REVIEW OF NEAR-FORWARD πN SCATTERING AMPLITUDES AT HIGH ENERGIES

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1 Introduction

πN scattering at high energies and small momentum transfer has been investigated by many authors from different points of view. It is the aim of the present article to summarize the information following from experimental data and general principles as unitarity, analyticity and charge independence. The treatment is based on recent work of the Karlsruhe group and it has not been attempted to give a complete survey of the existing literature.

The invariant amplitudes are defined in the usual way(1)

$$T = - A(\nu,t) + \frac{i}{2}\,\gamma\cdot(q + q')\ B(\nu,t) \tag{1.1}$$

where $\nu = (s - u)/4M$ and s,t,u are the Mandelstam variables. M = nucleon mass, $\hbar = m_{\pi^+} = 1$ unless stated otherwise. In addition to A and B we introduce a combination

$$C(\nu,t) = A + \frac{\nu}{1 - \frac{t}{4M^2}}\ B \tag{1.2}$$

which occurs in the optical theorem. C and B are the t-channel helicity no flip and flip amplitudes. Some authors use the notation A' instead of C.

Denoting the amplitudes for the reactions $\pi^{\pm}p \to \pi^{\pm}p$ and $\pi^- p \to \pi^{\circ} n$ by $A_\pm$, $B_\pm$, $C_\pm$ and A_o, B_o, C_o respectively, we define the isospin even and odd combinations

$$A^{+} = \frac{1}{2}\ A_- + A_+), \quad A^{-} = \frac{1}{2}\,(A_- - A_+) = -\frac{1}{\sqrt{2}}\ A_o \quad \text{etc} \tag{1.3}$$

2. The forward amplitudes $C^{\pm}(\nu,0)$

2.1 Imaginary parts

The optical theorem allows to calculate $\mathrm{Im}\, C^{\pm}(\nu,0)$ from total cross section data[2], which are available up to 65 GeV/c

$$\mathrm{Im}\ C^{\pm}(\omega) = k\sigma^{\pm} \equiv \frac{k}{2}\left[\sigma(\pi^-p) \pm \sigma(\pi^+p)\right] \tag{2.1}$$

(Notation : $C(\nu,0) \equiv C(\omega)$, $\nu = \omega + t/4M$. $\omega = \sqrt{1+k^2}$ = pion lab.energy). σ^+ decreases from 27.8 mb at 5 GeV/c to 24.5 mb at 20 GeV/c and all published fits suggested a further decrease at higher energies. Therefore it was a surprise that in the Serpuchov experiment σ^+ was found to be practically constant between 25 and 65 GeV/c. Nevertheless the data can be fitted by a reasonable curve and the connection between the Brookhaven and Serpuchov data at 20 GeV/c is better than that of the two series of Brookhaven data at 8 GeV/c (Fig. 1). (At Serpuchov the $\sigma(\pi^+p)$ "data" have been obtained from π^-n scattering in a π^-d experiment).

In the 5-20 GeV/c range σ^- decreases according to a power law

$$\sigma^- = 0.33\ k^{-0.45} \tag{2.2}$$

Fig. 2a shows that the Serpukov data are lying as well as it could be on the same straight line. Of course one cannot exclude that σ^- tends to a constant in the high energy limit, but the present data do <u>not</u> give an indication in favour of this possibility.

One could think that Fig. 2a is in contradiction with another plot of the same data, showing $\sigma(\pi^{\pm}p)$ as a function of $k^{-1/2}$ (Fig. 2b) (see for instance Fig. 12 in ref.[3]. Plots of this type led many authors to the conclusion that the data suggest a finite σ^- in the high energy limit, encouraging investigations on possible violations of Pomeranchuk's theorem. Obviously one has to prefer a <u>direct</u> plot of the quantity of interest. The second plot is misleading as far as σ^- is concerned.

A smooth interpolation of all recent total cross section data including results presented at the Kiev Conference 1970 has been given in our "Tables of πN Forward Amplitudes[2]". Tables and references of experimental data available in Nov. 1969 can be found in the compilation of Giacomelli et al.[4].

2.2 Real parts

Experimental information on the real parts at high energies can be obtained in two ways :

i) Re $C_{\pm}$ (ω) follows from elastic scattering experiments in the Coulomb interference region[5].

ii) $|$Re C^{-} (ω)$|$ can be determined from an extrapolation of charge exchange angular distributions to the forward direction

$$\frac{d\sigma_o}{d\Omega_{lab}} (0°) = \frac{1}{8\pi^2} \{(\mathrm{Re}\ C^{-}(\omega))^2 + (\mathrm{Im}\ C^{-}(\omega))^2\} \tag{2.3}$$

if Im C^{-} (ω) is calculated from σ^{-}, using the optical theorem. The compilation of Giacomelli et al.[4] contains all data available in Nov. 1969. A collection of more recent data is given in Ref.[6].

A theoretical prediction for Re $C^{\pm}$ (ω) follows from forward dispersion relations, if an assumption on the high energy behaviour of $\sigma^{\pm}$ is made. The isospin even and odd cases will be discussed separately.

2.2.1 Re C^{+} (ω)

The dispersion relation reads

$$\mathrm{Re}\ C^{+}(\omega) = \mathrm{Re}\ C^{+}(1) + \frac{4\pi f^2 k^2}{M(1-\omega_B^2)(\omega^2-\omega_B^2)} + k^2 \cdot \frac{2}{\pi} \int_0^{\infty} \frac{\sigma^{+}(k')dk'}{k'^2 - k^2}, \tag{2.4}$$

where $\omega_B = 1/2M$. In order to show clearly the influence of the high energy contribution on the real part, we rewrite the dispersion relation in the following way[7]

$$I(k,k_o)_{exp} = I(K,k_o) \tag{2.5}$$

where

$$I(k,k_o)_{exp} = \frac{1}{4\pi}\{\mathrm{Re}\ C^+(1) - \mathrm{Re}\ C^+(\omega)_{exp}\} + \frac{f^2k^2}{M(1-\omega_B^2)(\omega^2-\omega_B^2)} \tag{2.6}$$

$$+ \frac{k^2}{2\pi^2}\int_0^{k_o} \frac{\sigma^+(k')-\sigma^+(k_o)}{k'^2 - k^2}\,dk',$$

$$I(k,k_o) = \frac{k^2}{2\pi^2}\int_{k_o}^{\infty} \frac{\sigma^+(k_o) - \sigma^+(k')}{k'^2 - k^2}\,dk'. \tag{2.7}$$

If $k^2 \ll k_o^2$, the integral in (2.7) can be approximated by the first term of a power series

$$I(k,k_o) = a_2k^2 + \ldots, \quad a_2 = \frac{1}{2\pi^2}\int_{k_o}^{\infty} \frac{dk}{k^2}\left[\sigma^+(k_o) - \sigma^+(k)\right]. \tag{2.8}$$

Under rather weak conditions on $\sigma^+(k')$ the first term should be a good approximation up to at least $k_o/4$.

If (2.8) is inserted, (2.5) contains 3 parameters : Re C^+ (1), a_2 and f^2. It turns out that the dependence on f^2 is so weak that its value has to be determined from another dispersion relation (cf. Ref.[3,9]). Inserting $f^2 = 0.081$ and Re C^+ $(\omega)_{exp}$ from phase shifts and Coulomb interference experiments, the parameters Re C^+ (1) and a_2 can be determined from (2.5), (2.8).

The quantity Re C^+ (1) is essentially the s-wave scattering length

$$\mathrm{Re}\ C^+(1) = 4\pi\frac{M+1}{M}\,a^+_{o+}. \tag{2.9}$$

Its determination has recently been discussed in Ref.[10]. The numerical value is small and not even the sign is well-determined at present (see Ref. [8] for earlier determinations).

If $k_o \hat{=} 20$ GeV/c, $I(k,k_o)_{exp}$ is comparable with the uncertainty of Re C^+ (ω) in the energy region, where phase shifts are available. In the region of the Coulomb interference data, a plot of $I(k,k_o)_{exp}$ shows that the first term of the expansion (2.8) is a reasonable approximation up to about 15 GeV/c or even further. This result is confirmed by the fact that the evaluation of equ. (2.7) according to models which are in agreement with the Serpuchov data gives a straight line in the above momentum interval (Fig. 3).

Unfortunately this result is in contradiction to Lindenbaum's hope that Re $C_{\pm}$ -data and dispersion relations correspond to a "crystal ball that can view the road to Asymptopia"(11). Even if the data were more accurate, our discussion shows that one obtains information mainly on the first term of the expansion (2.8). In fact the "crystal ball" did not show that the decrease of σ^+ changes to a constant behaviour already a few GeV/c above Lindenbaum's highest momentum.

Furthermore it seems that the result of the very difficult Coulomb interference experiment has a systematic error(7), which was not noticed in Lindenbaum's discussion. The data below 15 GeV/c give $a_{2,exp} = (2 \pm 0.5)\ 10^{-5}$, which is appreciably larger than the contribution to the integral (2.8) from the interval 20...65 GeV/c, if the Serpuchov data are inserted. It is hard to see, how the missing part could be obtained from a reasonable extrapolation of σ^+ to higher momenta. Another aspect of the same discrepancy is the deviation in Fig. 17 of Ref.(12)

Originally the Coulomb interference experiment was motivated as an attempt to test microscopic causality. This problem is discussed from our point of view in Ref.(7).

It is clear from (2.5) and Fig. 3 how a model for the high energy behaviour can be tested and how it has to be constructed in order to be compatible with all data and analyticity. The main point is to introduce one parameter which can be adjusted to give the correct value of a_2 in (2.8).

Recently several authors have proposed other methods for testing high energy models without showing that there is any advantage in comparison with our simple procedure. In fact, if a model fulfils (2.5) (Fig. 3), it cannot be ruled out unless one imposes additional conditions. If (2.5) is violated, it can happen that other tests are fulfilled, but this would only indicate that these tests use only part of the information.

In particular it is hard to see, why it should be better to use contour integrals (Refs.[3,13]) or the reciprocal amplitude (Ref.[14]). But it is easy to notice disadvantages of these methods. Since the first term of the expansion (2.8) dominates strongly in the region of interest, it does not help to consider other expansions which have a better convergence (Ref.[15]).

Usually it is expected that σ^+ will show an appreciable energy dependence in the energy range of the Batavia machine (Ref.[12]). There remains the possibility that one will find essentially the same constancy as in the range of the Serpuchov machine. This would suggest that the integral

$$\int_0^\infty \left[\sigma^+(\omega) - \sigma^+(\infty)\right] dk \tag{2.10}$$

exists. One could think that a serious difficulty would occur in this case, remembering Igi's famous argument, which led to the introduction of the P' Regge pole[16]. However Igi's conclusion was based on a tacit assumption, which has no theoretical foundation. Obviously he had not seen an earlier paper, in which Lehmann's sum rule was derived[17]

$$\mathrm{Re}\ C^+(\infty) = \mathrm{Re}\ C^+(1) + \frac{4\pi f^2}{M(1-\omega_B^2)} - \frac{2}{\pi}\int_0^\infty dk\ \left[\sigma^+(\omega)-\sigma^+(\infty)\right] . \tag{2.11}$$

In this case the asymptotic behaviour of Im C^+ is not simply related to that of Re C^+, since the low energy contribution to the dispersion integral is important even for Re $C^+(\omega)$ at very large ω.

2.2.2 Re $C^-(\omega)$

The unsubtracted dispersion relation reads

$$\frac{\mathrm{Re}\, C^-(\omega)}{\omega} = \frac{8\pi f^2}{\omega^2 - \omega_B^2} + \frac{2}{\pi}\int_0^\infty \frac{dk'\, k'^2}{(k'^2 - k^2)\,\omega'}\,\sigma^-(\omega') \tag{2.12}$$

In order to treat the unknown high energy behaviour of σ^- carefully, we perform a subtraction in the integral[18] and write the dispersion relation in the following way

$$Y \equiv \left\{\frac{\mathrm{Re}\, C^-(\omega)}{8\pi\,\omega}\,\mathrm{exp} - \frac{k^2}{4\pi^2}\int_0^\infty \frac{dk'}{\omega'}\,\frac{\sigma^-(\omega')}{k'^2 - k^2}\right\}(\omega^2 - \omega_B^2)$$

$$= f^2 + (\omega^2 - \omega_B^2)\left\{\frac{J}{2} + \frac{k^2}{4\pi^2}\int_{k_0}^\infty \frac{dk'}{\omega'}\,\frac{\Delta\,\sigma^-(\omega')}{k'^2 - k^2}\right\} \tag{2.13}$$

The fit (2.2) has been used for σ^- up to infinity. In order to correct for a possible difference $\Delta\,\sigma^-$ between the fit and the true σ^-, the integral on the r.h.s. was added. $|\mathrm{Re}\, C^-(\omega)|_{\mathrm{exp}}$ follows from charge exchange data as described in § 2.2. The choice of the sign is discussed in detail in Refs.[6,18].

Fig. 4 shows a plot of y as a function of $\omega^2 - \omega_B^2$. We expect a straight line at low energies and, if $\Delta\,\sigma^-$ is large enough, a curvature at higher energies. Fig. 4 shows that not even the sign of a curvature can be detected. Small deviations can be ascribed to errors in the analysis of the charge exchange data as discussed in Ref.[6](*)

We conclude that there are no indications for a violation of charge-independence or of our analyticity assumption. Possible deviations from the power law (2.2) (including a finite high energy limit of σ^-) occur in such a way that the integral on the r.h.s. of equ. (2.13) is negligible in the energy range of the experimental data.

The slope of the straight line in Fig. 4 gives a very accurate determination of J : J = - 0.0485 ± 0.0005. If the unsubtracted dispersion

(*) However one should notice that the separation of strong and electromagnetic contributions to the amplitude is not yet well understood at low energies (see, for instance § 7 in Ref.[6]).

integral exists J is an integral over total cross sections

$$J = \frac{1}{2\pi^2} \int_0^\infty \frac{dk}{\omega} \sigma^- (\omega) \tag{2.14}$$

The interesting point is now that an accurate determination of J is already possible from Re C^- -data in the 1-6 GeV/c region, where the high energy contribution (k > 20 GeV/c) to the subtracted integral on both sides of equ. (2.13) is negligible. Subtracting from J (2.14) the known part ($0 \leq k \leq k_o \hat{=} 20$ GeV), one obtains a value for the high energy part of (2.14)

$$\frac{1}{2\pi^2} \int_{k_o}^\infty \frac{dk}{\omega} \sigma^- (\omega) = (40 \pm 5) \cdot 10^{-4} \tag{2.15}$$

is analogy to (2.8).

It is very interesting to compare the result (2.15) with a direct evaluation of the integral, assuming that the power law (2.2) is valid up to infinity. The two values agree within the errors and we conclude that this is a good argument in favour of the validity of the unsubtracted dispersion relation and of the Pomeranchuk theorem. If the integral (2.15) would be divergent, the agreement of the two numbers would have to be considered as fortuitous.

Another discussion of the real parts has recently been published by Horn and Yahil[(19)]. The authors came to a different conclusion, but this is obviously due to the fact that they ignored most of the information following from charge-exchange experiments. They used only one of the nine experiments on which Fig. 4 is based.

Our treatment is also at variance with a recent preprint by Wit[(20)], who started with the remark that the usual dispersion relation is not a powerful tool for analyzing high energy data. He proposed another method, claiming that it would be helpful in rejecting otherwise acceptable models. However it is hard to see, how a function $C^-(\omega)$, which fulfills the usual dispersion relation and is a good interpolation of all relevant data could be rejected by any other method, unless new conditions are imposed in addition to analyticity. Furthermore Wit's calculation has the difficulty that his input for Re C^- is not quite consistent with the usual dispersion relation and the high energy model, which is being tested.

3. Derivative of the Amplitude $C^\pm$

3.1 Isospin even case[21]

Fixed-t dispersion relations are useful not only at t = 0 but also in the t-interval $0 < t < -26$, in which Im C can be calculated from phase shifts. However it is clear that the method is less powerful than at t = 0, where Im C follows from total cross sections.

Aside from $C^+(\nu,0)$ the most favourable case is the derivative $(\partial/\partial t)\, C^+$ at $t = 0$, since information on the imaginary part can be obtained from the slope of the diffraction peak and, even more directly, from the analysis of Coulomb interference experiment[5].

The dispersion relation reads

$$S^+(\omega) = S^+(0) + S^+_N(\omega) + \frac{2\omega^2}{\pi}\int_1^\infty \frac{\frac{\partial}{\partial t}\,\mathrm{Im}\, C^+(\omega',t)\Big|_{t=0}}{\omega'\,(\omega'^2-\omega^2)}\, d\omega' + \frac{\omega}{2\pi M}\int_1^\infty \frac{d\omega'}{\omega'^2}\,\frac{2\,\omega'+\omega}{(\omega'+\omega)^2}\,\mathrm{Im}\, C^+(\omega',0) \tag{3.1}$$

where $C^\pm$ is now considered as a function of ω and t, the derivative being taken at fixed ω. $S^\pm(\omega)$ is defined by

$$S^\pm(\omega) = \frac{\partial}{\partial t}\, C^\pm(\omega,t)\Big|_{t=0} \tag{3.2}$$

and S_N denotes the nucleon Born term. At high energies it is convenient to introduce the slope parameter $b^{\pm}(\omega)$

$$\frac{\partial}{\partial t} \operatorname{Im} C^{\pm}(\omega,t)\Big|_{t=0} = \frac{k}{2} b^{\pm}(\omega)\, \sigma^{\pm}(\omega) \qquad (3.3)$$

The determination of the subtraction constant $S^{+}(0)$ has been discussed in Refs.[10,21]

$$S^{+}(0) = 1.13 \pm 0.10 \qquad (3.4)$$

The same fit gives information on a weighted average of b^{+} ($\bar{\omega} \simeq 2$ GeV)

$$\langle b^{+} \rangle = \int_{\bar{\omega}}^{\infty} b^{+}(\omega)\, \sigma^{+}(\omega) \frac{k\, d\omega}{\omega^3} \Bigg/ \int_{\bar{\omega}}^{\infty} \sigma^{+}(\omega) \frac{k\, d\omega}{\omega^3} = 6.0\ (\mathrm{GeV}/c)^{-2} \qquad (3.5)$$

which is in reasonable agreement with the experimental information (Fig.5)

The b^{+}-values following from phase shifts and Coulomb interference data[5] show an energy dependence between 2 and 20 GeV/c (Fig.5) which corresponds to a "shrinkage" of the diffraction peak as expected in the early days of Regge pole theory. The shrinkage has already been noticed by Lasinsky et al.[22] in their careful investigation of the slope of the diffraction peak.

Since the energy dependence of $b^{+}(\omega)$ is not known above 20 GeV we have considered two possibilities in our evaluation of the dispersion relation (3.1) :

i) the logarithmic increase of $b^{+}(\omega)$ continues up to infinity,

ii) $b^{+}(\omega)$ = const above 20 GeV/c.

Fig. 6 shows that the predicted $S^{+}(\omega)$ agrees well with the direct calculation of this quantity from phase shifts up to about 1 GeV/c. Of course part of the agreement is due to our choice of $S^{+}(0)$.

The discrepancy in the 1-2 GeV range is probably caused by a deficiency of the S^+-values, which were calculated from phase shifts as given in the table[23], assuming that the contributions of all higher partial waves ($\ell > 5$) are negligible. Instead of this assumption one should insert the "Born term" contributions to the real parts of the higher partial waves, which were used in the CERN phase shift analysis. Unfortunately these terms are not included in the table and since the phase shift analysis of the CERN group have never published a detailed account of their most recent analysis (1967/68), it was not possible for us to check, whether the discrepancy can be explained in this way[*].

We think that a careful evaluation of the dispersion relation (3.1) gives a more reliable information on $S^+(\omega)$ above 1 GeV/c than a phase shift analysis, in which the somewhat mysterious "Born terms" of partial wave dispersion relations play an important role. Therefore the prediction from (3.1) should be taken as an input in phase shift analysis together with the prediction for the forward amplitude[2], which is even more reliable.

It seems that the above mentioned "Born term contributions" to higher partial waves have been ignored in all evaluations of "Finite Energy Sum Rules" and "Continuous Moment Sum Rules". Therefore the discrepancy between our prediction and the Regge pole model fit of Barger and Phillips[24], in which CMSR were used, is not surprising.

It would be worthwhile to estimate the influence of $\ell > 5$ partial waves in backward scattering above 1 GeV/c, where the lower partial waves cancel each other to a considerable extent.

Finally it should be mentioned that the predicted $S^+(\omega)$ has a comparable magnitude but the opposite sign in comparison with the assumption made by Foley et al.[5] in their analysis of the Coulomb

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(*) I am grateful to Prof. DONNACHIE for a discussion on this question.

interference data. These authors assumed that the ratio of the real and imaginary parts is independent of t at small t, whereas our result predicts a strong-dependence. It is remarkable that our $|\mathrm{Re}\, C^+|$ is increasing, if t goes from zero to small negative values. Of course this quantity has to decrease at larger negative t.

Fig. 6 shows a positive background in $S^+(\omega)$, on which resonance structures are superimposed. This background is related to the subtraction constant $S^+(0)$, which came out even larger in the recent work of Cheng and Dashen[25](*). But their value is based on real parts at low energies only. It enlarges the discrepancy in Fig. 6 and extends it to the 0.5 - 1 GeV range (See also our discussion in Ref.[10].

3.2 Isospin odd case

The derivative $(\partial/\partial t)\, C^-(\omega,t)$ at $t = 0$ and high energies is of interest for the treatment of the "cross-over zero" of $d\sigma_-/dt - d\sigma_+/dt$ at $t \approx -0.1\ (\mathrm{GeV/c})^2$ and for the determination of the magnitude of $B^-(\omega,0)$ from the slope of $d\sigma_o/dt$ at $t = 0$.

The dispersion relation for $S^-(\omega)$, equ. (3.2)

$$S^-(\omega) = S_N^-(\omega) + \omega \frac{2}{\pi} \int_1^\infty \frac{\frac{\partial}{\partial t} \mathrm{Im}\, C^-(\omega',t)\big|_{t=0}}{\omega'^2 - \omega^2}\, d\omega' \, \frac{1}{2\pi M} \int_1^\infty \frac{k'\, \sigma^-(\omega')}{(\omega' + \omega)^2}\, d\omega' \tag{3.6}$$

was recently investigated by Jakob[26]. Using a similar method as in § 3.1 he obtained the average value ($\bar{\omega} = 2$ GeV).

$$< b^- > = \int_{\bar{\omega}}^\infty b^-(\omega)\, \sigma^-(\omega)\, \frac{k\, d\omega}{\omega^2} \Big/ \int_{\bar{\omega}}^\infty \sigma^-(\omega)\, \frac{k\, d\omega}{\omega^2} = 0.28 \simeq 14\ (\mathrm{GeV/c})^{-2} \tag{3.7}$$

(*) I am grateful to Prof. Dashen for a private communication.

and a reasonable consistency between the dispersion relation and the phase shift input up to about 1 GeV/c (Fig. 7).

The value of the slope is compatible with the result of Baacke[27] and with the application of Finite Energy Sum Rules by Dolen, Horn and Schmidt (see Fig. 6 of Ref. [28]).

The value (3.7) is larger than expected from the first detailed discussion of the cross-over effect in 1964 (Ref. [29]). It seems that there has been no serious attempt to obtain an improved value from more recent data. Our preliminary discussion of the Coulomb interference data[5] suggests that the slope at $t = 0$ is more significant than the cross-over zero, since the cross section difference starts with a large slope at $t = 0$ and turns over to a flat t-dependence already near $t \approx - 0.05\ (\mathrm{GeV}/c)^2$ (see the 16 GeV/c data in Fig. 2b of Ref. [30][*]). If theoretical predictions, for instance from Regge cut models, show a similar behaviour, one should be more interested in the slope of the cross section difference at $t = 0$ than in the exact t-value of the zero, the latter one being sensitive to small correction terms.

The consistency of Regge cut models with the dispersion relation (3.6) is an important test, since according to the absorption picture the main effect of the cut is expected in the no-flip amplitude. Presumably it will be difficult to reproduce the large slope b^-. In general the comparison of the predictions from Regge cut models with experimental data does not include the data of Ref.[5] except for the values of Re $C^{\pm}(\omega,0)$. (See for instance Ref.[31]).

(*) This work is being continued by E. Krubasik[33], who is also investigating several other points of the analysis in Ref.[5], which in our opinion should be reexamined in a critical way, for instance the influence of the parametrization of the strong amplitude and the determination of the slopes $b_{\pm}$.

The slope of $d\sigma_o/dt$ at $t = 0$ has not yet found much attention, although more accurate data would probably be the best source of information on the highest nucleon resonances. Fig. 8 shows indications for large structures even in the 3-4 GeV/c region. The background in this region and the high energy behaviour are of interest for Regge pole and cut model fits. One should notice that the attempts to separate no-flip and flip contributions at small t depend on more or less arbitrary assumptions for instance on the t-dependence of residue functions and should not be taken too seriously. Our result in Ref.[(32)] was a guess, based on the old value for the position of the cross-over zero.

A more reliable separation of no-flip and flip terms will be obtained in a new investigation, using (3.6) and also the data in Ref.[(5)]. Presumably $B^-(\omega,0)$ will come out appreciably larger than in Ref.[(32)] and similar analyses of other authors.

4. The Amplitude $A^+(\nu,t)$

In Regge pole models it is generally assumed that the amplitudes A^+ and νB^+ have the same high energy behaviour as the C^+ amplitude, the first terms of an asymptotic expansion belonging to the exchange of P and P' Regge poles. Several good fits to differential cross section and polarization data were obtained, in which P and P' gave rather large contributions to the A^+-amplitude[(*)].

However it was shown that some of these fits were not acceptable, if the combination of the data

$$P_+ \frac{d\sigma_+}{dt} + P_- \frac{d\sigma_-}{dt} - P_o \frac{d\sigma_o}{dt} \qquad (4.1)$$

was considered or the compatibility with the inverse dispersion relation was tested[(34)].

(*) A systematic investigation of the ambiguity of fits of this type has recently been performed by Daum et al.[(33a)]

In the more recent Regge model fit of Barger and Phillips[24] this difficulty was solved by introducing a third Regge pole P" and using continuous moment sum rules as an additional constraint. The authors found the remarkable result that $C^+ \approx \nu B^+$, i.e. A^+ came out considerably smaller than before. But in their opinion the relation was "evidently not exact". It should rather be considered as a useful starting-point for future analyses. According to Fig. 10 of Ref.[24] (first paper) the ratio of the Pomeron residues of the B^+ and C^+ amplitudes at t = 0 is 0.6, the value 1.0 being expected in the case of the equality $C^+ = \nu B^+$ for the leading term.

Therefore in the Barger-Phillips model the ratio of the s-channel helicity flip and no-flip amplitudes goes to a finite value in the high energy limit

$$\frac{f_{+-}}{f_{++}} \approx \frac{\sqrt{-t}}{2\,\omega} \left\{ 1 + \frac{\omega}{M} \frac{A}{C} \right\} \xrightarrow[\substack{t \text{ fixed} \\ S \to \infty}]{} \text{finite value} \qquad (4.2)$$

s-channel helicity is <u>not</u> conserved and a subtraction is required in the dispersion relation for A^+.

Starting from a comparison between phenomenological Lagrangians and dispersion relations[35], we have investigated in great detail the consequences of the conjoncture that the dispersion relation for A^+ is valid <u>without a subtraction</u>[36]. In this case $A^+(\omega,t) \to 0$ at fixed t if $\omega \to \infty$ and, in the usual Regge pole model, one has to assume a complete decoupling of the P and P' Regge poles from the A^+-amplitude. As pointed out in Ref.[36] <u>helicity conservation</u>.

$$\frac{f_{+-}}{f_{++}} \sim \frac{1}{\omega} \longrightarrow 0 \quad \text{if} \quad \omega \to \infty \qquad (4.3)$$

is a consequence, but if helicity is conserved, A^+ can still go to infinity in the high energy limit.

In the special case of the forward amplitude one obtains a new sum rule

$$A^{+}(0,0) = \frac{2}{\pi} \int_{1}^{\infty} \frac{d\omega}{\omega} \, \mathrm{Im}\, A^{+}(\omega,0) \qquad (4.4)$$

which is analogous to the famous Goldberger-Miyazawa-Oehme sum rule for the C^{-}-amplitude at threshold. If phase shifts are inserted, one obtains in both cases a saturation of the order of 80 % from the integral between threshold and 2 GeV.

The evaluation of the unsubtracted dispersion relation for A^{+}

$$\mathrm{Re}\, A^{+}(\nu,t) = \frac{2}{\pi} \int d\nu' \, \mathrm{Im}\, A^{+}(\nu',t) \, \frac{\nu}{\nu'^{2} - \nu^{2}} \qquad (4.5)$$

in the large t-interval $-26 < t < 4$ is also encouraging, since it leads to a prediction for $A^{+}(0,t)$ which agrees as good as it can be expected with an independent determination of this function from the subtracted dispersion relation[9]. The agreement would have to be considered as fortuitous, if the integral in (4.5) would not exist. It is remarkable that the integral is strongly dominated by the 33-contribution, which even gives a better approximation, if it is taken alone.

Since reliable phase shifts are available only up to 2 GeV, the dispersion integrals had to be cut off at this energy. Some information on A^{+} at higher energies was derived from the combination (4.1) of experimental data, which is proportional to the component $A^{+}_{\perp}$ of A^{+} orthogonal to C^{+} in the complex plane. It turns out that $A^{+}_{\perp}$ is decreasing as expected according to our conjoncture (Fig. 4 of Ref.[36]). In a Regge pole model the leading pole would be σ (or ε) and a very crude estimate gives $\alpha_{\sigma} \approx -0.7$ (Ref.[36]). However H.P. Jacob has shown that a model of this type is not consistent with the dispersion relation and CERN phase shifts[37]. It is difficult to draw a final conclusion, since the errors of the phase shifts are not known.

In principle the conjoncture $A^+ \to 0$ can be tested, if the energy dependence of the spin-rotation parameters is known in both reaction $\pi^\pm p \to \pi^\pm p$. A special diagram for this purpose has been proposed in Ref.[36]). However the ratio f_{+-}/f_{++} is already small ($\approx$ 0.2) at 2 GeV and, of course, it will never be possible to decide, whether it goes to zero or to a small finite value in the high energy limit.

One should notice that $\pi^- p$ data alone are of little help for our purpose, since the corresponding amplitude $A_- = A^+ + A^-$ is not expected to be dominated by A^+, which is probably small, but rather by A^- which goes to infinity at high energies. Furthermore A^- is not so well known that one could solve for A^+.

It is interesting to notice that one of the consequences of our conjoncture, namely the decoupling of the f-trajectory, can be tested in πN backward scattering. An analysis of the backward dispersion relation[38] led to results, which are compatible with the vanishing of the FNN-coupling to A^+, but the accuracy is still poor because of uncertainties of the phase shifts.[*]

The encouraging results in the case of πN scattering and indications for helicity conservation in the reaction $\gamma p \to \rho^\circ p$ led Gilman et al.[39] to the proposal that helicity conservation is a general property of diffraction scattering of hadrons. This paper found much attention and triggered a large number of further investigations.

The assumption $A^+ \to 0$ for the πN amplitude was also proposed by Pfeffer et al.[40], but these authors essentially repeated the arguments given in our earlier publication.

Sundermeyer[41] used CMSR for an investigation of the A^+-amplitude. However Jakob pointed out that this result is not reliable because of the poor consistency with the fixed-t dispersion relation.

(*) The possibility that the f-meson lies on the Pomeron trajectory has been reconsidered recently by Achuthan, Schlaile and Steiner[38a]. In this case the high energy behaviour $A^+ \to 0$ follows already from the decoupling of the Pomeron.

Carlitz et al[42] recently proposed a model, in which the Pomeron can only decouple together with the f-meson, i.e. $A^{+} \to 0$ is a consequence of helicity conservation.

Two attempts have been made to relate the decoupling of P and f to more basic assumptions :

i) Renner[43] started from Tensor Meson Dominance and derived the decoupling of the f-meson.

ii) Jones and Salam[44] assumed that the coupling vertices of the exchanged tensor meson are the same as those of the free field energy momentum tensor and derived helicity conservation.

Gross and Wess[45] have shown that conformal invariance requires the flip amplitude to vanish asymptotically in a domain, where all energy variables are large compared to the masses relevant to the theory. It seems that an extension of the result to our case of small $|t|$ is possible only, if additional assumptions are introduced(*).

s-channel helicity conservation leads to conditions for t-channel exchanges, which were studied by several authors[46].

The paper of Harari and Zarmi[47] was sometimes mentioned as containing an argument against the decoupling of the Pomeron from A^{+}, since these authors concluded that the P-contribution to B^{+} seemed to be "fairly small" in constrast with its contribution to C^{+}. However Mannheim[48] used another result of this paper as one of his argument in favour of "spin-dependence" of the Pomeron coupling, which means that P is decoupled from A^{+} in the leading order. Finally Harari and Zarmi tested a combination of the decoupling of P and their interesting ideas on duality[47] and found agreement with the experimental πN data [47a](**).

(*) I am grateful to Prof. J. Wess for a discussion

(**) After having completed the manuscript, I received a preprint of a review, which contains further references on helicity conservation.

5. Unitary Bounds

Up to now we have discussed consequences of dispersion relations. There is another class of results for the πN amplitude, which is based mainly on unitary and experimental data. There exist excellent reviews on this subject for instance Ref.[49] and I shall mention only two new results, which have recently been found by our group[50](see also[53].

5.1 McDowell-Martin Bound

Several years ago McDowell and Martin[51] have shown that from unitarity alone a lower bound could be obtained for the derivative with respect to momentum transfer of the absorptive part of the πN forward scattering amplitude. They noticed that this bound is remarkably close to the data above 7 GeV/c. During the last year similar methods were used in investigations of several authors, who assumed that the bound is of interest only in the case of a highly absorptive and spin-independent process.

In Ref.[50] the bound was investigated for π^+p elastic scattering at all energies above threshold, generalizing the method to the spin dependent case. It turned out that the strongest version of the McDowell Martin bound is almost saturated by the experimental data (from phase shifts) at 0.6 and 1.9 (GeV/c), where the non-resonant background dominates the scattering amplitude (Fig. 9). At 2 GeV/c the strongest version is about 30 % higher than the bound, which is usually applied at higher energies. Unfortunately the bound cannot be used in order to obtain a bound for the high energy part of the dispersion integral (3.1), since one would need information on the asymptotic behaviour of total elastic cross sections.

5.2 The Roy-Singh-bound for the charge-exchange amplitude[52]

Roy and Singh have derived a very interesting bound relating the difference of total πp cross sections and the total charge-exchange cross section.

$$\lim_{\omega \to \infty} (\sigma_- - \sigma_+)^2 \leq \lim_{\omega \to \infty} \frac{\pi^3}{2} \sigma_{ex} \qquad (5.1)$$

It was thought that the comparison with experimental data was of interest, although the proof is valid only for the high energy limit. Unfortunately it turned out that the inequality is violated at very high finite energies in the case of the usual models[50].

. Conclusion

Fixed-t dispersion relations and charge-independence are useful for an interpolation of experimental data, determinations of πN parameters and testing high energy models and phase shift sets.

As long as all πN data are compatible with the conjoncture that $A^+ \to 0$ in the high energy limit, it should be imposed as a condition on high energy models rather than the weaker condition of helicity conservation, since the number of adjustable functions is smaller in the first case.

It is a challenge to theoreticians to derive $A^+ \to 0$ from more basic assumptions. Furthermore one could hope that investigations on high energy bounds will help to disentangle the amplitudes.

The importance of two experiments has been stressed :

i) More accurate differential cross section data for $\pi^{\pm}p$ in the Coulomb interference region[*]

ii) More accurate data on the slope of the charge-exchange differential cross section at t = 0.

Of course the continuation of measurements of polarization and spin-rotation parameters will also be of great interest.

(*) Eden et al[54] have recently discussed another phenomenon, which could be detected by an experiment of this type.

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F I G U R E   C A P T I O N S

Fig. 1 Data for the total cross section σ^+. The fits are explained in Ref. (7)

Fig. 2a Data for σ^- (Sept. 1970)

Fig. 2b Plots of σ_+ as a function of $k^{-1/2}$

Fig. 3 Test of high energy models for C^+. $I(k,k_o)$ is defined in (2.6), (2.7). For details see Ref.(7)

Fig. 4 Test of the dispersion relation for C^- and determination of J. y is defined in (2.13). For details see Ref.(6)

Fig. 5 The slope $b^+(\omega)$ of Im C^+, equ. (3.3). x from CERN experimental phase shifts, o from Glasgow A phase shifts. $\overline{\underline{I}}$ from Coulomb interference data(5).

Fig. 6 Prediction for $S^+(\omega)$ from the dispersion relation (3.1). + $S^+(\omega)$ from CERN Experimental phase shifts, Δ from Glasgow A phase shifts. BP refers to Ref.[24], the value being very small between 2 and 8 GeV. "Foley" is the assumption in Ref.(5). The solid and dashed lines are predictions from the assumptions i) and ii) described in the text.

Fig. 7 Determination of $< b^- >$, eq. (3.7). The method is similar as in Figs. 3 and 4. A best straight line fit through the points from CERN phase sifts up to 1.5 GeV leads to the value (3.7). The consistency is reasonable.

Fig. 8 Slope of the differential cross section for charge-exchange scattering at t = 0. $d\sigma/dt = d\sigma/dt|_{t=0}\,(1 + b_o t)$. Solid line : helicity flip contribution, dashed line : Regge pole model(32). For detail see Ref.(6)

Fig. 9 Ratio of experimental slopes and McDowell-Martin bounds(50). Solid line : strongest version of the bound, calculation using CERN phase shifts. Dashed line : case of exponential diffraction peak. Experimental points from slopes of $d\sigma/dt$, using total elastic cross sections in the bound.

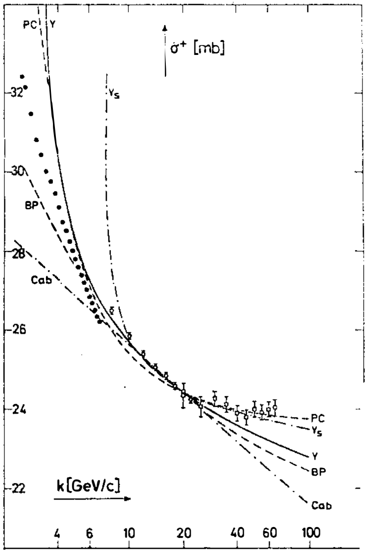

Fig. 1

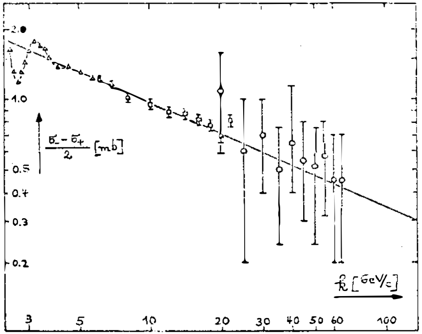

Fig. 2a

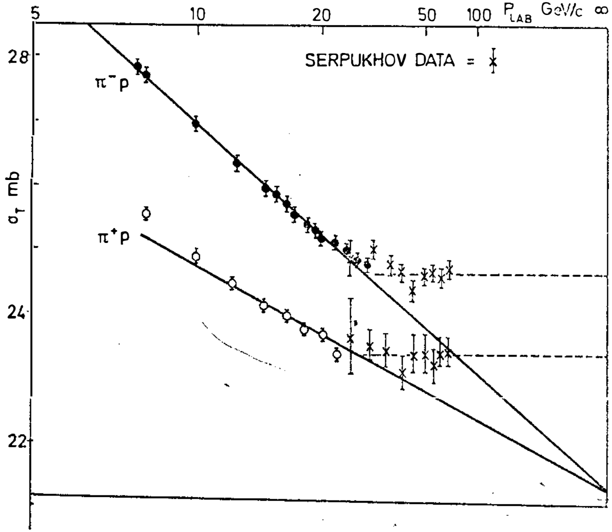

Fig. 2b

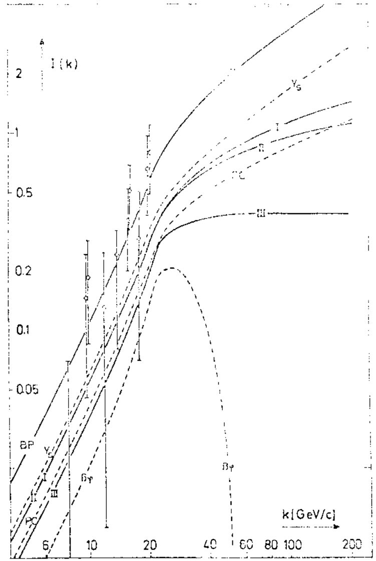

Fig. 3

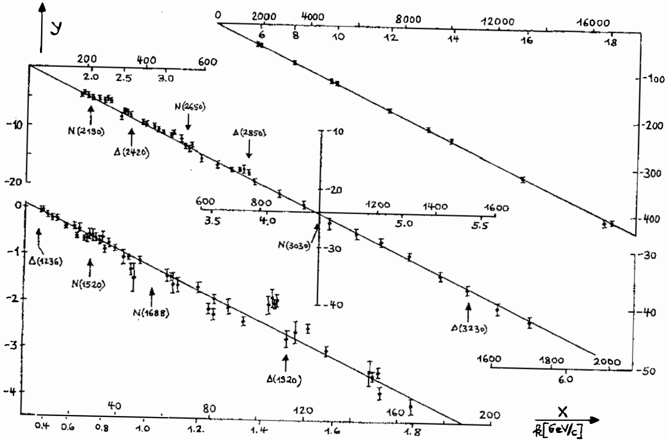

Fig.4

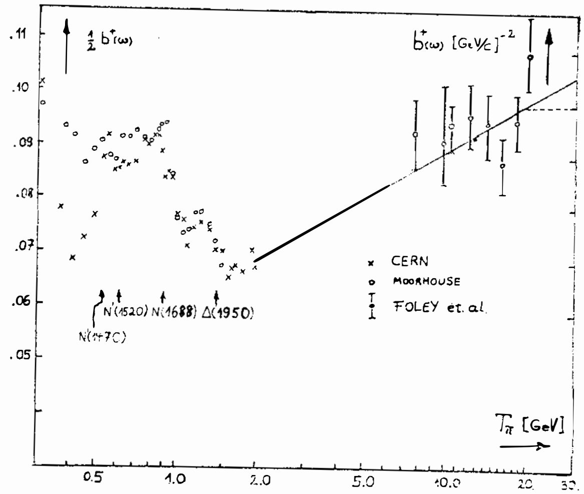

Fig. 5

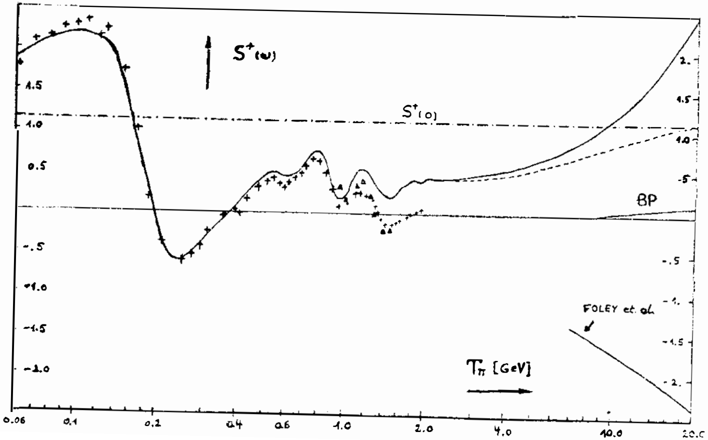

Fig. 6

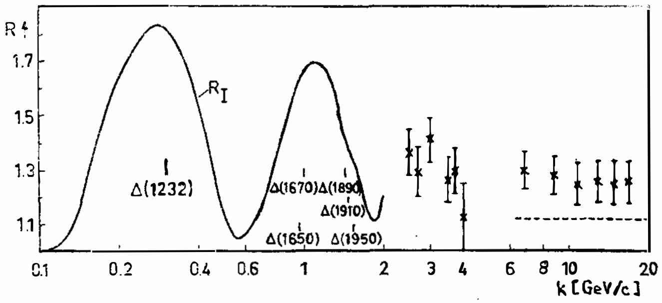

Fig. 9

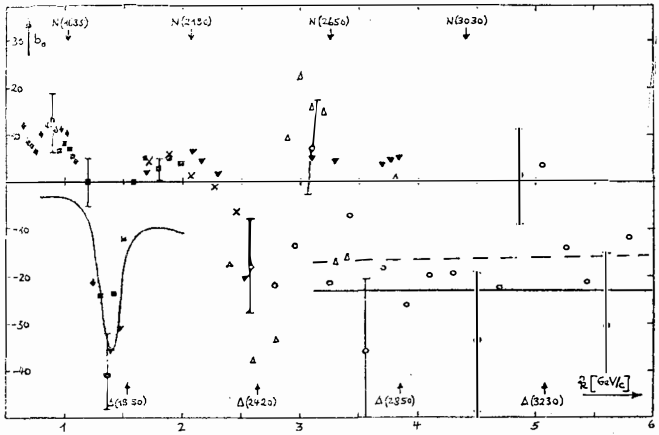

Fig. 8

Fig. 7