domain, by ϕ_i we denote all other fields, and the potential energy density is defined as

$$\mathcal{V}(\Psi) \equiv \frac{1}{\Omega} V(\Psi) = \frac{1}{\beta \Omega} \{ \Omega |\Psi|^2 [\ln (\Omega |\Psi|^2) - 1] + 1 \}, \quad (8)$$

up to an additive constant. Then at the “classical” level (replacing operators by their expectation values) one of the Euler–Lagrange equations can be always written as

$$\left[\frac{\delta \tilde{S}}{\delta \Psi^*} - \int \frac{d\mathcal{V}(\Psi)}{d(|\Psi|^2)} \Psi \right] \delta \Psi^* = 0, \quad (9)$$

which is equivalent to

$$\frac{\delta \tilde{\mathcal{L}}}{\delta \Psi^*} - \beta^{-1} \ln(\Omega |\Psi|^2) \Psi = 0, \quad (10)$$

where by $\delta \tilde{\mathcal{L}}/\delta \Psi^*$ we loosely mean the functional derivative of $\tilde{S}$ with the integration dropped. Thus, we readily recover the wave equation (1) upon a formal identification $\hat{\mathcal{H}}\Psi \Leftrightarrow \delta \tilde{\mathcal{L}}/\delta \Psi^*$.

Another way to see the fluidic features encoded in the logarithmic nonlinearity is to look for solutions of the quantum wave equation in the Madelung form

$$\Psi = \sqrt{\varrho} e^{iS}, \quad \varrho = |\Psi|^2, \quad \vec{v} = \frac{\hbar}{m} \vec{\nabla} S = \frac{\hbar}{im} \vec{\nabla} \ln \frac{\Psi}{|\Psi|}, \quad (11)$$

where m is the *inertial* mass of the condensate particle. Then the wave equation splits into two hydrodynamic ones — the equation of continuity for the condensate particle density $\varrho(x)$ and the equation of potential flow of superfluid for the velocity field $\vec{v}$ [15, 16]. From the latter, one immediately obtains that the zero-temperature (collisionless) equation of state of the logarithmic BEC in the first-order approximation is described by the Clapeyron–Mendeleev law,

$$p - p_0 = (m\beta)^{-1} \varrho + \mathcal{O}(\hbar^2) \propto T_\Psi \varrho, \quad (12)$$

where T_Ψ is in general a quantum (collisionless) kind of the temperature conjugated to the information entropy, $S_\Psi \equiv -k_B \int |\Psi|^2 \ln(\Omega |\Psi|^2) d^3x$, measuring the degree of spreading of a quantum object [4], see also Appendix. For comparison, the corresponding equation of state for the GP (quartic) condensate would be $p \propto \varrho^2$, thus, the logarithmic Bose liquid is more “ideal” than the Gross–Pitaevskii one yet non-trivial. Therefore, the logarithmic condensate can be added to any microscopical many-body system to serve as a calibrating background [17]. This confirms the usefulness of the logarithmic nonlinearity for describing the physical vacuum. It is interesting also that since the Gross–Pitaevskii potential can be perturbatively derived from the logarithmic one by expanding near minima and cutting the infinite series at the quartic term we have found another example of how the

essentially non-perturbative treatment, *i.e.*, taking into account the infinite number of powers of ϱ , can drastically change the physical picture.

In this connection one can also mention that the logarithmic terms (usually of the form $\varrho^m \ln^n \varrho$) commence to appear in higher orders of perturbation theory, *e.g.*, when taking into account certain combinations of loop diagrams, both in the relativistic scalar field theories [10] and condensed-matter Bose systems [13], where ϱ (modulo a dimensional scale factor) is the complex square of scalar field in the former case and the one of the condensate wave function in the latter. This duality-type interplay between the *relativistic* scalar field and *non-relativistic* Bose liquids has a profound origin and will be discussed in more details later, in the section devoted to the BEC/spacetime correspondence.

To conclude this section, we have shown that one can mimic vacuum effects by including the logarithmic nonlinearity into the quantum wave equation or, alternatively, by including into the full action the field with the potential (8). If we view the nonlinearity as a quantum gravity phenomenon then we prefer to deliberately call the Bose–Einstein condensate virtual because it cannot be physically separated from background and removed, in contrast to its condensed-matter counterparts. As a matter of fact, it *is* a background.

3. Mass generation

The exact form of the effective action $\tilde{S}$ is unknown to us but we can already guess the most obvious of its features. First, following the popular approach of taking into account vacuum effects by virtue of introducing an auxiliary scalar field, see for example Ref. [18], we can assume the psi-particle to be described by scalar field. At that, as long as here we are introducing this field as to account for the small fluctuations of the BEC vacuum and also we are going to describe objects with the quantum wave amplitude being much smaller than the background value of the condensate wave function amplitude, the field-theoretical models can be constructed in a covariant manner, for reasons which become clear below, in the section devoted to the BEC/spacetime correspondence. At the same time, we have to keep this field non-linearized as to account for the effects mentioned in the previous section. In principle, since we are dealing with low-energy effective models we are free to use any form of the covariant action for the psi-field — as long as it is physically transparent, self-consistent, mathematically manageable and the corresponding field equation contains the logarithmic nonlinearity. For instance, as to make the psi-particle field dynamical the minimal action must contain also the kinetic term which must be quadratic otherwise no proper wave equation can appear. Also, it is likely that $\tilde{S}$ will

contain couplings of the psi-particle to other fields. Thus, to get at least some idea about how the conventional dynamical systems might be affected by the logarithmic BEC vacuum, in this section we are going to construct few toy models complying with the above-mentioned requirements. The issue of renormalizability of such models is not a problem here because we do not require the Lorentz symmetry to be exact at the length scales shorter than ℓ_0 , *i.e.*, above the corresponding energy and momentum thresholds. Then these critical values serve as the natural UV cutoff making the upper limits of momentum-space integrals finite and no UV divergences arises. The infrared divergences are not a problem either because in the low-energy limit $E/E_0 \rightarrow 0$ the nontrivial structure of physical vacuum can be neglected and one arrives at the relativistic models which are well-studied in this regard.

3.1. Model with global symmetry breaking

The simplest toy model is just the self-interacting one — involving only the complex psi-field and no others. While not having much of physical relevance on its own, it will serve us as a good test-bed. In D -dimensional spacetime its Lagrangian can be written in the covariant form

$$\mathcal{L} = \ell_\Omega \partial_\mu \psi \partial^\mu \psi^* - \mathcal{V}(\psi), \quad (13)$$

where the potential is given by Eq. (8); here and below the factors like ℓ_Ω are introduced for dimensionality reasons, keeping in mind the original dimensionality of Ψ .

This model is invariant under a global change of phase of ψ but in the vacuum state the value of ψ must be non-zero, with a magnitude close to $1/\sqrt{\Omega}$ and arbitrary phase. In other words, there is a degenerate family of vacuum states. The latter circumstance together with the Goldstone theorem would suggest the presence of the Nambu–Goldstone bosons in the theory. To check this, we introduce the shifted real-valued fields φ_1 and φ_2 :

$$\psi = \Omega^{-\frac{1}{2}} + \frac{1}{\sqrt{2\ell_\Omega}}(\varphi_1 + i\varphi_2), \quad (14)$$

and expand the potential near the minimum. We obtain

$$\begin{aligned} \mathcal{L} = & \frac{1}{2} [(\partial\varphi_1)^2 + (\partial\varphi_2)^2] - \frac{1}{2} m_\psi^2 \varphi_1^2 - \frac{\sqrt{2}}{\beta} \ell_\Omega^{(D-4)/2} \varphi_1 (\varphi_1^2 + \varphi_2^2) \\ & - \frac{1}{4\beta} \ell_\Omega^{D-3} (\varphi_1^2 + \varphi_2^2)^2 + \mathcal{O}(\varphi^5), \end{aligned} \quad (15)$$

where the quantity

$$m_\psi = 2/\sqrt{\ell_\Omega \beta} \quad (16)$$

can be viewed as the effective mass of the fluctuation of the logarithmic condensate (not to be confused with the mass m of a bare condensate particle). If the running behavior of β turns out to be as derived in Ref. [4],

$$\beta \sim (E_0 - E)^{-1}, \quad (17)$$

then we expect

$$m_\psi \sqrt{\ell_\Omega} \sim \sqrt{E_0 - E}, \quad (18)$$

i.e., its mass is not determined solely by the Planck scale: for energy very small compared to E_0 it tends to the constant value,

$$m_\psi^{(0)} \equiv m_\psi(E = 0) \sim \sqrt{\frac{|E_0|}{\ell_\Omega}}, \quad (19)$$

but at higher energies it alters thus reflecting the dynamical nature of the physical vacuum.

Thus, in the broken symmetry regime this model describes two kinds of particles, one massive and one massless. The latter are the Nambu–Goldstone bosons which describe the spatial variations of the vacuum’s phase.

3.2. Model with gauge symmetry

Physically more useful toy model can be constructed by coupling the condensate to the Abelian gauge field. In D -dimensional spacetime its Lagrangian is

$$\mathcal{L} = \ell_\Omega D_\mu \psi^* D^\mu \psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \mathcal{V}(\psi), \quad (20)$$

with $D_\mu = \partial_\mu + ie\ell_\Omega^{\frac{D-4}{2}} A_\mu$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, as per usual, e is the elementary electrical charge.

In general, this Lagrangian is invariant under the $U(1)$ local gauge transformation and describes psi-particles and antiparticles interacting with massless photons. To see what happens in the regime of spontaneously broken symmetry, we make again the shift (14) to eventually obtain

$$\mathcal{L} = \frac{1}{2}(\partial\varphi_1)^2 - \frac{1}{2}m_\psi^2\varphi_1^2 - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}m_\gamma^2 B_\mu B^\mu + \dots, \quad (21)$$

where $B_\mu = A_\mu + \frac{1}{\sqrt{2}}\ell_\Omega e^{-1}\partial_\mu\varphi_2$ refers to the new gauge field of the mass

$$m_\gamma = \frac{\sqrt{2}e}{\ell_\Omega}, \quad (22)$$

which does not run with energy. We can see also that the masses of the photon and psi-particle and the elementary charge are related by the formula

$$\frac{em_\psi^2}{m_\gamma} = \frac{2^{3/2}}{\beta} \propto E - E_0 \quad (23)$$

which does not depend on D or ℓ_Ω . We remind that the Goldstone theorem is evaded here because one of its prerequisites, the Lorentz invariance, is violated in the logarithmic theory as was shown also in Ref. [4] in a different way.

Thus, we have established that the photon acquires mass m_γ and no massless Goldstone bosons appear. The models support the Coleman–Weinberg idea of the vacuum-induced spontaneous symmetry breaking [10] and show that the possible effect of the physical vacuum is that the photon becomes massive. Why its mass is so tiny small? The clue is that the correlation length scale ℓ_Ω can be very large — in fact, as long as the parameter $\Omega = \ell_\Omega^{D-1}$ has the dimensionality of the spatial volume and appears in the normalization condition of the dimensionless wave function $\sqrt{\Omega}\Psi$, it is tempting to conjecture the cosmological-scale value for it, say, the volume of the (observable part of the) Universe. At least, that would explain why the time-delay effects [1] are exactly as that small as to become visible precisely at the cosmological-scale distances. Then, for the current value of ℓ_Ω of about ten billion light years the above-mentioned characteristic masses can be estimated as

$$m_\psi^{(0)} \sim 10^{-3} \div 10^{-2} \text{ eV}, \quad m_\gamma \sim 10^{-35} \text{ eV}, \quad (24)$$

where for the former mass we imposed E_0 to be the Planck one (which is valid if the external fields are weak enough as not to change the vacuum energy significantly). These small, yet non-vanishing masses indicate that their gravitational effect and contributions to the density of matter in the Universe can be quite substantial, and can be phenomenologically estimated in the spirit of the works [19]. Another thing that comes to mind when looking at the formula (22) is that the appearance of e therein explains why it is the photon which mediates the long-range interactions between the electrically charged elementary particles. Recalling the analogy with superconductivity, the photons in this model can be interpreted as the pairs of virtual particles and antiparticles, see also Ref. [20] and references therein.

3.3. Other models

In our case, due to the interpretation of Ψ , it suffices to represent the complex-valued psi-field by two real scalars, φ_1 and φ_2 . In general (for instance, when the vacuum is required to be described by the multi-component

Bose liquid), one may wish to consider the multiplet of the scalar fields φ^a which belongs to a representation of the symmetry group G , non-Abelian in general. If the latter is spontaneously broken down to a subgroup H the fields acquire the non-zero expectation values φ_0 . Then the mass matrix for the gauge fields is given by $(M_A^2)_{ab} = g^2 \varphi_0^T T_a T_b \varphi_0$, where T_a are the group G 's generators, g is the gauge coupling constant. The elements of M_A^2 which correspond to the generators of H vanish, therefore, there appear $\dim(H)$ massless gauge bosons and $\dim(G/H)$ massive ones. The “survived” components of φ acquire the mass $(M_\varphi^2)_{ab} = \left(\frac{\partial^2 \mathcal{V}}{\partial \varphi^a \partial \varphi^b} \right)_{\varphi=\varphi_0}$, with $\mathcal{V}$ being the potential of the form (8).

The fermions, such as neutrinos, can be also included into this picture as nothing prevents them from interacting with the condensate. Thus, they could also acquire mass, although the question whether it would happen due to the condensate or due to the SM Higgs boson remains open.

4. Topology and solitons

The solitonic-type solutions of the logarithmic wave equations have been known for a long time [21]. However, at that time people were motivated by other things so they considered the potentials like (8) “upside down”, in which case no spontaneous symmetry breaking could arise. It came as a surprise to us that nobody actually considered other sector of the logarithmic theory — the one where multiple topological sectors can in principle appear. From the viewpoint of our theory, they were working with the “Wick-dual” theory — in a sense that the two theories can be transformed into one another either by inverting the sign of β or by the Wick-rotation of an appropriate variable, as in the Euclidean field-theoretical approach [22]. The well-known example of theories related by the Wick rotation is the quantum field theory at finite temperature β^{-1} and the statistical mechanics on the $\mathbb{R}^3 \times S^1$ manifold with the β -periodic imaginary time. In this connection, the relation between our β and certain kind of non-classical temperature was outlined in Ref. [4], see also the Appendix. Moreover, as long as β^{-1} itself is shown there to be proportional to $E - E_0$, the natural energy of vacuum E_0 plays the role of the critical parameter at which a phase transition happens (this can be seen from Eq. (18) as well), and the physical degrees of freedom in each of the phases $E < E_0$ and $E > E_0$ can be very distinct.

As an example, let us consider one-dimensional logarithmic Schrödinger equation. In the dimensionless form it can be written as

$$i\partial_t \psi + (\partial_{xx}^2 \pm \ln |\psi|^2) \psi = 0, \quad (25)$$

where the plus (minus) sign corresponds to the theory with the potential (6) open downwards (upwards); in practice this sign is associated with the sign

of β . For simplicity we impose the ansatz $\psi = \exp(-i\epsilon t)\phi(x)$, with $\phi(x)$ being real-valued, then the equation turns into the static one (the moving solutions can be always generated by performing the Galilean boost):

$$\phi''(x) - dU_{\pm} \frac{\phi}{d\phi} = 0, \quad (26)$$

where the potential is given by

$$U_{\pm}(\phi) \equiv \pm \frac{1}{2}\phi^2 (1 - \ln \phi^2) - \frac{1}{2}\epsilon\phi^2. \quad (27)$$

Let us consider first the “plus” case — where the symmetry $\phi \rightarrow -\phi$ stays unbroken because $\phi = 0$ is a stable local minimum of the potential $U_+(\phi)$. The corresponding normalized solutions are called *gaussons* (on the BEC language they would be called the *bright solitons*):

$$\phi_g(x) = \pi^{-1/4} e^{-(x-x_0)^2/2}, \quad (28)$$

with the eigenvalue $\epsilon = E_0 = 1 + \ln \sqrt{\pi}$. Their stability is ensured by the integrability conditions because E_0 is the lowest bound for the energies of all possible normalizable solutions (generally referred as the BPS bound).

Now we turn to the “minus” case — when the potential $U_-(\phi)$ has two degenerate minima, at $\phi = \pm \exp(\epsilon/2)$. Therefore, one should expect that all the non-singular and finite-energy static solutions can be cast into four topological sectors, according to the boundary conditions

$$e^{-\epsilon/2}[\phi(-\infty), \phi(\infty)] = \{[-1, 1], [1, -1], [-1, -1], [1, 1]\},$$

and $\phi'(\pm\infty) = 0$. The last two sectors contain the trivial solutions $\phi = -\exp(\epsilon/2)$ and $\phi = \exp(\epsilon/2)$, respectively, whereas the former two contain the kink and anti-kink solutions (*dark solitons*, in BEC terminology), with the non-vanishing topological charge. The latter is defined simply as the difference of the topological indexes

$$Q = \exp(-\epsilon/2) [\phi(\infty) - \phi(-\infty)]. \quad (29)$$

To find the analytic form of the kink solution, we solve the wave equation with the above-mentioned boundary conditions. We obtain the expression

$$\int \frac{d\phi}{\sqrt{\phi^2 (\ln \phi^2 - \epsilon - 1) + \exp \epsilon}} = x - x_0 \quad (30)$$

from which $\phi(x)$ can be found after taking the indefinite integral. Unfortunately, the latter cannot be expressed in known functions but simple numerical analysis confirms that Eq. (30) indeed represents the kink and anti-kink solutions.

Further generalizations are obvious, both in terms of considering more dimensions and other symmetries. If we relax the condition of real-valued $\phi(x)$ then the potential $U_-(\phi)$ takes the Mexican-hat shape on the plane of the real and imaginary components of ϕ . The topological classification is usually based on the homotopy groups $\pi_n(S_m)$ [23]. For instance, the homotopy group for the Abelian model (20) at $D = 3 + 1$ is $\pi_2(S_1) = 0$, *i.e.*, no nontrivial homotopy sectors of solutions can exist whereas at $D = 2 + 1$ its homotopy group is $\pi_1(S_1)$ which is a winding number group. The latter implies that in principle in effectively $(2 + 1)$ -dimensional Abelian gauge models with the condensate the magnetic flow becomes quantized and the vortex solutions can appear [24].

5. BEC vacuum vs. curved spacetime

Now, as long as the (quantum) gravity is concerned, how can one reconcile the BEC description of the physical vacuum with the concept of curved spacetime which is traditionally being used for describing the gravitational interaction?

5.1. Emergent spacetime

Let us first recall that in majority of physically meaningful cases one can establish a formal correspondence between the inviscid Bose liquids and manifolds of non-vanishing Riemann curvature. For instance, the following fluid/gravity correspondence is well-known [25]: the propagation of small perturbations inside an inviscid irrotational barotropic fluid, characterized by the background values of the density ϱ , pressure p and velocity $\vec{v}$, is analogous to propagation of test particles along the geodesics of the pseudo-Riemannian manifold with the metric

$$g_{\mu\nu} \propto \frac{\varrho}{c_s} \begin{bmatrix} -(c_s^2 - \vec{v}^2) & \vdots & -\vec{v} \\ \cdots \cdots & \cdot & \cdots \\ -\vec{v} & \vdots & \mathbf{I} \end{bmatrix}, \quad (31)$$

where $c_s = \sqrt{\partial p / \partial \varrho}$ is the speed of “sound” — the propagation speed of wave-like fluid fluctuations. This metric tensor is defined up to a constant factor which value is determined by measurement units and boundary conditions. Notice that while inside the background fluid the notions of space and time are clearly separated (such that one can assume the fluid being non-relativistic), the small perturbations themselves couple to the metric which treats space and time in a unified way. If we treat such fluid as a non-removable background then this metric describes the induced spacetime geometry. The latter should not be confused with the relativistic gravitational

effect of the ideal fluid as a source introduced via stress-energy tensor in the Einstein field equations (EFE). Instead, as long as the physical vacuum is concerned, for a given metric (31) one can always define the induced matter stress-energy tensor

$$T_{\mu\nu}^{\text{ind}} \equiv \kappa^{-1} \left[R_{\mu\nu}(g) - \frac{1}{2} g_{\mu\nu} R(g) \right], \quad (32)$$

thus, EFE are interpreted here not as the differential equations for the unknown metric but rather as an expression for the stress-energy tensor of the effective matter to which the small fluctuations and test particles couple. If an observer operates only with such fluctuations then this is the only matter s/he is going to “see” directly. Macroscopic (composite, finite-size) bodies also couple to the induced metric if they consist of the elementary particles which do not violate the small-fluctuation condition — such that the overall density is much less than the critical one.

Using Eq. (11) one can show that for the generic bulk Bose condensate described by the *non-relativistic* quantum wave equation

$$\left[-i\hbar \partial_t - \frac{\hbar^2}{2m} \vec{\nabla}^2 + V_{\text{ext}}(\vec{x}, t) + F(|\Psi|^2) \right] \Psi = 0, \quad (33)$$

the zero-temperature equation of state and hence velocity c_s can be determined from the differential equation

$$m \vec{\nabla} p - |\Psi|^2 \vec{\nabla} F = \mathcal{O}(\hbar^2), \quad (34)$$

the square of the BEC wave function yields the condensate density, as usual. By solving this equation we obtain

$$p - p_0 = m^{-1} \int_0^{|\Psi|^2} \varrho F'(\varrho) d\varrho + \mathcal{O}(\hbar^2), \quad (35)$$

$$c_s^2 \equiv \frac{\partial p}{\partial(|\Psi|^2)} = m^{-1} |\Psi|^2 F'(|\Psi|^2), \quad (36)$$

and the induced metric tensor takes the form:

$$g_{\mu\nu} \propto \frac{|\Psi|}{\sqrt{F'(|\Psi|^2)}} \begin{bmatrix} -\frac{1}{m} |\Psi|^2 F'(|\Psi|^2) - \frac{\hbar^2}{m^2} \left[\vec{\nabla} \ln \frac{\Psi}{|\Psi|} \right]^2 & \frac{i\hbar}{m} \vec{\nabla} \ln \frac{\Psi}{|\Psi|} \\ \dots\dots\dots & \vdots \quad \dots \\ \frac{i\hbar}{m} \vec{\nabla} \ln \frac{\Psi}{|\Psi|} & \vdots \quad \mathbf{I} \end{bmatrix}. \quad (37)$$

The value c_s thus becomes the maximum attainable propagation velocity of any object whose quantum wave amplitude is much smaller than the

magnitude of the background condensate wave function. For instance, when assuming the logarithmic condensate, $F(\varrho) \equiv \beta^{-1} \ln(\Omega\varrho)$, then in absence of any additional matter Eq. (35) yields Eq. (12) from which we obtain

$$c_s = \frac{1}{\sqrt{m\beta}} \equiv c_\beta, \quad (38)$$

provided $m\beta > 0$. One can immediately see that the maximal propagation velocity of small excitations in the long-wavelength approximation does not depend on density which makes the logarithmic BEC distinguished among other kinds of condensates. From the last formula one can derive also the second Einstein's postulate: if we recall Eq. (17) and assume an absence of extra fields so we can choose the proper BEC energy $|E_0| = mc^2$, with c playing the role of the units conversion factor, in the leading approximation. Then we indeed arrive at the fundamental velocity constant: $c_\beta \leq \sqrt{|E_0|/m} \leq c$.

Thus, in the BEC-vacuum approach the relativity is an emergent rather than a fundamental phenomenon¹, EFE and dependent concepts do not have any fundamental meaning on their own but rather represent an approximate long-wavelength description valid only within certain energy and length scale (after all, the Lorentzian geometry is what it is — a way of measuring distances, and the gravitational “field” in general relativity is known for not possessing a proper stress-energy tensor). In fact, some predicted quantum gravitational phenomena, such as the Hawking radiation, can be derived without the use of EFE [28] whereas others, such as gravitons and gravitational waves (at least, in current formulation), strongly rely upon EFE, and therefore, a careful treatment is needed there. The BEC-vacuum description of the black holes is also slightly different from general relativistic: while the analogue spacetimes may possess event horizons it is only long-wavelength excitations which follow geodesics and thus it is only them which might experience the irreversible properties of horizons. If a measuring apparatus operates with the objects which somehow do not satisfy the small-amplitude and long-wavelength conditions then no “canonical” event horizons can be detected. The space-time singularities, *i.e.*, the points where the Riemann tensor computed from the induced metric diverges, cannot be attributed to reality as the small-amplitude and long-wavelength conditions, main prerequisites of the induced relativity, are strongly violated there. It should be remembered also that due to the original relativistic time coordinate

¹ The question whether the general relativity is an effective theory has been raised long time ago [26]. Also, the early attempts to describe the physical vacuum as superfluid were dated as far back as 70s [27] (however, neither there nor in later works [29] any specific wave equations for the physical vacuum's wavefunction were proposed, to our best knowledge, and the debates about a specific expression for the vacuum energy density still continue).

being synchronized with the Newtonian time the BEC-induced geometries automatically fulfill the requirement of *stable causality* which prevents the appearance of the causal pathologies the general relativity suffers from.

Numerous examples of the fluid-gravity isomorphisms and further discussions can be found in the books [29]. In particular, the BEC-gravity analogue models have been already studied in Refs. [30], although they dealt with condensed-matter systems without referring to the physical vacuum and mass generation mechanism, an extensive bibliography can be found in Refs. [29, 31]. Moreover, the nonlinear wave equations in those models are not of the logarithmic type, therefore, they do not possess the above-mentioned Planck relation, energy additivity and constancy of c_s properties jointly which makes them less suitable for describing the fundamental background.

On a practical side, the BEC-gravity analogy² means that the (physical) observer operating at the length scale larger than the size of elementary fluid elements of quantum Bose liquid (which is of order ℓ_0) is not able to distinguish the propagation of small fluctuations in the fluid from the geodesic motion of test particles on an appropriately chosen manifold. To resolve the underlying microscopic structure of the liquid s/he has to input therein energy sufficient to reach the critical value $|E_0|$ which corresponds to the length resolution ℓ_0 . Then, as mentioned in previous section, the system “jumps” into other phase, with different physical degrees of freedom, the process which resembles the transition between the phonon and free-particle phases in Bose gases. But otherwise these two descriptions, Bose-liquid and geometrical one, are dual-equivalent and equally “effective” (and may be not the only possible), and the choice between them is purely a matter of taste and/or practicality. For example, while the simple superfluids (irrotational, barotropic, one-component) can be associated with simple pseudo-Riemannian manifolds (real, four-dimensional, torsion-free, metric-compatible, *etc.*) — such that one can employ the whole machinery of the Riemann geometry, the geometrical description of the liquids with any of the above-mentioned restrictions relaxed can easily go beyond the Riemann geometry and become complicated and/or physically non-transparent [32]. Besides, the applicability of either description depends on a concrete physical problem it is applied to. It seems that the propagation of test point-like objects inside the physical vacuum is more conveniently described within the framework of the relativistic approach (although, some corrections apply [4]) but the strong quantum processes such as the phase transitions related to

² In our case the term BEC/spacetime correspondence or duality would be more appropriate provided we assume the broader meaning of the condensate as the coherent ground state of superfluid described by a single wavefunction. In general, however, the notion of superfluid is more broad and complex than that of BEC.

the formation or depletion of the Bose liquids (“spacetimes”) themselves can be mathematically formulated only at the underlying BEC level (in fact, within the framework of the Lorentzian geometry such phenomena cannot be even well-defined).

Thus, the BEC/spacetime correspondence allows to look at some old problems at a different angle of view.

5.2. Mach’s principle and locality

In its most popular formulation the Mach’s principle states that the local inertial properties such as mass are determined by the total mass distribution in the Universe. While Einstein himself had this in mind when constructing general relativity the latter does not comply with the Mach’s principle favoring instead the strong equivalence one. The attempt of fixing that without breaking general covariance has been made in the theories of scalar-tensor gravity [33]. In those approaches the Mach’s principle is partially taken into account by making the gravitational constant a dynamical variable, at the cost of postulating the additional field — the scalar one. The origin of this hypothetical scalar remains unclear so far, moreover, being Lorentz-covariant the scalar-tensor gravitational models do not address the following two locality issues.

If the physical vacuum is trivial then an observer in the otherwise empty space would not be able to determine whether s/he has any inertia — due to the absence of any reference frame. The latter can be immediately created once a probe object appears somewhere else. Therefore, the observer is supposed to instantaneously find out own inertial properties with respect to that frame, no matter how far the probe is located or how “massive” it is. Another locality issue which arises in a theory with the trivial vacuum is the following: if we talk about interacting systems in general then what do we mean by energy of interaction, how can we differentiate “interacting” and “non-interacting” systems, how does a system “know” about the form of the potential it is supposed to obey when interacting with other system(s)?

To address all these questions in our approach, let us recall that the BEC vacuum is an essentially quantum object yet its correlation length ℓ_Ω can have the cosmological-scale value, as mentioned above, and the properties of its fluctuations are obviously determined by the whole matter distribution in the Universe. As a matter of fact, the condensate gives rise to masses of particles in a way similar to the gap generation mechanism in superconductors, as we have shown earlier. Therefore, the nontrivial vacuum can naturally serve as the physical realization of the Mach’s principle: it introduces the universal frame of reference and gives meaning to the “action-at-a-distance” processes in general and to the inertia in particular. In this framework the

Lorentz-covariant models involving the global scalar field is just a way to account for the BEC vacuum's effects in the (approximate) relativistic manner only for length scales larger than ℓ_0 and for transfer energies below $|E_0|$ — similarly to what we have done in Sec. 3. At that, one does not need to introduce any kind of “gravity” in the Euclidean space because the only mass parameter there, m , is the *inertial* mass of the condensate particle.

The issue of how to unambiguously define the concept of interaction under the conditions of the strong long-range correlations is resolved in the logarithmic BEC vacuum due to the above-mentioned energy additivity property which is preserved in the logarithmic quantum mechanics: an interacting energy of any two systems described by wave functions Ψ_1 and Ψ_2 (when taken separately from each other) still can be defined as the difference $E(\Psi) - E(\Psi_1) - E(\Psi_2)$ where Ψ is the wave function of the whole composite system. This definition naturally incorporates the quantum-mechanical nature of interactions: it preserves the notion of non-interacting systems whereas the interaction energy defined in such way is a measure of how much does the overall state vector $|\Psi\rangle$ differ from the plain product $|\Psi_1\rangle \otimes |\Psi_2\rangle$.

5.3. Cosmology

According to current cosmological paradigm, the early Universe's large-scale structure had a phase of the exponential expansion (inflation) followed by the reheating and, subsequently, radiation- and matter-dominated phases [34]. It is believed that without introducing the inflationary phase it would be difficult to explain the horizon, flatness and monopole problems. For the role of the agent driving the inflation one usually appoints the global scalar field called the inflaton and considers some kind of the scalar-tensor gravity rather than the original Einstein's theory.

Despite the overall success and popularity of the scalar-driven cosmological models, few questions remain unanswered. The main one is what is the physical nature of the inflaton, in particular, why did it appear in the early Universe before any other fields and particles we know so far, why its current vacuum expectation value is the way it is, why the current expectation value of its potential energy, known also as the (effective) cosmological constant, is so extremely small yet nonzero in present epoch. On top of that, if one associates this effective cosmological constant with the vacuum energy then one immediately arrives at the above-mentioned cosmological constant problem [35]. How would all these problems look from the viewpoint of the cosmology incorporating the BEC-vacuum idea?

First thing to notice, the notion of the cosmological constant makes sense in a relativistic theory only, therefore, within the framework of the BEC approach this constant can refer at most to the energy of small fluctuations

of the vacuum above a background value but not to the energy of vacuum itself [36]. Thus, in the BEC-vacuum cosmology this constant does not have any fundamental physical meaning and the related problems simply do not occur in first place.

Second, if typical energies of density fluctuations and masses of elementary particles are less than E_0 then the vacuum stays in the BEC phase and the Lorentz-symmetric cosmological models based on the spacetime metric tensor and scalar fields are obviously a good approximation, therefore, the physical conclusions based on the standard Friedmann–Lemaître–Robertson–Walker (FLRW) models remain unaltered. Moreover, in the BEC phase many of the conclusions based on scalar-driven models remain unaltered as well, as long as one adopts a suitable form of the scalar-tensor field-theoretical action. However, in the close vicinity of the threshold the relativistic description begins to fail: of course, as one approaches more and more early stages of the Universe’s evolution, one can still employ the relativistic fields but the price will be that this description will become more and more “effective” and less and less natural. In practice this means that one will need to adjust the form of the covariant field-theoretical action at each range of energy scale by hand.

Finally, let us discuss the problems which led to the inflation proposal and give them explanations based on the BEC-vacuum idea:

- The monopole problem is eliminated in the BEC-vacuum cosmology for the above-mentioned reasons: the stable GUT monopoles predicted so far are the solutions of relativistic field equations possessing a large mass. The latter circumstance violates the requirements for the BEC/spacetime correspondence’s validity and thus the relativistic monopole production in the early Universe is hardly justified even on theoretical grounds.
- The flatness or cosmological fine-tuning problem was motivated by the analysis of the Friedmann equations which are again intrinsically relativistic, therefore, they cannot be extrapolated to arbitrary short length scales and the genuine evolution of the density of matter and energy in the Universe did not have to obey them all the time. The reason why the density is so close to the critical one is that just an instant before the vacuum BEC was formed and its fluctuations became small enough there was no concept of curved Lorentzian spacetime available yet. Therefore, at that moment the total density had a critical value (corresponding to the flat space) and its large-scale average value could not change much since then — provided the BEC does not rarefy much. The latter can be achieved by self-sustainability due to

nonlinear effects [17], some sort of trapping potential, and/or boundary conditions for the wave equation the background BEC obeys. At that, one should not confuse, for instance, the spacetime (Hubble) expansion as viewed by the internal observer operating in the small-perturbation regime with the dynamics of the BEC background itself: below we demonstrate certain physical setup in which the BEC background flows with constant velocity (if viewed as an embedding in the fictitious Euclidean space) while the observer sees herself inside the FLRW-type universe.

- The problem of reconciling the early-Universe cosmology with the second law of thermodynamics which is closely related to the horizon problem (homogeneity and isotropy) and leads either to the inflation proposal or to the Weyl curvature hypothesis [37] can be reformulated in the BEC-vacuum cosmology as follows. During some epoch of the very early Universe when any conventional matter was absent the large-scale evolution was determined mainly by the vacuum, logarithmic condensate. The Weyl curvature hypothesis requires then that the induced metric (37) must be conformally flat during that epoch. Below we show that it is indeed the case. The horizon problem can be thus explained by the macroscopic size of the essentially quantum vacuum — as long as the latter is viewed as the BEC embedded into the Euclidean space with absolute time such that its particles tend to occupy the lowest state and any quantum exchanges happen instantly. In the case of the BEC-vacuum cosmology the correlation length ℓ_Ω can be interpreted as the size of the observable part of the Universe. Indeed, as long as an observer usually operates with the probe objects, such as photons and other elementary particles with energies less than E_0 , s/he is bound to the relativistic regime and thus unable to probe not only the distances smaller than ℓ_0 but also larger than ℓ_Ω . This also means that the regions relativistically disconnected from us can nevertheless affect our Universe — *e.g.*, by virtue of the large-amplitude density fluctuations for which $\delta(|\Psi|^2) \not\ll |\Psi|^2$. This fits the long-discussed idea of our Universe being a patch inside the much “larger” region, called the Multiverse, which may explain the dipole anisotropy of the cosmic microwave background and coherent large-scale flow of galaxy clusters [38]. Besides, since the Multiverse can contain many patches with different BEC vacua (or currents, if viewed as the Euclidean embeddings) which separately nucleated during the Bose condensation epoch, the chaotic inflationary scenarios [39] are compatible with the BEC-vacuum cosmology as well.

- The long-standing problem of how to formulate the early-Universe cosmology on quantum-mechanical grounds, commonly referred as the *quantum cosmology*, is treated in the following way: as long as the Lorentzian geometry is the induced effective phenomenon valid only for certain scales of length and energy, the metric tensor does not need to be quantized *per se* otherwise it leads to the double-counting similar to the one which appears when one attempts to (re)quantize phonons [36]. What happens actually to be quantum is the underlying background BEC vacuum, its ground-state wave function induces nontrivial geometry by virtue of the map (37). The effective metric thus emerges as one of the low-energy collective modes of the vacuum. Further, the metric defines the stress-energy tensor (32) which in turn determines the large-scale evolution of the Universe as well as the distribution of matter therein. The other SM-type interactions, chiral fermions and gauge fields, emerge as well as the different quasi-particle excitations of the quantum vacuum liquid (not to be confused with the bare particles of the latter), similarly to the mechanisms proposed in a theory of condensed matter [27, 29], although it might require adding the Fermi component to the Bose liquid describing the physical vacuum.

To give an analytical illustration of these statements, we consider the following physical setup which is the simplest one can imagine of yet can be realized in the “early” Universe at some stage: the just-formed BEC vacuum described by the logarithmic condensate is the predominating form of matter, any other kinds have not appeared yet. Then the induced metric (37) is completely determined by a solution of Eq. (33) with $F(x) \equiv \beta^{-1} \ln(\Omega x)$, namely

$$\left[-i\hbar \partial_t - \frac{\hbar^2}{2m} \vec{\nabla}^2 + V_{\text{ext}}(\vec{x}, t) + \beta^{-1} \ln(\Omega |\Psi|^2) \right] \Psi = 0, \quad (39)$$

under certain boundary conditions. While both those conditions and the trapping potential are still unknown to us, one can already deduce a very important general feature: due to the separability property of the logarithmic Schrödinger equation its simplest ground-state solutions have the phase which is linear with respect to the radius-vector [5],

$$i \ln \frac{\Psi_0(\vec{x}, t)}{|\Psi_0(\vec{x}, t)|} \propto \vec{v}^{(0)} \cdot \vec{x} + f(t), \quad (40)$$

which indicates, upon recalling Eq. (11), that the background condensate flows with a constant velocity $\vec{v}^{(0)}$ if viewed as an embedding into the Euclidean space. Together with Eq. (38) it means that the geometry induced

by such *panta rhei* solutions is conformally flat,

$$ds_{(\beta)}^2 \propto \Omega |\Psi_0(\vec{x}, t)|^2 \left[-c_\beta^2 dt^2 + \left(d\vec{x} - \vec{v}^{(0)} dt \right)^2 \right]. \quad (41)$$

At the level of metric, the value of $\vec{v}^{(0)}$ becomes irrelevant and can be set to zero by an appropriate coordinate transformation; at the level of the Euclidean observer this corresponds to selecting the Galilean frame of reference comoving with the background. Obviously, for manifolds with such metrics the Weyl tensor vanishes so they are of type **O** in the Petrov classification [40]. This is the class where all the FLRW spacetimes, including those expanding with an acceleration, belong to (in general relativity the manifolds corresponding to isolated gravitating objects belong to type **D**, spacetimes of other types involve gravitational waves of different kinds). Therefore, for our physical setup we will necessarily obtain one or another family of the FLRW spacetimes — just written in the conformally-flat coordinates, like in the kinematic cosmology [41].

Further, to derive the induced stress-energy tensor corresponding to our setup we use the definition (32) where assume that the metric is given by the last equation, $g_{\mu\nu} dx^\mu dx^\nu = ds_{(\beta)}^2$. With the help of the conformal rescaling technique we immediately obtain

$$\kappa T_{\mu\nu}^{(\beta)} = \tilde{D} \left[\nabla_\mu \nabla_\nu \Phi - \nabla_\mu \Phi \nabla_\nu \Phi - g_{\mu\nu} \left(\nabla_\lambda \nabla^\lambda \Phi + \frac{1}{2} (\tilde{D} - 1) \nabla_\lambda \Phi \nabla^\lambda \Phi \right) \right], \quad (42)$$

where $\tilde{D} \equiv D - 2 = 2$, ∇ is the covariant derivative with respect to the metric g , and we have designated

$$\Phi \equiv \ln \left(\Omega |\Psi_0(\vec{x}, t)|^2 \right), \quad (43)$$

up to an additive constant. This stress-energy tensor strongly resembles the one of some theory with scalar field, and indeed, one can check that it can be formally derived, by varying the metric, from the following scalar-tensor gravity action functional

$$S^{(\beta)}[g, \Phi] \propto \int d^D x \sqrt{-g} e^{\tilde{D}\Phi} \left[R + \tilde{D} (\tilde{D} + 1) (\nabla\Phi)^2 \right], \quad (44)$$

where the notation “ Φ ” reminds that the “dilaton field” Φ has been already fixed by the solution of the BEC quantum wave equation, in this case it would be Eq. (39). Being entirely formal and analogous (because in reality both the metric and “dilaton” are determined by the BEC vacuum which is in the state described by $\Psi_0(\vec{x}, t)$), this action nevertheless confirms what was written before about fundamental scalar field: it explains why the relativistic

models involving scalars, such as the scalar-tensor gravity or (bosonic sector of) supergravity, yield the expressions for metric tensors which seem to provide the good qualitative description of the large-scale evolution of the early Universe and agreement with current observational data yet no scalar partner has been detected. Moreover, this duality between the non-relativistic quantum BEC equation and relativistic classical scalar-tensor gravity also shows the already discussed limitations of the relativistic description alone: once the BEC vacuum goes into the different quantum state represented by other solution of Eq. (39) one gets a different expression for the induced metric and, therefore, for the induced stress-energy tensor and covariant action. In fact, for more complicated physical setups even the condition (40) leading to conformal flatness can be relaxed to the asymptotic one. Therefore, depending on a physical background (determined by external potential and boundary conditions) and the quantum state the vacuum stays in, the small fluctuations and test particles obey several covariant actions. The unified picture can be seen only at the level of the quantum wave equation for the background BEC.

To conclude, in this section we have shown that relativistic gravity can be viewed as the phenomenon which emerges due to the long-wavelength fluctuations of the quantum yet macroscopical object, the non-trivial BEC vacuum. In fact, it can be useful to think in terms of the duality rooted in some kind of uncertainty principle: one can view the physical vacuum either as the Lorentzian spacetime (which, as we know, can have the non-vanishing Riemann curvature but no well-defined microscopical structure) or as the flat Euclidean space, along with the Newtonian time parameter, filled with some kind of background quantum liquid (such that the microscopical structure is well-defined but no curved-spacetime description is possible).

6. Conclusions

It is shown that on the language of field theory the logarithmic nonlinear quantum wave equation can be interpreted in terms of the background Bose–Einstein condensate by analogy with the Bogoliubov–Ginzburg–Landau theory [12]. Recall that the latter is known as the effective mean-field theory of superconductivity which not only helped to figure out most of phenomenological implications long before the underlying microscopical model was formally written down [42] but also served as a guiding light on a crooked path of the theoretical constructing of the BCS theory. In our case the microscopical theory of the background BEC can be regarded as the quantum gravity itself so there is a hope that the non-axiomatic approach based on logarithmic wave equation will do its job here as well. As for the underlying microscopical theory then the presence of two length scales, ℓ_0 and

ℓ_Ω , points out at the possibility that the noncommutative-space extension of quantum mechanics (NCQM) is a strong candidate — and, indeed, the objects which resemble the Cooper pairs (and can be viewed as the dipole-order approximation of a fluid element) do arise there naturally [43, 44]. Another approach would be to leave the spatial commutators intact but instead treat the (bare) condensate particles and Euclidean space as the underlying entities, and construct the microscopical theory in the spirit of the conventional non-relativistic theories of superfluidity and superconductivity, and then use the maps like (37) and (43) to translate the results into the language of a physical (relativistic) observer. In any case, once the vacuum liquid is formed it can be regarded as the most fundamental object (due to its ground state being described by a single wave function only) whereas the particles and interactions observed by a physical observer are represented by its different modes — collective ones and excitations.

It is also worth mentioning that since the quantum gravity is concerned there exists the conceptual difference between the interpretation of our Bose–Einstein condensate and its condensed-matter counterparts: unlike the latter it represents the fundamental (non-removable) background. This essentially implies that not only the objects which are being observed are being immersed into the condensate but also are the observers themselves with their measuring apparatus. Thus, such condensate affects not only the “objective” motion of particles but also the process of measurement itself which results in the nonlinear corrections to the quantum wave equation, see some discussions in the Appendix and references therein. That is why the theory with the logarithmic nonlinearity [4] can be also viewed as (the nonlinear extension of) quantum mechanics [5, 45]. The latter is believed by many to be the consistent way of handling the difficult places of the conventional quantum mechanics — such as the measurement problem (wave-function collapse *vs.* many-worlds interpretation) [46].

Further, we demonstrated that this kind of nonlinearity can cause, in principle, the spontaneous symmetry breaking and mass generation phenomena. The mass generation mechanism based on vacuum fluctuations is universal in a sense that it may supplement the electroweak one (by generating the masses of the photon and Higgs boson, for instance) but also it is capable of enhancing or even replacing the latter, under certain physical circumstances. The role of BEC seems to be natural here because the mass generation by such a highly non-classical object naturally serves as a physical realization of the Mach’s principle. We proposed few toy models to estimate the values of the generated masses of the otherwise massless particles such as the photon. We wrote those models in a covariant form and also the above-mentioned effect of the vacuum upon the measurement procedure is neglected as well. These assumptions seem to be a good approximation

when one works in the energy range below the vacuum energy threshold E_0 and, therefore, deals with small perturbations of the vacuum and elementary particles being also small fluctuations.

The straightforward computation shows that the photon mass, gained due to its interaction with the quantum-gravitational vacuum represented by the logarithmic condensate, can be expressed as a ratio of the elementary electrical charge and the length related to one of the parameters of nonlinearity. We gave some phenomenological arguments for why this (coherent) length's scale can be related to the size of the (causally connected part of) Universe as well as why the electric charge appeared in the formula. It once again confirms the choice of the wave equation's nonlinearity to be of the logarithmic type.

The relation of the BEC description of the physical vacuum to the curved-spacetime one is established via the well-known fluid-gravity correspondence. The latter presumes the introduction of two types of observers — physical or relativistic, operating in the long-wavelength excitations regime, and mathematical or absolute one, acting in the fictitious Euclidean space. The latter is essentially unobservable yet allows to formulate certain phenomena in a more consistent way. The dictionary between the languages “spoken” by these two observers is still mostly unknown but it is already started: we formulate the map which relates the long-wavelength solutions of the non-relativistic nonlinear quantum wave equation to the metric manifolds arising in the relativistic classical scalar-tensor gravity.

To reconcile this description also with the current cosmological paradigm we advocated the idea that the curved-spacetime description of the Universe's large-scale evolution is valid only in the long-wavelength approximation, and it is not the only possible or most convenient: one can also describe it (hydro-)dynamically as the Bose liquid which flows in certain way when viewed as an embedding into the Euclidean space. Such description allows to take a different look at some long-standing problems of both the standard and inflationary cosmologies, and also hints at the possible ways of formulation the theory of quantum gravity. In particular, the generation of FLRW models and scalar field is shown explicitly for the physical setup involving the logarithmic BEC in the long-wavelength approximation.

Finally, the generic topological properties and corresponding solitonic solutions of the theories with “logarithmic” condensates related by the Wick rotation (or, alternatively, by inversion of the sign of the parameter β) were compared and discussed. The role of the natural energy of vacuum as a critical parameter for certain phase transition is outlined.

This paper is dedicated to the memory of Vitaly Ginzburg (1916–2009), a man of strong physical intuition whose research methods and style of thinking still remain in-demand nowadays. I acknowledge thoughtful discussions with Merab Gogberashvili and other participants of the Memorial Gamow’105 conference in Odessa as well as the comments by Alexander Avdeenkov and Dmitry Churochkin on the properties of Bose–Einstein condensates. The instructive debates with Frederik Scholtz about the problem of choice of proper physical degrees of freedom in quantum theory are acknowledged too. The correspondence from Vladimir Dzhunushaliev was very helpful as well. This work was supported under a grant of the National Research Foundation of South Africa.

Appendix

Logarithmic Schrödinger equation

There exist at least two ways of how the logarithmic Schrödinger equation (LogSE) can be introduced. The chronologically first one is based on the separability argument — the LogSE is the only local Schrödinger equation (apart from the conventional linear one) which preserves the separability of the product states: the solution of the LogSE for a composite system is a product of the solutions for uncorrelated subsystems [5]. The second way is based on the arguments closely related to open quantum systems and quantum information theory [47] which is relatively less known and thus deserves to be reminded here.

Consider a multi-particle (sub)system whose dynamics is described by the Hamiltonian-type operator $\hat{H}$. Besides, this subsystem is in a contact with its environment such that there is an exchange of energy and information. The state of the system is described by the vector $|\Psi\rangle$. If the Hamiltonian does not depend on wave function then in the Schrödinger coordinate representation we recover the linear differential equation for Ψ .

However, in general the interactions between the particles comprising the subsystem depend on the distribution $|\Psi|^2$ of the particles in the configuration space. To determine this distribution, *i.e.*, to extract, transfer and store the information in a particular configuration of matter, one requires certain amount of energy per bit, call it ε . The information acquired upon measurement of the state is proportional to the logarithm of the probability of an outcome Ψ , *i.e.*,

$$I_\Psi = -\log_2 (\Omega|\Psi|^2) = -\frac{\ln \Omega|\Psi|^2}{\ln 2}, \quad (45)$$

and the associated entropy of the subsystem is given by

$$S_\Psi = -k_B \langle \Psi | \ln (\Omega|\Psi|^2) | \Psi \rangle,$$

where k_B is the Boltzmann constant. This entropy minimizes on delta-like distributions and maximizes on uniform ones. Here the normalization factor Ω defines a measurement reference for the entropy because for continuous systems the latter is not absolute. For instance, one could establish the reference entropy as that for a uniform distribution hence if the subsystem has fixed volume and the states are box-normalized then Ω equals to this volume.

The above-mentioned energy thus brings the contribution to the Hamiltonian of the form

$$\hat{H} \rightarrow \hat{H}' = \hat{H} - \varepsilon \log_2 (\Omega |\Psi|^2), \quad (46)$$

and the effective temperature which can be formally associated with this kind of entropy is given by $T_\Psi \equiv (k_B \beta)^{-1} = (\partial E' / \partial S_\Psi)_\Omega = \varepsilon / (k_B \ln 2)$, where $E' = \langle \Psi | \hat{H}' | \Psi \rangle$ is the total energy of the system. Rewriting ε in terms of β , we recover LogSE in our notations (1). For stationary states one can write it in the form

$$\left[\hat{H} - \beta^{-1} \ln (\Omega |\Psi|^2) \right] \Psi = E' \Psi, \quad (47)$$

whereas the free energy is given by $E = \langle \Psi | \hat{H} | \Psi \rangle = E' - T_\Psi S_\Psi$. Unlike the free energy, the energy $T_\Psi S_\Psi$ is engaged in handling the information I_Ψ and thus unavailable to do dynamical work.

The Schrödinger equations of such type are suitable for describing subsystems in which the information is not conserved but being exchanged with environment. Therefore, they cannot be naively applied to systems without any kind of irreversibility hence the negative results of the experiments [48] are not surprising. On the other hand, in a theory of quantum gravity this question is still far from being settled [49]. Besides, one can notice that the logarithmic term describing the information exchange between a system and its environment plays the role similar to that of the chemical potential in condensed matter systems. This fulfills the condition for the condensed-matter-type approach being eligible for description of the physical vacuum [36].

To conclude, we write down the most important properties of LogSE:

- Separability of noninteracting subsystems (as in the linear theory): the solution of the LogSE for the composite system is a product of the solutions for the uncorrelated subsystems;
- Energy is additive for noninteracting subsystems (as in the linear theory);
- Planck relation holds as in the linear theory;

- All symmetry properties of the many-body wave-functions with respect to permutations of the coordinates of identical particles are preserved in time, as in the linear theory;
- Superposition principle is relaxed to the weak one: the sum of solutions with negligible overlap is also a solution;
- Free-particle solutions, called *gaussons*, have the coherent-states form, and upon the Galilean boost they become the uniformly moving Gaussian wave packets modulated by the de Broglie plane waves;
- Expressions for the probability density and current are the same as in the linear theory.

All these properties except the last one and, perhaps, second last and third last ones, are unique to LogSE among all other local nonlinear Schrödinger equations. Besides, many of these features are pertinent to the linear Schrödinger equation which makes the logarithmic one a “minimal” nonlinear modification in a sense.

REFERENCES

- [1] A.A. Abdo *et al.* [Fermi LAT/GBM Collaborations], *Nature* **462**, 331 (2009); A.A. Abdo *et al.*, arXiv:0908.1832 [astro-ph.HE].
- [2] F.W. Stecker, S.T. Scully, *New J. Phys.* **11**, 085003 (2009).
- [3] G. Amelino-Camelia, arXiv:0806.0339 [gr-qc]; G. Amelino-Camelia, L. Smolin, *Phys. Rev. D* **80**, 084017 (2009); J.D. Scargle, S.N. Simić, arXiv:0912.3839 [astro-ph.IM].
- [4] K.G. Zloshchastiev, *Grav. Cosmol.* **16**, 288 (2010) [arXiv:0906.4282 [hep-th]].
- [5] I. Bialynicki-Birula, J. Mycielski, *Ann. Phys.* **100**, 62 (1976).
- [6] T. Kifune, *Astrophys. J.* **518**, L21 (1999); R.J. Protheroe, H. Meyer, *Phys. Lett. B* **493**, 1 (2000); J. Albert *et al.* [MAGIC Collaboration and Other Contributors Collaboration], *Phys. Lett. B* **668**, 253 (2008); J. Chang *et al.*, *Nature* **456**, 362 (2008); G. Amelino-Camelia, *New J. Phys.* **6**, 188 (2004); U. Jacob, T. Piran, *Phys. Rev. D* **78**, 124010 (2008).
- [7] B.M. Bolotovskii, V.L. Ginzburg, *Usp. Fiz. Nauk* **106**, 577 (1972).
- [8] F. Englert, R. Brout, *Phys. Rev. Lett.* **13**, 321 (1964); G.S. Guralnik, C.R. Hagen, T.W.B. Kibble, *Phys. Rev. Lett.* **13**, 585 (1964); Y. Nambu, *Phys. Rev.* **117**, 648 (1960); Y. Nambu, G. Jona-Lasinio, *Phys. Rev.* **122**, 345 (1961); P.W. Anderson, *Phys. Rev.* **130**, 439 (1963).

- [9] S.L. Glashow, *Nucl. Phys.* **22**, 579 (1961); A. Salam, J.C. Ward, *Phys. Lett.* **13**, 168 (1964); T.W.B. Kibble, *Phys. Rev.* **55**, 1554 (1967); S. Weinberg, *Phys. Rev. Lett.* **19**, 1264 (1967); full bibliography can be found in S.F. Novaes, arXiv:hep-ph/0001283.
- [10] S.R. Coleman, E.J. Weinberg, *Phys. Rev.* **D7**, 1888 (1973).
- [11] W. Nernst, *Verhandl. der Deutschen Phys. Gesellschaften* **18**, 83 (1916).
- [12] N.N. Bogoliubov, *Izv. Acad. Nauk USSR* **11**, 77 (1947); *J. Phys.* **11**, 23 (1947); V.L. Ginzburg, L.D. Landau, *Zh. Eksp. Teor. Fiz.* **20**, 1064 (1950); E.P. Gross, *Nuovo Cim.* **20**, 454 (1961); L.P. Pitaevskii, *Sov. Phys. JETP* **13**, 451 (1961).
- [13] M. Schick, *Phys. Rev.* **A3**, 1067 (1971); E.B. Kolomeisky, J.P. Straley, *Phys. Rev.* **B46**, 11749 (1992); S.I. Shevchenko, *Sov. J. Low Temp. Phys.* **18**, 223 (1992); E.B. Kolomeisky, T.J. Newman, J.P. Straley, X. Qi, *Phys. Rev. Lett.* **85**, 1146 (2000); S.T. Chui, V.N. Ryzhov, *Phys. Rev.* **A69**, 043607 (2004).
- [14] S.T. Beliaev, *Zh. Eksp. Teor. Fiz.* **34**, 418 (1958); *ibid.* 433 (1958) [*Soviet Phys. JETP* **3**, 299 (1957)].
- [15] P. Nozières, D. Pines, *Theory of Quantum Liquids*, vol. II, Benjamin, New York 1966.
- [16] F. Dalfovo, S. Giorgini, L.P. Pitaevskii, S. Stringari, *Rev. Mod. Phys.* **71**, 463 (1999).
- [17] A.V. Avdeenkov, K.G. Zloshchastiev, in preparation.
- [18] N. Graham, R.L. Jaffe, V. Khemani, M. Quandt, M. Scandurra, H. Weigel, *Phys. Lett.* **B572**, 196 (2003).
- [19] J.A. Frieman, C.T. Hill, R. Watkins, *Phys. Rev.* **D46**, 1226 (1992); S.J. Sin, *Phys. Rev.* **D50**, 3650 (1994).
- [20] W. Heisenberg, *Z. Naturforsch.* **9a**, 292 (1934); *Rev. Mod. Phys.* **29**, 269 (1957); L. de Broglie, *Théorie Générale des Particules à Spin*, Paris: Gauthier-Villars (1943); I. Bialynicki-Birula, *Phys. Rev.* **130**, 465 (1963); J. Des Cloizeaux, *J. Phys.* **A6**, 597 (1973).
- [21] I. Bialynicki-Birula, J. Mycielski, *Phys. Scr.* **20**, 539 (1979); P. Schick, *Nucl. Phys.* **B104**, 137 (1976); G.C. Marques, I. Ventura, *Phys. Rev.* **D14**, 1056 (1976).
- [22] R. Rajaraman, *Solitons and Instantons*, North-Holland, Amsterdam, Netherlands, 1982, p. 409.
- [23] N. Steenrod, *The Topology of Fibre Bundles*, Princeton University Press, Princeton, NJ, USA, 1999, p. 230.
- [24] A.A. Abrikosov, *Sov. Phys. JETP* **5**, 1174 (1957) [*Zh. Eksp. Teor. Fiz.* **32**, 1442 (1957)]; H.B. Nielsen, P. Olesen, *Nucl. Phys.* **B61**, 45 (1973); S.R. Coleman, *Classical Lumps and Their Quantum Descendants in Aspects of Symmetry*, p. 185.
- [25] W.G. Unruh, *Phys. Rev. Lett.* **46**, 1351 (1981); *Phys. Rev.* **D51**, 2827 (1995); T. Jacobson, *Phys. Rev.* **D44**, 1731 (1991); *Phys. Rev.* **D48**, 728 (1993); M. Visser, arXiv:gr-qc/9311028; G.L. Comer, arXiv:gr-qc/0505005.

- [26] A.D. Sakharov, *Sov. Phys. Dokl.* **12**, 1040 (1968) [reprinted in *Gen. Rel. Grav.* **32**, 365 (2000), commented in M. Visser, *Mod. Phys. Lett.* **A17**, 977 (2002)].
- [27] K.P. Sinha, C. Sivaram, E.C.G. Sudarshan, *Found. Phys.* **6**, 65 (1976); *ibid.* 717 (1976); K.P. Sinha, E.C.G. Sudarshan, *Found. Phys.* **8**, 823 (1978).
- [28] S.W. Hawking, *Nature* **248**, 30 (1974).
- [29] M. Novello, M. Visser, G. Volovik, “Artificial Black Holes,” World Scientific, River Edge, USA, 2002, p. 391; G.E. Volovik, “The Universe in a Helium Droplet,” *Int. Ser. Monogr. Phys.* **117**, 1 (2003).
- [30] L.J. Garay, J.R. Anglin, J.I. Cirac, P. Zoller, *Phys. Rev. Lett.* **85**, 4643 (2000); *Phys. Rev.* **A63**, 023611 (2001); C. Barcelo, S. Liberati, M. Visser, *Class. Quantum Grav.* **18**, 1137 (2001).
- [31] C. Barcelo, S. Liberati, M. Visser, *Living Rev. Rel.* **8**, 12 (2005).
- [32] M. Visser, *Class. Quantum Grav.* **15**, 1767 (1998); S.E. Perez Bergliaffa, K. Hibberd, M. Stone, M. Visser, *Physica D* **191**, 121 (2001); L.C. Garcia de Andrade, *Phys. Rev.* **D70**, 064004 (2004) [Erratum-*ibid.* 129903 (2004)]; *Phys. Lett.* **A346**, 327 (2005); M. Rangamani, *Class. Quantum Grav.* **26**, 224003 (2009).
- [33] D.W. Sciama, *Mon. Not. Roy. Astron. Soc.* **113**, 34 (1953); P. Jordan, *Z. Phys.* **157**, 112 (1959); C. Brans, R.H. Dicke, *Phys. Rev.* **124**, 925 (1961).
- [34] E.B. Gliner, *Sov. Phys. JETP* **22**, 378 (1965); K. Sato, *Mon. Not. Roy. Astron. Soc.* **195**, 467 (1981); *Phys. Lett.* **B99**, 66 (1981); A.H. Guth, *Phys. Rev.* **D23**, 347 (1981); A.H. Guth, S.Y. Pi, *Phys. Rev. Lett.* **49**, 1110 (1982); A.D. Linde, *Phys. Lett.* **B108**, 389 (1982).
- [35] Y.B. Zeldovich, *JETP Lett.* **6**, 316 (1967) [*Pisma Zh. Eksp. Teor. Fiz.* **6**, 883 (1967)]; *Sov. Phys. Usp.* **11**, 381 (1968).
- [36] G.E. Volovik, *Int. J. Mod. Phys.* **D15**, 1987 (2006) [arXiv:gr-qc/0604062].
- [37] R. Penrose, *Singularities and Time Asymmetry*, in S.W. Hawking and W. Israel (eds.), *General Relativity*, Cambridge, CUP, 1979, pp. 581-638; R. Penrose, *Ann. NY Acad. Sci.* **571**, 249 (1989); Ø. Grøn, S. Hervik, arXiv:gr-qc/0205026.
- [38] M.S. Turner, *Phys. Rev.* **D44**, 3737 (1991); A. Kashlinsky, F. Atrio-Barandela, H. Ebeling, A. Edge, D. Kocevski, *Astrophys. J.* **712**, L81 (2010).
- [39] A.D. Linde, *Phys. Lett.* **B175**, 395 (1986); *Mod. Phys. Lett.* **A1**, 81 (1986).
- [40] A.Z. Petrov, *Uch. Zapiski Kazan Gos. Univ.* **144**, 55 (1954) [reprinted in, *Gen. Relativ. Gravitation* **32**, 1661 (2000)].
- [41] L. Infeld, A. Schild, *Phys. Rev.* **68**, 250 (1945); G.E. Tauber, *J. Math. Phys.* **8**, 118 (1967); G. Endean, *Astrophys. J.* **434**, 397 (1994); L. Querella, *Astrophys. J.* **508**, 129 (1998); M. Iihoshi, S.V. Ketov, A. Morishita, *Prog. Theor. Phys.* **118**, 475 (2007); G.U. Varieschi, *Gen. Relativ. Gravitation* **42**, 929 (2010) [arXiv:0809.4729 [gr-qc]]; Ø. Grøn, S. Johannesen, arXiv:0911.3310 [gr-qc].

- [42] J. Bardeen, L.N. Cooper, J.R. Schrieffer, *Phys. Rev.* **106**, 162 (1957); *Phys. Rev.* **108**, 1175 (1957).
- [43] D. Bigatti, L. Susskind, *Phys. Rev.* **D62**, 066004 (2000).
- [44] C.M. Rohwer, K.G. Zloshchastiev, L. Gouba, F.G. Scholtz, *J. Phys. A: Math. Gen.* **43**, 345302 (2010) [[arXiv:1004.1984v1](#) [math-ph]].
- [45] M. Czachor, *Phys. Rev.* **A57**, 4122 (1998); M. Czachor, H.-D. Doebner, *Phys. Lett.* **A301**, 139 (2002).
- [46] P.M. Pearle, *Phys. Rev.* **D13**, 857 (1976); L. Diosi, *Phys. Lett.* **A105**, 199 (1984); A.N. Grigorenko, *J. Phys. A: Math. Gen.* **28**, 1459 (1995); R. Penrose, *Gen. Relativ. Gravitation* **28**, 581 (1996); T.P. Singh, *J. Phys. Conf. Ser.* **174**, 012024 (2009).
- [47] K. Yasue, *Ann. Phys.* **114**, 479 (1978); J.D. Brasher, *Int. J. Theor. Phys.* **30**, 979 (1991).
- [48] A. Shimony, *Phys. Rev.* **A20**, 394 (1979); C.G. Shull, D.K. Atwood, J. Arthur, M.A. Horne, *Phys. Rev. Lett.* **44**, 765 (1980); R. Gaehler, A.G. Klein, A. Zeilinger, *Phys. Rev.* **A23**, 1611 (1981).
- [49] J. Preskill, [arXiv:hep-th/9209058](#).