

Construction and commissioning of the tracker for the SuperNEMO Demonstrator Module and unfolding the $2\nu\beta\beta$ spectrum of ^{100}Mo from the NEMO-3 experiment

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I, Ashwin Kumar Chopra, confirm that the work presented in this thesis is my own. Where information has been derived from other sources, I confirm that this has been indicated in the work.

Abstract

The SuperNEMO experiment will search for the neutrinoless double-beta decay of ^{82}Se , with an expected half-life sensitivity of $> 10^{26}$ years. The first stage of the experiment, the SuperNEMO Demonstrator Module, is expected to achieve a half-life sensitivity of 6.6×10^{24} years after running for 2.5 years with 7 kg of source isotope. Key results from the construction, and commissioning of the wire tracker for the Demonstrator Module are described, and the drift distance-time relationship of each of the first three quarters measured.

The two-neutrino double-beta decay half-life of ^{100}Mo has been measured as $7.01 \pm 0.01(\text{stat}) \pm 0.46(\text{syst}) \times 10^{18}$ years using a sample of 6914 g of isotope measured for 5.25 years by the precursor experiment, NEMO-3. The summed electron energy spectrum was unfolded, allowing for a limit of $-1.12 \times 10^{-1} < \hat{a}_{of}^{(3)} < 6.95 \times 10^{-2}$ MeV to be placed on the Lorentz Violating operator $\hat{a}_{of}^{(3)}$. A further unfolded distribution in the three dimensions of summed energy, energy asymmetry, and the opening angle of the emitted electrons has been produced, allowing subsequent experiment-agnostic analyses.

Impact Statement

The unfolding technique and results presented in this thesis allow for the comparison of empirical data with theory by those with limited information on the experimental set-up during data collection. By disseminating results in this way, the data can continue to be used for testing and validating models of the decay in question long after the maintenance of experiment-specific software and infrastructure lapses.

This stands in contrast to the more common form of plots and distributions used to convey experimental results which can be difficult or impossible to independently validate without access to propriety software and systems. The cost of this more democratic model of data dissemination is a reduction in the precision of the final distributions, so while this does not represent the primary output from the NEMO-3 experiment, as a supplementary data source it may be able to inform a wider pool of subsequent analyses.

The construction and commissioning of the SuperNEMO experiment has played a part in pioneering a unique experimental technique in the field of neutrinoless double-beta decay and helps to establish an experiment which may yield new insights into the field. Furthermore, the skills developed during data analysis and during the STFC funded industry placement have facilitated both the career transition of the author into the burgeoning field of data science and artificial intelligence, and that of an undergraduate student supervised during the project into a prestigious PhD programme in the same field.

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Preface

This thesis describes the author's work as a member of the NEMO-3 and SuperNEMO collaborations between 2015 and 2019. The structure of the text is described in Chapter 1, and the author's specific contributions were as follows:

SuperNEMO

- Construction of three of the four tracker subsections including assembly and insertion of tracking cells, internal cabling and repair, and gas sealing before transport.
- Commissioning of all four tracker subsections including the creation of the commissioning electronic and trigger systems, the creation of software for commissioning data analysis and visualisation, tracker operation for the collection of commissioning data, and supplementary electrical testing.
- Analysis of commissioning data to report to the collaboration the performance of each tracker section, and to provide the first models of the drift times for electromagnetic cascades within the tracking detector.
- In-situ integration and repair of the tracker sections, the calorimeter module and the gas delivery system in operational configuration, with further gas sealing and initial data collection carried out with the calorimeter and tracker operating in tandem.

NEMO-3

- The one and three-dimensional unfolding of ^{100}Mo $2\nu\beta\beta$ decay data from the NEMO-3 experiment, extracting from this a limit on a Lorentz Violating operator, and a measurement of the half life for the decay.

Chapter 1

Introduction

The field of neutrino physics, is a field with which the Standard Model of particle physics has been repeatedly tested, and on occasion challenged. The study of double-beta decay allows further testing of the underlying assumptions of modern physics. The neutrinoless version of this decay, if detected, would provide evidence for a new mass generation mechanism and an associated class of fermion, while the version of the decay with associated neutrino emission is sensitive to multiple classes of beyond Standard Model phenomena.

Described in the following chapters are an overview of the history and current state of the field (Chapter 2), experimental techniques used in the analysis of the double-beta decay with the NEMO-3 experiment (Chapter 4), and the construction status of the state-of-the art experiment, SuperNEMO (Chapter 5). Presented are measurements of the half life of the two-neutrino double-beta decay of ^{100}Mo , and a three-dimensional distribution covering the three major kinematic descriptors of this decay, having been unfolded from detector effects to allow generalisability of the result. An example of using an unfolded distribution to test the Standard Model is carried out, and limits placed on the associated Lorentz Violating parameter. The description of this technique can be seen in Chapter 3, with the results presented in Chapter 6.

Chapter 2

Theoretical Background

In this chapter the history of the neutrino from discovery to state-of-the-art measurements is reviewed, with a focus on both its place within and the challenges it poses to the Standard Model of particle physics. Two key questions relating to beyond the Standard Model physics on which neutrino experiments can offer insight are discussed: the existence of Majorana fermions and the possibility of Lorentz Violation in fundamental interactions, with the varying experimental approaches to the former introduced.

2.1 History of the Neutrino

The discovery of the neutrino arose from the study of radioactivity, first discovered in 1896 by Henri Becquerel [1]. Radioactive emissions were later categorised into three distinct types later termed alpha, beta and gamma rays. Observation of the energy spectra from alpha and gamma emissions showed a characteristic narrow shape corresponding to alpha and gamma rays that carried an energy precisely matching the energy difference between the initial and final states of the decaying nucleus. The beta decay spectrum, however, was found to have a different shape entirely - a continuous spectrum with an upper bound [2].

Due to the well defined energies of the initial and final state nuclei undergoing beta decay this observation represented an apparent violation of the principle of energy conservation. Wolfgang Pauli postulated the existence of a new undetectable particle, emitted in conjunction with the electron in beta decay, to resolve this prob-

lem [3]. By re-framing beta decay as a three-body process the observed energies of beta particles would no longer be anomalous, and would indeed be bounded continuous spectra as observed.

Although Pauli was deeply uncomfortable with the implications of proposing a particle he considered undetectable, observation by the Cowan-Reines experiment of the inverse beta decay process $\bar{\nu}_e + p \rightarrow e^+ + n$ - where the $\bar{\nu}_e$ is an anti-neutrino - showed that he was indeed correct [4].

2.2 The Neutrino in the Standard Model

The Standard Model describes the fields and particles that make up all known matter and the strong, weak and electromagnetic forces. Discoveries of the muon neutrino [5] and tau neutrino [6] demonstrated the existence of a neutrino for each generation of charged lepton in the Standard Model. These remain the most recent fundamental fermions to be discovered, and combined with the 2012 discovery of the Higgs Boson [7], complete the particle content of the Standard Model of particle physics as illustrated in Figure 2.1.

Neutrinos have only been seen to interact via the weak interaction, and carry no electromagnetic or colour charge, so when Chien-Shiung Wu showed in 1956 that the weak interaction was maximally parity violating [8] it was deduced that neutrinos must all be produced with the same helicity, as confirmed in 1958 [9].

The Higgs boson discovery provided evidence of spontaneous symmetry breaking in the electroweak sector of the Standard Model providing masses to the W and Z bosons [11]. Yukawa couplings [12] of the Higgs field to the relevant fermion fields allow for the generation of masses for the charged leptons and quarks but the Standard Model neutrino, with only one permitted helicity, remained massless.

The Vector-Axial (V-A) description of weak interactions outlined by Feynman and Gell-Mann [13] and the assumption of a massless neutrino allowed the weak sector of the Standard Model to be constructed consisting of three SU(2) weak isospin doublets, each comprised of a (left handed) neutrino and the corresponding charged lepton. Each right handed charged lepton forms an SU(2) singlet which

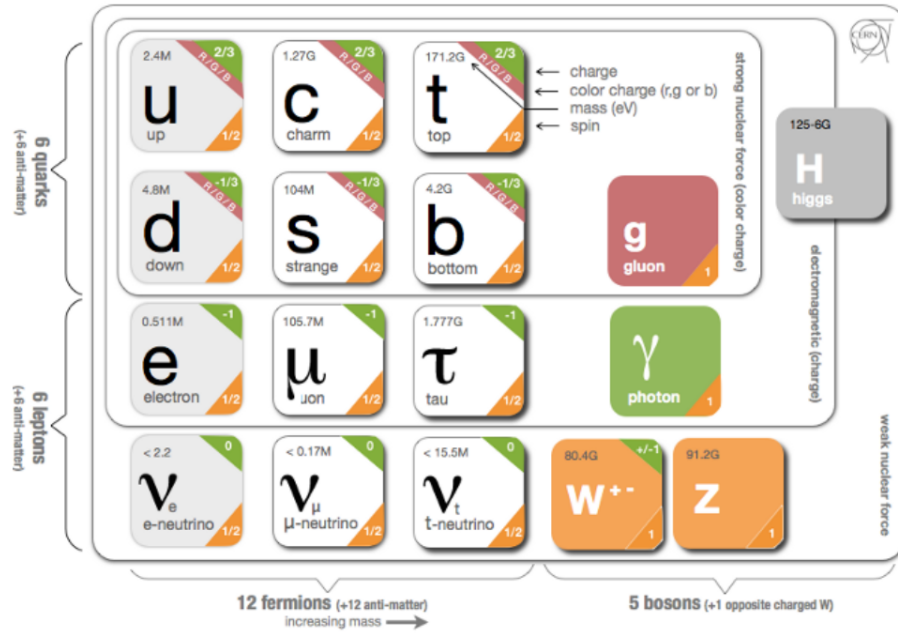


Figure 2.1: Particle content of the Standard Model showing quantum numbers, masses and families of the constituent particles [10].

does not directly interact via the weak force, and right handed neutrinos - which would not interact through any of the Standard Model forces - are not contained in the model.

2.2.1 Interactions

Since the Standard Model neutrino interacts only through the weak force, there are only two categories of interaction vertex. These are known as charged current and neutral current interactions, depending on whether the associated gauge boson is a (charged) $W^\pm$ or Z. These can be seen in Figure 2.2.

2.3 Beyond the Standard Model

2.3.1 Discovery of Neutrino Oscillations

The first evidence of neutrino phenomena contrary to the Standard Model arose in the 1960s when the Homestake experiment measured the flux of solar neutrinos and found that it was approximately 1/3 of the value predicted by the best solar models of the time [15]. This search detected neutrinos through the process $\nu_e + {}^{37}\text{Cl} \rightarrow {}^{37}\text{Ar} + e^-$.

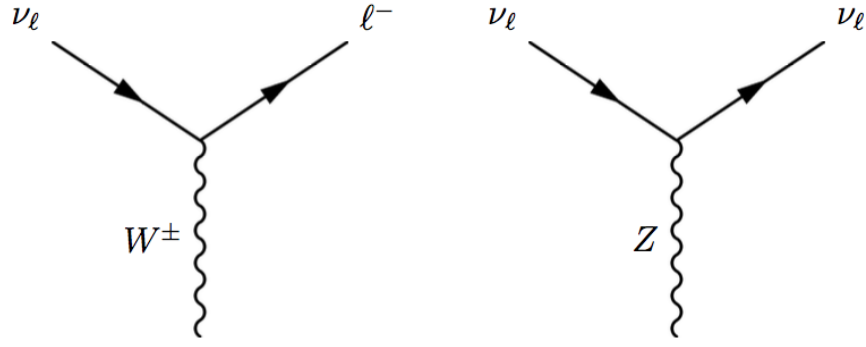


Figure 2.2: Charged current (left) and neutral current (right) Standard Model neutrino vertices [14].

$\text{Ar} + e^-$, where the ^{37}Ar atoms were counted after a period of detector exposure to the solar neutrino flux. This process was particularly sensitive to the neutrinos emitted from decays of ^8B in the sun, but due to large uncertainties on predictions of the flux the measurement was considered an anomalous result rather than firm evidence of new physics.

The Kamiokande-II experiment performed an independent measurement of the solar neutrino flux using the Cherenkov radiation caused by fast-moving charged leptons emitted from elastic scattering between a neutrino and a nucleus. The Cherenkov radiation is detected as a ring of photons coaxial with the lepton momentum vector, providing a directionality measurement of the lepton and thus the (highly correlated) incoming direction of the neutrino; allowing for the rejection of events not originating from the sun. The incoming solar neutrino flux was once again measured to be significantly below the predicted value, which combined with the direction measurements provided confirmation that the deficit measured by the Homestake experiment was a real phenomenon [16].

A model capable of explaining these measurements had already been proposed in 1957 by Pontecorvo [17], who suggested that neutrinos may oscillate between a neutrino and anti-neutrino state as they propagate in analogy with neutral Kaon mixing. Similar models involving oscillation between the neutrino flavours were also proposed and subsequently tested, notably in the Sudbury Neutrino Observa-

tory (SNO) and Super-Kamiokande experiments.

The Super-Kamiokande detector was built as the successor to the Kamiokande-II detector, using the same detection principle. The flux of muon neutrinos produced in cosmic ray hadron interactions in the upper atmosphere were measured as a function of the zenith angle from which they travelled to the detector. Observation of the patterns in Cherenkov radiation, such as those shown in Figure 2.3, allowed discrimination between muons and electrons, and therefore muon and electron neutrinos.

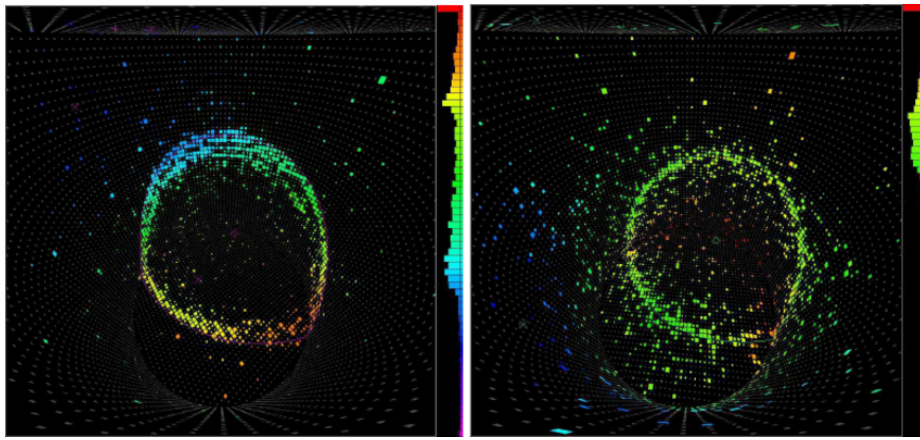


Figure 2.3: Muon-like (left) and electron-like (right) event simulations in the Super-Kamiokande water Cherenkov detector [18]. Depicted is the simulated pattern of measured energy deposits from photomultiplier tubes surrounding the detector, with colour representing the size of the measured signal. The difference in ring shapes is a result of the larger electromagnetic shower generated by the electron.

A measured deficit in the flux from muon neutrinos travelling upwards through the earth, relative to those travelling the much shorter downwards path from the atmosphere supported the oscillation model, where the changing path length of the neutrinos resulted in a varying, angular dependent, incoming flux as the neutrinos oscillated during propagation [19]. An example distribution measured by the Super-Kamiokande experiment is shown in Figure 2.4.

This hypothesis was further confirmed by measurements made by the SNO experiment. The SNO detector was sensitive to multiple types of neutrino-nucleus interaction including the charged current interaction described above - for electron

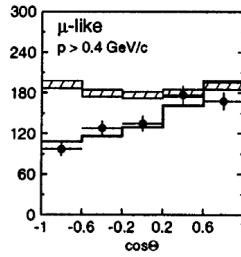


Figure 2.4: Measured (filled circles) and oscillation-free expectation (hatched) zenith angle $-\cos(\theta) < 0$ - distribution for μ -like events in the Super-Kamiokande detector. Upwards travelling particles have $\cos(\theta) < 0$. The variability of the deficit provides evidence of a path-dependent oscillation probability [19].

neutrinos only - and neutral current interactions, which is blind to the flavour of the inbound neutrino. A third detection channel measured elastic scattering events, where a neutrino scatters off of an electron without changing flavour. This process can proceed through a neutral current interaction for any flavour of neutrino, and through a charged current interaction for an electron neutrino, enhancing the cross-section for the ν_e . Through these three channels a robust measurement of both the electron neutrino and total neutrino flux was made. While SNO found the same deficit in electron neutrinos that had been seen previously, through measurement of the neutral current events it was determined that the total neutrino flux was in line with the best models for solar neutrino production, providing strong evidence that neutrinos were changing flavour as they propagated through space [20].

The combined Super-Kamiokande and SNO results were seen as unambiguous confirmation of the oscillation model, and lead researchers Takaaki Kajita and Arthur McDonald - pictured in Figure 2.5 - were consequently awarded the Nobel Prize in Physics for 2015.

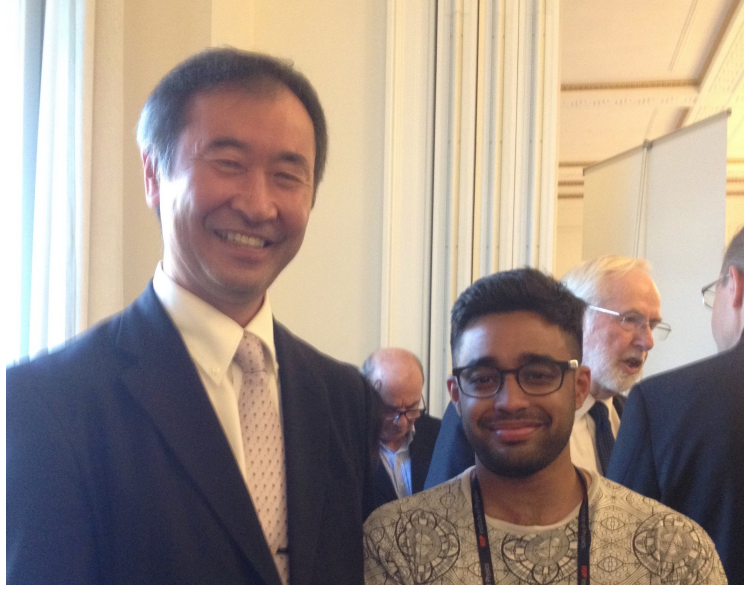


Figure 2.5: Nobel laureates Profs. Takaaki Kajita (left) and Arthur McDonald (right, background) pictured with the author.

2.3.2 Mathematical Formalism

The three-neutrino oscillation model posits the existence of three neutrino mass states each of which are eigenstates of the Hamiltonian which mix with the flavour eigenstates as follows:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}, \quad (2.1)$$

where the $U_{\alpha j}$ form a unitary matrix describing the proportions of each mass eigenstate ν_j in each flavour eigenstate ν_α [21]. This is known as the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix. Since the weak bosons couple to the flavour eigenstates this is the form in which neutrinos are experimentally detected, but they propagate in the form of a mass eigenstate. In the ultrarelativistic limit where $p \gg m$, for neutrino momentum p and mass m , the probability of a neutrino of

flavour α will be observed as having flavour β can be written as

$$\begin{aligned}
 P_{\alpha \rightarrow \beta} &= \left| \sum_i U_{\alpha i}^* U_{\beta i} e^{-im_i^2 L/2E} \right|^2 \\
 &= \delta_{\alpha\beta} - 4 \sum_{i>j} \text{Re}(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin^2 \left(\frac{\Delta m_{ij}^2 L}{4E} \right) \\
 &\quad + 2 \sum_{i>j} \text{Im}(U_{\alpha i}^* U_{\beta i} U_{\alpha j} U_{\beta j}^*) \sin^2 \left(\frac{\Delta m_{ij}^2 L}{4E} \right),
 \end{aligned} \tag{2.2}$$

where $\delta_{\alpha\beta}$ is the Dirac delta function, $U_{\alpha j}$ are elements of the PMNS matrix, Δm_{ij}^2 is the squared mass difference between the ν_i and ν_j states, E is the energy of the neutrino, and L is the distance across which it has propagated. For this model to adequately describe the SNO and Super-Kamiokande data at least one Δm_{ij}^2 is required to be non-zero implying that at least one neutrino has a non-zero mass, contrary to the Standard Model.

2.3.3 Oscillation Parameters

The PMNS matrix in Equation 2.1 is often parametrised as

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -s_{13} e^{-i\delta_{CP}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2.3}$$

where $s_{ij} = \sin(\theta_{ij})$, $c_{ij} = \cos(\theta_{ij})$, and θ_{ij} is an angle defined by the mixing between mass states i and j . The term δ_{CP} quantifies the amount of CP violation in the neutrino sector, which manifests in differing oscillation probabilities for anti-neutrinos and neutrinos.

When the generated flux from a neutrino source is known, by measuring the flux of neutrinos of each flavour detected far from their source, it is possible to estimate both the angles θ_{ij} and the mass differences Δm_{ij}^2 . Furthermore, flavour discriminating matter effects affecting the oscillation of neutrinos travelling through a dense medium can be used to constrain the sign of the Δm_{ij} [22]. This technique has been successfully applied to solar neutrinos to constrain Δm_{21} to be positive, though

the sign of Δm_{32} is as yet unknown. The current best fit oscillation parameters are summarised in Table 2.1 [23].

Parameter	Value ($\Delta m_{31,32} > 0$)	Value ($\Delta m_{31,32} < 0$)
$\sin^2(\theta_{12})$	0.304 ± 0.014	
Δm_{21}^2	$(7.53 \pm 0.18) \times 10^{-5} \text{ eV}^2$	
$\sin^2(\theta_{23})$	0.51 ± 0.05	0.50 ± 0.05
Δm_{32}^2	$(2.44 \pm 0.06) \times 10^{-3} \text{ eV}^2$	$(2.51 \pm 0.06) \times 10^{-3} \text{ eV}^2$
$\sin^2(\theta_{13})$	$(2.19 \pm 0.12) \times 10^{-2}$	

Table 2.1: Summary of best fit values for neutrino oscillation parameters

2.4 Neutrino Mass

The measured (non-zero) values for Δm_{ij} require the presence of at least two light massive neutrinos in direct contradiction to the Standard Model which assumes massless neutrinos. However the ambiguity in the sign of Δm_{32} allows for two possibilities in the ordering of these masses $m_{1,2,3}$ known as the Normal Hierarchy ($\Delta m_{31,32} > 0$) and Inverted Hierarchy ($\Delta m_{31,32} < 0$), illustrated in Figure 2.6.

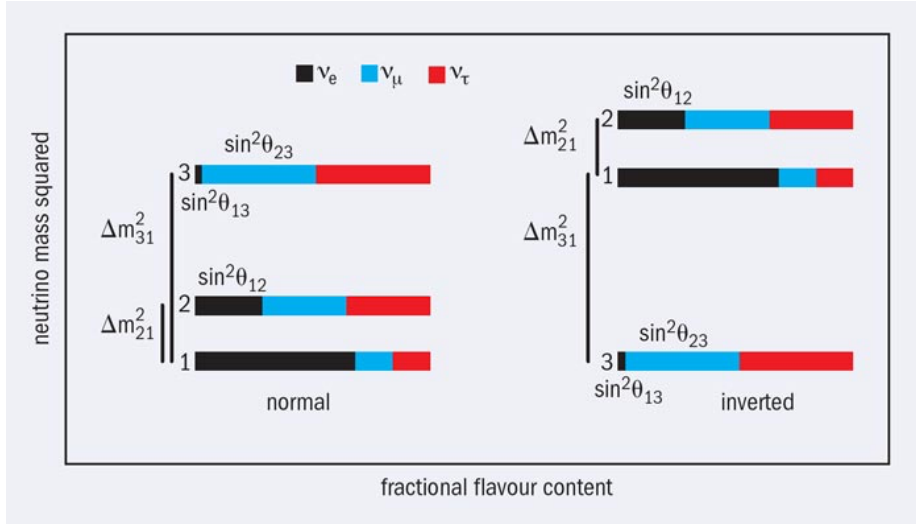


Figure 2.6: The two possible hierarchies of neutrino mass states, as mixtures of weak eigenstates, with corresponding mass differences and mixing parameters shown [24].

The absolute scale of these masses are bounded above by cosmological observations (described in Section 2.7) to be many times lower than the mass of existing particles in the theory as is illustrated in Figure 2.7.

2.4.1 Dirac Mass

Extending the Standard Model Lagrangian to include neutrino masses can be achieved through the introduction of right handed neutrino fields which couple to the left handed neutrino fields through a Higgs coupling m_D , producing a Dirac mass term in the same form as for the charged leptons, i.e.

$$\mathcal{L} \supset -m_D \bar{\nu}_L \nu_R + h.c. \quad (2.4)$$

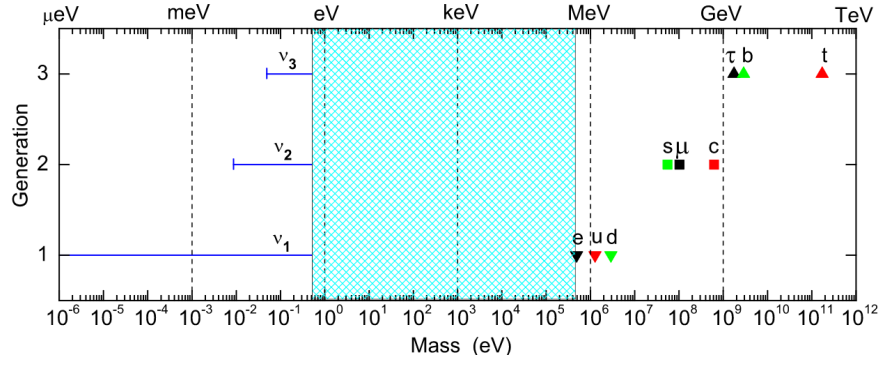


Figure 2.7: Masses of Standard Model fermions and neutrino mass eigenstates (Normal Hierarchy). Approximately six orders of magnitude lie between the heaviest possible neutrino mass and the electron mass (blue hatching). Fermion generation, matching the columns in Figure 2.1, is indicated by the coloured markers and vertical separation [25].

This approach has the benefit of minimally extending the form of new terms within the Lagrangian, but posits the existence of three right handed neutrino fields which, having no electric or weak charge, would be completely non-interacting in the Standard Model. This model postulates an undetectable right-handed neutrino to partner the Standard Model neutrino, where the empirical mass bounds require a Higgs coupling significantly smaller in magnitude than those already measured (shown in Figure 2.8), which may be an indication that a distinct process is involved in neutrino mass generation.

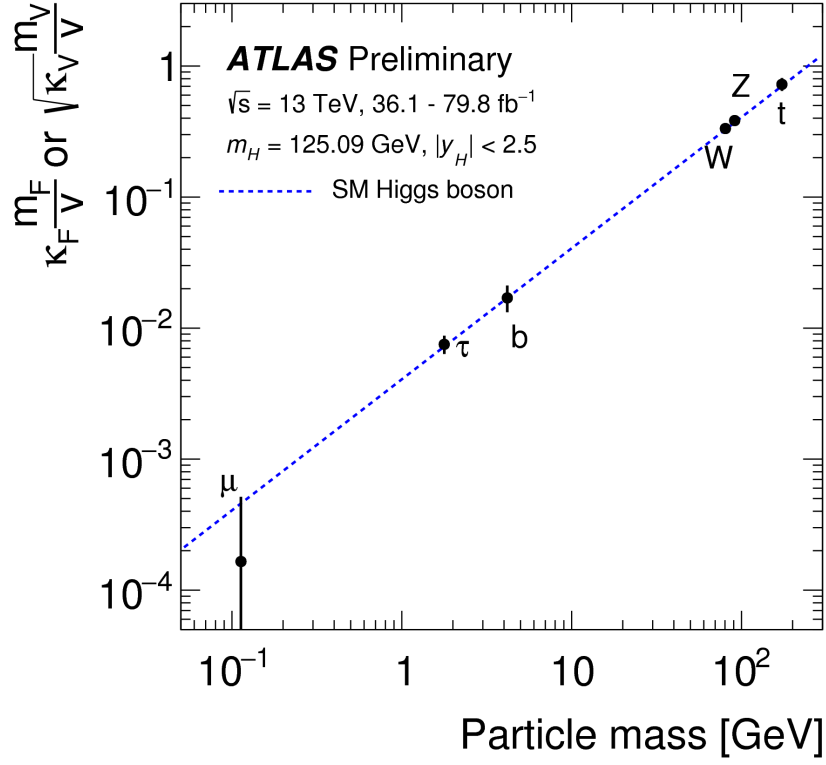


Figure 2.8: Masses of Standard Model fermions plotted against their Higgs coupling strength (error bars) as measured by the ATLAS collaboration, compared to the Standard Model prediction (blue line). Extending the relationship to neutrino mass scales would indicate a coupling far weaker than any yet measured [26].

2.4.2 Majorana Mass

A further form of mass term can be added to the Standard Model if the assumption of conservation of lepton number is relaxed. Here the usual left-handed neutrino field can be coupled to the anti-neutrino field in a term analogous to the usual Higgs coupling as in Equation 2.4. This term takes the following form:

$$\mathcal{L} \supset -m_M \bar{\nu}_R^C \nu_R + h.c. \quad (2.5)$$

where $\bar{\nu}^C$ is the charge conjugate of field $\bar{\nu}$ and m_M is a coupling constant. A non zero value for m_M ensures that a mass eigenstate

$$\nu_i = \nu_R + \nu_R^C \quad (2.6)$$

would satisfy

$$\nu_i^C = \nu_i, \quad (2.7)$$

and therefore imply that the neutrino and anti-neutrino are the same particle. Particles which satisfy this relationship are called Majorana particles, after Ettore Majorana who first proposed this form of mass [27]. Only the neutrino of all Standard Model fermions is a candidate Majorana fermion due to its lack of electric charge.

A Majorana mass term for the neutrino would introduce further degrees of freedom to the mixing between mass and flavour eigenstates [28] which may be parametrised as a post-multiplication of the standard PMNS matrix from Equation 2.3 by the matrix:

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{\frac{i\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{\frac{i\alpha_{31}}{2}} \end{pmatrix}, \quad (2.8)$$

where α_{21} and α_{31} describe the magnitude of CP violation arising from the Majorana fields.

2.4.3 The See-Saw Mechanism

A combination of Dirac and Majorana mass terms in the Lagrangian results in the general form:

$$\begin{aligned} \mathcal{L} &\supset -\frac{m_D}{2}(\bar{\nu}_L \nu_R + \bar{\nu}_R \nu_L) - \frac{m_M}{2} \bar{\nu}_R^C \nu_R \\ &= -\frac{1}{2} \begin{pmatrix} \bar{\nu}_L & \bar{\nu}_R^C \end{pmatrix} \begin{pmatrix} 0 & m_D \\ m_D & m_M \end{pmatrix} \begin{pmatrix} \nu_L^C \\ \nu_R \end{pmatrix}. \end{aligned} \quad (2.9)$$

In the oscillation formalism discussed in Section 2.3.3 the neutrino mass states were eigenvectors of the Hamiltonian, with the corresponding eigenvalues as masses. The eigenvalues of the mass matrix in 2.9 are

$$m_{1,2} = \frac{1}{2} \left(m_M \pm \sqrt{m_M^2 + 4m_D^2} \right), \quad (2.10)$$

which in the limit $m_M \gg m_D$ reduce to

$$m_1 \approx m_M, \quad (2.11)$$

and

$$m_{1,2} \approx \frac{m_D^2}{m_M}. \quad (2.12)$$

This approximation can be theoretically motivated by considering a model with a spontaneously broken symmetry group containing the Standard Model symmetry subgroup. Here the value m_M can be expected to be on the order of magnitude of the symmetry breaking scale, often thought to be $\sim 10^{15}$ GeV [29] resulting in an extremely heavy and an extremely light neutrino mass state when m_D is taken to be on the order of other fermion masses.

Mechanisms such as this, known as see-saw mechanisms, provide a natural explanation for the low neutrino masses and can introduce heavy, mostly sterile, neutrinos providing candidate particles for dark matter theories [30]. The potential for insight into mass generation, CP violation, dark matter and Grand Unification Theories have ensured that the search for a Majorana mass for the neutrino has remained an active one.

2.5 Beta Decay

Beta decay is a type of radioactive decay mediated by the weak force, through the interaction of $W^\pm$ bosons with baryons bound inside a nucleus. There are three types of beta decay each conserving baryon and lepton number but all resulting in a change in flavour of a baryon in the decaying nucleus. The most common form is β^- decay in which a neutron decays into a proton, releasing an electron and electron anti-neutrino. In β^+ decay a proton decays into a neutron through the emission of a positron and electron neutrino. In these processes the emitted positron or electron is known as a beta particle or beta ray. The final type of beta decay occurs when an electron is captured by a proton causing the pair to decay to a neutron and electron neutrino, and is therefore termed electron capture. The decay equations can be summarised as:

$$n \rightarrow p + e^- + \bar{\nu}_e \quad (\beta^- \text{ decay}), \quad (2.13)$$

$$p \rightarrow n + e^+ + \nu_e \quad (\beta^+ \text{ decay}), \quad (2.14)$$

$${}_Z^AX + e^- \rightarrow {}_{Z-1}^AX' + \nu_e \quad (\text{electron capture}). \quad (2.15)$$

$\beta^\pm$ decays can only occur in atomic nuclei when the daughter nucleus has an energy lower than that of the parent nucleus. This energy difference must be larger than the combined rest mass of the emitted beta electron and neutrino to ensure that their creation is not energetically forbidden. Additional energy released in these decays provides the beta rays and neutrinos with kinetic energy; therefore with a knowledge of the transition energy of the nuclear decay and a measurement of the electron kinetic energy the total energy of the released neutrino can be inferred without a direct measurement. It is for this reason that the endpoint of the beta decay energy spectrum is sensitive to the absolute value of the mass of the neutrino.

The binding energy of nuclei can be approximated well by the Semi-Empirical Mass Formula (SEMF):

$$E_B = a_V A - a_S A^{2/3} - a_C \frac{Z(Z-1)}{A^{1/3}} - a_A \frac{(A-2Z)^2}{A} + \delta(A, Z); \quad (2.16)$$

$$\delta(A, Z) = \begin{cases} +\frac{a_p}{A^{1/2}} & A, Z \text{ even} \\ 0 & A \text{ odd} \\ -\frac{a_p}{A^{1/2}} & A \text{ odd, } Z \text{ even,} \end{cases} \quad (2.17)$$

where A and Z are the mass and atomic numbers respectively. The terms in Equation 2.16 - reading from left to right - arise from the volume, surface, Coloumb force, neutron/proton asymmetry and spin-coupling of the nucleus with relative strengths described by the coefficients $a_{V,S,C,A,P}$.

The final term in the SEMF described lower energies for even- Z nuclei than odd- Z nuclei of the same mass number, allowing for the possibility of beta decay. The parabolae defined by this equation and allowed decays are shown in Figure 2.9. All $\beta^\pm$ decays transition between even and odd- Z nuclei due to the conversion between protons and neutrons.

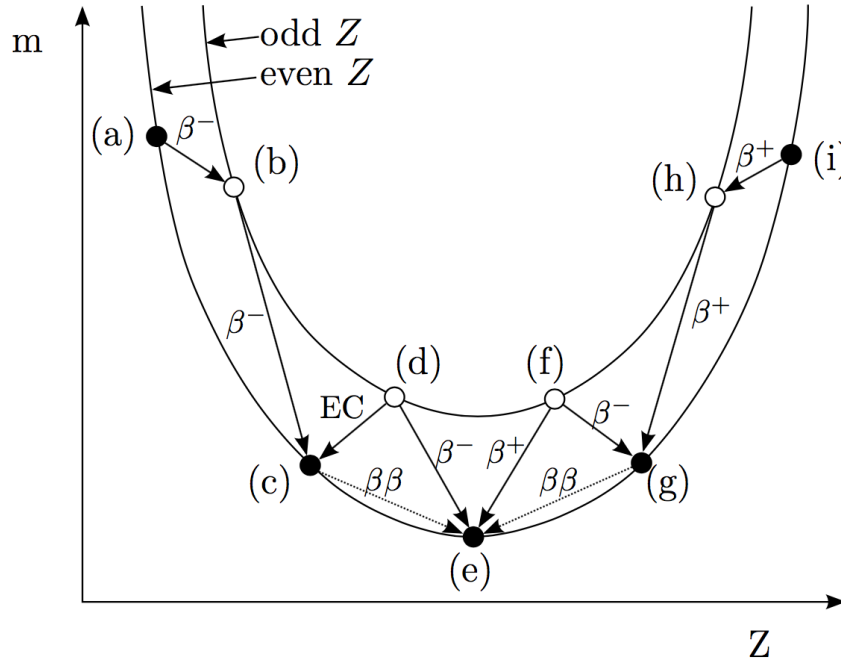


Figure 2.9: Total nuclear mass (m) is plotted against the atomic number (Z), where the splitting between odd (hollow circles) and even Z (solid circles) nuclei is visible. Atomic mass of isotopes are shown, with allowed beta decays indicated by arrows - double-beta decays are allowed from nuclei labelled (c) and (g) [31].

2.6 Double-Beta Decay

Also shown in Figure 2.9 are allowed decays between parent and daughter isotopes both with an even- Z nucleus. In these decays two neutrons simultaneously undergo β^- decay resulting in a daughter nucleus of atomic number two greater than the decaying nucleus. This process was proposed in 1935 [32] and first directly detected in ^{82}Se in 1987 [33]. It is a process allowed by the Standard Model for many isotopes, though it is practically only observable for those isotopes which have β^- decay energetically forbidden or strongly suppressed. The Feynman diagram for this process can be seen in Figure 2.10.

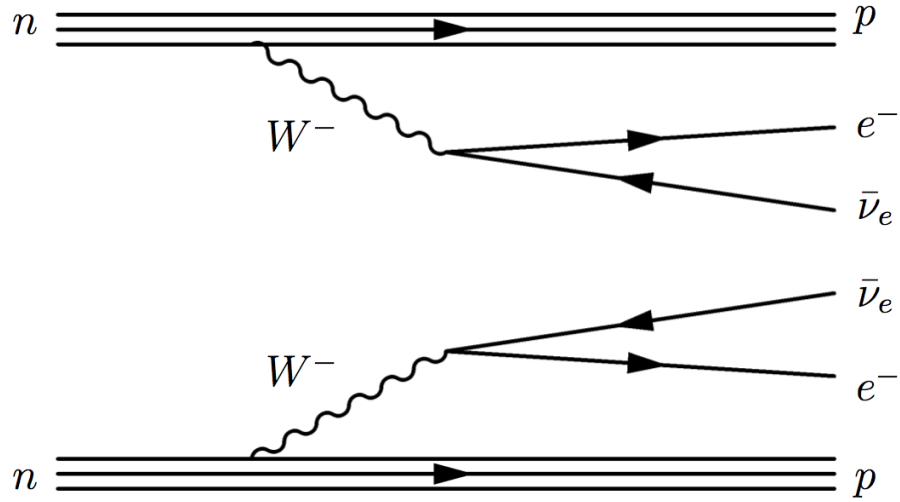


Figure 2.10: Feynman diagram for Standard Model double-beta decay.

To date this process has been observed in 11 isotopes, 7 of which were studied by the NEMO-3 experiment [34].

2.6.1 Neutrinoless Double-Beta Decay

If the neutrino is a Majorana particle a form of double-beta decay without neutrino emission is possible. In this process the beta particles are emitted with a combined energy equal to the difference between the decaying mother and daughter atoms. This energy difference, known as $Q_{\beta\beta}$, is an upper bound of the combined energy of the electrons in traditional double-beta decay and thus the experimental signature of the neutrinoless decay is a peak in the summed electron energy spectrum at the endpoint of the broad distribution measured from the Standard Model double-beta decay process as shown in Figure 2.11.

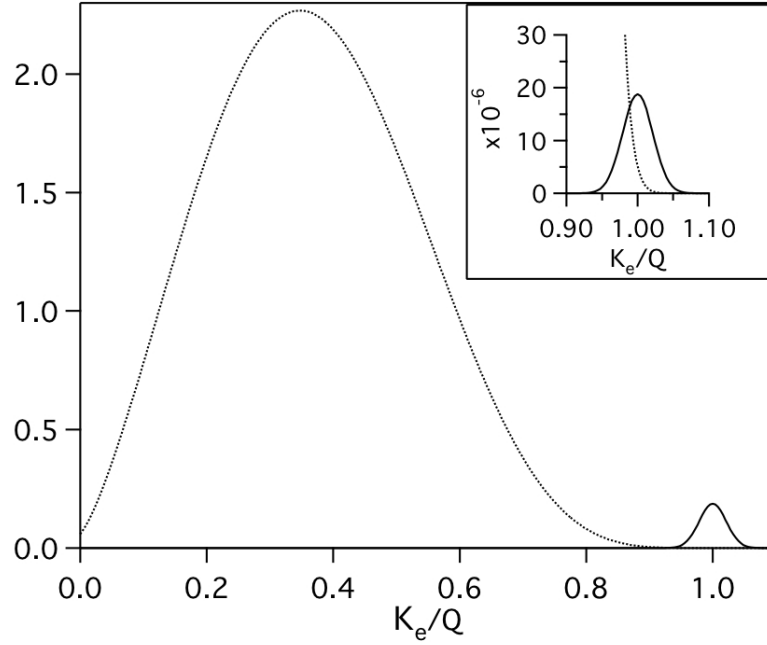


Figure 2.11: Summed reconstructed energy spectra for the measured electrons (K_e) against expected number of events (arbitrary units) for Standard Model (dotted line) and neutrinoless double-beta decay (solid line), with an exaggerated decay rate for the neutrinoless mode [35]. The x-axis has been scaled to multiples of the predicted decay energy, with exaggerated separation between the signals from the neutrinoless and two-neutrino decays. The inset shows a more realistic signal profile, with the two spectra significantly overlapping due to detector resolution limits.

While multiple Beyond Standard Model processes may contribute to the neutrinoless double-beta decay rate, including those which do not explicitly require a

Majorana neutrino, it has been shown that the observation of this decay would result in an effective Majorana mass for the neutrino regardless of the mediating process, as seen in Figure 2.12 [36].

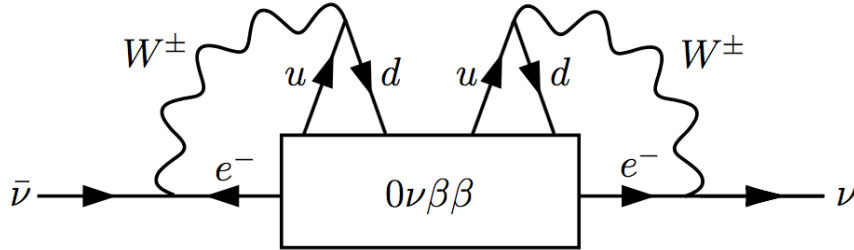


Figure 2.12: Generic Feynman diagram for neutrinoless double-beta decay, where the coupling of the ν and $\bar{\nu}$ fermion lines results in an effective Majorana mass term for the neutrino [36].

2.6.2 Mass Mechanism

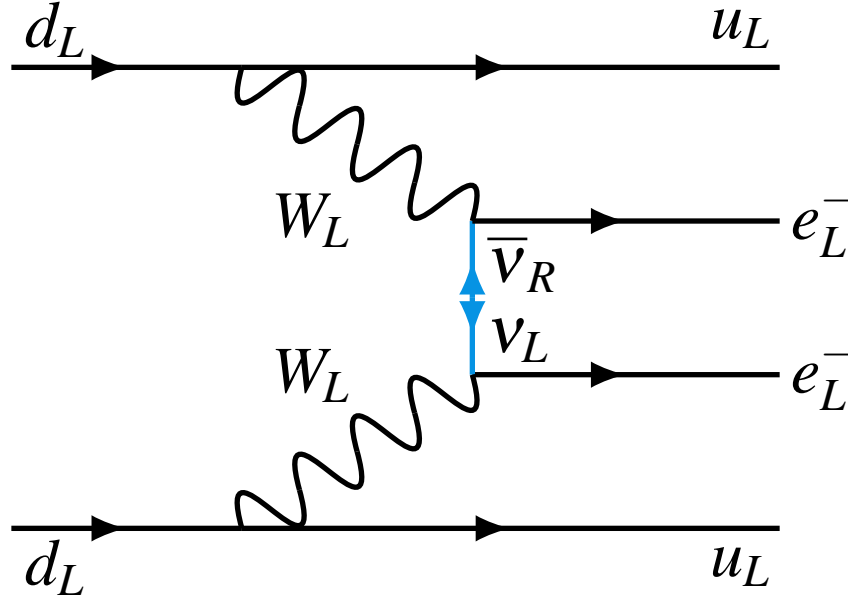


Figure 2.13: Feynman diagram for mass-mechanism neutrinoless double-beta decay.

In the case of a simple Majorana mass mechanism as described in Section 2.4.2 the corresponding double-beta decay process involves the exchange of a light neutrino as an internal line in the Feynman diagram, as seen in Figure 2.13. The decay rate for this process is described by

$$\Gamma_{0\nu\beta\beta} = (t_{0\nu}^{1/2})^{-1} = G^{0\nu} g_A^4 |M^{0\nu}|^2 \langle m_{\beta\beta} \rangle^2, \quad (2.18)$$

where $G^{0\nu} \propto Q_{\beta\beta}^5$ describes the phase space, g_A is the axial coupling constant, $M^{0\nu}$ is the nuclear matrix element (NME) for the process and $\langle m_{\beta\beta} \rangle$ is the effective Majorana neutrino mass. In a model containing 3 light active neutrinos this is defined as

$$\langle m_{\beta\beta} \rangle = \left| \sum_{i=1}^3 U_{ei}^2 m_i \right| = \left| (m_1 c_{12}^2 + m_2 s_{12}^2 e^{i\alpha_{21}}) c_{13}^2 + m_3 s_{13}^2 e^{i(\alpha_{31} - 2\delta_{CP})} \right| \quad (2.19)$$

where U_{ei} is the i th element of the first row of the PMNS matrix parametrised by the matrices in Equation 2.3 and the Majorana phases α_{21} and α_{31} from Equation 2.8, and m_i is the mass of the corresponding eigenstate. From this equation it can be

seen that measurements of the neutrinoless double-beta decay rate are sensitive to the absolute scale of the neutrino mass eigenstates.

One of the largest sources of uncertainty when modelling this decay is the underlying nuclear model used to calculate the nuclear matrix element. Competing models are based on different simplifications of the nuclear dynamics of the decay and consequently produce disparate predictions for the NME for a given isotope, which when propagated to a predicted half-life through Equation 2.18 can produce values up to an order of magnitude apart for a given isotope, shown in Figure 2.14.

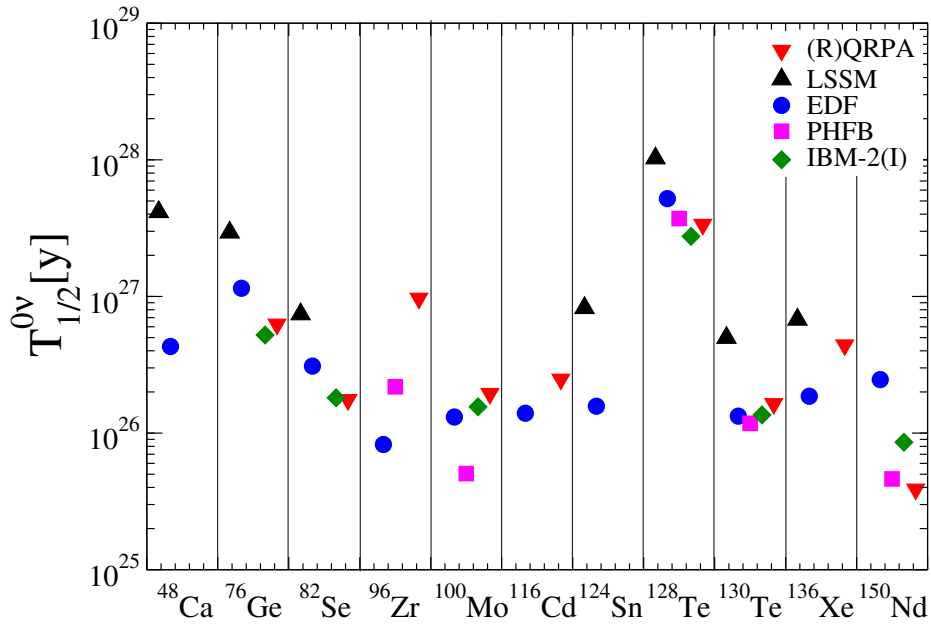


Figure 2.14: Predicted neutrinoless double-beta decay half-lives for a set of isotopes across five nuclear models, with $\langle m_{\beta\beta} \rangle = 50$ meV. Depicted calculations from the (renormalised) quasiparticle random phase approximation (QRPA), large scale shell model (LSSM), the energy density functional method (EDF), the projected Hartree-Fock Bogoliubov method (PHFB), and a version of the interacting boson model (IBM-2I). Each model makes different simplifying assumptions on the nuclear dynamics, and the spread of predictions between these models leads to an uncertainty of up to an order of magnitude on the predicted neutrinoless double-beta decay half life [37].

2.6.3 Majoron Emission

In the search for symmetries that may form the basis of a Grand Unification Theory the quantity $(B - L)$ - the difference between baryon and lepton numbers - has been theorised to be conserved, as in the Georgi–Glashow model based on the $SU(5)$ symmetry group [38]. The combination of such a symmetry and lepton number violating processes results in a universe in which Baryon number is no longer a conserved quantity, a plausible model for the large matter/antimatter asymmetry as observed in the universe today [39]. The spontaneous breaking of this symmetry, in which the neutrino acquires mass, results in associated bosons (termed Majorons).

In a neutrinoless double-beta decay process with associated Majoron emission, the combined electron energy is no longer required by energy conservation to be $Q_{\beta\beta}$ and therefore would take on a continuous range of values. This can be disentangled from Standard Model double-beta decay by analysing the spectral index of the summed energy spectrum.

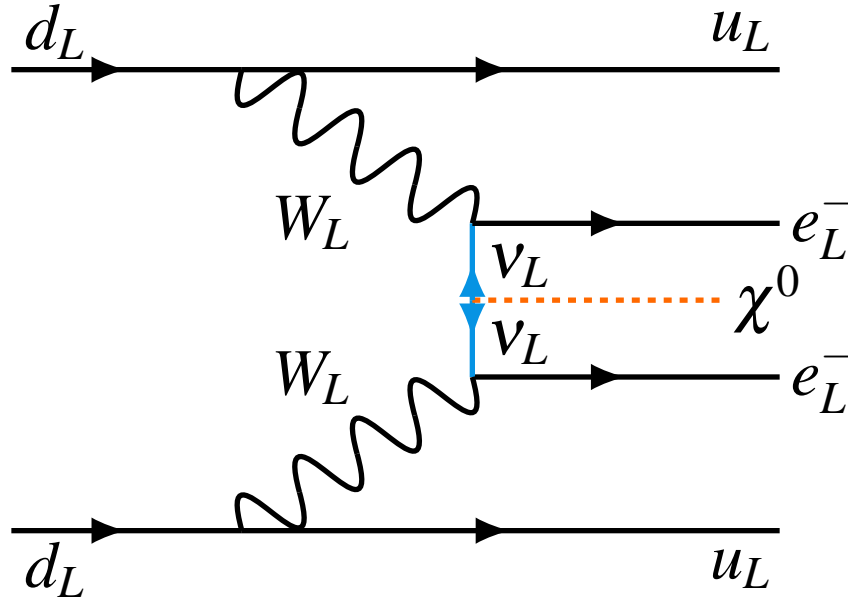


Figure 2.15: Feynman diagram for neutrinoless double-beta decay with associated Majoron (χ^0) emission.

2.6.4 Right Handed Current

Another symmetry is described by the left-right symmetric model which posits a right handed component to the weak force manifesting in right handed weak bosons, either appearing independently from the observed W and Z bosons, or as part of an admixture of states that comprise them [40].

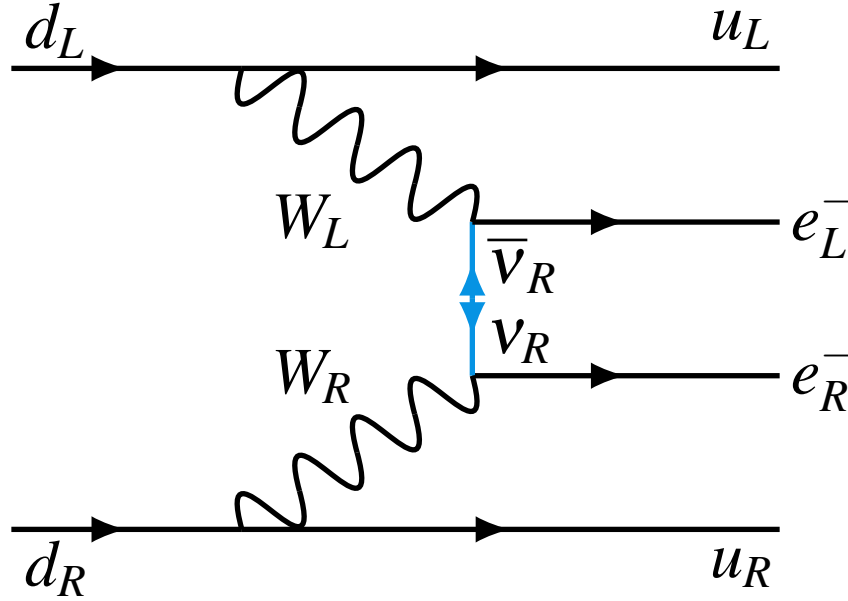


Figure 2.16: Feynman diagram for neutrinoless double-beta decay mediated by a right-handed current.

This model supposes the existence of right handed neutrinos, and so allows for neutrinoless double-beta decay to proceed as in Figure 2.16. The modified kinematics may be observed in both the opening-angle distribution of the emitted electrons and the asymmetric energy partitioning between them, as shown in Figure 2.17.

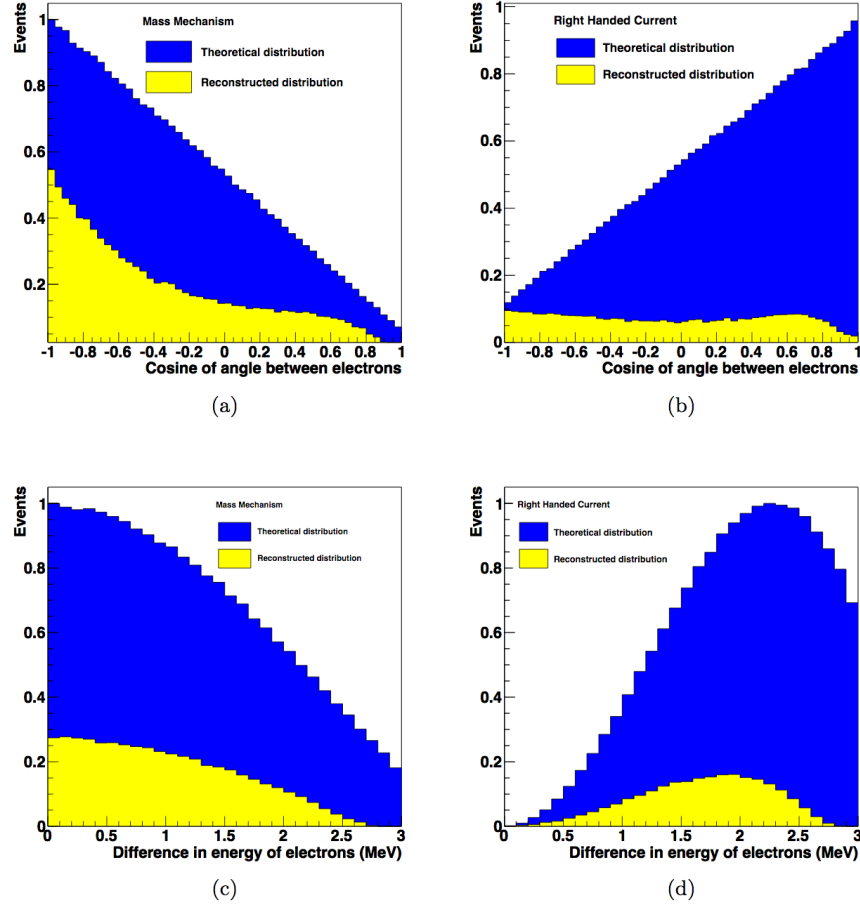


Figure 2.17: Monte-Carlo truth and reconstructed distributions for electron opening angle (top) and electron energy asymmetry (bottom) for mass mechanism (a, c) and pure right-handed current (b, d) models of neutrinoless double-beta decay [40].

2.7 Neutrino Mass Constraints

2.7.1 Neutrinoless Double-Beta Decay

If the neutrino is a Majorana particle, neutrinoless double-beta decay provides the possibility of measuring the absolute scale of neutrino masses through the dependency on the effective Majorana neutrino mass defined in Equation 2.19. The true value of this quantity also constrains the allowed values for the mass of the lightest neutrino mass eigenstate given knowledge of the hierarchy of these states. Figure 2.18 shows the form of these dependencies with the best constraints on $\langle m_{\beta\beta} \rangle$ from the double-beta decay process shown.

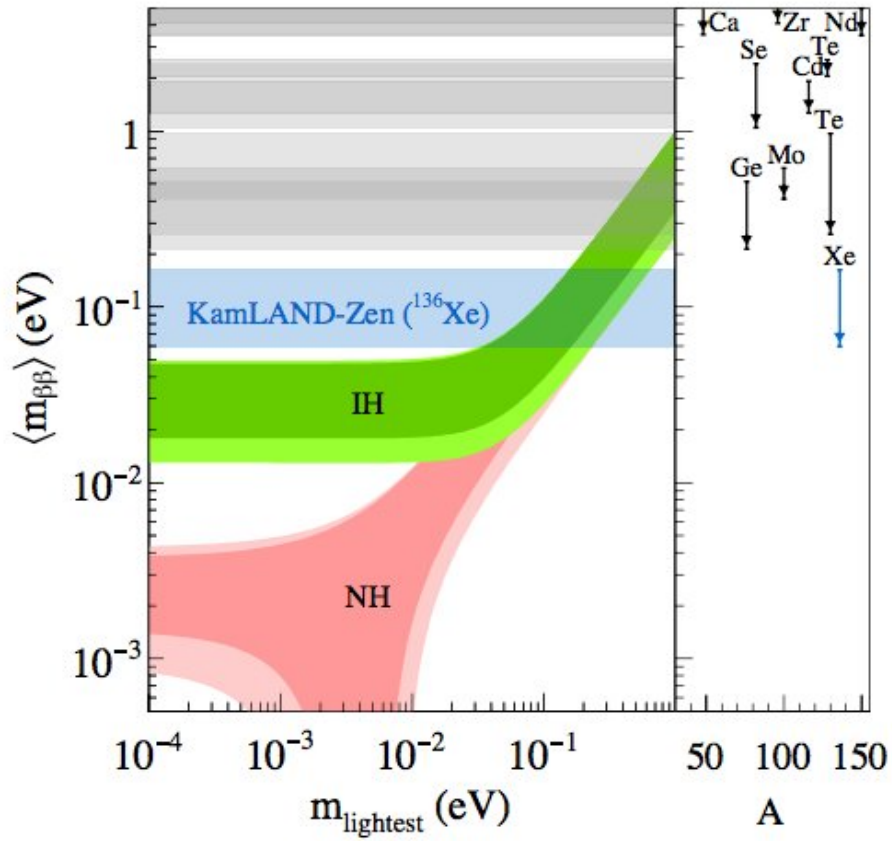


Figure 2.18: $\langle m_{\beta\beta} \rangle$ as a function of the lightest neutrino mass given best fit (dark) and 3σ ranges (light) for neutrino oscillation parameters in the normal hierarchy (NH) and inverted hierarchy (IH). Best limits on $\langle m_{\beta\beta} \rangle$ from measurements of the double-beta decay of the isotopes on the right are shown by the grey bands, with the 90% confidence range on the best limit in blue [41].

In the case of the normal mass hierarchy the Majorana phases α_{ij} could result

in cancellation between the terms of Equation 2.19 resulting in $\langle m_{\beta\beta} \rangle$ arbitrarily close to 0. This is the origin of the unbounded lower region of the plot in Figure 2.18. For this reason even arbitrarily small limits on $\langle m_{\beta\beta} \rangle$ cannot rule out the Majorana mass mechanism in the case of the normal hierarchy, but a low enough limit could show that the Majorana mass mechanism and the inverted hierarchy are mutually exclusive.

The current best limit on $\langle m_{\beta\beta} \rangle$ from double-beta decay comes from the KamLAND-Zen experiment searching for neutrinoless double-beta decay in Xenon. Their limit on the half life of neutrinoless double-beta decay translates to a limit of $\langle m_{\beta\beta} \rangle < 61 - 165$ meV [41] with the range primarily a result of uncertainty on the nuclear matrix element for the decay. As can be seen in Figure 2.18 this almost completely rules out the “quasi-degenerate” region of parameter space where the predictions of the normal and inverted hierarchies overlap.

By improving sensitivity to this parameter by a further order of magnitude, the next generation of neutrinoless double-beta decay experiments hope to either make a discovery in, or entirely rule out, the area of parameter space covered by the inverted hierarchy predictions.

2.7.2 Beta Decay

An analogous limit to that obtained on $\langle m_{\beta\beta} \rangle$ from double-beta decay can be set by observing the endpoint of the electron energy spectrum in Standard Model beta decay. The endpoint is sensitive to an effective electron neutrino mass similar to that in Equation 2.19 defined as [42]:

$$\langle m_{\beta} \rangle^2 = \sum_{i=1}^3 |U_{ei}|^2 m_i^2, \quad (2.20)$$

where the coherent sum ensures that Majorana phases have no impact, making this an observable that is independent of the possible Majorana nature of neutrino mass. This results in a complementarity between measurements of neutrino mass from beta and double-beta decay which allows constraints to be placed on the Majorana phases α_{21} and α_{31} through the relationship [42]:

$$\langle m_{\beta\beta} \rangle < \langle m_{\beta} \rangle < \frac{\langle m_{\beta\beta} \rangle}{\left| \cos(2\theta_{12})|(1 - |U_{e3}|^2) - |U_{e3}|^2 \right|}. \quad (2.21)$$

From Equation 2.21 and the definitions of $\langle m_{\beta\beta} \rangle$ and $\langle m_{\beta} \rangle$ it can be seen that measurements of these two effective masses provide information on both the absolute scale of neutrino mass and the scale of CP violating Majorana phases when combined with measurements of θ_{12} and $|U_{e3}|$ from oscillation experiments.

In beta decay, the larger $\langle m_{\beta} \rangle$ the less energy available to the emitted electron thereby lowering the endpoint of the recorded energy spectrum, as shown in Figure 2.19.

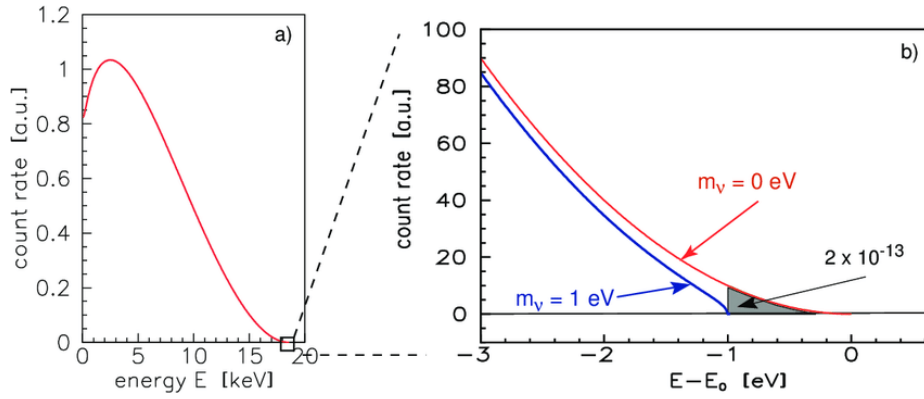


Figure 2.19: The Tritium beta decay energy spectrum (E) with respect to the Q -value (E_0) for a massless (red) and 1 eV neutrino (blue) [42].

As the size of this shift is independent of the Q -value of the decay this effect is of a larger relative size for beta decays of low Q -value. A particularly suitable isotope is Tritium - or Hydrogen-3 - which decays with a Q -value of 18.6 keV and has a simple atomic structure compared to other beta-decay isotopes with low Q -value, such as ^{115}In and ^{187}Rh .

The Troitsk experiment is a Tritium decay experiment attempting to measure the endpoint of the decay spectrum through the use of magnetic fields to collimate and filter beta decay electrons. A tunable spectrometer allowed selection of only those electrons above a set energy - determined by tuning the magnetic field within the collimator - which were then counted by Si detectors. By scanning through values of this set energy and counting the selected electrons an integrated spectrum

was measured. This has produced the best limit on $\langle m_\beta \rangle$ of < 2.2 eV at a 95% confidence level [43].

A larger experiment based on the same detection principle, the Karlsruhe Tritium Neutrino (KATRIN) experiment, is currently undergoing commissioning and hopes to increase sensitivity an order of magnitude to 0.2 eV [42]. A schematic showing the design of this experiment and demonstrating the detection technique is shown in Figure 2.20.

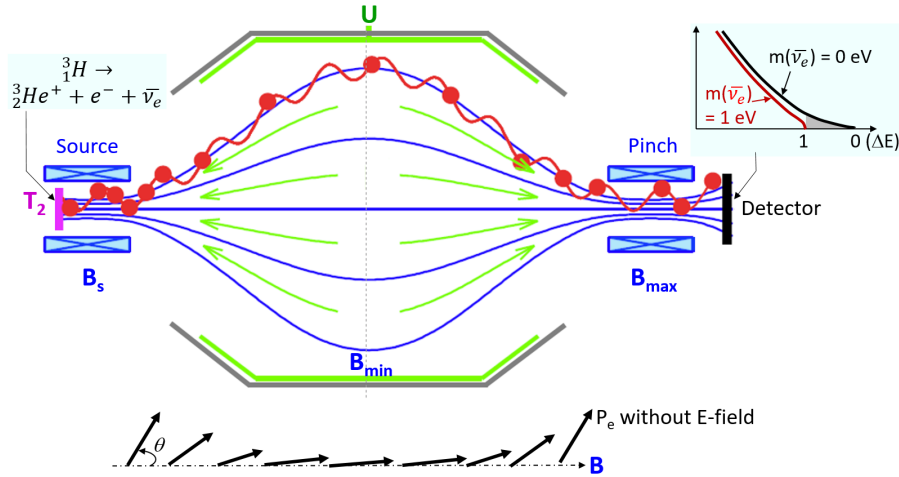


Figure 2.20: KATRIN and Troitsk detection principle. Electrons from the source are guided adiabatically via the magnetic fields (B , blue) through an electromagnetic field (U , green) to the detector where they are counted to measure an integrated energy spectrum. The momentum transfer of a typical electron traversing the magnetic field with no electrostatic field is shown by the black arrows (bottom) while a typical path is shown in red [44].

2.7.3 Cosmological Observations

The possible parameter space can be further narrowed by considering constraints on the mass of the lightest neutrino mass state. The most stringent of constraints on this parameter come from cosmological observations, where massive neutrinos impact upon the large scale structure of the universe. One proposed effect of massive neutrinos is the washing out of structure in the early universe as neutrinos stream from high to low density regions of space, suppressing density fluctuations [45]. Simulations of massive neutrinos, as shown in Figure 2.21, have been used to predict the magnitude of this effect on cosmological observables such as the power spectrum of

the Cosmic Microwave Background (CMB) and the modern day matter distribution of the observable universe.

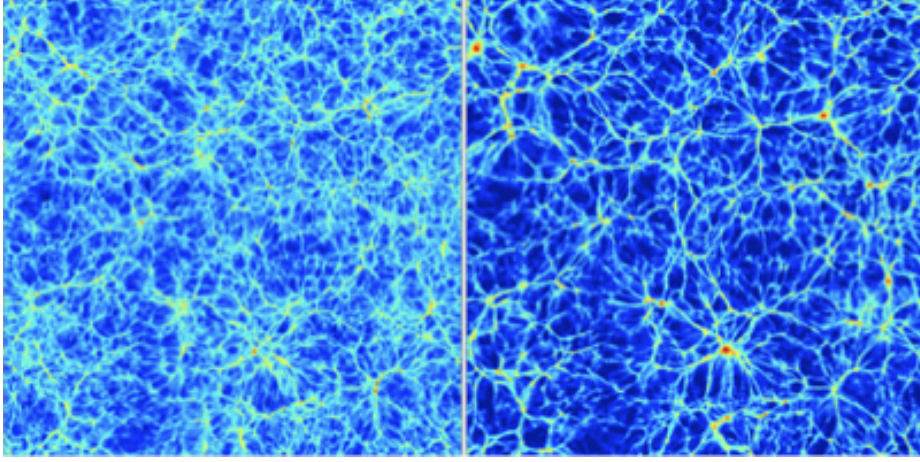


Figure 2.21: Baryon density in a simulated universe with $\Sigma m_\nu = 1.9$ eV (left) and $\Sigma m_\nu = 0$ eV (right) [45].

Such observations limit the summed neutrino mass to values on the order of 0.1 eV depending on the assumed cosmological model [46]. For a flat Λ CDM model the most stringent limit is derived from a combined analysis of the CMB power spectrum, baryon acoustic oscillations observed in the CMB, and a Lyman-alpha map of the modern day universe acting as a proxy for matter density. Such a limit restricts the sum of light active neutrino masses to be < 0.14 eV [47].

2.8 The Standard Model Extension

The Standard Model Extension (SME) is an effective field theory containing the Standard Model and Lorentz violating operators of generalised form. Interactions have been proposed [48] which result in a spontaneous breaking of CPT symmetry - and therefore Lorentz symmetry - in the form of operators that the SME was designed to describe [49].

This theory treats Lorentz symmetry as a global symmetry which is spontaneously broken at an energy below the Planck scale and describes with its Lagrangian all possible interactions between particle fields and background tensors which could arise from such a process. The minimal Standard Model Extension (mSME) constrains the form of these operators to those which preserve the Standard Model symmetry group and particle content, preserve translation invariance, and have dimension less than 5. These constraints ensure existing assumptions regarding causality and energy conservation are retained and that the Standard Model can be viewed as the low energy limit of the SME.

2.8.1 Lorentz Violation in the Neutrino Sector

Several of the Lorentz violating terms in the mSME Lagrangian, if their coefficients are sufficiently large, would give rise to experimentally observable perturbations in studies of neutrino oscillations. These perturbations include spacial or temporal anisotropies, non-standard energy dependence of oscillation parameters, neutrino/anti-neutrino mixing, and explicit CPT violation [50]. The neutrino sector can be parametrised in the SME Hamiltonian as [51]:

$$h_{ab} = |\mathbf{p}| \delta_{ab} + \frac{m_{ab}^2}{2|\mathbf{p}|} + (a_L)_{ab}^{\alpha} \hat{p}_{\alpha} - (c_L)_{ab}^{\alpha\beta} \hat{p}_{\alpha} \hat{p}_{\beta} |\mathbf{p}|, \quad (2.22)$$

where the indices $a, b = e, \mu, \tau$; $|\mathbf{p}| \approx E$ is the magnitude of the neutrino four-momentum; $\hat{p}$ is a unit four-vector in the direction of momentum and c_L , a_L are complex matrices describing the magnitude of Lorentz violating fields.

2.8.1.1 Neutrino Oscillation Probes

Neutrino oscillation experiments have been used to probe several of the components of these matrices. In short baseline experiments no neutrino oscillations are expected from the standard mass-mixing effects, leaving only the Lorentz violating terms of the SME to contribute to a transition probability:

$$P_{\nu_b \rightarrow \nu_a} \approx L^2 \left| (a_L)_{ab}^{\alpha} \hat{p}_{\alpha} - (c_L)_{ab}^{\alpha\beta} \hat{p}_{\alpha} \hat{p}_{\beta} E \right|^2 \quad (a \neq b), \quad (2.23)$$

where L is the baseline and E is the neutrino energy.

These transition probabilities of a neutrino oscillation can depend on the relative orientation of the neutrino with the fields described by c_L and a_L . For this reason variations in transition probabilities as a sinusoidal function of sidereal time are searched for, representing the rotation of the earth and neutrino beam relative to the field. Unlike short baseline experiments, in long baseline experiments this is a second order perturbation on top of expected neutrino oscillations.

In this formulation the last two terms of Equation 2.22 form an effective Lorentz violating Hamiltonian which may be written as [52]

$$\begin{aligned} (h_{eff})_{ab} = & (\mathcal{C})_{ab} + (\mathcal{A}_s)_{ab} \sin \omega_{\oplus} T_{\oplus} + (\mathcal{A}_c)_{ab} \cos \omega_{\oplus} T_{\oplus} \\ & + (\mathcal{B}_s)_{ab} \sin 2\omega_{\oplus} T_{\oplus} + (\mathcal{B}_c)_{ab} \cos 2\omega_{\oplus} T_{\oplus} \end{aligned} \quad (2.24)$$

where $\omega_{\oplus} T_{\oplus} \approx 2\pi/t_{days}$ is the sidereal angle and $(\mathcal{A})$, $(\mathcal{B})$, and $(\mathcal{C})$ can be defined in terms of c_L , a_L and experimental constants [52]. Remarkably, the LSND experiment found a non-zero best fit value for $|(\mathcal{C})_{\bar{e}\mu}|^2$ of order 10^{-19}GeV^2 [53] although this value is disfavoured by data from the MiniBooNE experiment [54].

Further constraints on components of these tensors, in both the short and long baseline limits, have been set by the LSND, MiniBooNE and other neutrino oscillation experiments [55].

2.8.1.2 Oscillation-Free Probes

Lorentz violating operators that act in a flavour blind way may produce effects undetectable in oscillation measurements. These include uniform energy shifts for all neutrino mass eigenstates leaving the differences between them, and therefore oscillation parameters, unchanged. Such an effect would, however, result in a modified neutrino velocity with the following form [56]:

$$v_v = 1 - \frac{|m|^2}{2E^2} + \sum_{d \geq 3, jm} (d-3)E^{d-4}Y_{jm}(\hat{p})[(a_{of}^{(d)})_{jm} - (c_{of}^{(d)})_{jm}], \quad (2.25)$$

for oscillation-free Lorentz violating operators a_{of} and c_{of} of dimension up to d , where Y_{jm} is a spherical harmonic function. Some component values of the operators a and c result in a prediction of superluminal neutrinos, a phenomenon famously thought to have been measured at the OPERA detector [57] before being revised [58].

Other experimental probes into these parameters include measuring the neutrino-anti-neutrino velocity asymmetry described by negating $(a_{of}^{(d)})_{jm}$ in Equation 2.25, and by measuring dispersive and timing patterns in supernova neutrinos where the large distances travelled can serve to amplify small corrections to the neutrino velocity.

It can be noted from Equation 2.25 that for operators of $d = 3$ the correction to the neutrino velocity is zero. These parameters therefore cannot be measured by experiments sensitive to oscillation or velocity of neutrinos only. Their experimental handle comes from weak decays where the operator $a_{of}^{(3)}$ can have measurable effects on, for example, the electron energy spectrum of double-beta decay. It can be shown that in the Standard Model double-beta decay process the anti-neutrinos will have an effective four-momentum [59]

$$\tilde{q}^\alpha = (\omega, \mathbf{q} + \mathbf{a}_{of}^{(3)} - \hat{a}_{of}^{(3)}\hat{\mathbf{q}}), \quad (2.26)$$

where $\hat{a}_{of}^{(3)}$ is the timelike, or isotropic, component of the four-vector $a_{of}^{(3)} =$

$((a_{of}^{(3)})_0, \mathbf{a}_{of}^{(3)})$ divided by $\sqrt{4\pi}$. This results in a modification to the decay rate as:

$$\frac{d\Gamma}{dK} \propto (K^5 + 10K^4 + 40K^3 + 60K^2 + 30K)[(K_0 - K)^5 + 10\mathring{a}_{of}^{(3)}(K_0 - K)^4], \quad (2.27)$$

where K is the summed kinetic energy of the electrons and K_0 is the total available in the decay. The Lorentz violating contribution is represented by the final term of Equation 2.27 and produces a distortion of the spectral shape of the summed electron energy distribution which can be seen in Figure 2.22.

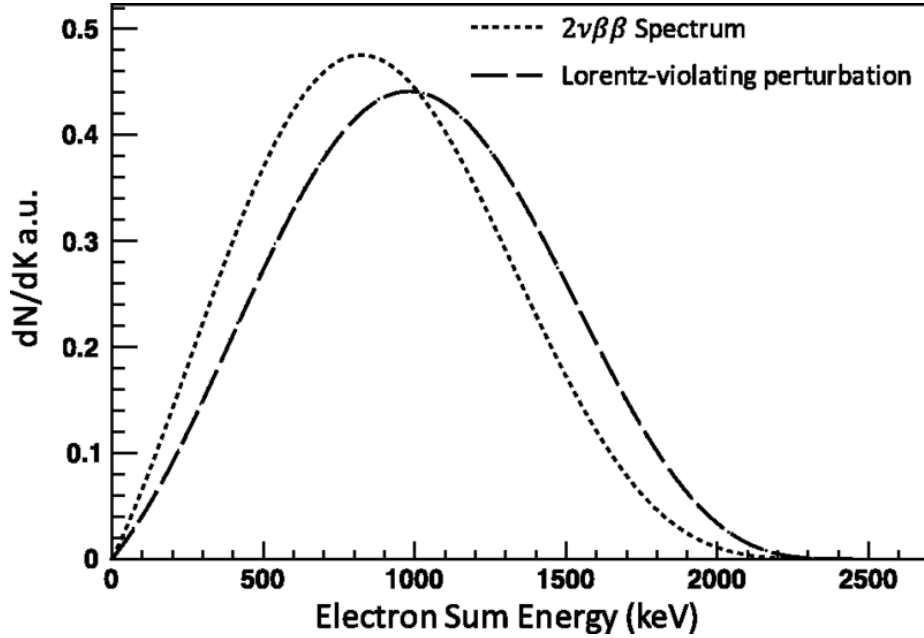


Figure 2.22: A summed electron energy spectrum from double-beta decay of ^{136}Xe , and the shape of the Lorentz Violating perturbation caused by $\mathring{a}_{of}^{(3)}$, where the distributions are scaled so that both are normalised to one [60].

The best two-sided limit on this parameter was produced by the EXO-200 experiment (see Section 2.9.1.3), constraining $-2.65 \times 10^{-5} \text{ GeV} < \mathring{a}_{of}^{(3)} < 7.60 \times 10^{-6} \text{ GeV}$ at a 90% confidence level [60]. An analysis of data from the GERDA experiment (see Section 2.9.1.1) purports to decrease this upper bound by two orders of magnitude, claiming a limit of $\mathring{a}_{of}^{(3)} < 7.4 \times 10^{-8} \text{ GeV}$ at 90% confidence level [61].

2.9 Searches for Double-Beta Decay

2.9.1 Experimental Techniques and Limits

The NEMO-3 and SuperNEMO detectors are exemplary of a class of double-beta decay experiments in which the decaying source isotope is physically separated from the main detector materials. This experimental approach is described in the context of the NEMO-3 and SuperNEMO detectors in Chapter 4. Other, complementary, experimental approaches involve the calorimeter itself containing the source isotope.

The advantages of the latter approach are that it allows for large quantities of source isotope to be present in a relatively compact detector and the consequent possibility of achieving very high energy resolutions due to reduced scattering of the emitted electrons. Furthermore the total containment of the source within the calorimeter results in a high detection efficiency when compared to an external source as in the NEMO-3 and SuperNEMO detectors. The primary advantage of the NEMO-3 and SuperNEMO design is the ability to reconstruct event topologies, which acts as a powerful technique for identifying background events. Topological information on any double-beta decay detection provides further insight into the mechanism mediating the decay, but the price paid for this detector design is a reduced source mass, energy resolution and efficiency.

Current experiments following the integrated source and calorimeter approach broadly rely on one of four technologies:

- Germanium Semiconductor Detectors
- Bolometers
- Liquid or Gaseous Xenon Time Projection Chambers
- Loaded Liquid Scintillators

2.9.1.1 Germanium Semiconductor Detectors

In a semiconductor detector the traversal of a sufficiently energetic charged particle produces electron-hole pairs across the band gap of the semiconductor. The detector

is placed under an electric field resulting in these newly created charge carriers drifting towards the electrodes and producing a measurable electric signal. The number of such pairs, proportional to the energy of the ionising particle, can be calculated from the size of the measured pulse and the known band gap of the semiconductor.

While any semiconductor can be used for such a detector the isotope ^{76}Ge is of particular interest for the construction of these detectors as it is an isotope capable of double-beta decay. Although Germanium semiconductor detectors must be cooled to cryogenic temperatures to reduce thermal noise, they have been widely used in nuclear gamma and x-ray spectroscopy resulting in established industrial processes for their production and purification.

The current best limits from this type of detector come from the Germanium Detector Array (GERDA) experiment. The first phase of this experiment ran between 2011 and 2013 for 21.6 kg·yr exposure, with the subsequent second phase currently taking data having reduced the number of background events to 10^{-3} counts/(keV·kg·yr) - representing less than one expected count in the region of interest over the lifetime of the experiment. Such low backgrounds will allow a sensitivity of 4×10^{25} years with the current best limit from phase 1 and partial phase 2 data, $T_{0\nu}^{1/2} > 5.3 \times 10^{25}$ years [62] corresponding to an effective Majorana mass of 0.15-0.39 eV.

2.9.1.2 Bolometers

Bolometers measure the change of electrical resistance of a material when heated, in the case of double-beta decay, by the emitted electron. ^{130}Te is a double-beta decay isotope which in oxide form, TeO_2 , is a suitable construction material for a bolometer. The current state of the art Tellurium bolometer experiment is the Cryogenic Underground Observatory for Rare Events (CUORE) experiment currently taking data in the Gran Sasso laboratory alongside GERDA-II. Combined data from the two precursor experiments CUORICINO and CUORE0 provided limits for ^{130}Te of $T_{0\nu}^{1/2} > 4 \times 10^{24}$ years[63]. The CUORE experiment expects to achieve a 90% confidence level exclusion sensitivity of 9×10^{25} years after 5 years of data collection, probing the inverted hierarchy region of parameter space.

2.9.1.3 Xenon Time Projection Chambers

Time projection chambers involve the application of an electric field across a detection medium which becomes ionised by incident radiation. These ions then drift to the anode and cathode regions of the detector where the size and timing of the signals can be used to track the original ionising particle. Such an approach suffers from a poorer energy resolution than the bolometric or semiconductor experiments which is compensated for through the tracking ability which, while not as accurate as the NEMO-3/SuperNEMO tracker, allows for significant background event rejection. Using Xenon in a time projection chamber allows this technique to be used to study the double-beta decay of ^{136}Xe but also provides enhanced time resolution capabilities through the detection of Xenon scintillation light.

The Enhanced Xenon Observatory (EXO-200) is a 200 kg liquid Xenon TPC enriched to 80.6% ^{136}Xe producing a limit of $T_{0\nu}^{1/2} > 1 \times 10^{25}$ years corresponding to a neutrino mass of 0.2 - 0.4 eV [64]. A next-generation experiment, nEXO (next-EXO) has been proposed by the collaboration, containing 5000 kg Xenon allowing the sensitivity to $T_{0\nu}^{1/2}$ to be increased to above 10^{27} years [65].

2.9.1.4 Liquid Scintillators

Large volumes of liquid scintillator surrounded by photomultiplier tubes can be loaded with a double-beta decay isotope to allow them to be used as detectors of neutrinoless double-beta decay. While this technique does not have the high energy resolution of bolometric or semiconductor experiments it does allow for high sensitivity to be achieved through the measurement of a large total mass of source isotope. The KamLAND-Zen is such an experiment and as discussed in Section 2.7 has produced the worlds best limit on the effective neutrino Majorana mass of 61-165 meV from a half life limit of $T_{0\nu}^{1/2} > 1.07 \times 10^{26}$ years [41].

Chapter 3

Data Analysis

To test aspects of the Standard Model Extension (see Section 2.8), and to allow future use of the NEMO-3 ^{100}Mo data without access to experiment specific Monte-Carlo models several key distributions, later measured in Chapter 6, are unfolded. The unfolding technique chosen is the Iterative Bayesian Unfolding method as introduced by D’Agostini [66], and this chapter describes the application of this technique. To illustrate the process, in Section 3.2 the procedure is applied to a series of toy models and the performance is investigated.

3.1 Iterative Bayesian Unfolding

To compare the observed distribution of any experimental quantity with a given prediction, effects within the detector that modify the measured distribution of the quantity must be modelled and accounted for. These effects arise from finite detector resolution and acceptance, noise, and systematic and statistical error. Producing an accurate comparison to theory can therefore be achieved in two ways - by applying these corrections to the predicted distribution or by undoing these effects in the measured distribution, a technique known as unfolding.

The primary advantage of producing an unfolded distribution of a measured experimental quantity is that subsequent comparisons to the observed spectra can be made without knowledge of the specific characteristics of the detector or experiment that made the observations. It provides a measurement of a distribution affected only by the underlying physics of the process and can therefore be used to constrain

physical parameters.

Iterative Bayesian Unfolding is an unfolding method first proposed by D'Agostini in 1995 involving the repeated application of Bayes' theorem to unfold an observed distribution of a measured quantity [66].

Bayes' theorem states:

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}, \quad (3.1)$$

where A and B are events and $P(B) \neq 0$. To apply this to binned distributions we rename the variables $A \mapsto C_i$ and $B \mapsto E_j$ where the C_i are causes, corresponding to the existence of $n(C_i)$ events in the underlying true distribution bin i , and the E_j are effects, corresponding to the existence of $n(E_j)$ events in the measured distribution bin j . Thus Bayes' Theorem states:

$$P(C_i|E_j) = \frac{P(E_j|C_i)P_0(C_i)}{\sum_{l=1}^{n_c} P(E_j|C_l)P_0(C_l)}, \quad (3.2)$$

where $P_0(C_i)$ is the initial probability of C_i and n_c is the total number of causes C , defined when choosing the number of bins for the true underlying distribution.

3.1.1 Calculating the Response Matrix and Efficiencies

Through the use of Monte Carlo simulation of events with an arbitrary distribution $P_0(C)$, observation of the bins these events are measured in when passed through analysis cuts and a detector simulation can define the matrix $P(E_j|C_i)$ and therefore, using Equation 3.2, the matrix of values $P(C_i|E_j)$. This matrix is known as a response or migration matrix, and defines the migration of events between bins from the generated distribution after cuts and reconstruction to the measured distribution.

From the Monte-Carlo sample, one can also calculate an estimate for the efficiency of measurement for each bin - that is to say the probability that a cause C_i produces an effect E_j at all. This is defined by:

$$\hat{\epsilon}_i = \frac{n(C_i)|_{observed}}{n(C_i)}, \quad (3.3)$$

which is the ratio of events in bin i of the Monte Carlo truth distribution before and after events have been dropped due to detection inefficiencies or event selection cuts.

3.1.1.1 Terminology

The following set of definitions are used to describe elements of the unfolding procedure in this chapter:

- Truth distribution: The inherent distribution of a quantity of interest,
- Training/Pseudodata truth: The underlying distribution used to generate Monte-Carlo/pseudodata for one of the following toy experiments,
- Reconstructed distribution: The distribution of a quantity of interest as measured by a detector,
- Response matrix $P(C_i|E_j)$: the unitary matrix describing migration of events between a truth distribution and a reconstructed distribution,
- Unfolding matrix M_{ij} : the response matrix after correcting for the efficiency of the detector,
- Detector response: The measurement error, including detection inefficiency, of a detector across the distribution of a quantity of interest, and
- Detector acceptance: The detection efficiency of a detector across the distribution of a quantity of interest.

3.1.2 Unfolding Data

Once the $\hat{\epsilon}_i$ and response matrix have been produced from Monte Carlo samples and a detector simulation, one has all the tools to unfold a distribution measured in experimental data. The partially unfolded distribution without efficiency correction, is then given as:

$$n(C_i)|_{observed} = \sum_{j=1}^{n_E} n(E_j)P(C_i|E_j), \quad (3.4)$$

where the $n(C_i)|_{observed}$ correspond to the unfolded histogram bins (without efficiency correction) and the $n(E_j)$ correspond to the measured histogram bins, the

number of which is defined to be n_E . While the initial prior distribution $P_0(C)$ can be arbitrarily chosen, its shape will impact the elements of the response matrix, and thus the final unfolded distribution. The unfolded distribution will therefore lie between the prior $P_0(C)$ and the true distribution. A typical choice of prior describes the predicted shape of the final distribution based on pre-existing beliefs, for example a Monte Carlo truth distribution.

To reduce the effect of bias towards the prior distribution an iterative approach is taken, whereby the unfolded distribution $n(C)|_{observed}$, normalised to unity, is substituted as the prior $P_0(C)$ in Equation 3.2 to produce a new prediction. This process is repeated until the differences between the latest unfolded $n(C)|_{observed}$ and the previous prior $P_0(C)$ become sufficiently small - as defined by the experimenter. The stopping criterion in the case of the NEMO-3 $2\nu\beta\beta$ unfolding is described in Section 6.5.1.

After convergence a fully unfolded histogram is constructed from the $n(C)|_{observed}$ and bin efficiencies by combining 3.4 and 3.3:

$$\hat{n}(C_i) = \sum_{j=1}^{n_E} M_{ij} n(E_j), \quad (3.5)$$

where

$$M_{ij} = \frac{1}{\hat{\epsilon}_i} P(C_i|E_j) = \frac{P(E_j|C_i)P_0(C_i)}{\hat{\epsilon}_i \sum_{l=1}^{n_c} P(E_j|C_l)P_0(C_l)} \quad (3.6)$$

is known as the unfolding matrix.

3.1.3 Error Propagation

D'Agostini calculates the covariance matrix of the final distribution to be defined by two inputs: uncertainty on the true number of events $\hat{N}_{true}$, and uncertainty of the response matrix due to finite Monte-Carlo statistics used in its generation. These are respectively parametrised by:

$$V(n_E) = \sum_{j=1}^{n_E} M_{kj} M_{lj} N(E_j) \left(1 - \frac{n(E_j)}{\hat{N}_{true}} \right) - \sum_{\substack{i,j=1 \\ i \neq j}}^{n_E} M_{ki} M_{lj} N(E_j) \frac{n(E_i) n(E_j)}{\hat{N}_{true}}, \quad (3.7)$$

for n_E effects - defined by the number of bins in the measured histogram - and:

$$V(\mathbf{M}) = \sum_{i,j=1}^{n_E} n(E_i)n(E_j)\text{Cov}(M_{ki},M_{lj}). \quad (3.8)$$

In the original formulation of the algorithm, the sum of these two components formed the final covariance matrix of the unfolded distribution, with the diagonal elements defining the histogram errors [66].

This formulation assumes the independence of the response matrix from the measured distribution n_{E_j} . While this is a valid assumption for the first iteration of the process, the use of an iteration's output distribution as the prior for the subsequent iteration breaks this independence. Consequently, an updated propagation formula was proposed by Tim Adye [67], where the propagation matrix:

$$\frac{\partial \hat{n}(C_i)}{\partial \hat{n}(E_j)} = M_{ij} + \frac{\hat{n}(C_i)\partial n_0(C_i)}{n_0(C_i)\partial n(E_j)} - \sum_{k,l=1}^{n_E,n_C} \frac{n(E_k)\hat{e}_l}{n_0(C_l)} M_{ik} M_{lk} \frac{\partial n_0(C_l)}{\partial n(E_j)}, \quad (3.9)$$

allows for the definition of the covariance matrix on the unfolded distribution as:

$$V(\hat{n}(C_k), \hat{n}(C_l)) = \sum_{i,j=1}^{n_E} \frac{\partial \hat{n}(C_k)}{\partial n(E_i)} V(n(E_i), n(E_j)) \frac{\partial \hat{n}(C_l)}{\partial n(E_j)}, \quad (3.10)$$

where $V(n(E_i), n(E_j))$ is the covariance matrix of the measured distribution. Assuming only Poisson errors on each bin, as in a typical counting experiment, this is the diagonal matrix $n(E_i)\delta_{ij}$.

3.2 Toy Model Examples

3.2.1 A Lossless Detector

This technique can be demonstrated on a toy model with measured and true distributions of an arbitrary quantity of interest. 1,000,000 events with a value for this quantity (x) in the interval $[-1,1]$ were generated with a probability $\cos^2(x)$ to form a Monte Carlo dataset used to create the response matrix described above.

This dataset was convolved with a function describing the effects of a hypothetical detector encompassing terms accounting for systematic errors producing measured quantities parametrised by $g(x,0.2) + g(0.5,0.2)$ where $g(x,\sigma)$ denotes sampling from a Gaussian with mean x and variance σ . Here a systematic linear shift of $+0.5$ and a detector resolution of 0.2 in the quantity of interest have been chosen. How this transforms the Monte Carlo distribution of the quantity of interest is shown in Figure 3.1.

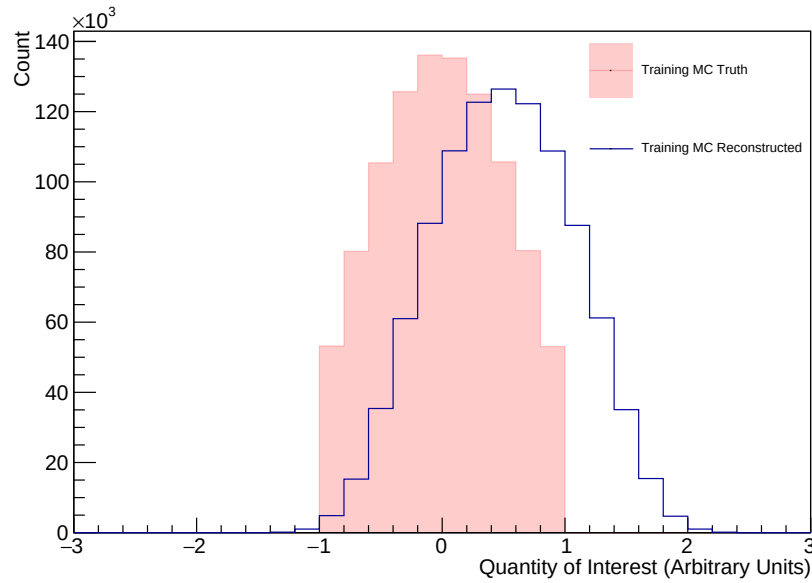


Figure 3.1: Lossless detector response: a distribution generated according to a training truth distribution (filled) is reconstructed as a shifted distribution (line) due to detector effects.

In this example the detector acceptance is chosen to be 100% representing a lossless detector that measures every event taking place, resulting in an unfolding

matrix $P(E_j|C_i)$ that satisfies the condition

$$\sum_j P(E_j|C_i) = 1, \quad (3.11)$$

i.e. each column sums to unity.

An independent set of 1,000,000 pseudodata events was generated with the same distribution as the Monte Carlo set and unfolded using this response matrix. One iteration of the unfolding algorithm recovers the pseudodata truth distribution, as shown in Figure 3.2, the immediate convergence being a consequence of the Monte-Carlo truth distribution (used as the prior $P_0(C)$ from Equation 3.4) being identical to the pseudodata truth distribution. If an alternative prior is used the unfolded distribution is biased towards the prior. This bias is reduced on each subsequent iteration of the unfolding procedure but ultimately represents a source of systematic error on the final distribution.

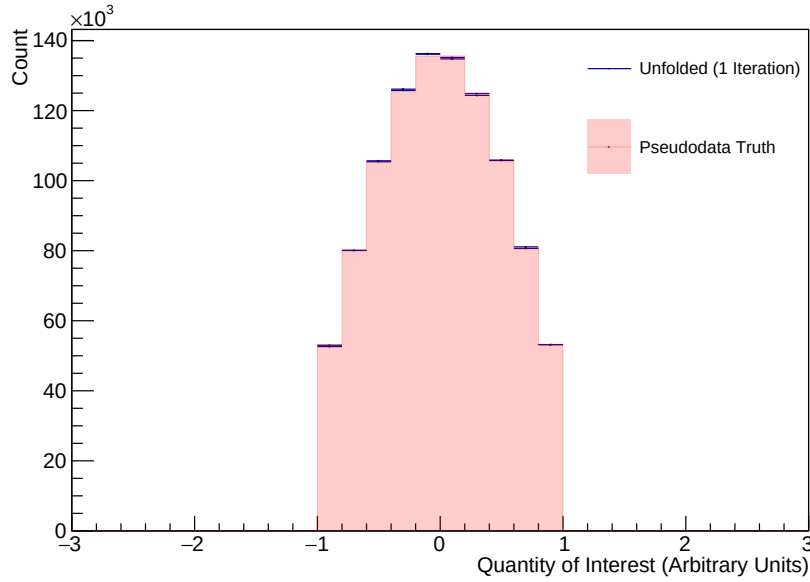


Figure 3.2: Unfolded pseudodata after one iteration - convergent when MC and pseudodata have the same underlying truth distribution.

3.2.2 A Realistic Detector

3.2.2.1 Pseudodata and Monte-Carlo Generation

To demonstrate the process in a generalisable case, a more realistic toy model can be constructed by defining two distinct distributions of the quantity of interest, namely:

- A distribution for building the response matrix (Monte-Carlo truth distribution),
- A distribution for generating pseudodata (pseudodata truth distribution).

Distributions in a quantity of interest generated according to these distributions were convolved with a function describing the effects of a more realistic hypothetical detector, encompassing terms accounting for systematic, statistical and efficiency corrections, producing Monte-Carlo reconstructions and pseudodata. Table 3.1 describes the functions and distributions chosen for this example. 1,000,000 Monte-Carlo and pseudodata events were generated.

	Parametrisation
Quantity of Interest	$x \in [-1, 1]$
Monte-Carlo Truth Distribution	$\cos^2(x)$
Pseudodata Truth Distribution	$1 - x^2$
Detector Acceptance	$\propto (1/2 + x/3)$
Measurement Errors	$g(x, 0.2) + g(0.5, 0.2)$

Table 3.1: Parametrisation of unfolding toy model, where $g(x, \sigma)$ denotes sampling from a Gaussian with mean x and variance σ .

The quantity of interest for this toy model was measured across the interval $[-3, 3]$ with Monte-Carlo and pseudodata truth distributions chosen to have similar but distinct shapes, analogous to a spectral shift in the double-beta decay summed energy distribution between data and Standard Model Monte-Carlo. The shape of the detector acceptance was chosen to produce an asymmetry in the measured pseudodata mimicking a non-linear detector response in the quantity of interest, and the

measurement error parametrisation terms simulate a detector resolution of 0.2 and a systematic linear error of $+0.5$. A visualisation of this detector response on pseudodata uniformly generated on $[-1,1]$ can be seen in Figure 3.3, with bin widths chosen to match the detector resolution.

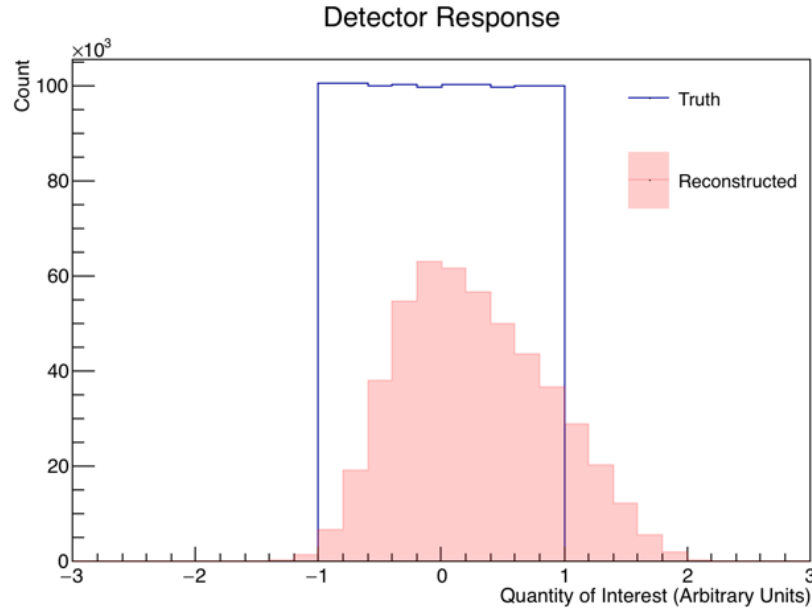


Figure 3.3: Hypothetical detector response on uniform pseudodata.

Monte-Carlo truth distributions of the quantity of interest, as shown in Figure 3.4 were compared before and after applying the detector acceptance simulation. The ratio of these histograms forms the vector of bin efficiencies presented in Figure 3.5.

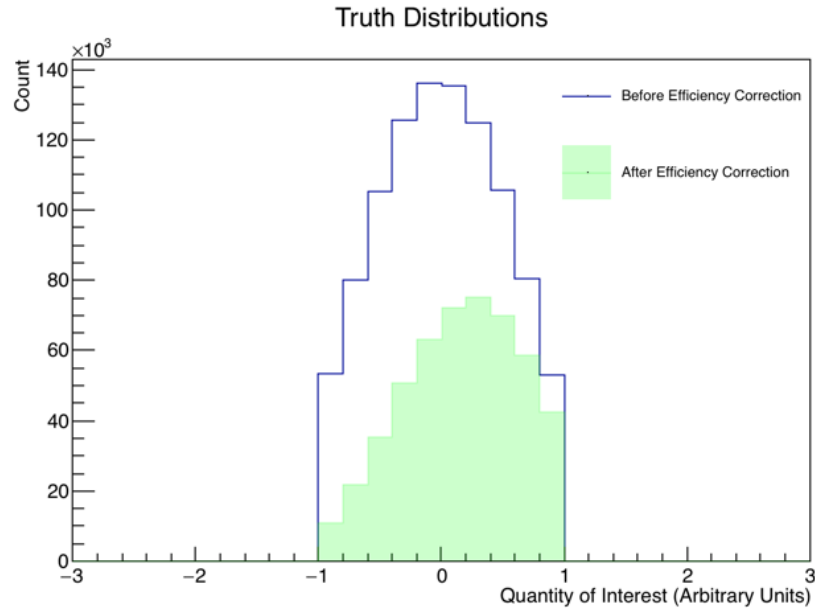


Figure 3.4: Truth distributions before and after efficiency correction.

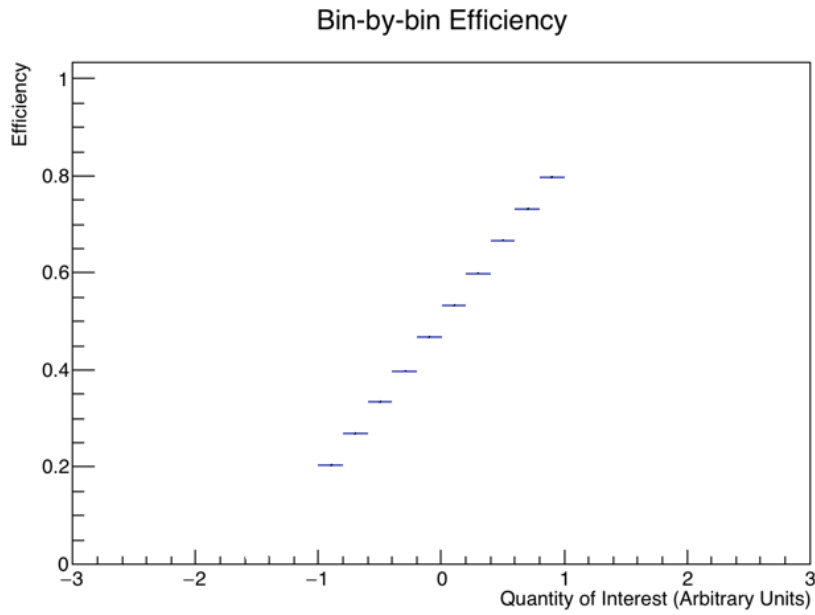


Figure 3.5: Efficiency per bin.

Monte-Carlo truth and reconstructed distributions of the quantity of interest, as shown in Figure 3.6, were compared after applying the detector acceptance simulation. The probability of migration between bins of these events due to measurement error forms the response matrix, presented in Figure 3.7. The choice of a bin

width comparable with the detector resolution results in an approximately diagonal response matrix, balancing the simplification of the unfolding process against maximising the granularity of the resulting distribution.

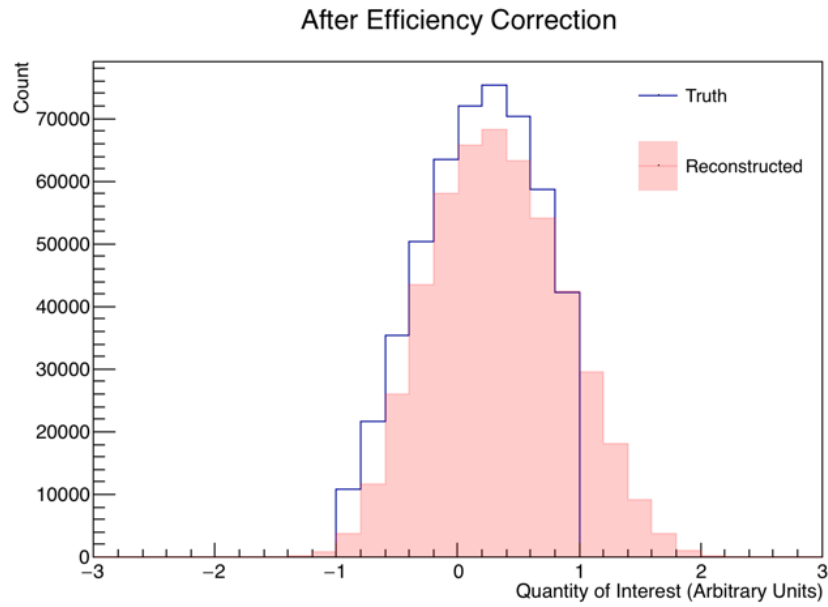


Figure 3.6: Truth and reconstructed distributions after efficiency correction of the Monte-Carlo truth distribution.

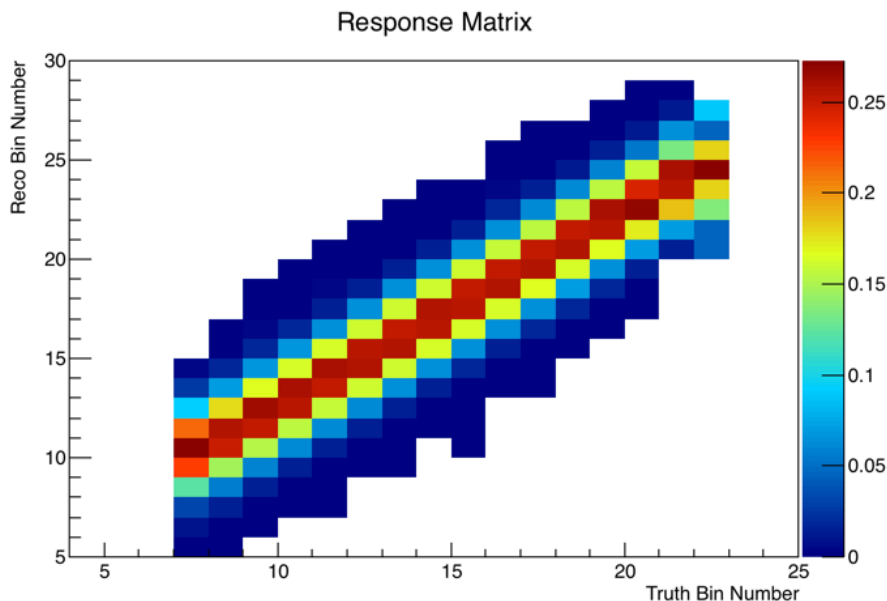


Figure 3.7: Response matrix, where off-diagonal elements correspond to measurement errors.

3.2.2.2 Unfolding Pseudodata

The response matrix and vector of efficiencies were then used to unfold pseudodata generated with a $1 - x^2$ distribution over the range $[-1,1]$. The impact of simulated detector acceptance, bias and resolution on this distribution can be seen in Figure 3.8.

Figure 3.9 shows the unfolded distribution after a single iteration of the Bayesian unfolding algorithm, and after two subsequent iterations. It can be seen that from the first iteration the unfolded spectrum lies in between the pseudodata and Monte-Carlo truth spectra, with the subsequent two iterations showing convergence towards the pseudodata truth, as the bias from the broader $\cos^2$ distribution used to construct the response matrix diminishes.

After 100 iterations the $1 - x^2$ shape of the true distribution of the pseudodata has been largely recovered by the unfolding process as shown in Figure 3.10. When considering the systematic error introduced by the unfolding procedure, described in Section 6.5.1, such a large number of iterations will induce a significant uncertainty on the output, and so the process is typically truncated before this point.

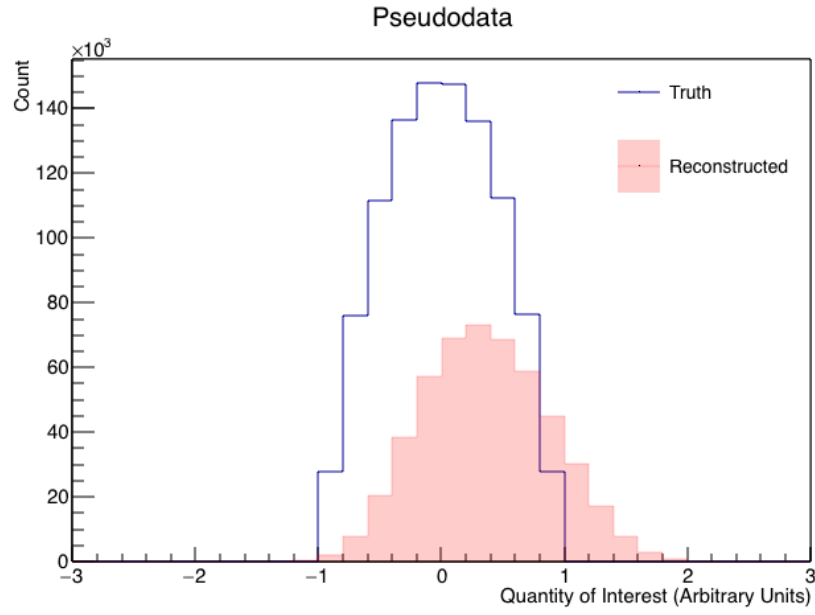


Figure 3.8: True and reconstructed distributions of toy model pseudodata.

In practice the optimum number of iterations may be tuned through Monte-Carlo studies, where a truncation of the process to only a few iterations regularises the final distribution. D’Agostini has noted that further iteration beyond this point often has the effect of amplification of noise within both the measured and Monte-Carlo truth input distributions [68].

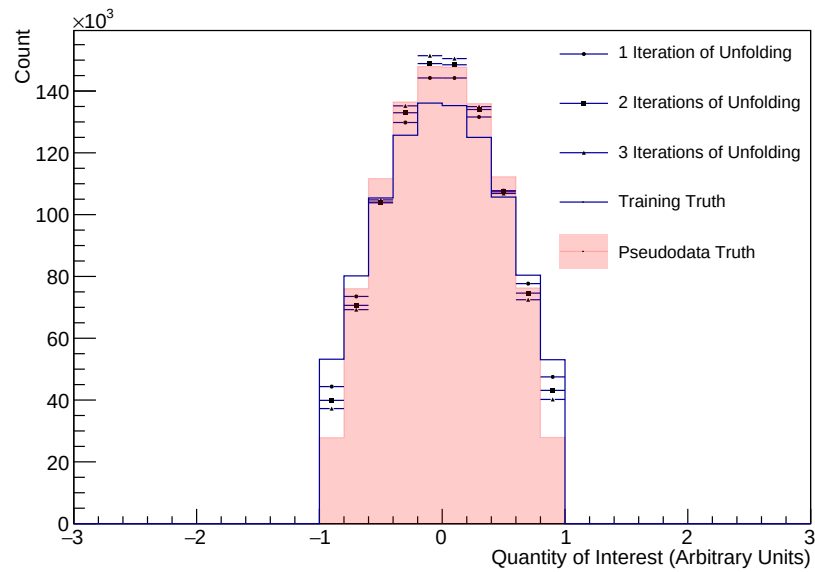


Figure 3.9: 1, 2 and 3 iterations of the Bayesian unfolding algorithm, propagated statistical errors only.

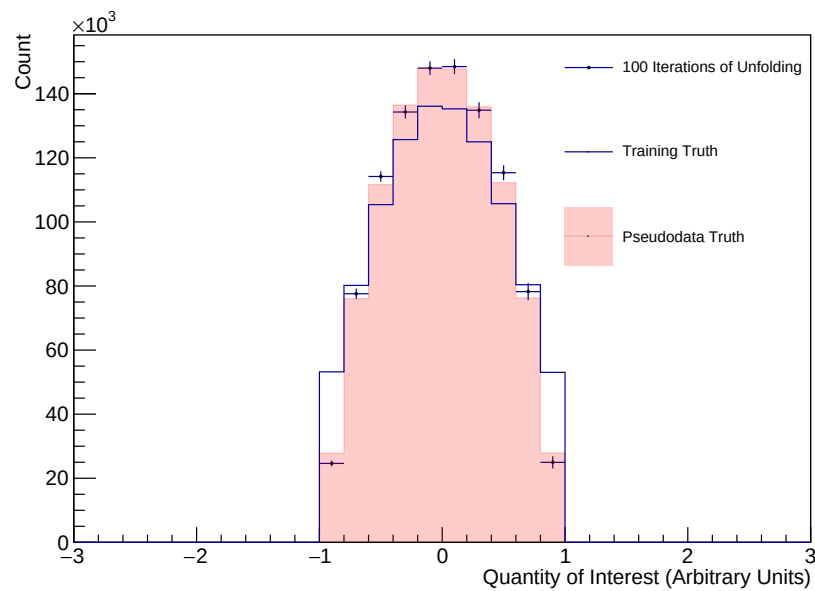


Figure 3.10: 100 iterations of the Bayesian unfolding algorithm, propagated statistical errors only.

Chapter 4

The NEMO-3 and SuperNEMO Detectors

In this chapter the detectors which produced the double-beta decay data analysed in the final two chapters of this work are introduced. The NEMO-3 and the SuperNEMO detectors are both designed to use the same detection technique for measuring double-beta decay parameters: the tracking of each emitted electron through an ionisation chamber containing wire anode and cathodes, and the subsequent energy measurement of the particle through plastic scintillator based calorimeters surrounding the ionisation chamber. The NEMO-3 detector demonstrated the effectiveness of this technique, and the SuperNEMO demonstrator module was built to refine it through improvements to electron acceptance, energy resolution, spatial resolution, and radiopurity.

4.1 Detection Principle

4.1.1 Drift Chamber Trackers

The NEMO-3 and SuperNEMO experiments are both comprised of two major detector components, the tracker and the calorimeter. Each tracker is a wire drift chamber filled primarily with Helium. When a charged particle traverses the tracking detector electromagnetic interactions with the tracking gas atoms produce He^+/e^- ion/electron pairs along the path of the particle if it has energy greater than the ionisation potential of the tracker gas. By producing a static electric field within

the detector, through the use of wires at voltage within the tracking gas, these pairs of charged particles will drift apart towards the anodes and cathodes of the field.

As the electrons drift towards the anode they gain energy from the electric field and begin to ionise additional atoms within the tracking gas creating an exponentially growing avalanche of electron-ion pairs drifting to the poles of the electric field. To ensure the field is strong enough to produce this amplification effect a thin anodic wire operating at a high voltage is used. When the avalanche nears the anode wire a measurable current in the wire is induced producing the tracker signals that are measured by the electronics of the experiments.

For low field strengths the magnitude of this current is proportional to the number of ions created by the original charged particle traversing the detector but as the field is increased a phenomenon known as burnout can occur, whereby the region of gas surrounding the anode wire is saturated by the growing avalanche as the newly created ions decrease the field strength in the region and thus suppress further avalanching. In this regime the magnitude of the recorded current is independent of the energy of the original ionising particle and it is in this mode that the NEMO-3 and SuperNEMO trackers operate, as it provides a robust signal with a significant gain across a wide range of incident particle energies. The relationship between applied voltage and charge collected for a typical Geiger counter, and its evolution due to ageing, is shown in Figure 4.1.

The time the first avalanche takes to reach the anode wire is dependent on the electric field in the cell and the distance at which the initial ionising particle passed. In the NEMO-3 and SuperNEMO experiments this drift time is measured as a timing offset from the calorimeter trigger signal and the electric field is shaped using grounded wires within each tracking cell.

As electrons and ions recombine in the avalanche, particularly in the region of strong field close to the anode wire, de-excitation and recombination photons are emitted isotropically and can themselves ionise tracking gas atoms after travelling some way through the detector causing secondary avalanches along the length of the entire anode wire as shown in Figure 4.2. When one side of this cascade of

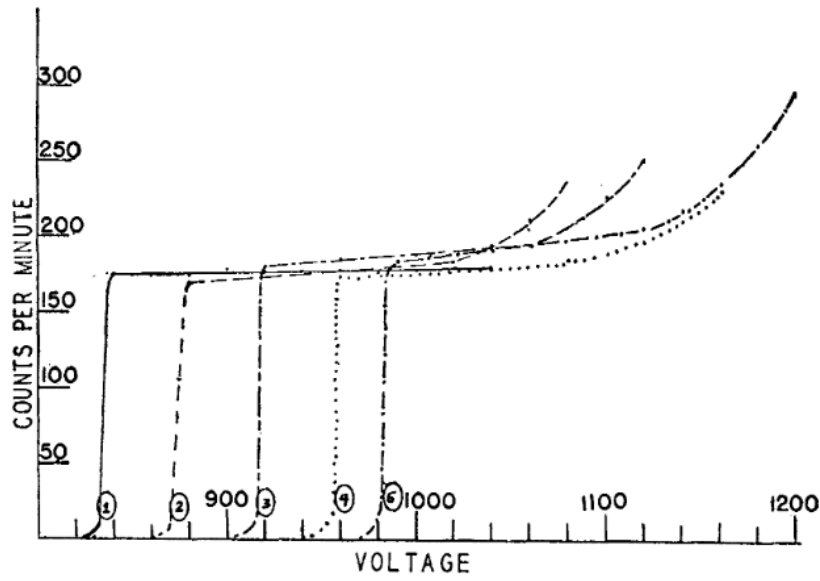


Figure 4.1: Geiger plateau of a typical counter in an argon and alcohol environment. (1) is the response before high voltage operation and (2)-(5) demonstrate plateau shifting due to ageing [69].

avalanches reaches the end of the wire, having caused burnout in the gas close to the anode, the avalanches on this front no longer self propagate and the amount of current being generated in the anode wire drops by half; when both ends have been reached and the region near to the wire is entirely burnt out the signal ceases. By the timing difference these two events the position along the wire of the original cascade can be deduced.

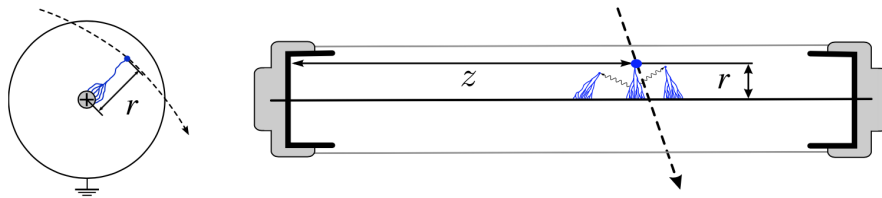


Figure 4.2: Avalanche generation and propagation within a tracker cell (blue), with an associated measured drift radius (r) and longitudinal distance (z) in a cylindrical coordinate system coaxial with the cell [14].

4.1.2 Scintillator Calorimeters

The NEMO-3 and SuperNEMO experiments make use of plastic scintillator blocks coupled to photomultiplier tubes to measure the energy of detected particles. Electrons entering a scintillator block slow down and eventually stop within the block due to electromagnetic interactions with the block's molecules, leaving those molecules in an excited state. As these molecules de-excite, they release some of the energy in the form of optical photons, and the total yield of these photons is measured by the voltage output from the coupled photomultiplier tube. From this signal the time and energy of the initial particle can be calculated.

4.2 The NEMO-3 Detector

4.2.1 Geometry

The NEMO-3 detector was arranged in a cylindrical geometry divided into 20 sectors, designed to be identical in all aspects but the isotope contained within the source foil. The layout of the detector can be seen in Figure 4.3. In total seven double-beta decay isotopes were studied across the 20 sectors, mounted within vertical foils arranged in a ring within the centre of the tracking detector. One sector contained only inert copper for calibration and background studies. By ensuring an identical detector geometry across sectors multiple isotopes could be studied with minimal variation in background and systematic error. The reference for the following section is the NEMO-3 technical design report [70].

4.2.2 Isotope Selection

The choice of isotopes used in the detector was strongly constrained by two considerations. Firstly, a natural isotopic abundance of $>2\%$ was required to ensure enrichment was practical. Secondly, the transition energy of the $0\nu\beta\beta$ decay was required to be above 2.615 MeV to ensure the gamma ray from the decay of ^{208}Tl did not produce background within the region of interest. Five isotopes met these criteria, ^{116}Cd , ^{82}Se , ^{100}Mo , ^{96}Zr and ^{150}Nd . Two further isotopes not meeting this criteria were also included: ^{48}Ca , which has a low isotopic abundance of 0.187% but a high transition energy (4.272 MeV) ensuring any potential signal would lie

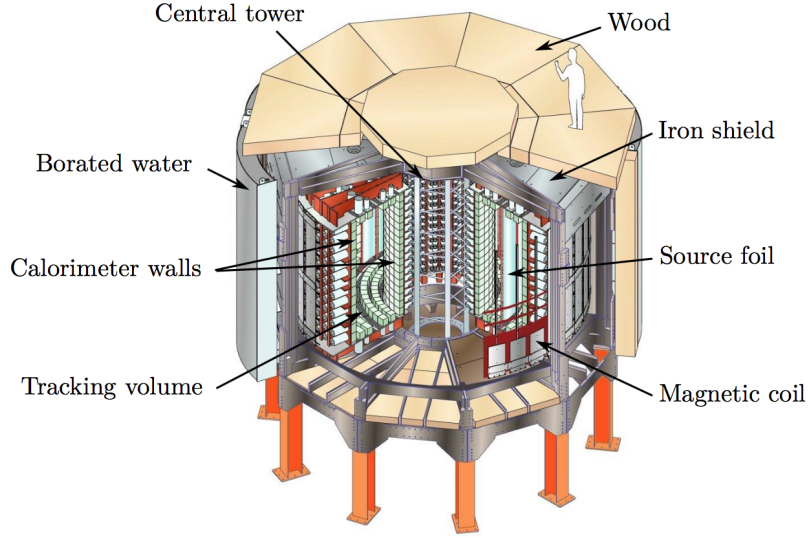


Figure 4.3: Cut-away view of the NEMO-3 detector with key components labelled [70].

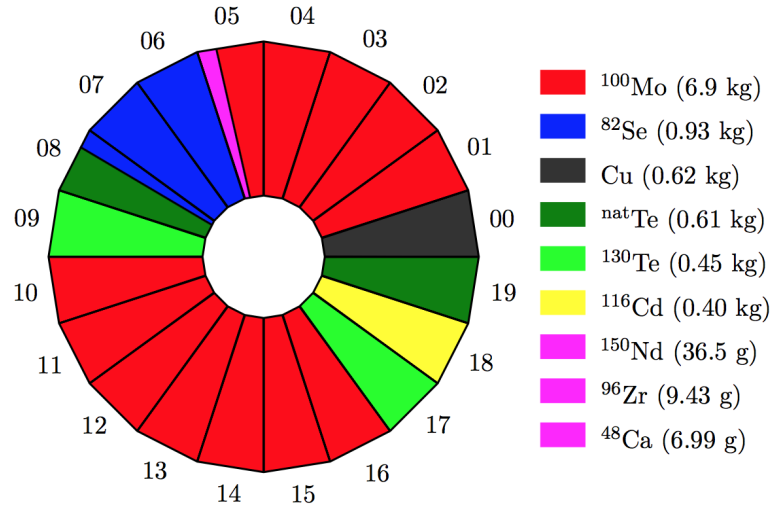


Figure 4.4: Double-beta decay isotopes within the NEMO-3 sectors, with their locations and approximate total masses [70].

above most expected backgrounds; and ^{130}Te , with a low transition energy (2.5289 MeV) but which had previous $2\nu\beta\beta$ half-life measurements in tension. Of these isotopes, ^{100}Mo proved to be the most readily available and consequently made up 6.9 kg of the 10 kg capacity of the detector.

4.2.3 Source Foils

Each sector contained seven strips approximately 2480 x 63 mm in size containing the source isotopes. The strips were produced in one of two formats, metallic or composite. While metallic strips contained the source isotope in a crystalline form the composite strips comprised a mixture of the source in powdered form and organic glue, sandwiched between sheets of mylar film for mechanical rigidity. For ^{100}Mo , 4396 g of the source was in the form of composite foils, with 2519 g in metallic foils. Monte Carlo studies showed that a foil surface density of less than $60 \text{ mg}\cdot\text{cm}^{-2}$ improved energy resolution, resulting in foils being produced with a thickness of $<60 \mu\text{m}$ for metallic and $<300 \mu\text{m}$ for composite foils.

4.2.4 Tracker

The tracking detector was comprised of 6180 drift cells operating in Geiger mode comprising a total of 39,820 wires vertically strung in concentric layers on both sides of the source foil. Nine layers of drift cells were situated on each side of the foil in the geometry shown in Figure 4.6. Each drift cell was an octagonal arrangement, roughly 3 cm in diameter, of eight ground wires surrounding a central anodic wire. Extra ground wires were added between adjacent layers to minimise electrostatic cross talk between cells. At the top and bottom of each of these cells was a hollow copper cylinder, coaxial with the anode wire, functioning as the cathodes (at 0 V) for each cell. The operating voltage for the anode wires was close to +1620 V, creating a roughly cylindrical electric field typically producing six secondary ionisation electrons in the surrounding gas per primary ionising electron. The layout of these cells was optimised for track reconstruction through Monte Carlo simulations - the layers close to the source provide good vertex reconstruction with the rest providing trajectory and curvature measurements. Large gaps between layers were required to facilitate the plastic scintillator modules on the top and bottom endcaps as seen at the bottom of Figure 4.5.

The gas mixture within the tracking volume was tuned in conjunction with the wire diameter and material to provide good tracker performance whilst minimising ageing of the tracking wires, with the gas mixture being further adjusted during



Figure 4.5: A NEMO-3 sector containing a Tellurium source. Optical modules consisting of coupled scintillator blocks and photomultiplier tubes are visible, along with the copper tubes comprising the cathode for the tracking cells [70].

the experiment’s initial commissioning period. Stainless steel wires of $50\ \mu\text{m}$ were chosen, operating in a gas mixture of Helium (95%), Argon (1%), alcohol (4%), and water vapour (1500 ppm) at an overpressure of 10 mbar. Helium was chosen as the primary tracker gas due to its low ionisation potential, therefore minimising energy losses in particles traversing the tracking volume.

By recording the time offset of the initial electromagnetic cascade on the anode wire with respect to the trigger from the calorimeter, measured as a negative electrical pulse on the wire, it was possible to extrapolate the radial distance of closest approach of the ionising particle. By the timing difference between the arrival of the Geiger plasma propagation at each cathode ring the longitudinal position of closest approach along the wire could be measured, with the recorded traversal of the Geiger cell by an ionising particle termed a “hit”. Further ambiguities in hit

positioning were resolved by fitting tracks to sequences of hits and finding the helix which fit best to these measurements. Through these techniques a resolution of 0.5 mm was achieved on the radial coordinate of a hit and 0.8 cm on the longitudinal.

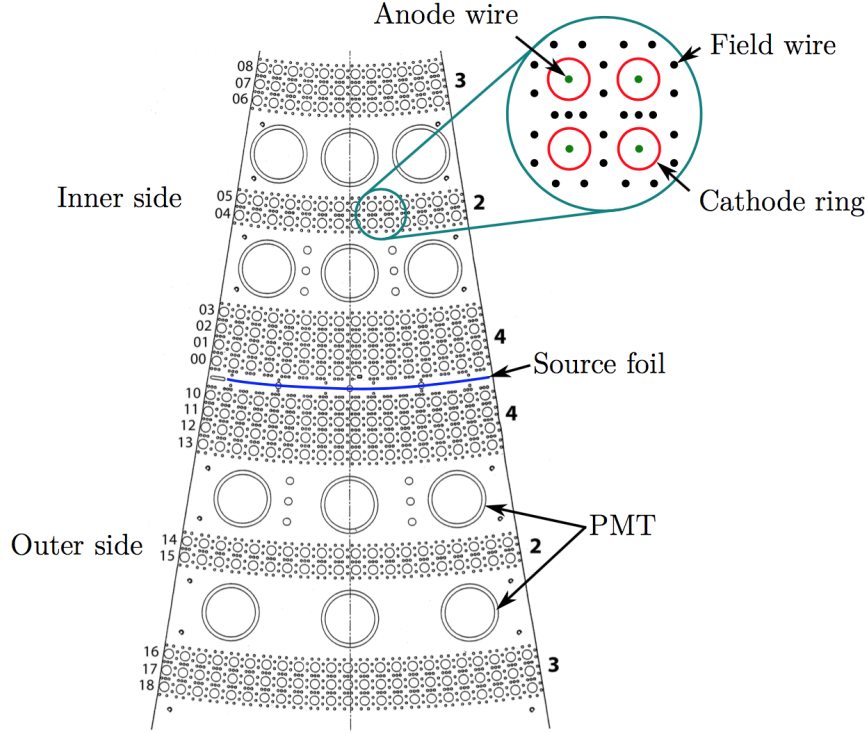


Figure 4.6: Plan view of a NEMO-3 sector [70]. The top and bottom plates of each sector are termed the “petals”.

4.2.5 Magnetic Field

To distinguish electrons from positrons within the tracker, the entire experiment was placed within a solenoidal magnetic field of 2.5 mT generated by a coil wrapped around the external wall of the calorimeter. This allowed e^+/e^- pairs produced within the detector to be rejected with an efficiency of 95%.

4.2.6 Calorimeter

The NEMO-3 calorimeter served three primary purposes in the experiment: to measure the energy of detected particles, to measure their time of flight, and to serve

as the trigger for the experiment. The calorimeter was composed of 1940 modules, each composed of an aluminised mylar wrapped block of plastic scintillator coupled to a photomultiplier tube through a light guide. These were primarily used to cover the outer and inner walls of the detector, with additional modules on the top and bottom endcaps as can be seen in Figure 4.6.

Each scintillator block was composed primarily of polystyrene, for which a few centimetres is sufficient to stop electrons of energies up to 12 MeV. However to ensure a high efficiency for photon detection, cubes of width 10 cm were chosen to ensure sufficient Compton scattering was possible within the low density polystyrene. A small percentage of p-Terphenyl was added to the polystyrene as a scintillating agent, and POPOP as a wavelength shifter. Two sizes of PMT were used in the experiment, developed by the collaboration in conjunction with the photomultiplier manufacturer Hamamatsu, using glass up to 1000 times less radioactive than standard glass. 5 inch PMTs were used in the outer wall and outer endcap ring modules, and 3 inch PMTs on the inner wall and rings. Each calorimeter module had an energy resolution between 14% and 17% FWHM at 1 MeV, and a time resolution of approximately 25 ps.

4.2.7 Calibrations

Two main techniques were used to perform regular calibrations of the detector during operation. Every 40 days, insertion of radioactive sources into copper calibration tubes located on the source foil plane in each sector allowed for time and energy scale calibration of the calorimeter modules and efficiency measurements of the detector as a whole, while daily laser light injection into each calorimeter module allowed the stability of these calibrations to be monitored.

^{207}Bi sources were used to calibrate the energy scale of a calorimeter module at low energies by detection of the 0.48 MeV and 0.98 MeV conversion electrons emitted during the isotope's decay to ^{207}Pb through electron capture. A higher energy reference point for this calibration was obtained by measuring the endpoint of the beta decay spectrum of ^{90}Y , at 2.3 MeV, through the insertion of the parent isotope ^{90}Sr . Measuring relative timing offsets between modules was achieved with ^{60}Co

sources, which decay through beta radiation to a daughter nucleus that concurrently emits two gamma rays with energies of approximately 1.3 and 1.2 MeV.

Laser calibration used an N₂ laser with a wavelength of 420 nm sent along optical fibres to each calorimeter module, and recorded a measure of the absolute energy and time calibrations along with the linearity of PMT responses.

4.2.8 Data Acquisition (DAQ) and High Voltage (HV) Electronics

Each PMT was powered with a positive voltage from a CAEN HV power supply, with a mean value of 1350 V for the 5" PMTs and 1800 V for the 3" PMTs. Voltage for each group of 3 PMTs was defined by the resistance on the associated output channel on the HV distribution board, with further adjustment by the selection of resistance values for resistors on the individual PMT bases.

Each PMT base was also connected to a DAQ system through a daughter board mounted on a mother-board using the Versa Module Europa (VME) bus standard, corresponding to the half-sector (divided by the foil plane) in which it is located. The daughter boards would begin a time measurement and open an 80 ns charge integration window if a signal above 23 keV was recorded, however this would not be read out unless a level 1 trigger signal was received. When a PMT recorded an integrated charge corresponding to an electron of energy greater than 150 keV a signal was sent to the motherboard acting as a level 1 trigger for all calorimeter channels. The time and energy resolutions possible with these electronics were respectively 53 ps and 3 keV per PMT.

Each group of 8 tracking cells were connected to one VME mother-board, with fifteen daughter boards, 5 for providing voltage and decoupling signals from the anode wires, 5 for the bottom cathode rings, and 5 for the top cathode rings. Each decoupled signal was fed to a channel on a corresponding acquisition board containing one analog and one digital integrated circuit (ASIC) for each signal channel. Each analog ASIC amplified the signal and compared it to a programmed threshold, anode signals exceeding the thresholds would begin four time counters in the digital ASIC - two for the anodic signal and one for each cathodic signal - and send a

signal to the level 2 trigger system. The cathodic counters would stop when signals were received above threshold, while the anodic counters waited for signals from the trigger system.

4.2.9 Trigger

Two levels of trigger were used during normal data taking of the experiment. The first comes from the calorimeter system as described above. The second level responded to signals in the tracking detector using basic pattern recognition algorithms to make a triggering decision. For normal data runs this level required Geiger hits in at least two neighbouring layers with the full track activating at least three Geiger cells in total within a half sector. After several inbuilt delay windows the level 2 trigger would stop the Geiger cell anodic time counters, each with different time windows to capture both prompt and delayed hits. The trigger rate in this mode was originally ~ 7.4 Hz which dropped to 5.7 Hz when the Radon trapping facility was implemented in the laboratory.

4.2.10 Radon Trapping Facility

To reduce the diffusion of Radon into the detector volume from the surrounding laboratory air a Radon trapping facility was constructed a year into the lifetime of the experiment, due to the measurement of significant numbers of background events identified as originating from Radon daughter nuclei. To achieve this a Radon trapping facility was constructed to produce low Radon air for flushing the volume of the lab surrounding the detector. A polyethelene tent was constructed around the detector with two columns containing 500 kg of activated charcoal at -50°C , placed upstream of the air supply for this volume. Radon within the air was trapped within the activated carbon resulting in a reduction of ^{222}Rn activity within the detector by a factor of 5 when the tent was flushed with low radon air at $150\text{ m}^3/\text{hr}$. Data taken before and after the construction of this facility are referred to as phase 1 and phase 2 data respectively.

4.2.11 Passive Shielding

Several layers of shielding surrounded the detector to further reduce background events within the detector. The laboratory itself is situated inside the Fréjus Mountain, under 1200 m of rock (4800 m water equivalent depth) reducing the rate of cosmogenic muons to 5.4 per m² per day. A layer of 20 cm of iron was placed around the magnetic coil and over the endcaps of the detector to block gamma rays, with a tripartite outer shield of wood, water and borated water to slow and capture neutrons from natural radioactivity in the mountain rock.

4.3 The SuperNEMO Demonstrator Module

The full SuperNEMO experiment is designed to detect or to set a limit on the neutrinoless double-beta decay of ⁸²Se of order 10²⁶ years (corresponding to an effective neutrino mass of order 50-100 meV) through the construction of 20 independent modules identical in design. The first such module, the demonstrator module, has been assembled in the LSM and will serve as a proof of concept for the design and background suppression techniques. The physics goals for the full SuperNEMO experiment and the demonstrator module are shown in Table 4.1.

	Full SuperNEMO	Demonstrator module
Source mass	100 kg	7 kg
Energy resolution	4% at 3 MeV (FWHM)	
²²² Rn in tracker volume	<0.15 mBq/m ³	
²⁰⁸ Tl in source foil	<2 μ Bq/kg	
²¹⁴ Bi in source foil	<10 μ Bq/kg	
Sensitivity to $0\nu\beta\beta$ half life	>10 ²⁶ yr	>6.6 $\times$ 10 ²⁵ yr
Sensitivity to $\langle m_{\beta\beta} \rangle$	<40-100 meV	<200-400 meV

Table 4.1: The SuperNEMO and demonstrator module goals [14].

4.3.1 Geometry

The SuperNEMO demonstrator module is planar in geometry, allowing the calorimeter an increased geometrical acceptance of electrons when compared to the NEMO-3 detector. The source foils are vertically strung, with equally spaced layers of tracker cells on each side, with portions of the calorimeter placed behind the cathode rings of each cell, allowing cathode and tracker coverage to overlap on the top and bottom planes of the experiment. Surrounding each half of the tracking detector are the two main calorimeter blocks comprising a 6 x 2 x 4 m cuboidal detector before addition of the magnetic coil and shielding.

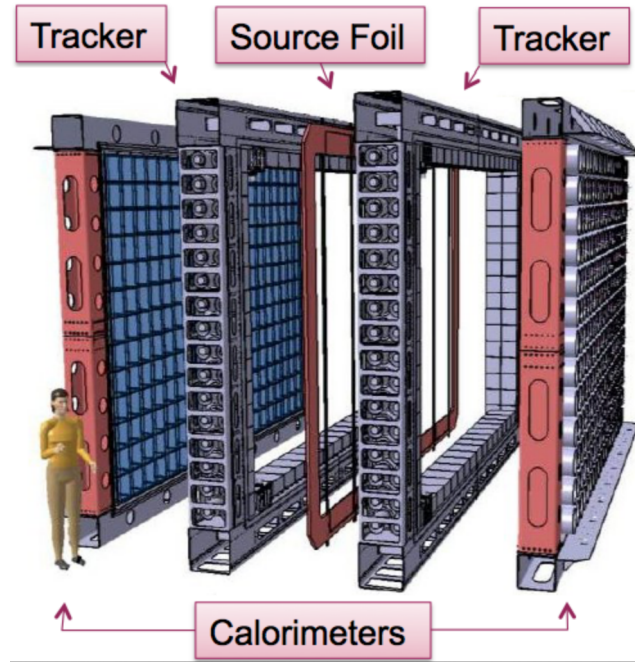


Figure 4.7: Exploded view of the SuperNEMO demonstrator module.

4.3.2 Source Foils

The isotope to be used in the SuperNEMO demonstrator module is ^{82}Se chosen for its high double-beta decay transition energy (≈ 3 MeV), as in the NEMO-3 experiment, and for the feasibility of enrichment and production of the source foils at a reasonable cost.

The source foils were produced in a form similar to the composite foils described for the NEMO-3 detector. They consist of powdered ^{82}Se mixed with a

PVA binding agent sandwiched between layers of Mylar. 34 Selenium foils have been produced with two radiopure copper foils to be mounted either side acting, as in NEMO-3, as proxy foils for background measurements. Each foil has a surface density of 50 mg/cm^2 and a thickness of 0.3 mm to minimise scattering of beta particles within the foil, resulting in an energy resolution limit of 6% (FWHM) in the region of interest.

4.3.3 Tracker

The SuperNEMO tracker is divided into two halves by the source foils, with each half comprising of 113 columns and 9 layers of tracking cells laid out in a grid. Each cell is 3 m long and has a diameter of 4.4 cm with 4 cm copper cathode rings composing cells similar to the NEMO-3 detector design. The larger radius of each tracker cell when compared to the NEMO-3 detector was chosen to minimise the amount of material, and therefore possible sources of background, within the detector. Unlike the NEMO-3 design the electric feedthroughs and a portion of the cabling for these cells are placed within the tracking volume, allowing calorimeter modules to be placed behind the cathode rings and therefore increasing calorimeter coverage within the detector. Each neighbouring pair of tracking cells is separated by 3 grounded wires to shape the electric fields and minimise cross talk between cells. The layout of tracker cells and wires is shown in Figure 4.8.

The tracker volume gas will be the same as in the NEMO-3 detector excluding the water vapour, which had not been shown to significantly affect tracker performance. The typical operating voltage for the SuperNEMO tracker cell is $1750 - 1800 \text{ V}$, with the specific voltage for subgroups of cells to be tuned according to cell performance during commissioning.

4.3.4 Calorimeter

The calorimeter is comprised of two main walls of calorimeter modules on each side of the detector with additional modules around the top, bottom and sides of the tracking volume for full geometrical coverage. The modules comprise plastic scintillator blocks (a combination of polystyrene and polyvinyl-toluene) and photomul-

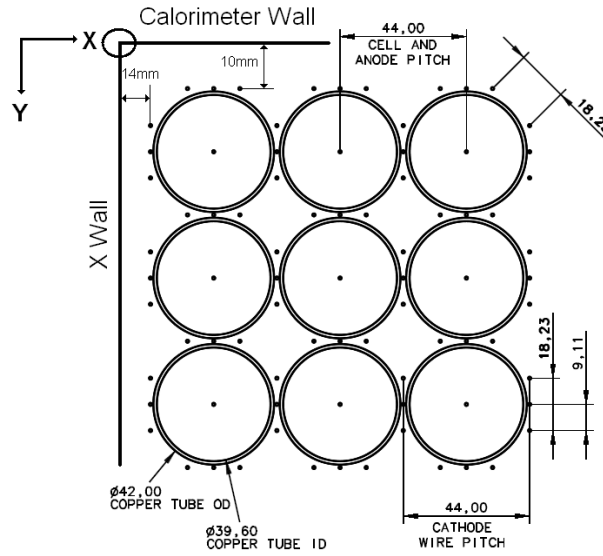


Figure 4.8: Layout of SuperNEMO tracker wires.

tiplier tubes as in the NEMO-3 experiment, however for the SuperNEMO demonstrator the main wall calorimeter bricks are directly coupled to the photomultiplier tubes without the use of a light guide, increasing the energy resolution. A mixture of low radiation 8” and 5” photomultiplier tubes are used, where the edges of the tracking volume were equipped with recycled NEMO-3 photomultiplier tubes, and the main calorimeter walls used new tubes developed in conjunction with Hamamatsu.

The use of the new 8” photomultiplier tubes and the direct coupling technique shown in Figure 4.9 allowed for a significant increase in energy resolution compared to the NEMO-3 detector with the main wall modules having a resolution of $\sim 8\%$ (FWHM) and a timing resolution of 400 ps at 1 MeV.



Figure 4.9: Photomultiplier tube directly coupled to a scintillator block.

4.3.5 Calibration Source Deployment System

As for the NEMO-3 experiment ^{207}Bi sources will be used for calibrations of the calorimeter. A new deployment system has been developed to allow these sources to be inserted and removed from the detector in a fully automated manner allowing for more regular calibration runs, planned to occur on a weekly basis. The system consists of electric stepper motors mounted above the source foil plane which open a valve and lower the sources, mounted within copper envelopes suspended by vertical wires, into the detector. A copper plumb bob underneath the sources will also descend until lowered to the bottom of the source frame where the deployment system will detect its presence and shut off the motors. After a calibration run, usually lasting a few hours, this process is run in reverse to remove the plumb bobs and calibration sources from the tracking volume.

4.3.6 DAQ and HV Electronics

Each column of main-wall optical modules will be cabled to a single VME readout board in a crate with a further four boards for the top and bottom optical modules and eight for the side wall modules of the tracker. Each readout board consists

of constant fraction discriminators, analog to digital converters and time to digital converters to record charge and timing information. This is subsequently passed onto a single board acting as a controller for the VME crate for preprocessing, which is itself connected to the trigger board of the full detector, responsible for implementation of the trigger logic. A trigger signal causes the readout of charge and timing information from the appropriate calorimeter modules and (optionally) a sampled waveform of the received pulse. Although the full trigger logic is yet to be defined, due to the size of memory buffers on the front end boards, the trigger signal must arrive at the front-end boards within 250 ns of the triggering charge. It is expected that a 1 kHz acquisition rate can be achieved, rising to 10 kHz if the waveform is not required. A separate crate of HV supply units provides power to each optical module.

The tracker DAQ electronics are organised in a similar fashion, with an HV crate supplying power for each cell, and passing the decoupled and differentiated signals to the readout front end boards. These then transmit up to seven timestamps to the control boards defined by the incoming signal and four voltage thresholds. A typical signal and the recorded timestamps are shown in Figure 4.10. The control board communicates with the trigger board to initiate readout of the front end memory buffers as in the case of the calorimeter DAQ system.

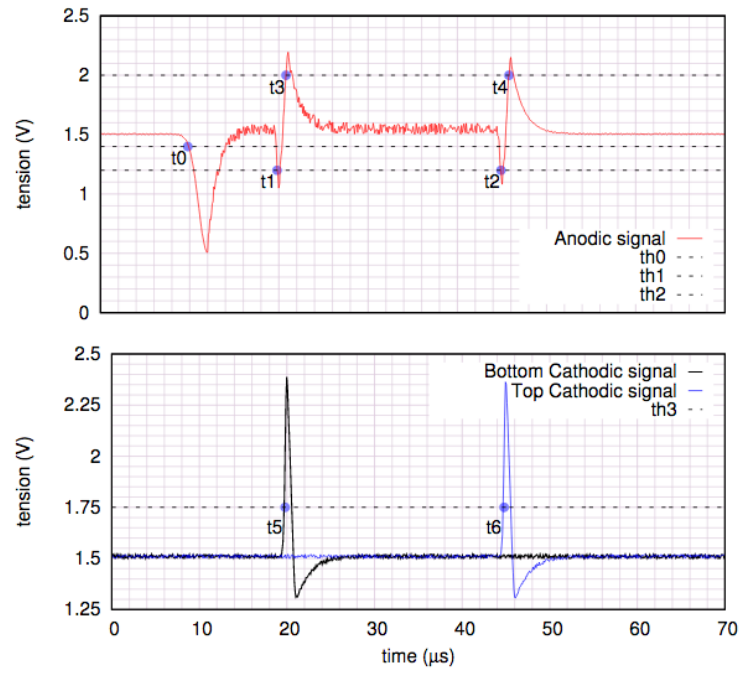


Figure 4.10: Voltage thresholds and recorded timestamps for a typical SuperNEMO tracker signal. Top: Simulated (differentiated) anodic signal, with 5 corresponding timestamps defined by 3 voltage thresholds. Bottom: Simulated (differentiated) cathodic signal, with 2 timestamps defined by 1 voltage threshold.

4.3.7 Shielding and Magnetic Field

A magnetic field of the same strength as in the NEMO-3 experiment (2.5 mT) will permeate the tracking volume, created by a coil to be wrapped around the sides of the calorimeter and tracker modules. Surrounding this will be passive shielding, primarily in the form of water and iron, with the 1200 m depth of the lab once again reducing the cosmic muon activity. The full composition and structure of the shielding has yet to be determined.

Chapter 5

SuperNEMO Tracker Construction and Commissioning

The assembly of the tracker for the SuperNEMO experiment took place over four years from 2015 to 2018 at the Mullard Space Science Laboratory. Here the construction process is briefly outlined, along with the set-up and results from the initial commissioning prior to transport of the tracker to the final installation site, the Laboratoire Souterrain de Modane.

5.1 Tracker Construction

The tracking chamber for the SuperNEMO Demonstrator Module was constructed in four sections, labelled C0-3, in a clean room at UCL's Mullard Space Science Laboratory. Construction of each section began with the assembly of three sides of the metal frame consisting of two iron beams (eventually comprising the top and bottom walls of the tracker) and one stainless steel beam (to be the vertical side wall of the tracker). Each remaining open face of the tracker section was closed with temporary stainless steel structures - to provide dust protection and gas tightness during commissioning - to be eventually removed at the LSM during assembly of the full Demonstrator Module.

Each beam contained a series of openings for insertion of electrical feedthroughs and optical modules consisting of a coupled PMT and Mylar-wrapped scintillator block. Two layers of optical modules were inserted into each long stain-

less steel beam and one layer in each iron beam. Unlike the optical module shown in Figure 4.9 the tracker optical modules were constructed using only PMTs recovered from the NEMO-3 experiment with light guides between the scintillator and PMT, providing a lower energy resolution than the modules in the main calorimeter walls. After optical module insertion into the iron end beams, a stainless steel and copper structure was placed over the blocks to act as a mechanical support and positioning guide for the tracker cells.

The tracking cells were inserted onto this frame two columns at a time as each set of two columns of nine tracking cells (a cassette) formed a coupled set of cells due to the sharing of cabling and feedthroughs. Before insertion each cassette was tested for electrical connectivity throughout the cabling and thoroughly inspected under ultraviolet light for dust and debris on the tracker wires. Each cassette was given a final clean and dusting with compressed nitrogen before being mounted onto the cassette insertion apparatus, which supported and positioned the cassette, during the remaining stages of the insertion operation. A cassette being inserted into the tracker is shown in Figure 5.1.



Figure 5.1: A cassette of SuperNEMO Geiger cells being inserted into a tracker section.

5.2 Tracker Commissioning Setup

Prior to being shipped to the LSM each tracker section was independently sealed and operated using customised electronics and software. This commissioning procedure provided the first opportunity to assess tracker performance through the measurement of cosmogenic muons passing through the tracking volume.

5.2.1 Gas System

To provide the tracking volume of the detector with the stable and well defined mixture of Helium, Argon and ethanol required for optimal operation, the gas system described in Figure 5.2 was constructed.

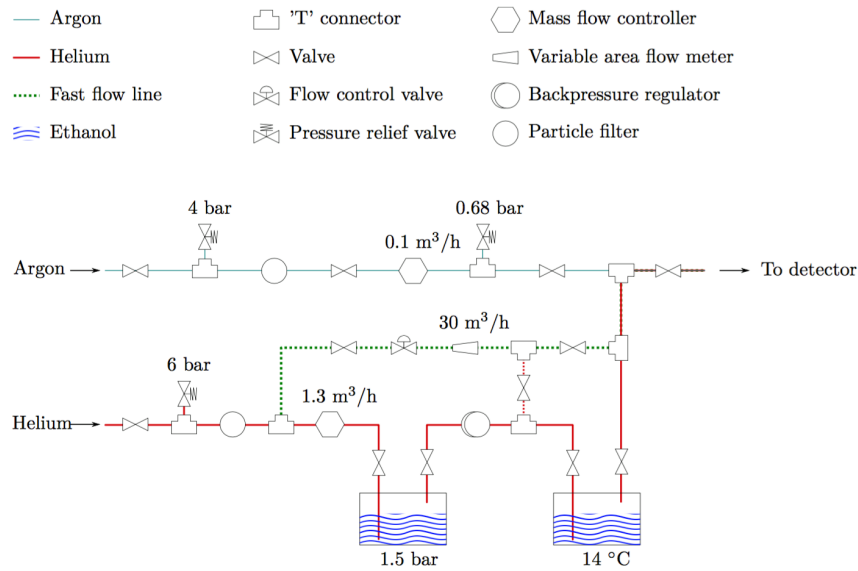


Figure 5.2: Schematic of the SuperNEMO gas system: Ethanol baths are controlled at the labelled pressure or temperature, with maximum operational flow rates and pressures indicated above the gas line components [14].

Argon and Helium are provided along two gas intake lines, each equipped with release valves in case of exceptional overpressure in the system. Each gas line is equipped with a filter to remove particulate matter, then each line is passed through a mass flow controller (MFC) to regulate flow rates. These MFCs are electronically controlled by a centralised unit such that the ratio of flow rates ensures the required He/Ar ratio. The Helium is subsequently passed through two ethanol baths, one with a controlled pressure and one with a controlled temperature.

By modifying the values of these pressure and temperature set points, and therefore the pressure of the ethanol within, the ethanol content of the downstream helium gas can be controlled. Granular control of the temperature of the secondary bubbler ensures that this remained at 4% for the SuperNEMO tracker commissioning. This gas is then mixed with the Argon and flows into the tracker.

5.2.2 Trigger Electronics

An external set of electronics was designed to emulate the trigger signal of the NEMO-3 detector upon the transit of a charged particle through two parallel planes of plastic scintillator. Each scintillator plane was connected via a light guide to a recovered NEMO-3 photomultiplier tube, with the output of each converted to a square wave of width 20 ns using a constant fraction discriminator. When these signals overlapped, as determined by a coincidence logic unit, a square pulse was sent to the NEMO-3 Trigger Board, a custom FPGA array designed to emulate the timing and shape of the NEMO-3 trigger system signals as described in Section 4.2.9.

With the scintillator planes fully overlapping a trigger rate of approximately 50 Hz was measured, consistent with the expected muon flux [71], and a false coincidence rate of 1.5 Hz was observed when the planes were fully separated.

5.2.3 HV & DAQ

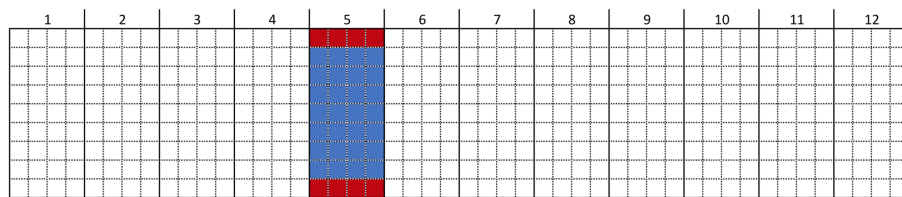


Figure 5.3: End view schematic of one tracker section (C0) in commissioning orientation. One tracker sector, consisting of four columns of nine Geiger cells, is highlighted in red and blue corresponding to the two independently tunable voltages supplied by the HV system.

The HV and DAQ boards used in the commissioning were recovered from the NEMO-3 tracker electronics as described in Section 4.2.8. Connectors were attached to the recovered cables to provide an interface to the SuperNEMO

tracker feedthroughs with custom DAQ logic, including channel mapping, provided through software created for the SuperNEMO commissioning installed on a dedicated commissioning server. The HV channel mapping allowed for voltage control at the level of granularity shown in Figure 5.3, with the bulk and edge cells of each sector (4 columns of 9 Geiger cells) independently tunable. With the available recovered DAQ and HV hardware, a maximum of four such sectors could be powered at any one time.

5.3 Time Domain Reflectometry

A further method to investigate the electrical connectivity of each cell was developed using a technique termed Time Domain Reflectometry (TDR). In this technique a signal generator is used to inject a narrow pulse into the feedthrough pins, with the reflection recorded on an oscilloscope and analysed as shown in Figure 5.4.

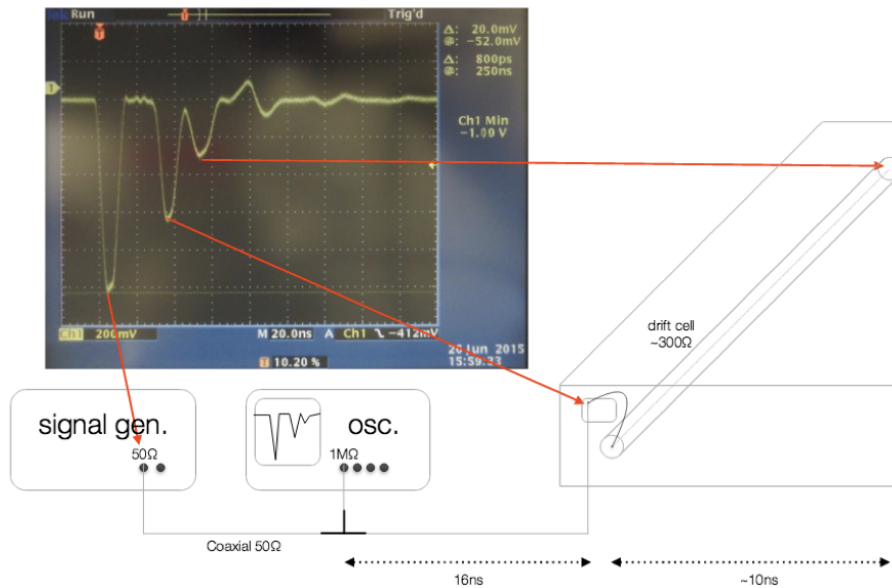


Figure 5.4: Setup and expected signal for Time Domain Reflectometry testing. A pulse is injected into a tracker cell with reflections generated by impedance mismatches in the detector. Points on the recorded signal are linked by arrow to the surfaces within the detector that generate the recorded reflections.

Observation of the structure of the reflected pulse depends on the electrical connectivity of the feedthrough, the internal cabling, and anode wire, along which a pulse will travel in a fully operational tracker cell. The reflected pulses from

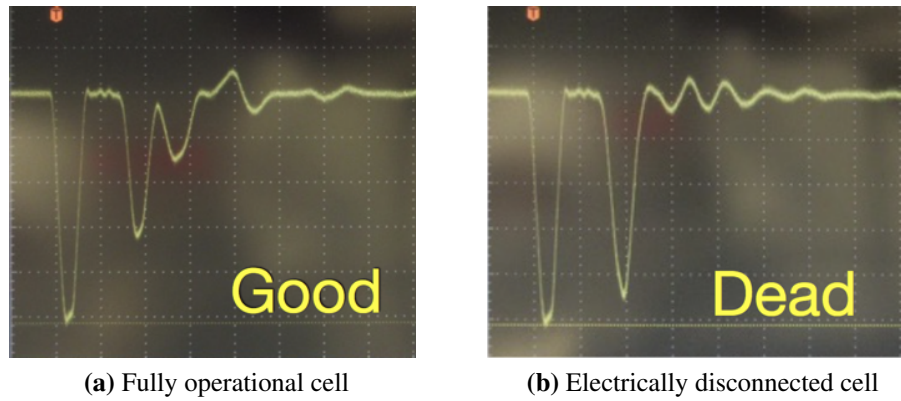


Figure 5.5: TDR reflection structure.

the connection of the internal cabling to the anode wire and from the far end of the anode wire were expected to arrive approximately 16 ns and 10 ns after the signal injection for a healthy cell, and an observation of this structure was deemed sufficient to conclude that the electrical connectivity of the cell is unimpeded, and therefore that full operation of the cell was likely to be possible.

A typical failure mode which would be detected by this technique is when the crimp connecting the internal cabling of the cell to the anode wire has loosened enough for the connection to break. Such a failure would produce a TDR signal missing the second reflection as can be seen in Figure 5.5. Due to the fragile nature of these crimps this failure mode was expected to be among the most common, distinguishable from failure modes originating in other parts of the cell through observation of the time taken for the first reflection to arrive at the oscilloscope.

All cells which did not produce data from commissioning with the standard DAQ electronics were investigated using the TDR method and categorised as either connected or disconnected accordingly.

5.4 Results

5.4.1 Anode Status

The primary metric used for assessing the condition of anode wires within the tracker cells was the relative occupancy of each cell; that is the number of hits seen from that cell as a percentage of overall triggers. An abnormally low number

indicated a cell that often failed to generate a signal when muons passed near the anode, while an abnormally high number indicated a self-triggering cell, generating spurious signals not associated with a passing particle.

Relative occupancies for each Geiger cell in the first tracker section (C0) can be seen in Figure 5.6. Gaps represent anode channels for which no data was recorded due to a fault in the Geiger cell or HV and readout systems, several of which were known before data collection. Such cells were further investigated using the TDR method outlined in Section 5.3 and categorised as connected or disconnected.

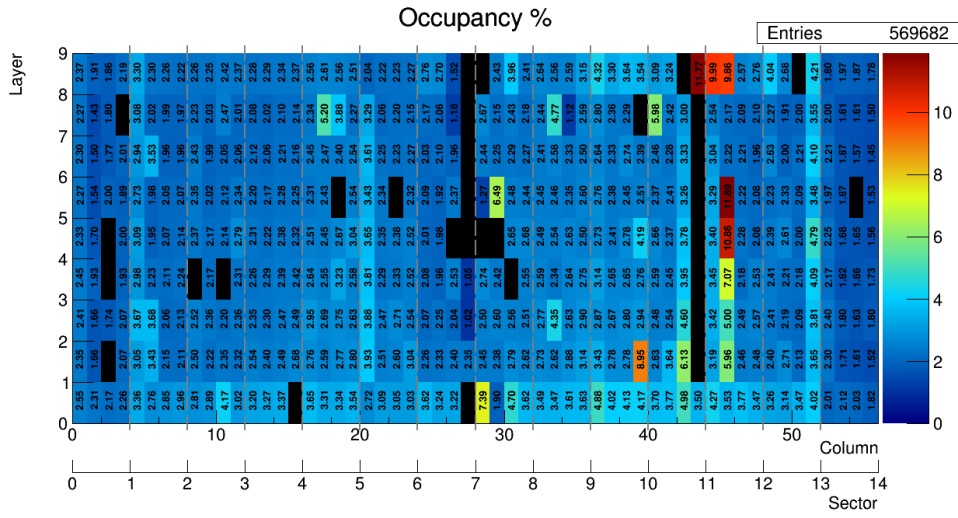


Figure 5.6: Percentage occupancy for anode wires in C0 as measured during commissioning. Orientation as in Figure 5.3.

Several patterns are apparent in Figure 5.6, most prominently higher occupancies for cells located at the edge of a tracker sector. These cells had fewer connected neighbouring cells during data collection, resulting in an extended magnetic field due to the reduced number of adjacent grounded field-shaping wires. This was partially compensated by operating the edge cells at a lower voltage to those in the bulk of the tracker, and by independently grounding adjacent sectors, however the inconsistent application of such measures during commissioning resulted in a non-uniform pattern of occupancies across the data collection process.

It can be seen that cell occupancies lay within the 1-5% range for a general cell, with higher occupancies at the edges of sectors a result of the extended electric field

in the absence of active neighbour cells. Cells with an occupancy above this range were considered to be self-triggering or “hot” cells. Combining occupancy data with TDR measurements of cells allowed a categorisation of each cell according to its status, where normal, hot and electrically connected cells were expected to be fully operational or recoverable. Electrically disconnected cells were flagged for future inspection and repair. C0 cell status is shown in Figure 5.7, including a final category of cell for those anodes which drew a current above the safety limits implemented in the HV system, indicating a short circuit inside the detector. A list of cell status categories is presented in Table 5.1.

Status	Explanation	Colour
Good	Operating as expected.	
Good, hot	Operational, but with high occupancy.	
Connected	No commissioning data, but electrically connected.	
Disconnected	No commissioning data, not electrically connected.	
High Current	Drew a current above the safety threshold.	
Undetermined	To be tested at the LSM.	

Table 5.1: SuperNEMO tracking cell status categories.

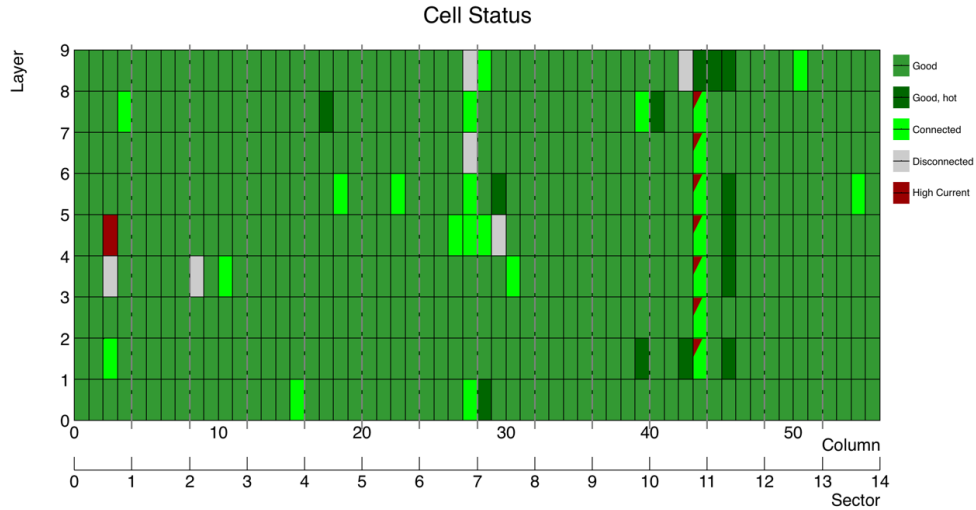


Figure 5.7: Status of C0 anode cells. Partially shaded cells in column 43 indicate that at least one cell in the indicated group was responsible for drawing a high current.

This process was repeated for each tracker section in turn to build up a picture of the overall status of the tracker anode wires, with the final section C3 measured using the TDR technique only due to time constraints. The overall cell status can be seen in Figure 5.8 with the number of cells in each category summarised in Table 5.2

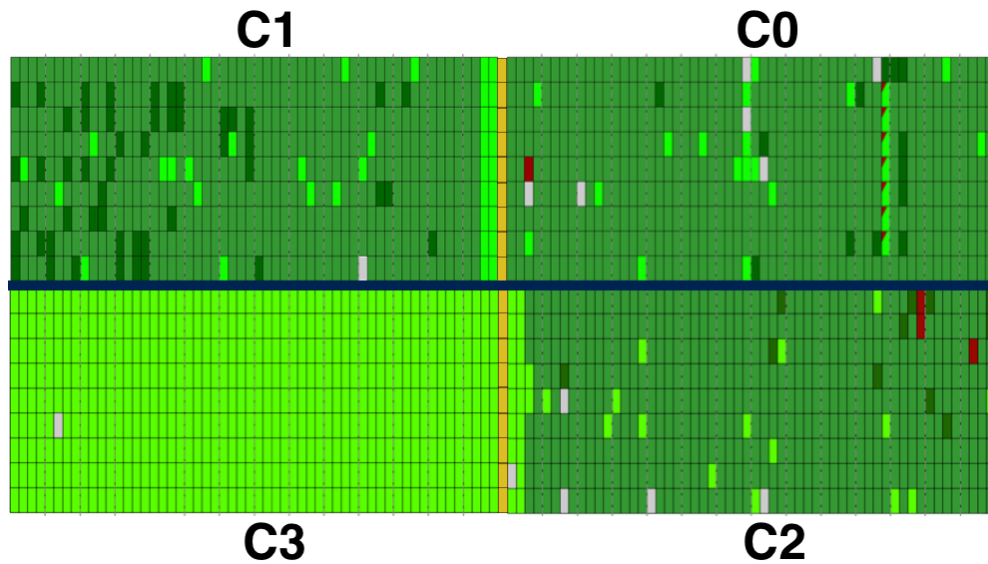


Figure 5.8: Status of all tracker anode cells, top-down view in LSM orientation. Cells marked in yellow were installed into the tracker after commissioning and will be checked in-situ.

Status	Number of Cells	Percentage of Tracker
Good	1328	65.3%
Good (hot)	74	3.7%
Connected	599	29.4%
Undetermined	18	0.9%
Disconnected	12	0.6%
High Current	5	0.2%

Table 5.2: Summary of tracker cell status measurement.

Up to three timing signals were recorded for each Geiger cell traversed by a muon, the arrival times of signals on the anode wire and the two cathode end-caps relative to the trigger. The former is referred to as the “drift time” and relates to the radial distance of closest approach of the muon to the tracking wire. The sum of cathode timing signals is referred to as “plasma propagation time”, as described in Section 4.1.1, with at least one cathode signal required to resolve the measurement of muon position in the longitudinal direction of the wire.

The distribution of recorded drift times for a cell can be used in addition to the average anode occupancy as a metric for distinguishing a hot cell from a cell operating normally, as shown in Figure 5.9. Hot cells were observed to have a larger noise floor, consisting of a larger right hand tail to their drift time distribution, representing a continual emission of signals independent of the trigger window. These noise floors resulted in measuring mean drift times for each cell higher than the expected typical drift time for a well reconstructed hit, due to the tail disproportionately weighting the mean, however the comparatively larger impact this had on hot cells made these long mean drift times useful as a further indicator of a self-triggering anode.

The distributions of mean drift times for each good cell, and mean plasma propagation times for hits with two associated cathode signals - shown in Figure 5.10 - were recorded and found to be in broad agreement with measurements made on

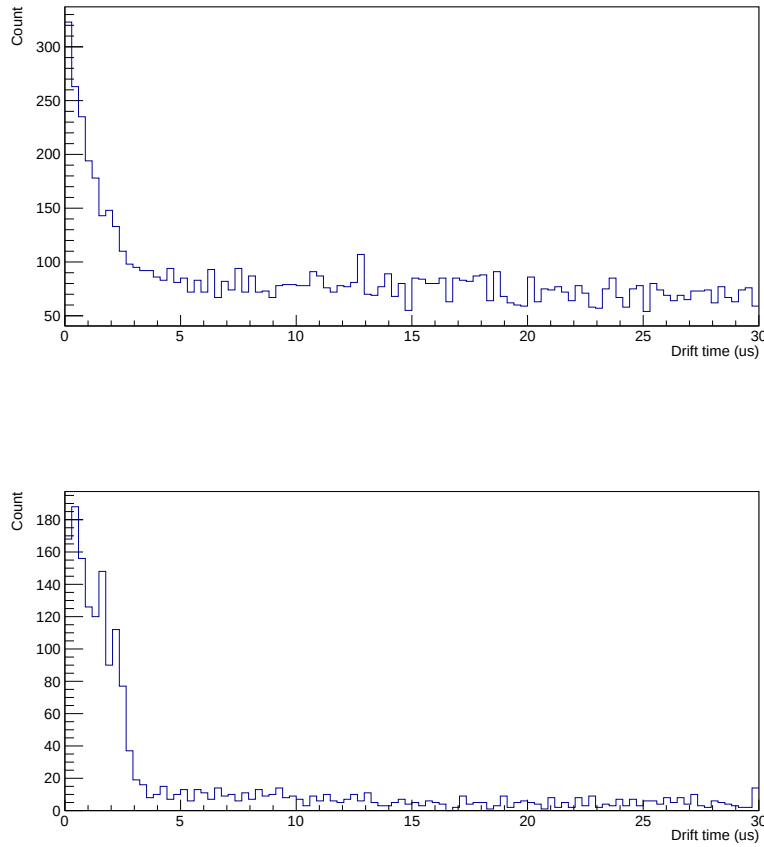


Figure 5.9: Histograms of drift times recorded for two tracker cells. Top: hot cell; Bottom: good cell.

each cell before tracker assembly [72], indicating no degradation of performance during the assembly process for measured cells.

These metrics allowed like-for-like comparison between the first three tracker sections, summarised in Table 5.3. Due to the variability in operating conditions between the commissioning of each tracker section, the variability seen in these metrics is thought to be entirely attributable to differences in grounding, operating voltage, and scintillator positioning between data collection periods.

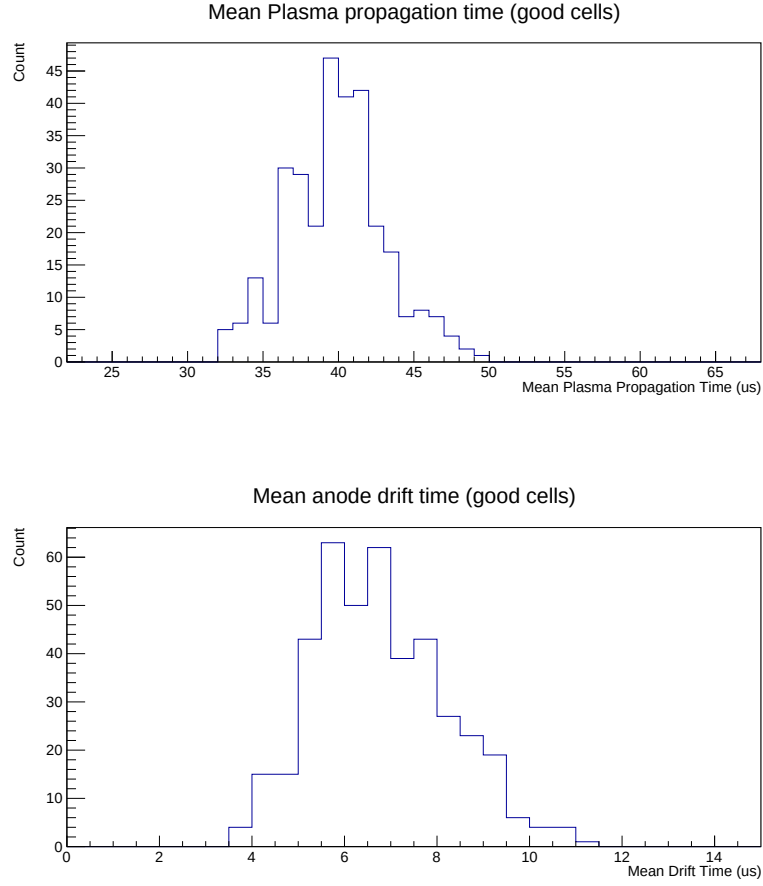


Figure 5.10: Histograms of mean plasma propagation times (top) and mean drift times (bottom) for good cells in C1.

Metric	C2	C1	C0
Proportion of good & connected cells	99.0%	99.8%	98.4%
Proportion of good cells with at least one active cathode	97.8%	98.3%	98.7%
Mean anode occupancy w.r.t. number of triggers	2.58%	1.99%	2.47%
Mean cathode occupancy w.r.t. number of anode hits	89.0%	88.6%	88.7%
Mean drift time (good cells)	6.51 μ s	6.57 μ s	5.78 μ s
Mean plasma propagation time (good cells)	38.8 μ s	39.8 μ s	42.0 μ s

Table 5.3: Summary of tracker metrics by section.

5.4.2 Drift Model Measurement

The drift model characterises the relationship between the radial distance of closest approach of a charged particle to an anode wire (known as the drift distance) and the time at which the signal was recorded relative to the trigger. This is affected by the electrostatic and gas conditions within a tracker cell, which are generally cylindrically symmetric near the anode wire of a cell. Although the square cross section of a tracker implies that this symmetry does not hold close to the edges of the cells, for the purposes of commissioning, the approximation of a radially symmetric drift model was considered sufficient. During the NEMO-3 experiment the following functional form was used:

$$r = \frac{At}{t^B + C} + D, \quad (5.1)$$

where t is the measured drift time, r is the drift distance and A , B , C and D are empirical constants. This form can produce a family of monotonically increasing, concave functions corresponding to the monotonic fall in the electric field at increasing distances away from the anode wire. Best fit values measured with a single SuperNEMO tracker cell were found to be:

$$A = 0.68, \quad B = 0.84, \quad C = 287.09, \quad D = 0.02, \quad (5.2)$$

for t measured in ns and r in cm [73].

A first attempt at fitting new drift models for the first two SuperNEMO tracker sections was carried out using data from the commissioning period at MSSL. High energy cosmic muons were expected to produce straight vertical tracks through the detector. The search for this topology and the desire to remove events containing spurious detector activity motivated the following selection cuts on candidate events:

- after discarding hits with a drift time of >3 μs , events must have
- more than 8 anode hits,
- at least one hit in the top two layers of cells,

- at least one hit in the bottom two layers of cells,
- a vertical gap of at most one layer between hits in candidate track,
- a horizontal gap of at most one column between hits in candidate track,
- more than $3 \times$ the number of hits in the candidate track than in neighbouring unassociated cells.

Initially drift distances for hits in these events were calculated using the NEMO-3 drift model in 5.1. The initial cut discarding events with long drift times was intended to ignore spurious hits, eliminating the tail in the drift time distribution seen in Figure 5.9.

A track was fit to each of these events in the xy plane (as viewed in Figure 5.3) through SIMPLEX minimisation [74] of the the distances between the track and each calibrated hit in the event, where a calibrated hit is defined as a circle around a hit anode wire in the xy plane, with a radius equal to the drift distance. The result of such a fit on a commissioning event is shown in Figure 5.11.

The average distance of closest approach between the best-fit track and each calibrated hit in the event served as a measure of goodness of fit for the individual track, and the sum of these averages over the population of candidate track events served as a measure of goodness of fit for the drift model. The model parameters that minimised this quantity served as an initial measurement of the drift model for the tracker section in question.

For the commissioning drift model a new functional form was devised to avoid a drawback of the functional form from Equation 5.1 whereby values for the pair of parameters (A, B) spanning multiple orders of magnitude result in almost degenerate drift models, producing multiple disparate local minima when attempting to fit model parameters. Considering the electric field within a cylindrical drift cell, an alternative parametrisation of the following functional form

$$r = A \ln \left(Bx^D + e^{-\frac{C}{A}} \right) + C \quad (5.3)$$

allows for a drift model of similar shape while reducing the degeneracy. An ap-

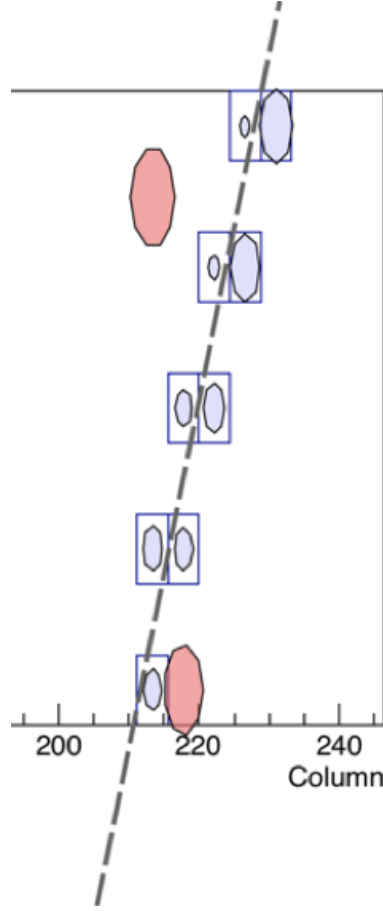


Figure 5.11: A track fit to a C0 commissioning event containing 9 calibrated hits in the candidate track and 2 unassociated hits.

proximation to the drift model parametrised by Equation 5.1 with the constants in Equation 5.2 can be constructed from the logarithmic drift model functional form with the values

$$A = 0.7, \quad B = 0.1, \quad C = -2.2, \quad D = 1.0, \quad (5.4)$$

as shown in Figure 5.12.

Track and drift model fitting were both carried out using the MINUIT algorithm implemented in the ROOT software package. The procedure used the SIMPLEX minimiser to locate a local minimum in the (A, B, C, D) parameter space which was then used as the starting point for a second pass of minimisation using the MIGRAD minimiser. Details of these algorithms are provided in the MINUIT

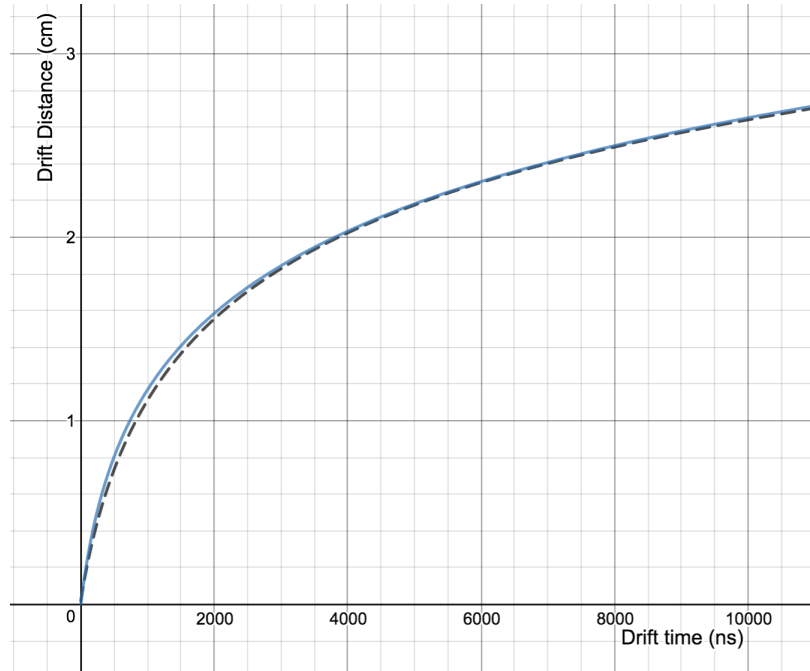


Figure 5.12: The original NEMO-3 drift model (dashed line) and an approximation using the logarithmic parametrisation (blue).

user guide [74].

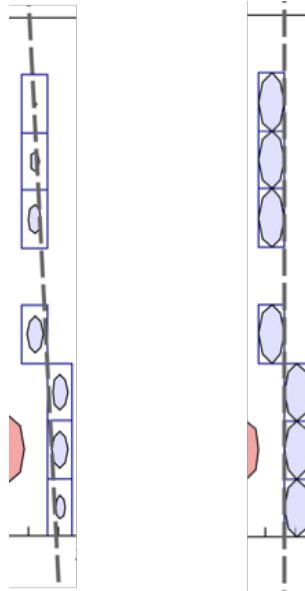


Figure 5.13: A track fit to a two-column commissioning event using a realistic (left) and constant (right) drift model.

Due to the topology of the commissioning events there existed artificial local

minima in the (A, B, C, D) lying on the plane defined by $A = 0$, $C = 2.2$; corresponding to a model where drift distance is always equal to the cell radius, independent of the measured hit time. The origin of this phenomenon is illustrated in Figure 5.13 showing how such a model can produce a perfect fit to any single or two-column track regardless of the track angle defined by a more physical drift model. The orientation of the tracker columns relative to the cosmic flux ensured that one and two-column events made up in excess of 95% of the total population of events passing cuts. To avoid fitting these unphysical drift models parameter limits were introduced to the fitting procedure. These limits and best fit values can be seen in Table 5.4 with the corresponding drift models shown in Figure 5.14.

	Range	C0	C1	C2
A	[0.5, 1.5]	0.8093 ± 0.0006	0.7971 ± 0.0006	0.5849 ± 0.0005
B	[0.5, 1.5]	1.130 ± 0.003	0.991 ± 0.003	1.026 ± 0.103
C	$[-5.0, -1.0]$	-3.370 ± 0.003	-3.298 ± 0.004	-1.081 ± 0.008
D	[0.5, 1.5]	0.8727 ± 0.0007	0.8506 ± 0.0007	0.7122 ± 0.0002

Table 5.4: Best fit drift model parameters (to 4 s.f.) for first the three tracker sections.

The most apparent feature of the fitted drift models when compared to the NEMO-3 model is that for any given distance from the anode at which an avalanche is generated it reaches the centre of a cell in a shorter time, i.e. the SuperNEMO tracker cells have faster drift velocities than the NEMO-3 cells. This is in agreement with measurements on the SuperNEMO 90-cell prototype [73] and is partly attributable to the higher operating voltages of the SuperNEMO cells. The differences between drift models for each section of the tracking detector are thought to be attributable entirely to varying operating conditions during commissioning, though this will be further tested during full-detector commissioning.

These fitted drift models served as an initial verification of the tracker and commissioning procedure, which were validated by the observation of a population of well defined track-like events corresponding well to drift models of the expected

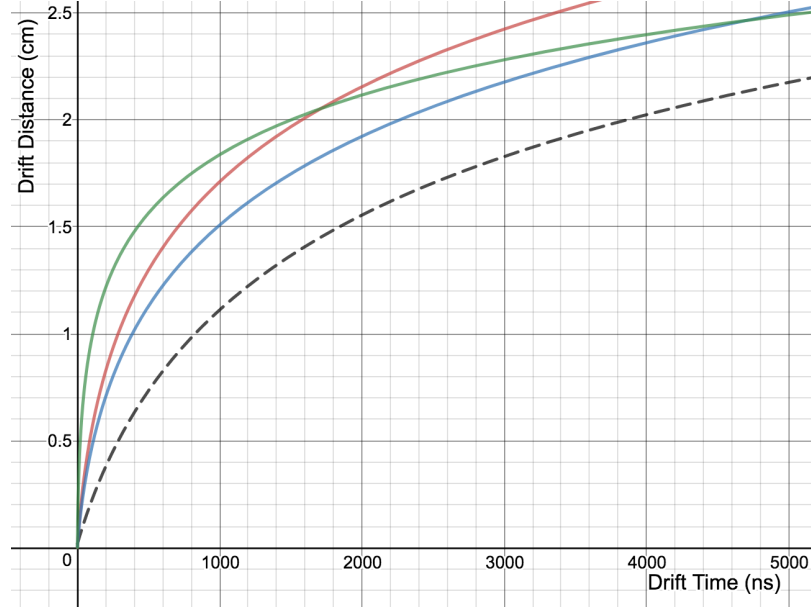


Figure 5.14: The C0 (red), C1 (blue) and C2 (green) drift models compared to the NEMO-3 model (dashed). Y-axis is truncated shortly above the Geiger cell radius.

functional shape. While they are not descriptions of the expected drift model in full-detector, in-situ, operating conditions, they serve as a starting reference point for such a measurement.

A more precise drift model can be obtained using this procedure through the use of a point source of events at a known location relative to the tracker wires and trigger system, oriented such that a larger population of events cross multiple columns, avoiding the degeneracy illustrated in Figure 5.13. A drift model corresponding more accurately to operating conditions at the LSM will be measured during full detector commissioning using the main calorimeter as a trigger and the finalised HV system, allowing for voltage tuning of individual cells. Finally, a more sophisticated analysis accounting for radial asymmetry of the electric field within a cell and local conditions within the tracker - including broken wires - is required for a reliable drift model.

Chapter 6

Unfolding the NEMO-3 ^{100}Mo $2\nu\beta\beta$ Spectrum

To allow the NEMO-3 ^{100}Mo data to be used to test theories by researchers without access to the collaboration's Monte-Carlo models, the $2\nu\beta\beta$ spectrum was unfolded over a combination of several key kinematic variables. In the following chapter this procedure is described, the half life of this process measured, and the unfolded distributions presented, with limits found for the Standard Model Extension operator $\hat{a}_{of}^{(3)}$ using one such distribution.

6.1 Event Selection

Several stages of cuts are applied to the NEMO-3 data to ensure we have a good data sample for the analysis. The first stage is a selection of which data collection runs are included in our sample, consisting of runs which meet the following criteria:

- the PMT DAQ system was switched on,
- PMT laser time calibrations are available for the run,
- at least one event has been reconstructed, and
- data was collected for over $>90\%$ of the time of the run,

where the final criterion is intended to remove runs where the DAQ system was inoperational. Each run passing these criteria is then reconstructed, with the next stage of cuts applied at the reconstruction level. This stage of cuts is designed to

ensure only events which can be reliably reconstructed pass, and so consist of a series of criteria on the PMT and scintillator blocks in the event. Events involving PMTs which are known to be faulty, noisy or dead are not reconstructed, and the same for PMTs without any available energy calibration or laser time correction. Furthermore, events reporting unphysical values such as a negative reconstructed energy or a timestamp before the trigger time are also removed. The final stage of the cuts refines the data sample in three main ways. Firstly by further ensuring that our events are well reconstructed using the following cuts:

- electrons must have an energy above 300 keV,
- tracks must begin in the layer of Geiger cells closest to the foil,
- tracks must be longer than 50 cm,
- tracks must hit the front face of a scintillator block,
- each track is associated to a different scintillator block,
- scintillator hits associated with tracks are not accompanied by spurious hits in neighbouring blocks,
- each PMT that was hit was successfully calibrated during the last calibration run (as described in Section 4.2.7),
- a vertex location was successfully reconstructed,
- an internal or external probability was successfully calculated (see equations 6.1 and 6.2), and
- low resolution and low light-yield scintillator blocks are not associated to the tracks;

secondly by requiring an event topology consistent with ^{100}Mo double-beta decay using the following cuts:

- exactly two tracks have been reconstructed,
- the tracks share a common vertex,
- the vertex is located on a ^{100}Mo foil, and

- the tracks have negative curvature;

and finally by rejecting background events with the following cuts:

- no reconstructed gammas* in the event,
- no reconstructed alphas† in the event,
- there is a maximum of 1 Geiger hit near the vertex if not associated to a track,
- if tracks are on the same side of the foil no unassociated Geiger hits are within 15 cm of the vertex on the opposite side,
- internal probability of the event is $>4\%$, and
- external probabilities of the event were $<1\%$.

The internal and external probabilities were constructed to test the hypothesis of an internal or external origin of the event, where an external electron entering the detector could cause a double-beta decay topology through scattering within the source foil. To construct these metrics the difference in measured time of flight and that calculated for an electron of the measured energy traversing the reconstructed path ($t^{meas} - t^{calc} = \Delta t$) was computed. By subtracting these, and normalising to the errors in each, a goodness of fit measure was calculated under the presumption of simultaneous emission of the electrons, as in Equation 6.1:

$$\chi_{int}^2 = \frac{(\Delta t_{e_1} - \Delta t_{e_2})^2}{(\sigma t_{e_1}^{meas} + \sigma t_{e_1}^{calc})^2 + (\sigma t_{e_2}^{meas} + \sigma t_{e_2}^{calc})^2}. \quad (6.1)$$

Summing the Δt with the same normalisation, as in Equation 6.2:

$$\chi_{ext}^2 = \frac{((t_2^{meas} - t_1^{meas}) - (t_2^{calc} + t_1^{calc}))^2}{(\sigma t_{e_1}^{meas} + \sigma t_{e_1}^{calc})^2 + (\sigma t_{e_2}^{meas} + \sigma t_{e_2}^{calc})^2}, \quad (6.2)$$

produces a goodness of fit measure under the presumption that one of the electrons first entered the detector before scattering in the foil and producing the second measured electron. Both possible time orderings of electron tracks were considered and

* A reconstructed gamma is defined as an isolated cluster of scintillator hits without a track.

† A reconstructed alpha is defined as a delayed <50 cm track without an associated scintillator.

the maximum χ_{ext}^2 selected. These values were subsequently used to compute the internal and external probabilities assigned to an event through integration of the χ^2 distribution with one degree of freedom.

The number of events passing each cut are shown in Table 6.1.

Cut	# Events
≥ 1 scintillator hit from a calibrated photomultiplier (PM) in the event	943543387
Number of scintillators hit ≤ 10	943494455
Number of tracks ≥ 1	907282099
Number of tracks ≤ 2	810971402
≥ 1 tracker hit in one of first 4 tracker planes	565093821
Tracks hitting “petal” scintillators hit the front face	531945142
Tracks are associated to different scintillators	524282505
Tracks are each associated to a calibrated PM	26538367
No unassociated PM hits (γ s) are recorded	14648469
Only one PM is associated to each track	14062376
A vertex location was reconstructed	14062370
No unassociated tracker activity close to the opposite side of the foil for same-side events ^a	14006168
Exclude a single damaged source strip	13510714
An external or internal probability was calculated	12588400
Track lengths are >50 cm	11477515
≥ 1 tracker hit in the first tracker plane	9769669
Reconstructed track origin vertices are separated by <4 cm in the foil plane	8427170
Reconstructed track origin vertices are separated by <8 cm in total	8152174
Reconstructed electron energies are >300 keV	3407884
Reconstructed tracks have negative curvature	912400
Exclude events hitting the innermost layer of “petal” scintillators	877955
Hit scintillator blocks had been successfully calibrated in the last calibration run	651836
Internal probability $>4\%$	549089
External probability $<1\%$	549086
≤ 1 tracker hit unassociated to a track near the vertex	537309
No <50 cm tracks unassociated to a scintillator (α s)	534129
Source foil of reconstructed vertex is ^{100}Mo	507365

Table 6.1: Cut flow for NEMO-3 $2\nu\beta\beta$ analysis.

^aA same-side event is where both tracks emerge from the same face of the source foil.

6.2 Variable Selection

The primary quantity of interest is the summed electron energy distribution, which is sensitive to the type of double-beta decay process happening, and beyond standard model physics. This is unfolded and used to set a limit on the Lorentz Violating parameter $\hat{a}_{of}^{(3)}$ described in Section 2.8.1.1. This parameter is not expected to modify the angular distributions of emitted electrons.

This one-dimensional unfolding, however, does not capture the full event kinematics and is therefore insufficient for providing a generalised unfolded spectrum for model evaluation. Each event in this spectrum is the combination of two electrons with possibly different energies and could be unfolded into different bins if the individual electron energy spectra were considered. To measure this a further dimension is added to the spectrum to be unfolded: electron energy asymmetry, defined as

$$A = \frac{|E_{e_1} - E_{e_2}|}{E_{e_1} + E_{e_2}}, \quad (6.3)$$

where E_{e_1} and E_{e_2} are the energies of the individual electrons in the event. To capture the topology of the event a third dimension is added to the spectrum to be unfolded, $\cos(\theta)$ where θ is the angle between the two electrons at emission. Including all three dimensions ensures that assumptions on the relative electron energies or opening angles, both of which affect detector acceptance of the event, are not inherent in the final unfolded histogram. Both the summed energy spectrum alone and three-dimensional unfolded distributions were produced, with the latter providing less granular but more generalised output distributions.

6.3 Binning

The bin size for the one-dimensional unfolding was chosen to be of the order of the calorimeter resolution, maximising the sensitivity to the shape of the distribution, and consequently the limits placed on $\hat{a}_{of}^{(3)}$ in Section 6.7.2.

The binning for the three-dimensional distribution was chosen in consideration of several conflicting requirements. The number of bins in the full distribution is

on the order of n^3 and the number of entries in the unfolding matrix n^6 , where n is the number of bins in the projections onto the summed energy, energy asymmetry and $\cos(\theta)$ axes. As in the one-dimensional case more numerous, smaller, bins capture more of the shape of the distribution but rapidly increase the complexity of determining the order n^6 unfolding matrix. Five equally sized and spaced bins were chosen to cover the relevant phase space in each of the three variables, producing a full distribution containing 125 bins, and an unfolding matrix with 15,625 entries. This captures the key elements of the shape of each dimension without producing many highly correlated adjacent bins: Table A.2 shows only one off-diagonal pair of bins with a correlation above 0.9.

For the purposes of visualisation, background fitting, and measurement of systematic uncertainty, this three-dimensional distribution was manipulated in unrolled form. To construct this one-dimensional histogram the distribution was plot on a single axis, where the integer value at each bin's low edge maps to a unique index defining the bin position in the three-dimensional axes. This mapping is shown in the headers of Table A.2.

6.4 Background Events

Table 6.2: Background MC Samples

Decay (neutrinos omitted)	Source	Location
$^{60}\text{Co} \rightarrow ^{60}\text{Ni} + e^- (+\gamma)$	^{59}Co Neutron Capture	Petals (Cu), Tower (Cu), Frame (Steel), Mu-Metal, Internal to Foils
$^{40}\text{K} \rightarrow ^{40}\text{Ca} + e^- (+\gamma)$ $^{40}\text{K} + e^- \rightarrow ^{40}\text{Ar} (+\gamma)$	Natural Contaminant	PMTs, Scintillators, Frame (Steel), Internal to foils
$^{228}\text{Ac} \rightarrow ^{228}\text{Th} + e^- (+\gamma)$	^{232}Th chain	PMTs, Frame (Steel), Shielding (Fe), Internal to foils
$^{208}\text{Tl} \rightarrow ^{208}\text{Pb} + e^- (+\gamma)$	^{232}Th chain	PMTs, Frame (Steel), Shielding (Fe), Tracker gas, Tracker wire surfaces, Internal to foils
$^{212}\text{Bi} \rightarrow ^{212}\text{Po} + e^- (+\gamma)$	^{232}Th chain	Internal to foils
$^{214}\text{Pb} \rightarrow ^{214}\text{Bi} + e^- (+\gamma)$	^{238}U chain	Tracker gas, Tracker wire surfaces, Foil surfaces, Scintillator surfaces, Scintillator Mylar, Internal to foils
$^{214}\text{Bi} \rightarrow ^{214}\text{Po} + e^- (+\gamma)$	^{238}U chain	PMTs, Frame (Steel), Shielding (Fe), Tracker gas, Tracker wire surfaces, Foil surfaces, Scintillator surfaces, Scintillator Mylar, Internal to foils
$^{210}\text{Bi} \rightarrow ^{210}\text{Po} + e^- (+\gamma)$	^{238}U chain	Tracker wire surfaces, Foil surfaces
$^{234m}\text{Pa} \rightarrow ^{234}\text{U} + e^- (+\gamma)$	^{238}U chain	Scintillator surfaces, Internal to foils
Other γ	Neutron Capture	Mu-Metal
$^{207}\text{Tl} \rightarrow ^{207}\text{Pb} + e^- (+\gamma)$	Natural Contaminant	Internal to foils
$^{137}\text{Cs} \rightarrow ^{137(m)}\text{Ba} + e^- (+\gamma)$	Natural Contaminant	Internal to foils
$^{211}\text{Pb} \rightarrow ^{211}\text{Bi} + e^- (+\gamma)$	Natural Contaminant	Internal to foils
$^{88}\text{Y} \rightarrow ^{88}\text{Sr} + e^+ (+\gamma)$	Natural Contaminant	Internal to foils

Monte Carlo simulations of the background processes described in Table 6.2 were generated at the corresponding locations, assigned to runs, and processed through

the same cut flow as the data sample. Each sample was weighted to the measured activity of the background source for the relevant run which are constrained by analyses measuring the number of gamma, alpha and beta particles, their distributions through the detector, and their timings. An example of such an analysis is shown in Figure 6.1 where the quantity of ^{222}Rn in the tracker gas is constrained through measurements of the electron and gamma ray energies in events containing exactly one of each and originating within the tracking volume. These events correspond to ^{214}Bi beta decay - a ^{222}Rn daughter isotope - which is often accompanied by a high energy gamma ray, although a component originating from external radioactivity producing Compton scattering within the detector is also considered.

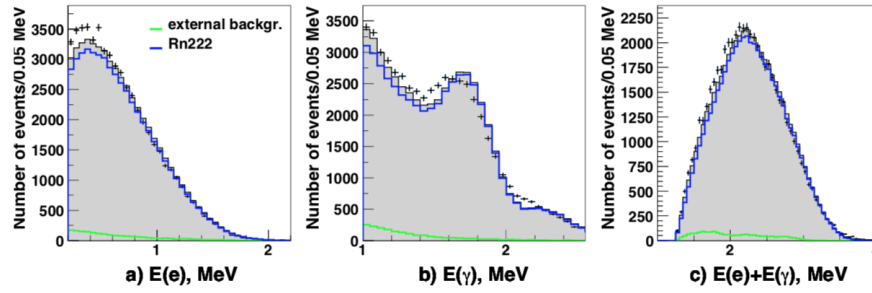


Figure 6.1: The a) electron, b) gamma-ray and c) total energy of $e\gamma$ events originating within the tracking volume, and best fit ^{222}Rn and external background activities [75].

6.4.0.1 Background Categorisation

The different sources of background are categorised into three classes, according to their provenance within the detector. These correspond to the Monte-Carlo simulated locations listed in Table 6.2, and are as follows:

- Internal: decaying contaminants within the source foils, corresponding to samples generated at locations “Internal to foils”,
- Radon: Radon daughter nuclei within the gas volume, corresponding to samples generated in the “Tracker gas”, and
- External: decaying contaminants on detector surfaces or activated detector materials, corresponding to samples generated at all other locations.

For internal backgrounds the emitted electron beta decay of a radioactive contaminant may be detected in conjunction with a second electron, ejected from the foils through a Compton or Møller scattering process, as shown in Figure 6.2.

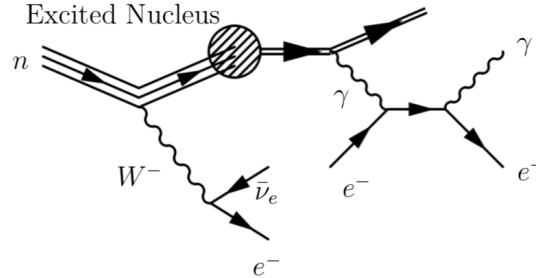


Figure 6.2: An internal background process, where a β decay of a foil-contaminant is followed by a nuclear de-excitation and Compton-scattered electron [14].

For external and Radon backgrounds, a photon released from a decaying nucleus can traverse the tracking volume undetected, before interacting in the foil to eject two electrons through a chain of Compton and Møller scattering events, such as the process depicted in Figure 6.3.

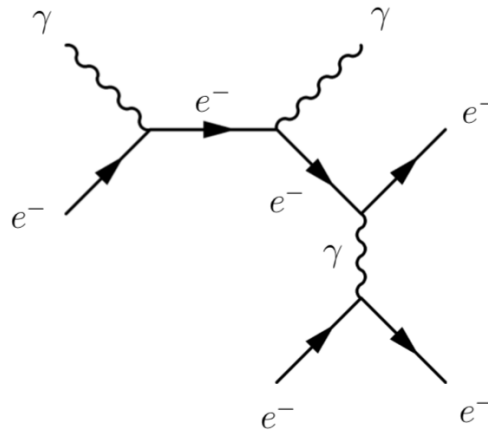


Figure 6.3: An external background process, where a photon ejects two electrons from the source foil through Compton and Møller scattering [14].

The production of an electron-positron pair from a photon near the foil may also mimic the double-beta decay topology, through misidentification of the positron as an electron. Each of these cases is detected as two electrons as being

emitted from a single location in the source foils; such events are difficult to disentangle from the double-beta decay signal through topological cuts alone.

6.4.0.2 Background Fitting

By performing a maximum likelihood fit to several measured quantities (including the summed energy, asymmetry, $\cos \theta$ distributions) after cuts, an estimate of background events in our data sample was obtained and subtracted from the 3 dimensional histogram described in Section 6.2 to produce a background-subtracted histogram. This background model provides a good description of the summed energy and asymmetry distributions, as shown in Figures 6.4 and 6.5, but a data-MC mismatch is evident in the $\cos \theta$ distribution in Figure 6.6.

The three categories of background events are coloured independently in these Figures, with the “Internal” category further subdivided into events originating in the metallic and the composite ^{100}Mo foils. Also shown are the estimated number of signal events from the metallic (MoMet) and composite (MoCom) foils in each phase of data taking (P1 and P2).

To first order, the background normalisations are constrained by measurements of activities across multiple decay channels repeated over years of operation and calibration. However the fine-tuning of these normalisations through the maximum likelihood fit described above suffers from an overestimation of the data constraints by treating each distribution as independent. It is likely that this results in an underestimation of the uncertainty on the background normalisations, although one which is rendered insignificant by the negligible level of these backgrounds and associated uncertainties in the final event sample.

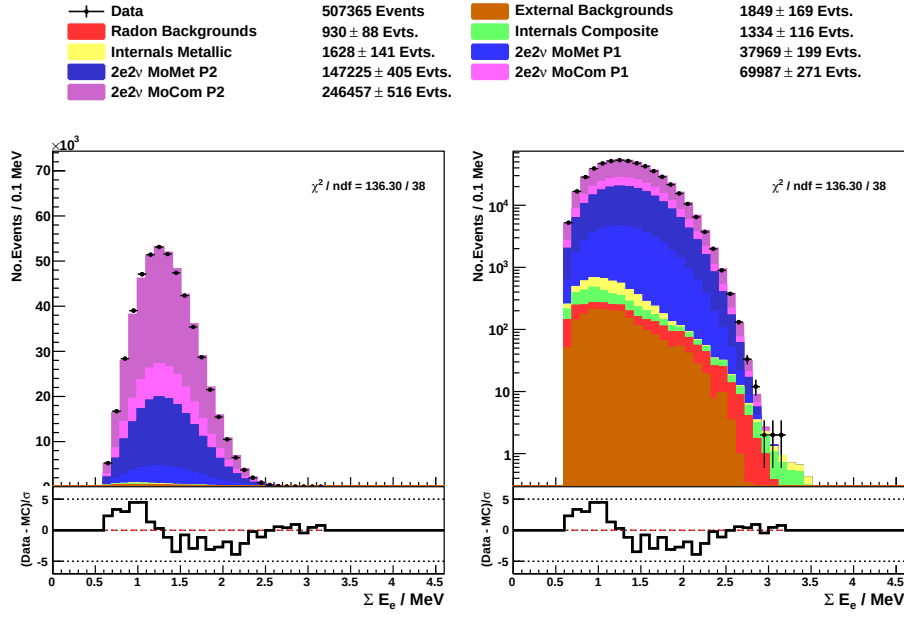


Figure 6.4: Background model and the summed electron energy distribution, with a linear (left) and log (right) scale.

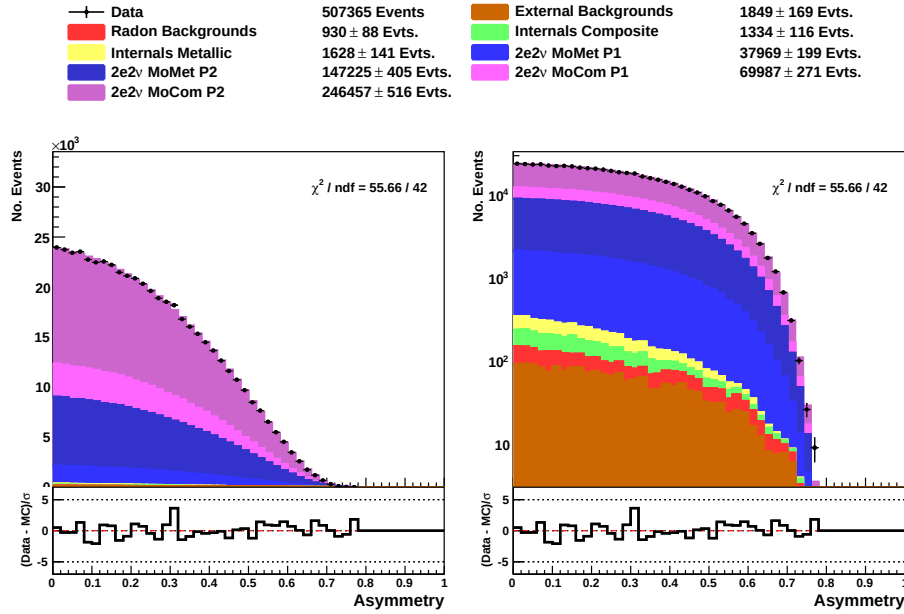


Figure 6.5: Background model and the electron energy asymmetry distribution, with a linear (left) and log (right) scale.

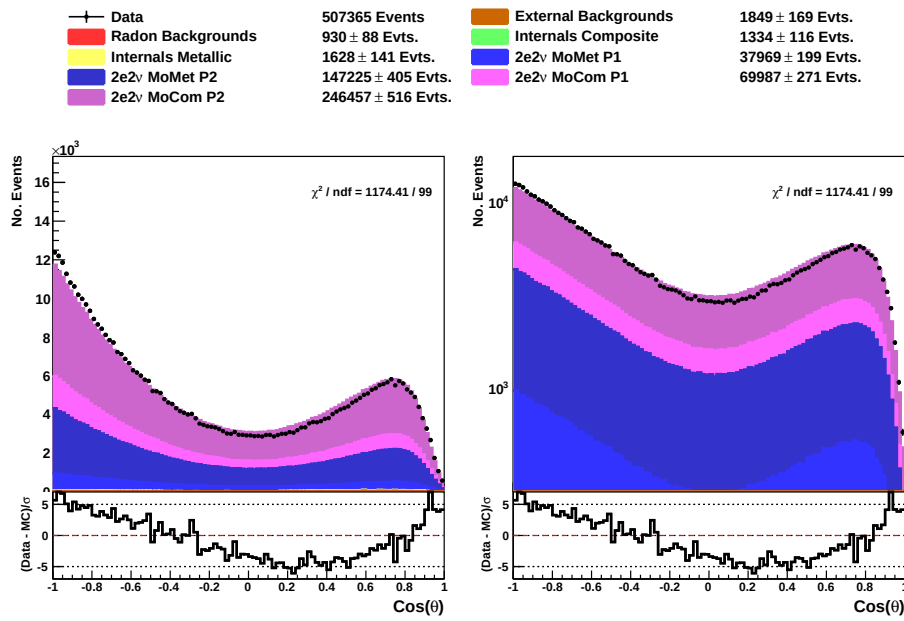


Figure 6.6: Background model and the $\cos(\theta)$ distribution, with a linear (left) and log (right) scale.

This mismodelling of the opening angle between the emitted electrons leads to a poor χ^2 between the overall three-dimensional distribution and the best-fit Monte-Carlo model, leading to the periodic structure in the ratio plot of Figure 6.7. The measurement of this uncertainty and its impact on the unfolded distribution is discussed in Section 6.5.2.6.

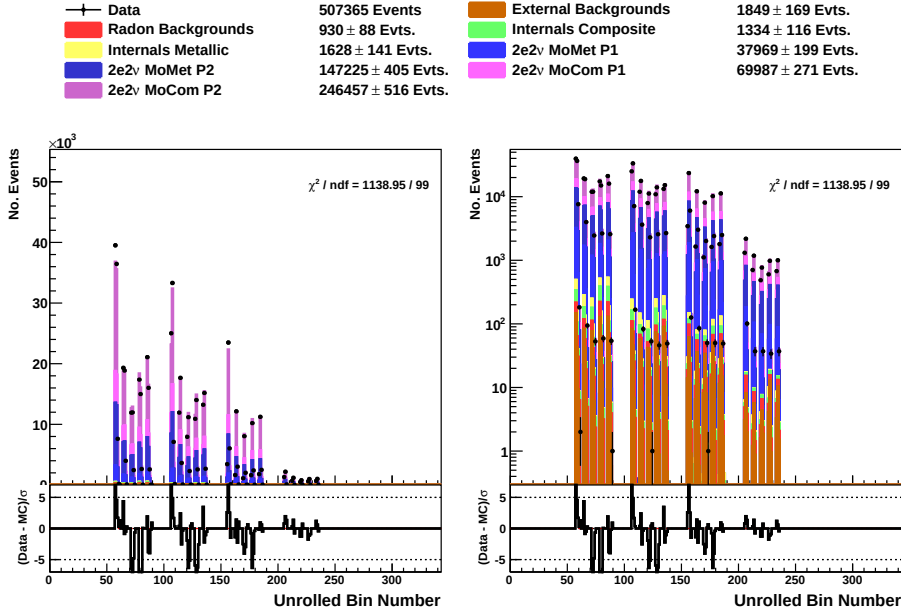


Figure 6.7: Histogram of summed energy, energy asymmetry and $\cos(\theta)$ with backgrounds. Mapping between the unrolled bin number, defined by the bin low edge, and values of the unfolding variables is shown in Table A.2.

6.5 Uncertainties

Two primary categories of systematic error were considered in this analysis: those arising from the unfolding procedure and those specific to the detector and physics modelling of the NEMO-3 experiment. The measurement of the overall ^{100}Mo $2\nu\beta\beta$ half-life is independent of the unfolding procedure and the associated errors, but all unfolded results presented have quantified errors across both categories.

6.5.1 Unfolding Systematics

Systematic error is inherent in the unfolding process in two forms: statistical errors on the Monte Carlo data used to populate the response matrix and bin efficiency vec-

tor, and a bias towards the prior used for the first iteration of the procedure, which in this case is the Monte Carlo truth distribution. While the former is inherently a statistical error, it may be reduced through generation of more Monte Carlo events, and is therefore categorised as a systematic error of the unfolding procedure. For this analysis Monte Carlo events were generated until this source of error was lower in magnitude than the errors corresponding to detector and mismodelling systematics, resulting in approximately 7 times as many Monte Carlo events as data events after reconstruction and cuts.

The bias towards the unfolding prior is measured by a data-driven closure test proposed for, and used by, data analyses by the ATLAS collaboration [76]. The ratio of the reconstructed data histogram to Monte Carlo is fit by a smooth function, in this case a polynomial, categorising the scale of data/MC disagreement over the phase space. This function is used to re-weight the response matrix in such a way as to reduce this disagreement, with the reweighted matrix then being projected to produce a reweighted Monte Carlo truth distribution. The reconstructed reweighted Monte Carlo distribution is then unfolded with the nominal response matrix, and the difference between this histogram and the reweighted Monte Carlo truth distribution is the unfolding error.

This is designed to measure the sensitivity of the unfolding to the distributions used to generate the response matrix - unfolding a reweighted distribution with a nominal response matrix built from a different Monte Carlo truth distribution captures the extent to which the unfolding procedure is biased by the Monte Carlo truth when it is inaccurate at a scale defined by the data/MC discrepancy. To avoid overfitting and washing out localised signals within the data, it is imperative to fit with a smooth function and not a bin-by-bin reweighting of the Monte Carlo data. This function is shown for the 1D and 3D cases in Figures 6.8 and 6.9.

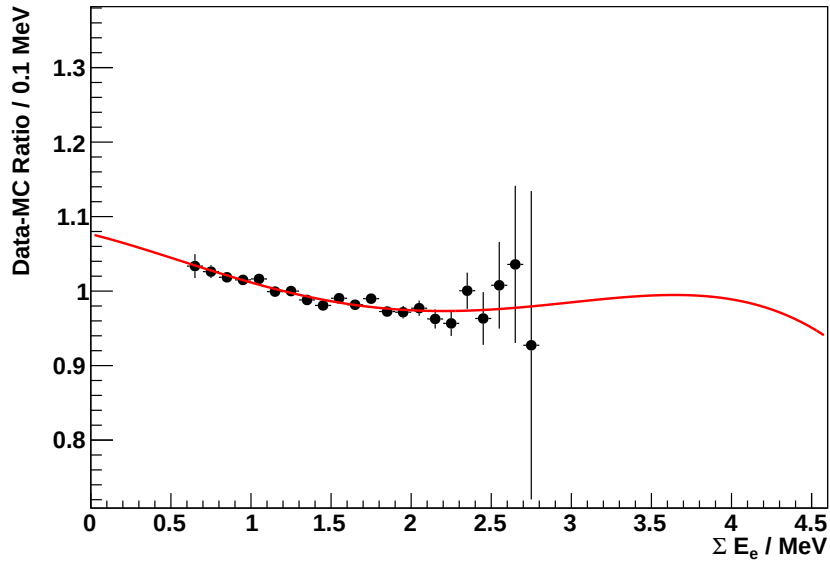


Figure 6.8: Reweighting function for 1D unfolding.

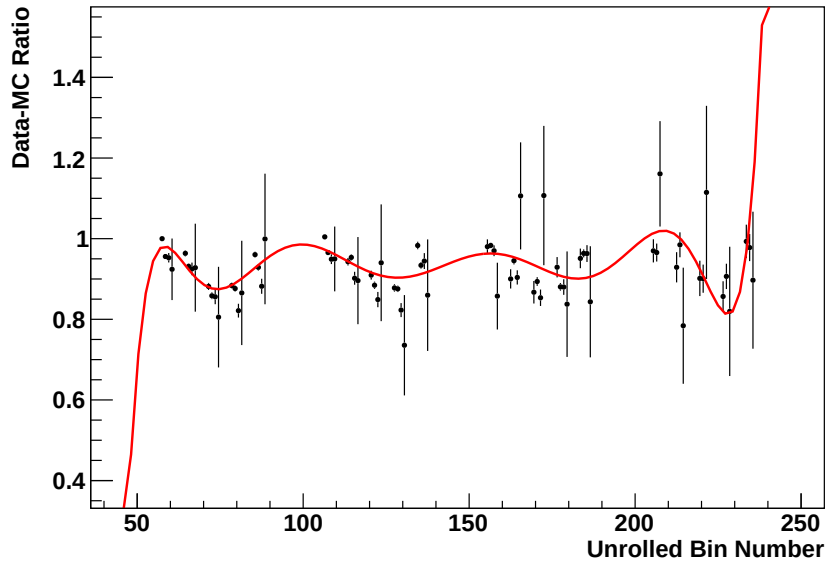


Figure 6.9: Reweighting function for 3D unfolding.

As seen in 6.9, the reweighting function for the three-dimensional distribution was determined by a fit to the unrolled, one-dimensional histogram. This introduces an arbitrary constraint to the fit function, requiring continuity across neighbouring bins which may be separated in three-dimensional parameter space. While such

a procedure contains sufficient flexibility to capture the local fluctuations in the data-MC ratio, a more principled approach would involve independently fitting a piecewise function to the four subsections of consecutive bins. It is not expected that a change of reweighting function would have a significant impact on the final error estimations in this analysis, which motivated the choice of a more simple polynomial fit, but in general the removal of this unphysical constraint is preferable.

The error defined by this quantity also plays a role in choosing the number of iterations for unfolding. If, after increasing the number of iterations, the change in output histograms is on the order of this unfolding error, no new information is being gained and continuing the process merely increases the error. Through this process it was found that 1 and 3 iterations were optimal for the 1D and 3D cases respectively. As discussed in Section 3.2.2.2, small numbers of iterations are typical for smooth distributions with sensible priors.

6.5.2 Detector Systematics

The NEMO-3 specific sources of uncertainty include:

- the total efficiency of the NEMO-3 detector,
- the exact mass of ^{100}Mo within the source foils,
- the number of background events in the data sample,
- energy misreconstruction due to known detector effects,
- general mismodelling of the detector energy response function, and
- general mismodelling of the detector angular response function.

The first three of these uncertainties impact the calculation of total number of ^{100}Mo double-beta decay events in the data sample, and therefore the measured half-life and the overall normalisation of the unfolded distributions. The final three uncertainties change the shape of the recorded, and therefore unfolded, distributions and are accordingly marked on the unfolded distribution legends. The methods for calculating the magnitude of their effects is presented below.

6.5.2.1 Detection Efficiency

The uncertainty on the NEMO-3 detection efficiency for double-beta decay was measured through the insertion of ^{207}Bi sources into the detector and selecting events where two electrons from internal conversion, Møller, or Compton scattering - are emitted from the source, providing a topology comparable with double-beta decay. With a known detection efficiency, the number of detected events can be used to measure the activity of the sources, with the result cross checked against an independent measurement to validate the estimate of detection efficiency.

This calibration process was carried out three times during the lifetime of the NEMO-3 experiment, in March 2004, June 2004 and April 2006. The activities of the sources were independently measured in two High Purity Germanium (HPGe) detectors (referred to as Geranium and Jasmin) in the LSM in April 2003, with the results shown in Table 6.3. The systematic error on this measurement is estimated at 5% [77].

Source	Geranium (Bq)	Jasmin (Bq)	Mean (Bq)
72	180 ± 3	177 ± 3	178.5 ± 2.1
76	179 ± 3	176 ± 2	177.5 ± 1.8

Table 6.3: High Purity Germanium (HPGe) activity measurements of ^{207}Bi calibration sources with statistical errors [77].

To estimate the activity of the sources with the NEMO-3 detector, a χ^2 fit was performed and the best fit value extrapolated back to the time of the HPGe measurements using the known ^{207}Bi half-life of 31.55 ± 0.04 yr. A comparison of these results with the HPGe measurements is shown in Figure 6.10. The combined source activity was measured by NEMO-3 to be 355.5 ± 3.2 Bq when extrapolated back to the time of the HPGe measurements, which is in good agreement with the activity of 356.0 ± 2.8 Bq measured by the HPGe detectors. Consequently, the 5% uncertainty of the HPGe measurements was taken as the best estimate for the uncertainty on the NEMO-3 detection efficiency for double-beta decay.

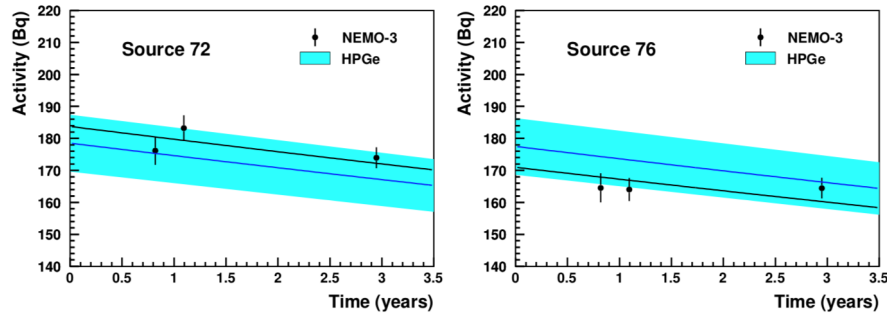


Figure 6.10: NEMO-3 and HPGe measurements of ^{207}Bi sources. The blue band represents a window of 5% uncertainty around the combined best fit activity [77].

6.5.2.2 ^{100}Mo Mass

Exact knowledge of the amount of source mass within the composite foils is limited primarily by an unknown quantity of residual PVA glue within the composite foils when the ^{100}Mo was being extracted for weighing. No such error existed for the metallic foils giving an error defined only by inherent inaccuracies of the mass measurement. The difference in masses as measured before construction and after decommissioning is used to constrain this error on the overall mass of ^{100}Mo to 0.3% [77].

6.5.2.3 Background Normalisation

The normalisation of the background Monte-Carlo samples is defined by the fitting described in Section 6.4.0.2. Errors on this normalisation are output from the fitting procedure, but due to the extremely low background nature of the experiment has a negligible impact on the half-life measurement. While the previous three uncertainties are expected to be flat across the unfolding variables, and therefore represent an error in overall normalisation only, background normalisation is expected to affect the shape of the final distribution and is calculated in the unfolding by reweighting the background subtracted data according to the uncertainties on the background fitting, and measuring the spread when unfolded using the nominal response matrix.

6.5.2.4 Energy Misreconstruction

Two effects limiting knowledge of the electron energies are calibration uncertainties on the PMTs and modelling of the rate of energy loss as electrons propagate through the tracking volume. Monte Carlo data has been generated for varying values of the parameters defining these models and their effect on the summed electron energy distribution studied. For calibration uncertainties there are two parameters, as an energy is calculated from the raw ADC counts through the equation:

$$E = \alpha \times \text{ADC} + \beta, \quad (6.4)$$

where α and β are calibration parameters. The effect of changing these parameters within their uncertainty - $\pm 0.2\%$ on α and ± 2.5 keV on β - is shown in Figure 6.11 [78]. The same effect has been calculated for the uncertainty on dE/dx as shown in Figure 6.12.

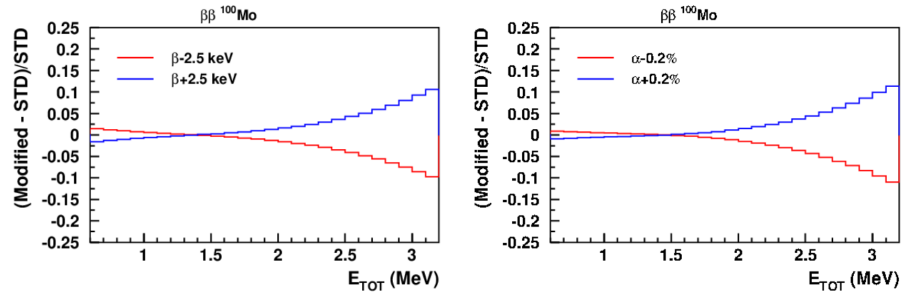


Figure 6.11: Effect of α and β uncertainty on the summed electron energy spectrum [78].

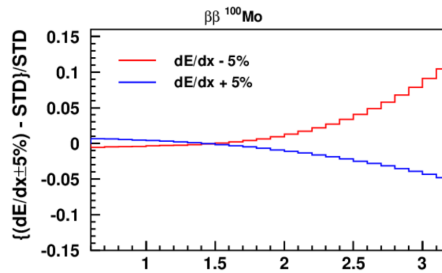


Figure 6.12: Effect of dE/dx uncertainty on the summed electron energy spectrum [78].

To account for these effects the data was perturbed subject to these uncertainties and unfolded, the difference from the nominal spectrum defining the error.

6.5.2.5 Energy Mismodelling

Generalised Monte-Carlo mismodelling of electron energies was measured independently through the use of a ^{90}Sr beta-decay calibration source of known activity within the detector. A data-MC mismatch was observed and parametrised as shown in Figure 6.13.

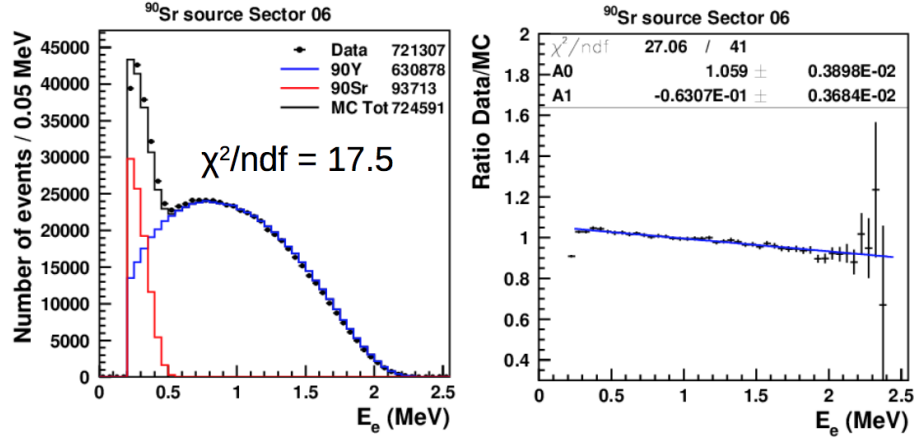


Figure 6.13: Data-MC mismatch for ^{90}Sr calibration source [79].

To account for this error the response matrices were reweighted using measured data-MC correction factors, and the difference between the unfolded spectra from the nominal and reweighted matrices was taken as the error.

6.5.2.6 Topological Mismodelling

The distribution of the opening angle between the two emitted electrons in the ^{100}Mo metallic foils were found to be in worse agreement with Monte Carlo simulations than those of the composite foils. The thinness of the foils requires a smaller than optimal number of GEANT step simulations within the foils, and therefore modelling of electron scattering within these foils is thought to be degraded [77]. An estimate of the size of this error on half-life measurement can be obtained by a comparison of the half-life measurements from composite and metallic foils, giving a value of $3.9 \pm 0.3\%$.

^{207}Bi calibration sources in the detector were used to model the total data-MC shape discrepancy, with the same correction factor extraction and matrix reweight-

ing process as in Section 6.5.2.5 defining the error on the unfolded histograms. The discrepancy and correction factors are shown in Figure 6.14.

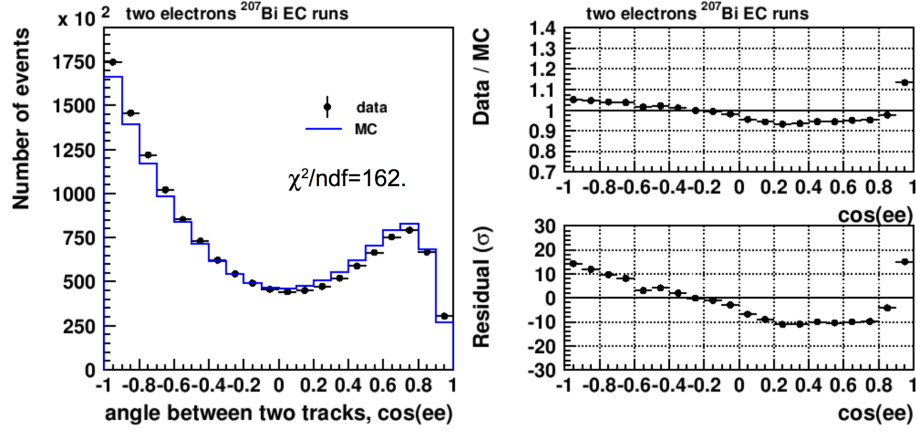


Figure 6.14: Data-MC mismatch for ^{207}Bi calibration source [80].

6.6 Half-life calculation

To calculate the measured half-life of the $2\nu\beta\beta$ decay of ^{100}Mo , one can perform a Taylor expansion of the exponential decay formula

$$N(t) = N_0 e^{-\lambda t} \approx N_0 (1 - \lambda t) \quad (6.5)$$

where N_0 is the number of initial nuclei which can undergo decay, N_t is the number of nuclei present at time t , and the decay constant λ is related to the half-life $T_{\frac{1}{2}}$ by

$$T_{\frac{1}{2}} = \frac{\ln(2)}{\lambda}. \quad (6.6)$$

This value can be expressed in terms of experimental quantities by noting that the number of decays observed during a given time t_{live} , for a detection efficiency ε is

$$N_{\text{obs}} = \varepsilon (N_0 - N(t_{\text{live}})) = \varepsilon N_0 \lambda t_{\text{live}}. \quad (6.7)$$

The number of initial nuclei, given an initial total mass of the decaying isotope m , its atomic mass M , and Avogadro's constant N_A , is:

$$N_0 = \frac{N_A m}{M}. \quad (6.8)$$

Substitution of 6.6 and 6.8 into 6.7 and rearranging then gives the following formula for calculation of the half-life:

$$T_{\frac{1}{2}} = \frac{\epsilon}{N_{obs}} \frac{N_A m}{M} \ln(2) t_{live}. \quad (6.9)$$

t_{live} can be found by adding together the data collection time of each run that passes the run selection described in Section 6.1, with N_{obs} being the number of data events passing the full selection. ϵ was determined through the Monte-Carlo generation of 40 million $2\nu\beta\beta$ events in the ^{100}Mo foils and observing the fraction of events passing all cuts. Table 6.4 shows the values of parameters required to generate a half-life calculation from our dataset.

Parameter	Value	Source
ϵ	2.32%	Monte-Carlo simulation
N_{obs}	501,624	Background-subtracted data
N_A	$6.022 \times 10^{23} \text{ mol}^{-1}$	Physical constant
$m_{^{100}\text{Mo}}$	6914 g	Foil measurements
M	$9990.7 \text{ g mol}^{-1}$	Physical constant
t_{live}	1918 days	Run list and measured dead time

Table 6.4: Input parameters for half-life calculation

Using these parameters, the half-life for ^{100}Mo $2\nu\beta\beta$ decay is measured to be $7.01 \pm 0.01(\text{stat}) \pm 0.46(\text{syst}) \times 10^{18}$ years, in agreement with the result of $7.11 \pm 0.02(\text{stat}) \pm 0.54(\text{syst}) \times 10^{18}$ years published after the first phase - 20 months - of detector operation [81]. This is compatible with, and more precise than, the leading published value of $6.90 \pm 0.15(\text{stat}) \pm 0.36(\text{syst}) \times 10^{18}$ years from the

CUPID collaboration [82], providing the world-leading measurement of the ^{100}Mo double-beta decay half-life.

6.7 Unfolded Spectra

6.7.1 Closure Test

An initial test of the validity of the unfolding procedure was carried out by generating a sample of $2\nu\beta\beta$ events with a perturbed summed energy spectrum, which was passed through the NEMO-3 detector simulation to provide a set of pseudodata of comparable statistics to the nominal dataset. This pseudodata was then unfolded using the nominal response matrix and the same procedure used on the real data. The unfolded summed energy spectrum was found to lie close to the (perturbed) truth Monte-Carlo distribution, and far from the nominal truth distribution used in training, after only two iterations as can be seen in Figure 6.15. The perturbed spectrum used for this test was the standard spectrum with a Lorentz Violating contribution as described in Section 2.8.1.1 with parameter $\hat{a}_{of}^{(3)} = 1 \text{ MeV}$.

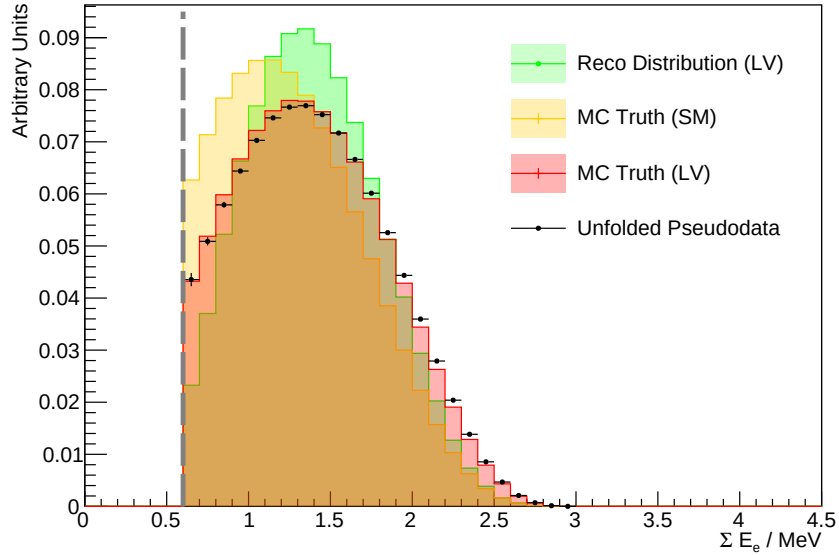


Figure 6.15: Closure check of unfolding technique. LV Parameter $\hat{a}_{of}^{(3)} = 1 \text{ MeV}$.

6.7.2 One-Dimensional Unfolding

While the unfolding procedure was carried out on real data across a wide range of phase space, due to the minimum energy cut described in Section 6.1 no information is contained in the data for a summed electron energy below 600 keV; as such plots are only presented for bins containing events above this threshold. The unfolded distribution in the summed electron energy dimension only is shown in Figure 6.16. To obtain an experiment agnostic quantity of interest, the number of events recorded after unfolding is divided by the total exposure of the experiment to obtain a differential decay rate $d\Gamma$ with units s^{-1}/bin .

Each point on the unfolded distribution is marked by three types of markers and a shaded region. They indicate respectively the magnitude of cumulative errors from propagated data statistical errors, unfolding systematics, detector shape systematics and detector normalisation systematics.

This histogram can be used to constrain models without the need for a simulation of the NEMO-3 detector. Such an analysis was carried out to set limits on the $\hat{a}_{of}^{(3)}$ parameter described in Section 2.8.1.2. Monte Carlo simulations of summed energy distributions generated according to the dispersion relation defined by the Lorentz Violating perturbation in Equation 2.26 were added to the nominal summed energy distribution with varying weights, where the weight corresponds to the $10 \cdot \hat{a}_{of}^{(3)}$ strength of the Lorentz Violating perturbation term in Equation 2.27. The weighting that minimised the χ^2 statistic between the LV-perturbed summed energy distribution and the unfolded distribution in 6.16, as found through SIMPLEX minimisation [74], was taken as the best fit value for $10 \cdot \hat{a}_{of}^{(3)}$, where the prefactor of 10 weights the relative size of the Lorentz Violating contribution as seen in Equation 2.27.

The χ^2 statistic was calculated on bins lying in the range $[0.6, 2.6]$ MeV to ensure only well-populated bins were included in the calculation, resulting in a total of 20 degrees of freedom. The $\chi^2/\text{d.o.f}$ statistic for various values of $\hat{a}_{of}^{(3)}$ and an approximately parabolic curve of best fit centred on the minimum value is shown in Figure 6.17, along with the upper bound on $\chi^2/\text{d.o.f}$ corresponding to a

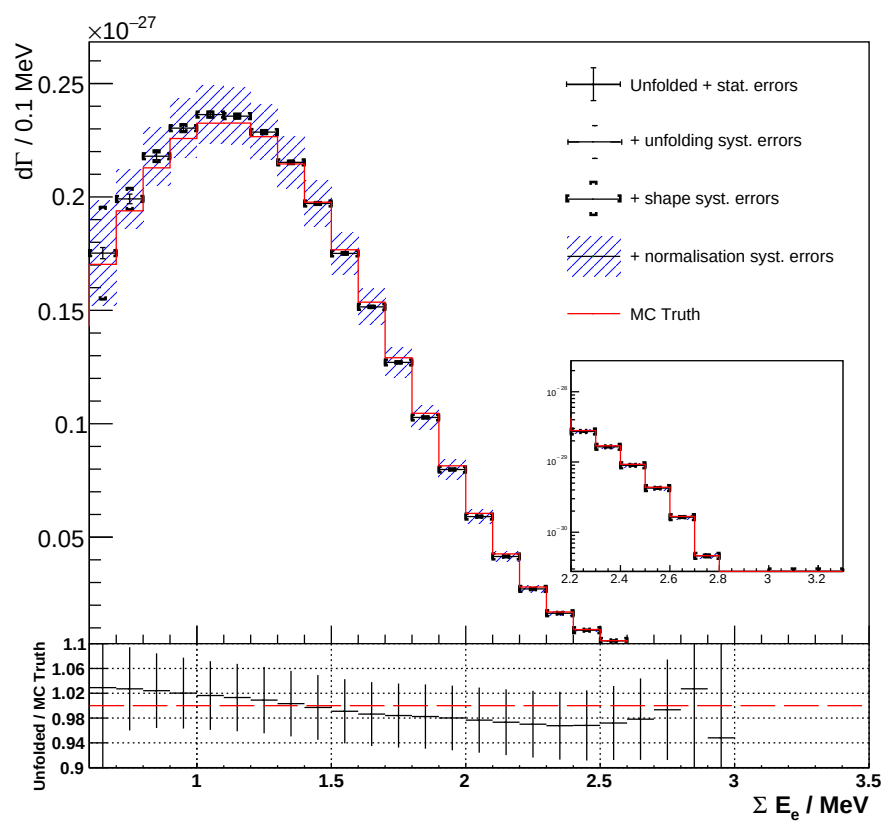


Figure 6.16: Above: Unfolded ^{100}Mo $d\Gamma/\text{summed electron energy}$ distribution (one-dimensional unfolding), with the tail of the distribution shown on a log-scale (inset). Below: Unfolded/MC Truth ratio, shown with total errors.

90% confidence level.

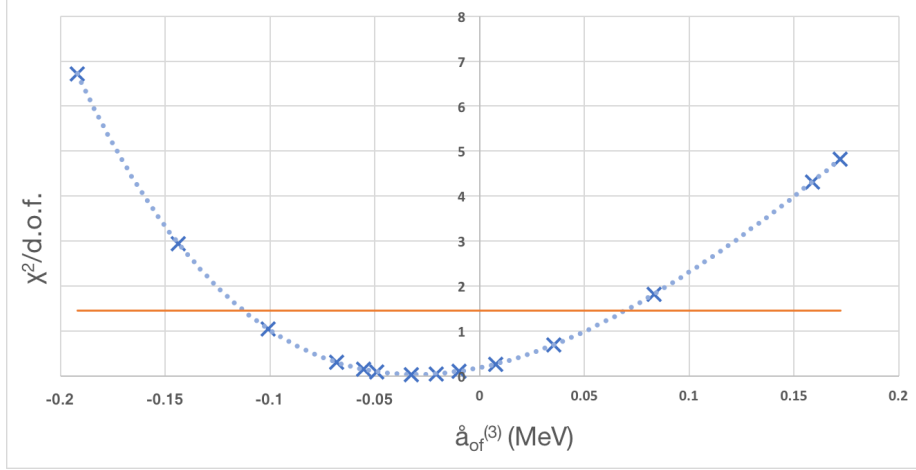


Figure 6.17: $\chi^2/\text{d.o.f.}$ for various values of $\dot{a}_{of}^{(3)}$ and 90% CL boundary.

From this distribution the best-fit value of $\dot{a}_{of}^{(3)} = -3.03 \times 10^{-2}$ MeV was obtained, with 90% CL bounds -1.12×10^{-1} MeV $< \dot{a}_{of}^{(3)} < 6.95 \times 10^{-2}$ MeV. These values are comparable to the strictest published bounds [60] on this parameter (-2.65×10^{-2} MeV $< \dot{a}_{of}^{(3)} < 7.60 \times 10^{-2}$ MeV) - slightly improving on the upper bound - and were set with no information on the NEMO-3 detector beyond that inherent to the distribution shown in Figure 6.16.

6.7.3 Three-Dimensional Unfolding

Subsequently, the technique was applied to the full three dimensional distribution. The projection of the unfolded distribution onto the three axes of summed electron energy, energy asymmetry and $\cos(\theta)$ are shown in the Figures 6.18-6.23, compared to the nominal Monte Carlo truth distribution of $2\nu\beta\beta$ events. In the following tables, errors are grouped into two categories: statistical and systematic errors that may affect the shape of the distribution - including those errors arising from the unfolding technique - are combined into the “shape + unfolding” category, while sources of error which affect the overall normalisation of the distribution, leaving the shape unchanged, are presented independently.

A list of the contributions from each individual source of error to a typical bin of a one-dimensional projection is shown in Table 6.8, where for illustrative purposes the bin with the largest relative error is chosen.

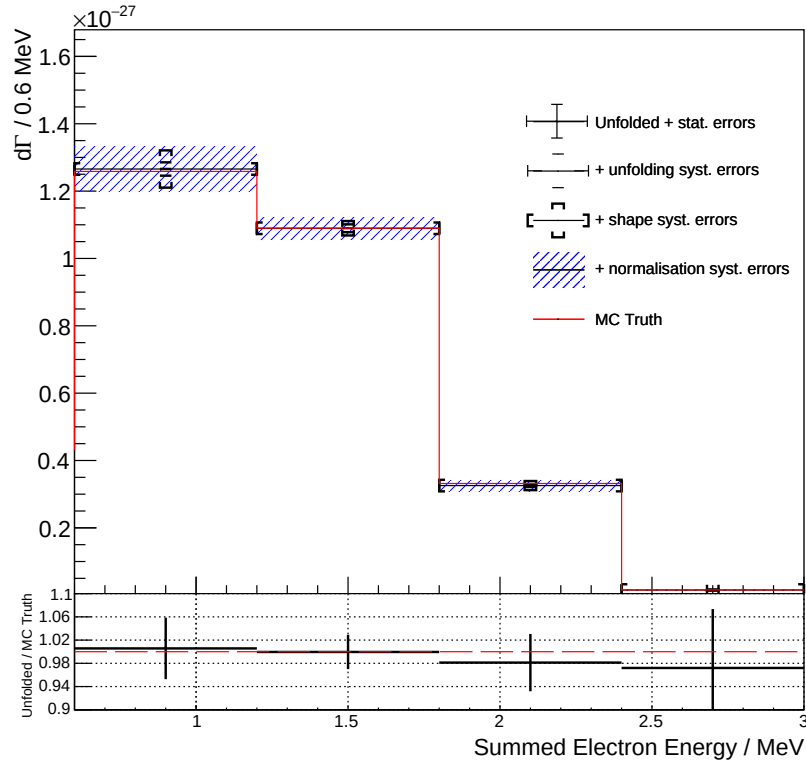


Figure 6.18: Unfolded ^{100}Mo $d\Gamma/\text{summed electron energy}$ distribution (3D unfolding).

Bin central value (MeV)	0.9	1.5	2.1	2.7
Unfolded distribution	1.265×10^{-27}	1.090×10^{-27}	3.255×10^{-28}	1.530×10^{-29}
Shape + unfolding errors	$\pm 5.533 \times 10^{-29}$	$\pm 2.138 \times 10^{-29}$	$\pm 1.351 \times 10^{-29}$	$\pm 1.465 \times 10^{-30}$
Normalisation errors	$\pm 1.267 \times 10^{-29}$	$\pm 1.177 \times 10^{-29}$	$\pm 3.522 \times 10^{-30}$	$\pm 1.570 \times 10^{-31}$

Table 6.5: Unfolded ^{100}Mo $d\Gamma/\text{summed electron energy}$ distribution (3D unfolding).

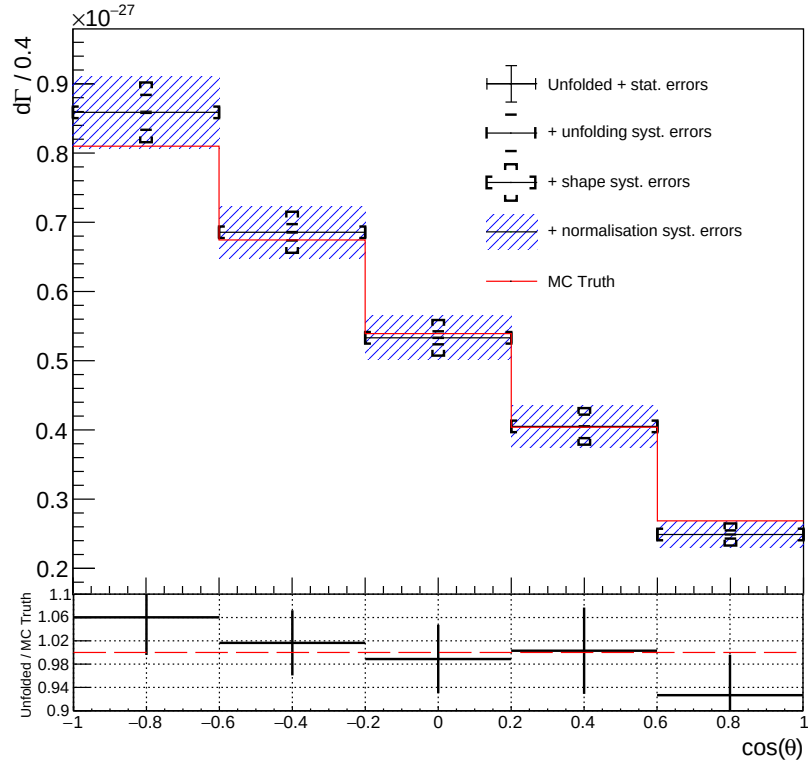


Figure 6.19: Unfolded ^{100}Mo $d\Gamma/\text{opening angle}$ distribution (3D unfolding).

Bin central value	-0.8	-0.4	0	0.4	0.8
Unfolded distribution	8.588×10^{-28}	6.855×10^{-28}	5.331×10^{-28}	4.051×10^{-28}	2.489×10^{-28}
Shape + unfolding errors	$\pm 4.315 \times 10^{-29}$	$\pm 2.963 \times 10^{-29}$	$\pm 2.570 \times 10^{-29}$	$\pm 2.642 \times 10^{-29}$	$\pm 1.593 \times 10^{-29}$
Normalisation errors	$\pm 1.162 \times 10^{-29}$	$\pm 9.331 \times 10^{-30}$	$\pm 7.195 \times 10^{-30}$	$\pm 5.169 \times 10^{-30}$	$\pm 3.293 \times 10^{-30}$

Table 6.6: Unfolded ^{100}Mo $d\Gamma/\text{opening angle}$ distribution (3D unfolding).

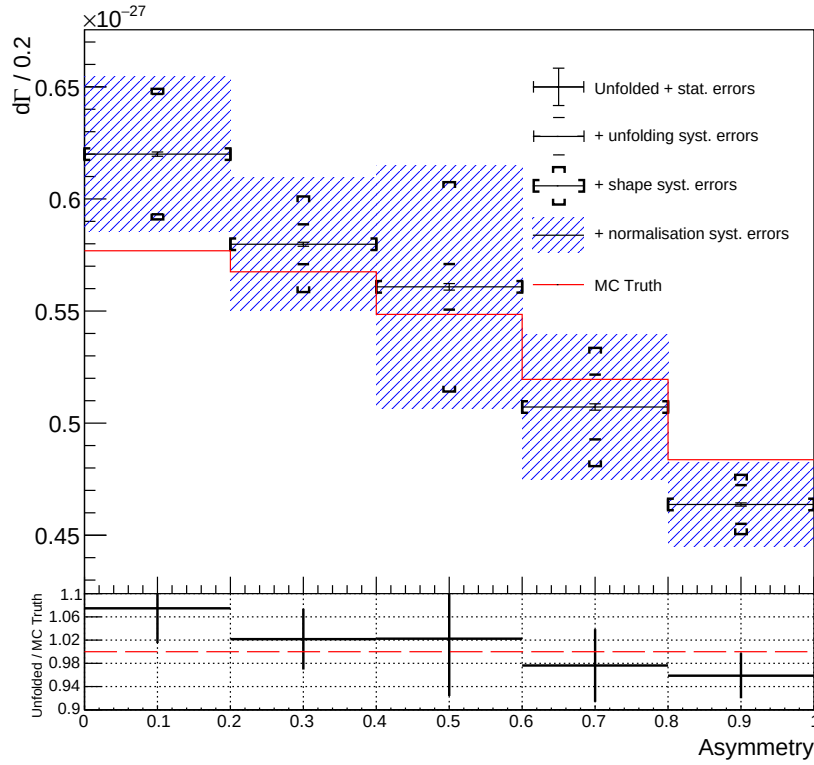


Figure 6.20: Unfolded ^{100}Mo $d\Gamma/\text{electron energy asymmetry}$ distribution (3D unfolding).

Bin central value	0.1	0.3	0.5	0.7	0.9
Unfolded distribution	6.200×10^{-28}	5.798×10^{-28}	5.608×10^{-28}	5.072×10^{-28}	4.637×10^{-28}
Shape + unfolding errors	$\pm 2.911 \times 10^{-29}$	$\pm 2.132 \times 10^{-29}$	$\pm 4.670 \times 10^{-29}$	$\pm 2.637 \times 10^{-29}$	$\pm 1.321 \times 10^{-29}$
Normalisation errors	$\pm 9.116 \times 10^{-30}$	$\pm 8.986 \times 10^{-30}$	$\pm 8.550 \times 10^{-30}$	$\pm 6.378 \times 10^{-30}$	$\pm 5.814 \times 10^{-30}$

Table 6.7: Unfolded ^{100}Mo $d\Gamma/\text{electron energy asymmetry}$ distribution (3D unfolding).

Group	Error source	Error
Unfolding errors	Propagated statistical errors on measured histogram	1.461×10^{-30}
	Statistical error on unfolding matrix	9.078×10^{-31}
	Unfolding systematic (see Section 6.5.1)	5.192×10^{-30}
Shape errors	Topological mismodelling	8.567×10^{-30}
	Energy mismodelling	4.554×10^{-29}
	Background normalisation	2.252×10^{-31}
	Energy misreconstruction	1.894×10^{-30}
	Normalisation errors	8.550×10^{-30}

Table 6.8: Breakdown of errors for the bin spanning $[0.4, 0.6]$ of the $d\Gamma/\text{electron energy asymmetry}$ distribution shown in Figure 6.20 and Table 6.7.

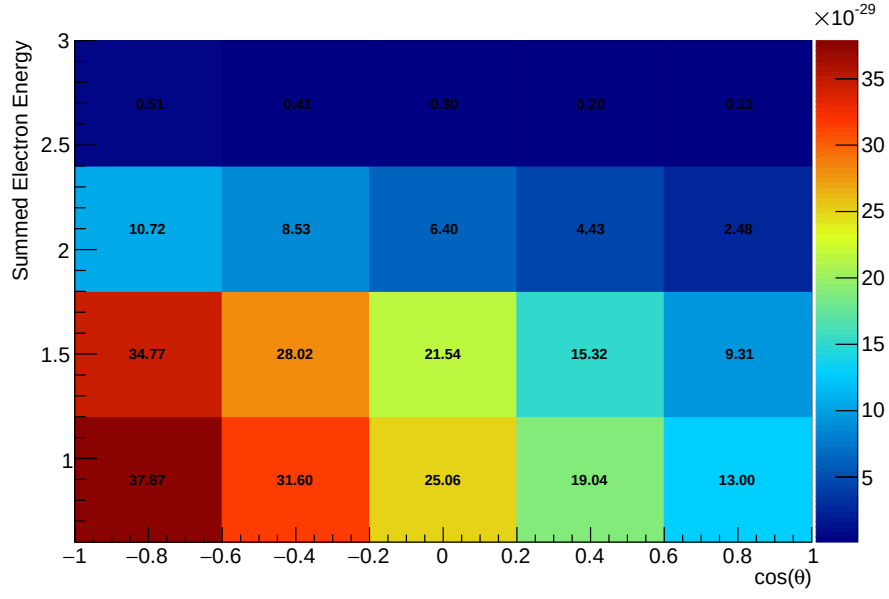


Figure 6.21: Unfolded $d\Gamma/(\text{summed electron energy}, \cos(\theta))$ distribution.

Bin Central Values ΣE_{e^-} (MeV)	$\cos(\theta)$				
	-0.8	-0.4	0	0.4	0.8
0.9	3.787×10^{-28}	3.160×10^{-28}	2.506×10^{-28}	1.904×10^{-28}	1.299×10^{-28}
	$\pm 4.136 \times 10^{-29}$	$\pm 3.300 \times 10^{-29}$	$\pm 2.818 \times 10^{-29}$	$\pm 2.345 \times 10^{-29}$	$\pm 1.725 \times 10^{-29}$
	$\pm 8.254 \times 10^{-30}$	$\pm 6.659 \times 10^{-30}$	$\pm 5.203 \times 10^{-30}$	$\pm 3.814 \times 10^{-30}$	$\pm 2.534 \times 10^{-30}$
1.5	3.477×10^{-28}	2.802×10^{-28}	2.154×10^{-28}	1.532×10^{-28}	9.308×10^{-29}
	$\pm 2.208 \times 10^{-29}$	$\pm 1.591 \times 10^{-29}$	$\pm 1.297 \times 10^{-29}$	$\pm 9.435 \times 10^{-30}$	$\pm 6.445 \times 10^{-30}$
	$\pm 7.821 \times 10^{-30}$	$\pm 6.258 \times 10^{-30}$	$\pm 4.765 \times 10^{-30}$	$\pm 3.353 \times 10^{-30}$	$\pm 2.034 \times 10^{-30}$
2.1	1.072×10^{-28}	8.528×10^{-29}	6.400×10^{-29}	4.428×10^{-29}	2.477×10^{-29}
	$\pm 1.109 \times 10^{-29}$	$\pm 8.697 \times 10^{-30}$	$\pm 6.402 \times 10^{-30}$	$\pm 4.636 \times 10^{-30}$	$\pm 2.565 \times 10^{-30}$
	$\pm 2.386 \times 10^{-30}$	$\pm 1.883 \times 10^{-30}$	$\pm 1.405 \times 10^{-30}$	$\pm 9.550 \times 10^{-31}$	$\pm 5.312 \times 10^{-31}$
2.7	5.11×10^{-30}	4.060×10^{-30}	3.007×10^{-30}	2.033×10^{-30}	1.092×10^{-30}
	$\pm 1.070 \times 10^{-30}$	$\pm 8.285 \times 10^{-31}$	$\pm 6.385 \times 10^{-31}$	$\pm 4.612 \times 10^{-31}$	$\pm 3.092 \times 10^{-31}$
	$\pm 1.070 \times 10^{-31}$	$\pm 8.350 \times 10^{-32}$	$\pm 6.339 \times 10^{-32}$	$\pm 4.061 \times 10^{-32}$	$\pm 2.319 \times 10^{-32}$

Table 6.9: Unfolded $d\Gamma/(\text{summed electron energy}, \cos(\theta))$ distribution with (shape + unfolding) and normalisation errors.

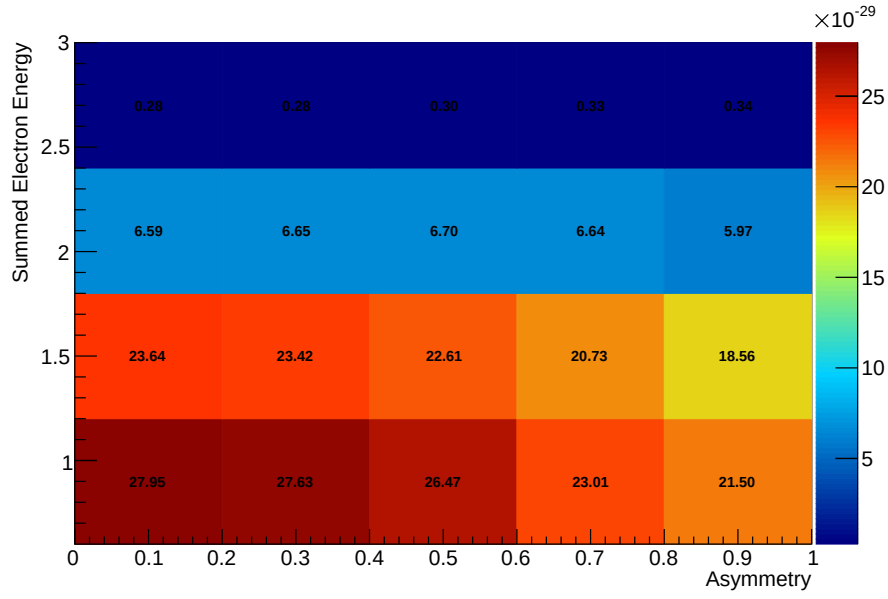


Figure 6.22: Unfolded $d\Gamma/(\text{summed electron energy, energy asymmetry})$ distribution.

Bin Central Values ΣE_{e^-} (MeV)	Asymmetry				
	0.1	0.3	0.5	0.7	0.9
0.9	2.795×10^{-28}	2.763×10^{-28}	2.647×10^{-28}	2.301×10^{-28}	2.150×10^{-28}
	$\pm 1.878 \times 10^{-29}$	$\pm 2.695 \times 10^{-29}$	$\pm 5.216 \times 10^{-29}$	$\pm 1.981 \times 10^{-29}$	$\pm 1.576 \times 10^{-29}$
	$\pm 6.777 \times 10^{-30}$	$\pm 6.667 \times 10^{-30}$	$\pm 6.301 \times 10^{-30}$	$\pm 3.752 \times 10^{-30}$	$\pm 4.043 \times 10^{-30}$
1.5	2.364×10^{-28}	2.342×10^{-28}	2.261×10^{-28}	2.073×10^{-28}	1.856×10^{-28}
	$\pm 1.255 \times 10^{-29}$	$\pm 1.174 \times 10^{-29}$	$\pm 1.358 \times 10^{-29}$	$\pm 2.153 \times 10^{-29}$	$\pm 9.787 \times 10^{-30}$
	$\pm 5.862 \times 10^{-30}$	$\pm 5.789 \times 10^{-30}$	$\pm 5.534 \times 10^{-30}$	$\pm 4.895 \times 10^{-30}$	$\pm 3.994 \times 10^{-30}$
2.1	6.592×10^{-29}	6.647×10^{-29}	6.695×10^{-29}	6.642×10^{-29}	5.971×10^{-29}
	$\pm 4.712 \times 10^{-30}$	$\pm 4.305 \times 10^{-30}$	$\pm 4.325 \times 10^{-30}$	$\pm 1.410 \times 10^{-29}$	$\pm 3.058 \times 10^{-30}$
	$\pm 1.661 \times 10^{-30}$	$\pm 1.664 \times 10^{-30}$	$\pm 1.660 \times 10^{-30}$	$\pm 1.618 \times 10^{-30}$	$\pm 1.223 \times 10^{-30}$
2.7	2.750×10^{-30}	2.781×10^{-30}	3.025×10^{-30}	3.344×10^{-30}	3.401×10^{-30}
	$\pm 2.581 \times 10^{-31}$	$\pm 3.050 \times 10^{-31}$	$\pm 5.256 \times 10^{-31}$	$\pm 1.403 \times 10^{-30}$	$\pm 3.767 \times 10^{-31}$
	$\pm 6.980 \times 10^{-32}$	$\pm 7.026 \times 10^{-32}$	$\pm 7.594 \times 10^{-32}$	$\pm 8.288 \times 10^{-32}$	$\pm 4.691 \times 10^{-32}$

Table 6.10: Unfolded $d\Gamma/(\text{summed electron energy, energy asymmetry})$ distribution with (shape + unfolding) and normalisation errors.

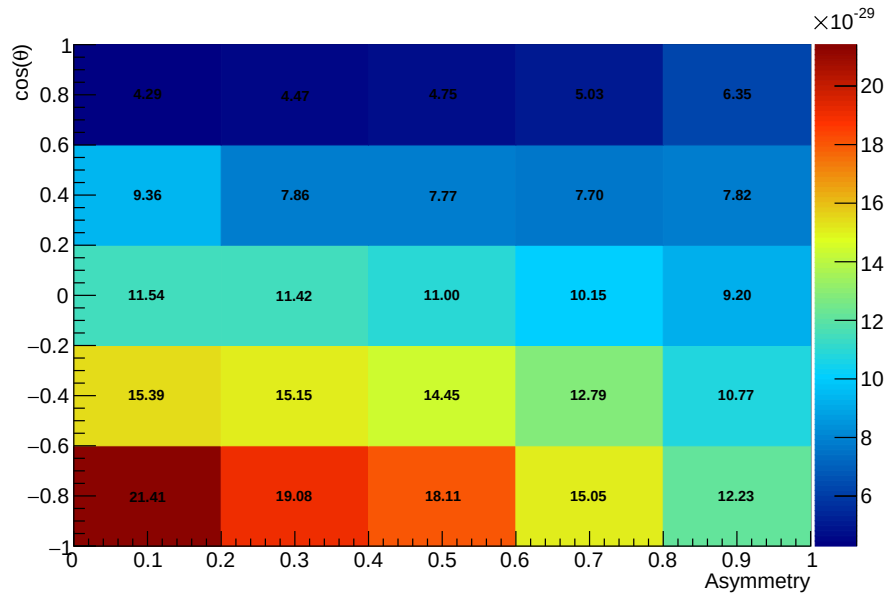


Figure 6.23: Unfolded $d\Gamma/(\cos(\theta), \text{energy asymmetry})$ distribution.

Bin Central Values $\cos(\theta)$	Asymmetry				
	0.1	0.3	0.5	0.7	0.9
-0.8	2.141×10^{-28}	1.908×10^{-28}	1.811×10^{-28}	1.505×10^{-28}	1.223×10^{-28}
	$\pm 2.620 \times 10^{-29}$	$\pm 2.010 \times 10^{-29}$	$\pm 3.349 \times 10^{-29}$	$\pm 2.038 \times 10^{-29}$	$\pm 1.076 \times 10^{-29}$
	$\pm 6.123 \times 10^{-30}$	$\pm 6.016 \times 10^{-30}$	$\pm 5.669 \times 10^{-30}$	$\pm 4.205 \times 10^{-30}$	$\pm 3.391 \times 10^{-30}$
-0.4	1.539×10^{-28}	1.515×10^{-28}	1.445×10^{-28}	1.279×10^{-28}	1.077×10^{-28}
	$\pm 1.207 \times 10^{-29}$	$\pm 1.560 \times 10^{-29}$	$\pm 2.628 \times 10^{-29}$	$\pm 1.592 \times 10^{-29}$	$\pm 9.204 \times 10^{-30}$
	$\pm 4.875 \times 10^{-30}$	$\pm 4.788 \times 10^{-30}$	$\pm 4.527 \times 10^{-30}$	$\pm 3.347 \times 10^{-30}$	$\pm 2.943 \times 10^{-30}$
0	1.154×10^{-28}	1.142×10^{-28}	1.100×10^{-28}	1.015×10^{-28}	9.204×10^{-29}
	$\pm 8.683 \times 10^{-30}$	$\pm 1.171 \times 10^{-29}$	$\pm 2.271 \times 10^{-29}$	$\pm 0.4516 \times 10^{-29}$	$\pm 8.070 \times 10^{-30}$
	$\pm 3.671 \times 10^{-30}$	$\pm 3.622 \times 10^{-30}$	$\pm 3.461 \times 10^{-30}$	$\pm 2.620 \times 10^{-30}$	$\pm 2.512 \times 10^{-30}$
0.4	9.363×10^{-29}	7.857×10^{-29}	7.773×10^{-29}	7.697×10^{-29}	7.818×10^{-29}
	$\pm 1.645 \times 10^{-29}$	$\pm 8.414 \times 10^{-30}$	$\pm 1.964 \times 10^{-29}$	$\pm 1.092 \times 10^{-29}$	$\pm 7.028 \times 10^{-30}$
	$\pm 2.518 \times 10^{-30}$	$\pm 2.515 \times 10^{-30}$	$\pm 2.467 \times 10^{-30}$	$\pm 1.860 \times 10^{-30}$	$\pm 2.119 \times 10^{-30}$
0.8	4.292×10^{-29}	4.471×10^{-29}	4.748×10^{-29}	5.026×10^{-29}	6.354×10^{-29}
	$\pm 3.442 \times 10^{-30}$	$\pm 5.246 \times 10^{-30}$	$\pm 1.448 \times 10^{-29}$	$\pm 7.607 \times 10^{-30}$	$\pm 6.220 \times 10^{-30}$
	$\pm 1.417 \times 10^{-30}$	$\pm 1.472 \times 10^{-30}$	$\pm 1.547 \times 10^{-30}$	$\pm 1.201 \times 10^{-30}$	$\pm 1.683 \times 10^{-30}$

Table 6.11: Unfolded $d\Gamma/(\text{summed electron energy}, \cos(\theta))$ distribution with (shape + unfolding) and normalisation errors.

Bin Central Values		Asymmetry				$\times 10^{-29}$
$\cos(\theta)$	ΣE_{e^-} (MeV)	0.1	0.3	0.5	0.7	0.9
-0.8	0.9	$8.985 \pm 0.850 \pm 0.449$	$8.802 \pm 1.349 \pm 0.439$	$8.266 \pm 2.782 \pm 0.411$	$6.333 \pm 0.848 \pm 0.251$	$5.477 \pm 0.664 \pm 0.230$
-0.8	1.5	$7.991 \pm 0.516 \pm 0.399$	$7.879 \pm 0.469 \pm 0.393$	$7.462 \pm 0.587 \pm 0.372$	$6.442 \pm 1.110 \pm 0.319$	$4.994 \pm 0.332 \pm 0.236$
-0.8	2.1	$2.318 \pm 0.222 \pm 0.115$	$2.300 \pm 0.192 \pm 0.114$	$2.275 \pm 0.197 \pm 0.113$	$2.162 \pm 0.835 \pm 0.108$	$1.660 \pm 0.104 \pm 0.075$
-0.8	2.7	$0.099 \pm 0.014 \pm 0.004$	$0.098 \pm 0.017 \pm 0.004$	$0.105 \pm 0.031 \pm 0.005$	$0.114 \pm 0.088 \pm 0.005$	$0.093 \pm 0.014 \pm 0.002$
-0.4	0.9	$7.222 \pm 0.626 \pm 0.360$	$7.069 \pm 1.066 \pm 0.353$	$6.665 \pm 2.187 \pm 0.331$	$5.746 \pm 0.799 \pm 0.196$	$4.896 \pm 0.552 \pm 0.200$
-0.4	1.5	$6.307 \pm 0.334 \pm 0.315$	$6.214 \pm 0.296 \pm 0.310$	$5.915 \pm 0.422 \pm 0.295$	$5.226 \pm 0.727 \pm 0.256$	$4.355 \pm 0.300 \pm 0.206$
-0.4	2.1	$1.788 \pm 0.157 \pm 0.089$	$1.793 \pm 0.137 \pm 0.089$	$1.784 \pm 0.136 \pm 0.089$	$1.730 \pm 0.668 \pm 0.086$	$1.432 \pm 0.090 \pm 0.063$
-0.4	2.7	$0.075 \pm 0.009 \pm 0.003$	$0.076 \pm 0.012 \pm 0.003$	$0.081 \pm 0.023 \pm 0.004$	$0.088 \pm 0.068 \pm 0.004$	$0.083 \pm 0.013 \pm 0.002$
0.0	0.9	$5.519 \pm 0.446 \pm 0.275$	$5.435 \pm 0.812 \pm 0.271$	$5.169 \pm 1.952 \pm 0.256$	$4.674 \pm 0.797 \pm 0.152$	$4.264 \pm 0.504 \pm 0.177$
0.0	1.5	$4.674 \pm 0.215 \pm 0.233$	$4.618 \pm 0.182 \pm 0.230$	$4.457 \pm 0.280 \pm 0.222$	$4.109 \pm 0.741 \pm 0.202$	$3.683 \pm 0.239 \pm 0.169$
0.0	2.1	$1.290 \pm 0.110 \pm 0.064$	$1.306 \pm 0.095 \pm 0.065$	$1.313 \pm 0.092 \pm 0.065$	$1.303 \pm 0.491 \pm 0.065$	$1.186 \pm 0.084 \pm 0.052$
0.0	2.7	$0.053 \pm 0.006 \pm 0.002$	$0.054 \pm 0.008 \pm 0.002$	$0.059 \pm 0.016 \pm 0.002$	$0.064 \pm 0.052 \pm 0.003$	$0.069 \pm 0.018 \pm 0.002$
0.4	0.9	$3.892 \pm 0.306 \pm 0.194$	$3.888 \pm 0.595 \pm 0.194$	$3.811 \pm 1.742 \pm 0.189$	$3.720 \pm 0.628 \pm 0.102$	$3.723 \pm 0.447 \pm 0.153$
0.4	1.5	$3.094 \pm 0.143 \pm 0.154$	$3.089 \pm 0.117 \pm 0.154$	$3.049 \pm 0.190 \pm 0.152$	$3.016 \pm 0.551 \pm 0.148$	$3.073 \pm 0.207 \pm 0.140$
0.4	2.1	$0.823 \pm 0.072 \pm 0.041$	$0.846 \pm 0.062 \pm 0.042$	$0.874 \pm 0.062 \pm 0.043$	$0.916 \pm 0.364 \pm 0.045$	$0.968 \pm 0.066 \pm 0.040$
0.4	2.7	$0.032 \pm 0.004 \pm 0.001$	$0.034 \pm 0.005 \pm 0.001$	$0.038 \pm 0.010 \pm 0.001$	$0.043 \pm 0.039 \pm 0.002$	$0.054 \pm 0.010 \pm 0.001$
0.8	0.9	$2.335 \pm 0.185 \pm 0.116$	$2.433 \pm 0.379 \pm 0.121$	$2.557 \pm 1.306 \pm 0.126$	$2.532 \pm 0.467 \pm 0.069$	$3.137 \pm 0.415 \pm 0.122$
0.8	1.5	$1.570 \pm 0.077 \pm 0.078$	$1.620 \pm 0.065 \pm 0.080$	$1.724 \pm 0.112 \pm 0.085$	$1.939 \pm 0.396 \pm 0.094$	$2.453 \pm 0.188 \pm 0.111$
0.8	2.1	$0.372 \pm 0.034 \pm 0.018$	$0.401 \pm 0.031 \pm 0.020$	$0.448 \pm 0.035 \pm 0.022$	$0.530 \pm 0.196 \pm 0.026$	$0.723 \pm 0.055 \pm 0.029$
0.8	2.7	$0.014 \pm 0.001 \pm 0.000$	$0.015 \pm 0.002 \pm 0.000$	$0.017 \pm 0.006 \pm 0.000$	$0.023 \pm 0.023 \pm 0.001$	$0.038 \pm 0.015 \pm 0.001$

Table 6.12: Three-Dimensional $d\Gamma$ unfolded distribution, with (shape + unfolding) and normalisation errors.

Chapter 7

Conclusion

The study of double-beta decay provides a rich variety of tools with which to probe physics of the Standard Model and beyond. The NEMO-3 experiment measured two-neutrino double-beta decay, and searched for the neutrinoless double-beta decay, of seven isotopes, with ^{100}Mo being the largest sample. The ability of the experiment to reconstruct the topology of decays allowed for ultra-low background data to be collected, from which the half-life of the two-neutrino double-beta decay of ^{100}Mo was measured as:

$$T_{\frac{1}{2}} = 7.01 \pm 0.01(\text{stat}) \pm 0.46(\text{syst}) \times 10^{18} \text{yr}, \quad (7.1)$$

the most precise measurement of this physical constant to date. The large data statistics and purity also allow for an unfolding of the measured distribution of the total energy carried by the emitted electrons in a double-beta decay event, providing the ability to compare models to the data without a NEMO-3 detector simulation. Such an analysis was carried out to constrain the Standard Model Extension parameter $\hat{a}_{of}^{(3)}$, representing a Lorentz Violating process in the electroweak sector inaccessible through measurements of neutrino oscillation. A best fit value of

$$\hat{a}_{of}^{(3)} = -3.03 \times 10^{-2} \text{MeV}, \quad (7.2)$$

was obtained, with limits of

$$-1.12 \times 10^{-1} \text{MeV} < \dot{a}_{of}^{(3)} < 6.95 \times 10^{-2} \text{MeV}, \quad (7.3)$$

at a 90% confidence level, providing an upper bound more precise than any yet published.

To provide a more generalisable model and exploit the unique tracking capabilities of the detector, the measured three dimensional distribution of the summed electron energy, the opening angle between electrons, and the energy asymmetry between them was also unfolded. The resulting distribution and the one and two dimensional projections were presented, and may be used to test models expected to affect the kinematics of the decay, such as the contribution of a right-handed component of the weak force.

The technologies and experimental techniques developed during the lifetime of the NEMO-3 experiment have informed the design and construction of the successor experiment, the SuperNEMO demonstrator module. The demonstrator module is the first phase of the SuperNEMO experiment and will demonstrate the precision and background suppression such a detector can achieve. In doing so, the breadth of the feasible physics programme will be apparent, with the ultimate goal of detecting neutrinoless double-beta decay, and therefore Majorana fermions. During construction and commissioning of the tracker sections, drift cell status and performance was assessed, with the results presented to inform later in-situ commissioning at the Laboratoire Souterrain de Modane. The first measurement of the relationship between measured electromagnetic drift time and distance of closest approach of an ionising particle was performed for each of the first three tracker sections, alongside channel maps describing the status of each cell.

Appendix A

Correlation Matrices

Presented below are the bin correlation matrices for the unfolded distributions presented in Chapter 6. Omitted values are zero or deducible by diagonal symmetry.

Bin Central Value ΣE_{e^-} (MeV)	0.65	0.75	0.85	0.95	1.05	1.15	1.25	1.35	1.45	1.55	1.65	1.75
0.65	1	0.9824	0.8388	0.5398	0.2778	0.1238	0.0566	0.02537	0.01232	0.006889	0.003626	0.002043
0.75	0.9824	1	0.9248	0.672	0.3862	0.1845	0.08636	0.03973	0.01896	0.01027	0.005163	0.002905
0.85	0.8388	0.9248	1	0.887	0.6052	0.3255	0.1599	0.07592	0.0359	0.01866	0.009016	0.004943
0.95	0.5398	0.672	0.887	1	0.8669	0.569	0.3101	0.1544	0.07398	0.03694	0.01769	0.009093
1.05	0.2778	0.3862	0.6052	0.8669	1	0.8624	0.5695	0.3148	0.1577	0.07736	0.03706	0.01807
1.15	0.1238	0.1845	0.3255	0.569	0.8624	1	0.8617	0.5751	0.3195	0.1616	0.07808	0.03735
1.25	0.0566	0.08636	0.1599	0.3101	0.5695	0.8617	1	0.867	0.5809	0.3253	0.1645	0.0791
1.35	0.02537	0.03973	0.07592	0.1544	0.3148	0.5751	0.867	1	0.8676	0.5864	0.3306	0.1664
1.45	0.01232	0.01896	0.0359	0.07398	0.1577	0.3195	0.5809	0.8676	1	0.8716	0.5932	0.334
1.55	0.006889	0.01027	0.01866	0.03694	0.07736	0.1616	0.3253	0.5864	0.8716	1	0.8734	0.5941
1.65	0.003626	0.005163	0.009016	0.01769	0.03706	0.07808	0.1645	0.3306	0.5932	0.8734	1	0.873
1.75	0.002043	0.002905	0.004943	0.009093	0.01807	0.03735	0.0791	0.1664	0.334	0.5941	0.873	1
1.85	0.001114	0.001555	0.002593	0.004701	0.009169	0.0184	0.03793	0.07973	0.1678	0.335	0.5972	0.8764
1.95	0.0008	0.00114	0.001871	0.003075	0.005307	0.009739	0.01908	0.03868	0.08037	0.1669	0.3342	0.596
2.05	0.0002	0.0003	0.0006	0.001358	0.002774	0.005189	0.009513	0.01849	0.03787	0.07872	0.166	0.3349
2.15	0.0002	0.0003	0.0005	0.001036	0.001774	0.002927	0.004966	0.009119	0.01789	0.03629	0.07668	0.164
2.25	0.0003	0.0003	0.0004	0.0006	0.001038	0.001723	0.002934	0.004967	0.008807	0.01688	0.03546	0.07677
2.35	0.0001	0.0002	0.0003	0.0004	0.0006	0.001038	0.00175	0.002831	0.004553	0.007957	0.01549	0.03257
2.45	0.0001	0.0001	0.0001	0.0002	0.0004	0.0007	0.001086	0.001589	0.002437	0.004063	0.007256	0.01397
2.55	0	0	0	0.0001	0.0002	0.0003	0.0005	0.0006	0.001076	0.001795	0.003188	0.006035
2.65	0.0003	0.0003	0.0003	0.0002	0.0002	0.0002	0.0004	0.0006	0.0008	0.001087	0.00142	0.002271
2.75	0	0	0	0	0	0	0	0	0	0.0001	0.0002	0.0006
2.85	0	0	0	0	0	0	0	0	0	0	0.0002	0.0005
2.95	0	0	0	0	0	0	0	0	0	0	0	0

Bin Central Value ΣE_{e^-} (MeV)	1.85	1.95	2.05	2.15	2.25	2.35	2.45	2.55	2.65	2.75	2.85	2.95
0.65	0.001114	0.0008	0.0002	0.0002	0.0003	0.0001	0.0001	0	0.0003	0	0	0
0.75	0.001555	0.00114	0.0003	0.0003	0.0003	0.0002	0.0001	0	0.0003	0	0	0
0.85	0.002593	0.001871	0.0006	0.0005	0.0004	0.0003	0.0001	0	0.0003	0	0	0
0.95	0.004701	0.003075	0.001358	0.001036	0.0006	0.0004	0.0002	0.0001	0.0002	0	0	0
1.05	0.009169	0.005307	0.002774	0.001774	0.001038	0.0006	0.0004	0.0002	0.0002	0	0	0
1.15	0.0184	0.009739	0.005189	0.002927	0.001723	0.001038	0.0007	0.0003	0.0002	0	0	0
1.25	0.03793	0.01908	0.009513	0.004966	0.002934	0.00175	0.001086	0.0005	0.0004	0	0	0
1.35	0.07973	0.03868	0.01849	0.009119	0.004967	0.002831	0.001589	0.0006	0.0006	0	0	0
1.45	0.1678	0.08037	0.03787	0.01789	0.008807	0.004553	0.002437	0.001076	0.0008	0	0	0
1.55	0.335	0.1669	0.07872	0.03629	0.01688	0.007957	0.004063	0.001795	0.001087	0.0001	0	0
1.65	0.5972	0.3342	0.166	0.07668	0.03546	0.01549	0.007256	0.003188	0.00142	0.0002	0.0002	0
1.75	0.8764	0.596	0.3349	0.164	0.07677	0.03257	0.01397	0.006035	0.002271	0.0006	0.0005	0
1.85	1	0.8734	0.5979	0.3351	0.1649	0.0721	0.02963	0.01233	0.004284	0.001361	0.0007	0
1.95	0.8734	1	0.8762	0.5997	0.3332	0.1587	0.06636	0.02673	0.009101	0.002909	0.001016	0
2.05	0.5979	0.8762	1	0.8765	0.5947	0.3289	0.1497	0.06081	0.02145	0.006357	0.001635	0
2.15	0.3351	0.5997	0.8765	1	0.8726	0.5939	0.3158	0.1386	0.05207	0.01461	0.003582	0
2.25	0.1649	0.3332	0.5947	0.8726	1	0.8759	0.5847	0.3034	0.1277	0.03806	0.009071	0.0001
2.35	0.0721	0.1587	0.3289	0.5939	0.8759	1	0.8722	0.5749	0.2933	0.1049	0.02709	0.003424
2.45	0.02963	0.06636	0.1497	0.3158	0.5847	0.8722	1	0.8627	0.5584	0.2526	0.07741	0.02101
2.55	0.01233	0.02673	0.06081	0.1386	0.3034	0.5749	0.8627	1	0.8557	0.5103	0.1941	0.07016
2.65	0.004284	0.009101	0.02145	0.05207	0.1277	0.2933	0.5584	0.8557	1	0.83	0.443	0.2375
2.75	0.001361	0.002909	0.006357	0.01461	0.03806	0.1049	0.2526	0.5103	0.83	1	0.7763	0.4513
2.85	0.0007	0.001016	0.001635	0.003582	0.009071	0.02709	0.07741	0.1941	0.443	0.7763	1	0.563
2.95	0	0	0	0	0.0001	0.003424	0.02101	0.07016	0.2375	0.4513	0.563	1

Table A.1: Correlation matrix from 1D unfolding.

Bin Central Value			Asym.	0.1									
			bin #	58	59	60	61	65	66	67	68	72	73
			$\cos(\theta)$	-0.8	-0.8	-0.8	-0.8	-0.4	-0.4	-0.4	-0.4	0.0	0.0
Asym.	bin #	$\cos(\theta)$	ΣE_{e^-} (MeV)	0.9	1.5	2.1	2.7	0.9	1.5	2.1	2.7	0.1	1.5
0.1	58	-0.8	0.9	1	0.2455	-0.1729	0.03392	0.9006	0.2505	-0.214	0.04289	0.7708	0.2022
	59	-0.8	1.5	0.2455	1	0.1792	-0.1032	0.2053	0.8857	0.1539	-0.1299	0.1415	0.6612
	60	-0.8	2.1	-0.1729	0.1792	1	0.2388	-0.1772	0.1324	0.8545	0.2183	-0.1671	0.06156
	61	-0.8	2.7	0.03392	-0.1032	0.2388	1	0.03508	-0.1114	0.1877	0.7863	0.03345	-0.1045
	65	-0.4	0.9	0.9006	0.2053	-0.1772	0.03508	1	0.2413	-0.2205	0.04437	0.8644	0.229
	66	-0.4	1.5	0.2505	0.8857	0.1324	-0.1114	0.2413	1	0.1612	-0.142	0.2097	0.8337
	67	-0.4	2.1	-0.214	0.1539	0.8545	0.1877	-0.2205	0.1612	1	0.2497	-0.2092	0.1454
	68	-0.4	2.7	0.04289	-0.1299	0.2183	0.7863	0.04437	-0.142	0.2497	1	0.04232	-0.1355
	72	0.0	0.9	0.7708	0.1415	-0.1671	0.03345	0.8644	0.2097	-0.2092	0.04232	1	0.2398
	73	0.0	1.5	0.2022	0.6612	0.06156	-0.1045	0.229	0.8337	0.1454	-0.1355	0.2398	1
	74	0.0	2.1	-0.1972	0.07732	0.5605	0.1082	-0.2036	0.1285	0.8216	0.2297	-0.1939	0.1707
	75	0.0	2.7	0.03768	-0.1107	0.1137	0.4875	0.03897	-0.1228	0.1978	0.7697	0.03718	-0.1195
	79	0.4	0.9	0.5829	0.06718	-0.1421	0.02877	0.7351	0.1562	-0.179	0.03644	0.8505	0.2243
	80	0.4	1.5	0.1359	0.3947	-0.002103	-0.08397	0.1889	0.6338	0.1059	-0.1106	0.2338	0.8374
	81	0.4	2.1	-0.1589	0.01035	0.2822	0.03445	-0.165	0.08547	0.5808	0.1548	-0.1582	0.169
	82	0.4	2.7	0.03025	-0.08915	0.0385	0.2768	0.03129	-0.1003	0.1379	0.5164	0.02985	-0.09934
	86	0.8	0.9	0.4223	0.01435	-0.115	0.02365	0.5968	0.1075	-0.1458	0.02998	0.753	0.193
	87	0.8	1.5	0.09431	0.1814	-0.04857	-0.05836	0.1622	0.4237	0.05068	-0.07751	0.228	0.6718
	88	0.8	2.1	-0.1149	-0.02473	0.108	-0.005625	-0.1203	0.04713	0.3722	0.08689	-0.1165	0.1352
	89	0.8	2.7	0.01966	-0.05674	0.002714	0.07765	0.02032	-0.06435	0.07001	0.2392	0.01938	-0.06465
0.1	58	-0.8	0.9	-0.1972	0.03768	0.5829	0.1359	-0.1589	0.03025	0.4223	0.09431	-0.1149	0.01966
	59	-0.8	1.5	0.07732	-0.1107	0.06718	0.3947	0.01035	-0.08915	0.01435	0.1814	-0.02473	-0.05674
	60	-0.8	2.1	0.5605	0.1137	-0.1421	-0.002103	0.2822	0.0385	-0.115	-0.04857	0.108	0.002714
	61	-0.8	2.7	0.1082	0.4875	0.02877	-0.08397	0.03445	0.2768	0.02365	-0.05836	-0.005625	0.07765
	65	-0.4	0.9	-0.2036	0.03897	0.7351	0.1889	-0.165	0.03129	0.5968	0.1622	-0.1203	0.02032
	66	-0.4	1.5	0.1285	-0.1228	0.1562	0.6338	0.08547	-0.1003	0.1075	0.4237	0.04713	-0.06435
	67	-0.4	2.1	0.8216	0.1978	-0.179	0.1059	0.5808	0.1379	-0.1458	0.05068	0.3722	0.07001
	68	-0.4	2.7	0.2297	0.7697	0.03644	-0.1106	0.1548	0.5164	0.02998	-0.07751	0.08689	0.2392
	72	0.0	0.9	-0.1939	0.03718	0.8505	0.2338	-0.1582	0.02985	0.753	0.228	-0.1165	0.01938
	73	0.0	1.5	0.1707	-0.1195	0.2243	0.8374	0.169	-0.09934	0.193	0.6718	0.1352	-0.06465
	74	0.0	2.1	1	0.2499	-0.1663	0.1749	0.8427	0.2448	-0.1361	0.1376	0.6527	0.1566
	75	0.0	2.7	0.2499	1	0.03198	-0.09979	0.2277	0.7627	0.02627	-0.07264	0.1764	0.5145
	79	0.4	0.9	-0.1663	0.03198	1	0.2549	-0.1366	0.02568	0.8551	0.2756	-0.1017	0.01666
	80	0.4	1.5	0.1749	-0.09979	0.2549	1	0.2172	-0.08406	0.2463	0.8315	0.2053	-0.0553
	81	0.4	2.1	0.8427	0.2277	-0.1366	0.2172	1	0.2896	-0.1127	0.2091	0.8481	0.2267
	82	0.4	2.7	0.2448	0.7627	0.02568	-0.08406	0.2896	1	0.02113	-0.06147	0.2594	0.7614
	86	0.8	0.9	-0.1361	0.02627	0.8551	0.2463	-0.1127	0.02113	1	0.2912	-0.08486	0.0137
	87	0.8	1.5	0.1376	-0.07264	0.2756	0.8315	0.2091	-0.06147	0.2912	1	0.2365	-0.04053
	88	0.8	2.1	0.6527	0.1764	-0.1017	0.2053	0.8481	0.2594	-0.08486	0.2365	1	0.2523
	89	0.8	2.7	0.1566	0.5145	0.01666	-0.0553	0.2267	0.7614	0.0137	-0.04053	0.2523	1

Table A.2: Correlation matrix from 3D unfolding.

Bin Central Value			Asym.	0.3									
			bin #	107	108	109	110	114	115	116	117	121	122
			$\cos(\theta)$	-0.8	-0.8	-0.8	-0.8	-0.4	-0.4	-0.4	-0.4	0.0	0.0
Asym.	bin #	$\cos(\theta)$	ΣE_{e^-} (MeV)	0.9	1.5	2.1	2.7	0.9	1.5	2.1	2.7	0.1	1.5
0.1	58	-0.8	0.9	0.3558	-0.06425	-0.06889	0.02286	0.3086	-0.08298	-0.08607	0.02599	0.2124	-0.09934
	59	-0.8	1.5	0.1137	0.0841	-0.0691	-0.03581	0.08457	0.03387	-0.09049	-0.04095	0.04075	-0.02754
	60	-0.8	2.1	-0.1394	-0.07019	0.1106	0.00536	-0.1402	-0.09213	0.0699	-0.007342	-0.1252	-0.103
	61	-0.8	2.7	0.02974	-0.03947	0.006191	0.1521	0.03022	-0.04245	-0.007941	0.1262	0.02735	-0.03959
	65	-0.4	0.9	0.313	-0.08789	-0.06967	0.02355	0.2996	-0.09582	-0.08754	0.02695	0.2385	-0.0979
	66	-0.4	1.5	0.1113	0.04439	-0.09408	-0.03844	0.101	0.03148	-0.1008	-0.04449	0.07424	0.006101
	67	-0.4	2.1	-0.1748	-0.08806	0.07261	-0.01358	-0.1768	-0.09795	0.0814	-0.01128	-0.1588	-0.09767
	68	-0.4	2.7	0.03777	-0.05177	0.007045	0.1285	0.03837	-0.05669	0.005553	0.1491	0.0347	-0.05395
	72	0.0	0.9	0.2426	-0.1051	-0.06491	0.02238	0.2657	-0.1011	-0.08212	0.0258	0.2476	-0.08811
	73	0.0	1.5	0.08406	-0.0186	-0.108	-0.03547	0.0956	0.01074	-0.09847	-0.04165	0.09377	0.03381
	74	0.0	2.1	-0.1612	-0.1041	0.01287	-0.02767	-0.1636	-0.09733	0.06214	-0.01155	-0.1475	-0.08004
	75	0.0	2.7	0.03293	-0.04297	-0.01603	0.08527	0.03338	-0.04765	-0.005916	0.1356	0.03011	-0.04588
	79	0.4	0.9	0.1583	-0.1119	-0.05458	0.01923	0.209	-0.09859	-0.06956	0.02239	0.2331	-0.07348
	80	0.4	1.5	0.03962	-0.0781	-0.1062	-0.02759	0.06584	-0.02468	-0.08825	-0.03285	0.08386	0.03414
	81	0.4	2.1	-0.1279	-0.09882	-0.0341	-0.03338	-0.1304	-0.08458	0.02527	-0.01455	-0.1181	-0.05927
	82	0.4	2.7	0.02626	-0.03115	-0.03255	0.0427	0.02665	-0.03627	-0.02199	0.09317	0.02407	-0.0363
	86	0.8	0.9	0.09355	-0.1085	-0.0434	0.01573	0.1537	-0.09092	-0.05575	0.01846	0.2036	-0.06035
	87	0.8	1.5	0.02667	-0.09486	-0.09317	-0.01832	0.06109	-0.03629	-0.07469	-0.02221	0.09264	0.03431
	88	0.8	2.1	-0.09464	-0.08246	-0.05293	-0.0314	-0.09714	-0.06808	-0.002263	-0.0147	0.08881	-0.04374
	89	0.8	2.7	0.0172	-0.02116	-0.03158	0.009032	0.01744	-0.02446	-0.01813	0.02954	0.01575	-0.0245
Bin Central Value			Asym.	0.3									
			bin #	123	124	128	129	130	131	135	136	137	138
			$\cos(\theta)$	0.0	0.0	0.4	0.4	0.4	0.4	0.8	0.8	0.8	0.8
Asym.	bin #	$\cos(\theta)$	ΣE_{e^-} (MeV)	2.1	2.7	0.9	1.5	2.1	2.7	0.9	1.5	2.1	2.7
0.1	58	-0.8	0.9	-0.07897	0.02517	0.1107	-0.1037	-0.06468	0.01905	0.0465	-0.09595	-0.04818	0.01169
	59	-0.8	1.5	-0.106	-0.03604	-0.002877	-0.07985	-0.1026	-0.02787	-0.02459	-0.1003	-0.08968	-0.01598
	60	-0.8	2.1	0.01311	-0.02658	-0.1002	-0.1008	-0.03436	-0.03043	-0.07755	-0.08991	-0.04925	-0.02753
	61	-0.8	2.7	-0.01591	0.05771	0.02237	-0.03191	-0.02318	0.01662	0.01765	-0.0224	-0.02331	-0.007315
	65	-0.4	0.9	-0.08057	0.02588	0.1649	-0.09293	-0.06642	0.01961	0.1065	-0.07881	-0.04957	0.01211
	66	-0.4	1.5	-0.104	-0.04072	0.0417	-0.02252	-0.09173	-0.03205	0.01956	-0.03934	-0.07825	-0.01911
	67	-0.4	2.1	0.06138	-0.01578	-0.1279	-0.08732	0.0307	-0.01458	-0.09952	-0.07784	0.006355	-0.01758
	68	-0.4	2.7	0.006673	0.1141	0.02834	-0.04462	0.003219	0.0706	0.02235	-0.03251	0.003491	0.03024
	72	0.0	0.9	-0.07589	0.02454	0.2107	-0.07336	-0.06301	0.01861	0.1642	-0.05345	-0.0471	0.01158
	73	0.0	1.5	-0.08551	-0.04005	0.08228	0.04413	-0.0632	-0.03214	0.06582	0.03735	-0.04979	-0.01996
	74	0.0	2.1	0.09246	0.001295	-0.1196	-0.05655	0.09791	0.01286	-0.0934	-0.04435	0.07497	0.002037
	75	0.0	2.7	0.01134	0.1545	0.02451	-0.03854	0.02233	0.1177	0.01924	-0.02878	0.01996	0.09661
	79	0.4	0.9	-0.06476	0.02108	0.237	-0.04877	-0.05406	0.01598	0.2124	-0.02359	-0.04021	0.01002
	80	0.4	1.5	-0.06339	-0.03336	0.09185	0.08391	-0.0331	-0.02713	0.0861	0.09827	-0.01664	-0.01741
	81	0.4	2.1	0.08969	0.009465	-0.09644	-0.02736	0.1362	0.02898	-0.07594	-0.01155	0.1341	0.02324
	82	0.4	2.7	0.004758	0.1592	0.01962	-0.03158	0.01902	0.1601	0.01541	-0.02389	0.01729	0.1936
	86	0.8	0.9	-0.05231	0.01723	0.2337	-0.02937	-0.04396	0.01307	0.2326	-0.00197	-0.0324	0.00821
	87	0.8	1.5	-0.04664	-0.02412	0.1144	0.1068	-0.01451	-0.0193	0.1215	0.1444	0.007341	-0.01269
	88	0.8	2.1	0.06607	0.007929	-0.07297	-0.009458	0.1329	0.03416	-0.05739	0.01113	0.1603	0.04052
	89	0.8	2.7	0.008597	0.1022	0.01282	-0.02146	0.03896	0.1133	0.01007	-0.01654	0.04578	0.2279

Table A.2: Correlation matrix from 3D unfolding (cont.).

Bin Central Value			Asym.	0.5									
			bin #	156	157	158	159	163	164	165	166	170	171
			$\cos(\theta)$	-0.8	-0.8	-0.8	-0.8	-0.4	-0.4	-0.4	-0.4	0.0	0.0
Asym.	bin #	$\cos(\theta)$	ΣE_{e^-} (MeV)	0.9	1.5	2.1	2.7	0.9	1.5	2.1	2.7	0.1	1.5
0.1	58	-0.8	0.9	0.01808	-0.04658	0.002792	0.008866	-0.002884	-0.05215	0.002102	0.00979	-0.009501	-0.04934
	59	-0.8	1.5	0.002957	-0.1117	-0.04252	0.007984	-0.002956	-0.1265	-0.05152	0.008668	-0.01401	-0.1256
	60	-0.8	2.1	-0.0512	-0.0563	-0.048	-0.01521	-0.04218	-0.06254	-0.05567	-0.01687	-0.03461	-0.06047
	61	-0.8	2.7	0.01214	-0.003422	-0.01732	-0.01189	0.01012	-0.004195	-0.01909	-0.01245	0.008611	-0.004362
	65	-0.4	0.9	0.006417	-0.04955	0.003097	0.009082	-0.009135	-0.05439	0.002586	0.01002	-0.006575	-0.05064
	66	-0.4	1.5	0.002625	-0.1271	-0.04944	0.008998	-0.000173	-0.1401	-0.05888	0.009554	-0.006884	-0.1368
	67	-0.4	2.1	-0.06498	-0.06459	-0.05681	-0.01825	-0.05418	-0.07182	-0.06482	-0.01993	-0.04528	-0.06931
	68	-0.4	2.7	0.01555	-0.006114	-0.02331	-0.01529	0.01299	-0.007215	-0.02598	-0.01595	0.01109	-0.00753
	72	0.0	0.9	-0.003785	-0.04861	0.003027	0.008596	-0.01384	-0.05245	0.002762	0.009476	-0.002491	-0.04799
	73	0.0	1.5	-0.004004	-0.1272	-0.04792	0.008867	-0.00221	-0.1383	-0.0565	0.009258	-0.003312	-0.133
	74	0.0	2.1	-0.05982	-0.06307	-0.05394	-0.01687	-0.04985	-0.06823	-0.0601	-0.01822	-0.04179	-0.06434
	75	0.0	2.7	0.01331	-0.005931	-0.02055	-0.01403	0.01112	-0.00664	-0.02256	-0.01441	0.009452	-0.006638
	79	0.4	0.9	-0.01109	-0.04305	0.002685	0.007362	-0.01749	-0.04576	0.002715	0.008135	0.001242	-0.04118
	80	0.4	1.5	-0.01339	-0.1073	-0.038	0.007602	-0.008197	-0.1147	-0.04454	0.007751	-0.002776	-0.1082
	81	0.4	2.1	-0.0472	-0.05127	-0.04294	-0.01321	-0.03947	-0.05514	-0.0483	-0.01434	-0.03303	-0.0514
	82	0.4	2.7	0.01054	-0.00306	-0.01518	-0.01223	0.008831	-0.003771	-0.01668	-0.01174	0.007506	-0.004208
	86	0.8	0.9	-0.01538	-0.03605	0.002435	0.005974	-0.02039	-0.03803	0.002713	0.006583	0.0009131	-0.03366
	87	0.8	1.5	-0.009367	-0.08537	-0.02905	0.006446	-0.004858	-0.09019	-0.03381	0.006409	0.003479	-0.08328
	88	0.8	2.1	-0.03478	-0.03725	-0.03166	-0.009786	-0.02952	-0.03952	-0.03525	-0.01032	-0.02509	-0.03621
	89	0.8	2.7	0.006945	-0.00167	-0.01105	-0.008399	0.005808	-0.002164	-0.01238	-0.00793	0.004955	-0.002289
Bin Central Value			Asym.	0.5									
			bin #	172	173	177	178	179	180	184	185	186	187
			$\cos(\theta)$	0.0	0.0	0.4	0.4	0.4	0.4	0.8	0.8	0.8	0.8
Asym.	bin #	$\cos(\theta)$	ΣE_{e^-} (MeV)	2.1	2.7	0.9	1.5	2.1	2.7	0.9	1.5	2.1	2.7
0.1	58	-0.8	0.9	0.001589	0.009404	-0.01977	-0.04395	0.00251	0.007482	-0.03474	-0.03388	0.001916	0.005445
	59	-0.8	1.5	-0.04797	0.008863	-0.02061	-0.1098	-0.03858	0.007594	-0.02585	-0.0895	-0.02967	0.005825
	60	-0.8	2.1	-0.05352	-0.0158	-0.02612	-0.05024	-0.04396	-0.01358	-0.02171	-0.04012	-0.03379	-0.008992
	61	-0.8	2.7	-0.01819	-0.01218	0.006769	-0.004256	-0.01467	-0.009659	0.005671	-0.003155	-0.01124	-0.008291
	65	-0.4	0.9	0.002189	0.009725	-0.01337	-0.04441	0.003094	0.007903	-0.02935	-0.03347	0.002602	0.005704
	66	-0.4	1.5	-0.05392	0.009477	-0.01326	-0.118	-0.04347	0.008426	-0.01891	-0.09588	-0.03359	0.006175
	67	-0.4	2.1	-0.0615	-0.01869	-0.03517	-0.05704	-0.0508	-0.01686	-0.02953	-0.04563	-0.03913	-0.01131
	68	-0.4	2.7	-0.02456	-0.01489	0.008646	-0.007133	-0.01857	-0.01062	0.007176	-0.005717	-0.01292	-0.009775
	72	0.0	0.9	0.002519	0.009312	-0.00496	-0.04126	0.00335	0.007734	-0.02088	-0.03033	0.003105	0.005552
	73	0.0	1.5	-0.0513	0.00881	-0.008575	-0.1135	-0.04156	0.008278	-0.01437	-0.09171	-0.03216	0.005718
	74	0.0	2.1	-0.05571	-0.01686	-0.03223	-0.05219	-0.04631	-0.01619	-0.02663	-0.04157	-0.03582	-0.01111
	75	0.0	2.7	-0.02128	-0.0135	0.007427	-0.006122	-0.01668	-0.009033	0.006169	-0.004604	-0.01266	-0.009706
	79	0.4	0.9	0.002507	0.008083	0.002734	-0.03445	0.003143	0.006841	-0.009295	-0.02428	0.003333	0.004926
	80	0.4	1.5	-0.04005	0.007203	-0.004338	-0.09072	-0.03265	0.007141	-0.005865	-0.07216	-0.02526	0.004711
	81	0.4	2.1	-0.04464	-0.01325	-0.02545	-0.04084	-0.03684	-0.01417	-0.02094	-0.03182	-0.02772	-0.009669
	82	0.4	2.7	-0.01553	-0.01186	0.005887	-0.004409	-0.0118	-0.002604	0.004865	-0.003865	-0.008923	-0.006244
	86	0.8	0.9	0.002331	0.006705	0.007601	-0.02745	0.002856	0.005712	0.000837	-0.01834	0.003399	0.004211
	87	0.8	1.5	-0.02987	0.006138	0.003978	-0.06813	-0.02452	0.006007	0.007327	-0.05126	-0.01821	0.003991
	88	0.8	2.1	-0.03204	-0.009595	-0.01967	-0.02866	-0.02628	-0.01114	-0.01675	-0.02101	-0.0181	-0.007369
	89	0.8	2.7	-0.01113	-0.008117	0.003884	-0.002321	-0.00765	-0.00375	0.003226	-0.001441	-0.005361	-0.004538

Table A.2: Correlation matrix from 3D unfolding (cont.).

Bin Central Value			Asym.	0.7									
			bin #	205	206	207	208	212	213	214	215	219	220
			$\cos(\theta)$	-0.8	-0.8	-0.8	-0.8	-0.4	-0.4	-0.4	-0.4	0.0	0.0
Asym.	bin #	$\cos(\theta)$	ΣE_{e^-} (MeV)	0.9	1.5	2.1	2.7	0.9	1.5	2.1	2.7	0.1	1.5
0.1	58	-0.8	0.9	0.07067	0.05584	0.002724	0.00173	0.06471	0.07486	0.00309	0.001673	0.04552	0.06759
	59	-0.8	1.5	-0.001562	-0.01325	0.01006	0.004789	-0.003079	-0.005493	0.009938	0.00571	-0.002604	0.001598
	60	-0.8	2.1	-0.02241	-0.03408	0.00503	0.003399	-0.01883	-0.03642	0.005231	0.003853	-0.01361	-0.03056
	61	-0.8	2.7	0.004631	0.003203	0.0006997	0.0007602	0.003904	0.003169	0.001003	0.0008233	0.002879	0.002971
	65	-0.4	0.9	0.1161	0.08253	0.002712	0.001642	0.1024	0.1015	0.003175	0.00168	0.07178	0.08483
	66	-0.4	1.5	0.02203	0.009433	0.01187	0.005159	0.01642	0.01681	0.01166	0.006181	0.01068	0.01955
	67	-0.4	2.1	-0.02864	-0.03823	0.00559	0.003989	-0.02405	-0.0419	0.005803	0.004348	-0.01744	-0.03449
	68	-0.4	2.7	0.005861	0.004829	0.0008751	0.0009239	0.00495	0.004176	0.000995	0.0008364	0.003641	0.003596
	72	0.0	0.9	0.157	0.1061	0.002481	0.001424	0.1385	0.1227	0.003038	0.001563	0.09636	0.09681
	73	0.0	1.5	0.04403	0.0338	0.01248	0.004776	0.035	0.03818	0.01211	0.005711	0.02401	0.03766
	74	0.0	2.1	-0.02684	-0.03226	0.005724	0.004008	-0.0226	-0.03632	0.005918	0.004237	-0.01635	-0.02856
	75	0.0	2.7	0.005181	0.003947	0.001734	0.001179	0.004348	0.003433	0.001729	0.001109	0.0032	0.00283
	79	0.4	0.9	0.1758	0.1195	0.001988	0.001128	0.1639	0.1285	0.002708	0.001316	0.1127	0.1
	80	0.4	1.5	0.05437	0.04854	0.01138	0.003689	0.04743	0.04947	0.01078	0.004449	0.03256	0.04875
	81	0.4	2.1	-0.02233	-0.02336	0.005243	0.003559	-0.01887	-0.02664	0.005243	0.003865	-0.0137	-0.01897
	82	0.4	2.7	0.004141	0.003271	0.0003287	0.0005985	0.003505	0.002883	0.0008073	0.0007382	0.002571	0.002161
	86	0.8	0.9	0.1585	0.118	0.001584	0.0008461	0.1735	0.1174	0.002341	0.00104	0.1169	0.09068
	87	0.8	1.5	0.05241	0.05546	0.009871	0.002743	0.05735	0.05293	0.008648	0.00331	0.03838	0.05378
	88	0.8	2.1	-0.01678	-0.01634	0.004524	0.002788	-0.01449	-0.01829	0.0042	0.003644	-0.01041	-0.0112
	89	0.8	2.7	0.002684	0.001446	0.0003925	0.0008563	0.00227	0.00145	0.0004681	0.0009514	0.001667	0.00133
Bin Central Value			Asym.	0.7									
			bin #	221	222	226	227	228	229	233	234	235	236
			$\cos(\theta)$	0.0	0.0	0.4	0.4	0.4	0.4	0.8	0.8	0.8	0.8
Asym.	bin #	$\cos(\theta)$	ΣE_{e^-} (MeV)	2.1	2.7	0.9	1.5	2.1	2.7	0.9	1.5	2.1	2.7
0.1	58	-0.8	0.9	0.002398	0.001052	0.05373	0.06532	0.002476	0.001043	0.04474	0.04644	0.002742	0.0003838
	59	-0.8	1.5	0.01138	0.003971	-0.0003815	0.00106	0.008554	0.003781	-0.001453	0.00196	0.007713	0.003369
	60	-0.8	2.1	0.005211	0.004468	-0.01409	-0.02804	0.004525	0.003124	-0.01255	-0.01891	0.003395	0.002543
	61	-0.8	2.7	0.0004277	0.00115	0.002893	0.003198	0.0005281	0.00075	0.002565	0.002297	0.0001391	0.0003303
	65	-0.4	0.9	0.002278	0.001085	0.08092	0.08461	0.002697	0.001061	0.07298	0.06255	0.002416	0.0005184
	66	-0.4	1.5	0.01335	0.004315	0.01374	0.02047	0.01022	0.004191	0.01308	0.01387	0.008735	0.003377
	67	-0.4	2.1	0.005591	0.004227	-0.01794	-0.03147	0.005269	0.00359	-0.01604	-0.02254	0.003415	0.002884
	68	-0.4	2.7	0.0001036	0.0009918	0.003661	0.003702	0.0005975	0.0008845	0.003241	0.002748	-5.251e-06	0.0004802
	72	0.0	0.9	0.001928	0.001033	0.1065	0.09965	0.002722	0.00098	0.09949	0.0758	0.001815	0.0006524
	73	0.0	1.5	0.01428	0.003987	0.02664	0.03974	0.01088	0.003863	0.02622	0.02593	0.008537	0.002867
	74	0.0	2.1	0.005584	0.003057	-0.01682	-0.02587	0.005845	0.002969	-0.01503	-0.01919	0.003045	0.00287
	75	0.0	2.7	0.0007267	0.0006434	0.003223	0.003092	0.001034	0.0006544	0.002871	0.002291	7.49e-05	0.0005993
	79	0.4	0.9	0.001549	0.0008742	0.1231	0.1053	0.002578	0.0007169	0.1136	0.08399	0.001003	0.0007808
	80	0.4	1.5	0.01342	0.003047	0.03528	0.04996	0.009832	0.002906	0.03391	0.03333	0.007092	0.002119
	81	0.4	2.1	0.005182	0.001991	-0.01406	-0.01738	0.005711	0.001975	-0.01259	-0.01432	0.002523	0.002851
	82	0.4	2.7	0.0003061	0.0002437	0.002594	0.002316	0.001941	0.0002748	0.002299	0.001759	0.001358	0.0008828
	86	0.8	0.9	0.001157	0.0006833	0.1299	0.0996	0.002119	0.0004644	0.1126	0.08303	0.0004623	0.0008802
	87	0.8	1.5	0.0117	0.002235	0.04258	0.05293	0.008271	0.0021	0.03695	0.03864	0.005555	0.001577
	88	0.8	2.1	0.004348	0.001238	-0.01072	-0.01152	0.004702	0.00116	-0.009552	-0.00981	0.001865	0.002958
	89	0.8	2.7	-0.0001335	0.0001932	0.00168	0.001509	0.0004824	0.0001275	0.001488	0.001198	-0.0002156	0.0006647

Table A.2: Correlation matrix from 3D unfolding (cont.).

Bin Central Value				Asym.	0.3									
				bin #	107	108	109	110	114	115	116	117	121	122
				cos(θ)	-0.8	-0.8	-0.8	-0.8	-0.4	-0.4	-0.4	-0.4	0.0	0.0
Asym.	bin #	cos(θ)		ΣE_{e^-} (MeV)	0.9	1.5	2.1	2.7	0.9	1.5	2.1	2.7	0.1	1.5
0.3	107	-0.8	0.9	1	0.2787	-0.1344	0.02859	0.8094	0.2757	-0.164	0.03199	0.6722	0.2254	
	108	-0.8	1.5	0.2787	1	0.1609	-0.08701	0.2458	0.8926	0.1344	-0.09758	0.1903	0.7033	
	109	-0.8	2.1	-0.1344	0.1609	1	0.2812	-0.1372	0.1199	0.8644	0.2497	-0.1261	0.06292	
	110	-0.8	2.7	0.02859	-0.08701	0.2812	1	0.02915	-0.09334	0.2393	0.7853	0.02652	-0.08765	
	114	-0.4	0.9	0.8094	0.2458	-0.1372	0.02915	1	0.2714	-0.1676	0.0328	0.7601	0.2618	
	115	-0.4	1.5	0.2757	0.8926	0.1199	-0.09334	0.2714	1	0.1392	-0.1056	0.2428	0.8561	
	116	-0.4	2.1	-0.164	0.1344	0.8644	0.2393	-0.1676	0.1392	1	0.2658	-0.1541	0.1213	
	117	-0.4	2.7	0.03199	-0.09758	0.2497	0.7853	0.0328	-0.1056	0.2658	1	0.03013	-0.09984	
	121	0.0	0.9	0.6722	0.1903	-0.1261	0.02652	0.7601	0.2428	-0.1541	0.03013	1	0.2774	
	122	0.0	1.5	0.2254	0.7033	0.06292	-0.08765	0.2618	0.8561	0.1213	-0.09984	0.2774	1	
	123	0.0	2.1	-0.1525	0.0686	0.6059	0.1616	-0.1564	0.1167	0.8191	0.2323	-0.1442	0.1503	
	124	0.0	2.7	0.03201	-0.0948	0.168	0.4825	0.03249	-0.1049	0.2459	0.729	0.02947	-0.1009	
	128	0.4	0.9	0.4831	0.1185	-0.105	0.02186	0.6291	0.1893	-0.1287	0.02512	0.7412	0.2634	
	129	0.4	1.5	0.1636	0.4551	0.005329	-0.07126	0.2205	0.6731	0.08362	-0.08209	0.2654	0.8441	
	130	0.4	2.1	-0.1215	0.01476	0.3272	0.08488	-0.1256	0.08108	0.5856	0.1671	-0.1167	0.1449	
	131	0.4	2.7	0.024	-0.0702	0.074	0.2512	0.02452	-0.07873	0.1563	0.5239	0.02239	-0.07654	
	135	0.8	0.9	0.3484	0.07145	-0.08413	0.01739	0.5013	0.1422	-0.1035	0.02002	0.6371	0.2273	
	136	0.8	1.5	0.1183	0.2696	-0.03031	-0.05294	0.1859	0.4906	0.0431	-0.06134	0.2471	0.7037	
	137	0.8	2.1	-0.09503	-0.01494	0.1705	0.0364	-0.09868	0.05607	0.3934	0.1043	-0.09196	0.133	
	138	0.8	2.7	0.01388	-0.03776	0.01747	0.09839	0.01417	-0.04329	0.06632	0.3091	0.0129	-0.04302	
Bin Central Value				Asym.	0.3									
				bin #	123	124	128	129	130	131	135	136	137	138
				cos(θ)	0.0	0.0	0.4	0.4	0.4	0.4	0.8	0.8	0.8	0.8
Asym.	bin #	cos(θ)		ΣE_{e^-} (MeV)	2.1	2.7	0.9	1.5	2.1	2.7	0.9	1.5	2.1	2.7
0.3	107	-0.8	0.9	-0.1525	0.03201	0.4831	0.1636	-0.1215	0.024	0.3484	0.1183	-0.09503	0.01388	
	108	-0.8	1.5	0.0686	-0.0948	0.1185	0.4551	0.01476	-0.0702	0.07145	0.2696	-0.01494	-0.03776	
	109	-0.8	2.1	0.6059	0.168	-0.105	0.005329	0.3272	0.074	-0.08413	-0.03031	0.1705	0.01747	
	110	-0.8	2.7	0.1616	0.4825	0.02186	-0.07126	0.08488	0.2512	0.01739	-0.05294	0.0364	0.09839	
	114	-0.4	0.9	-0.1564	0.03249	0.6291	0.2205	-0.1256	0.02452	0.5013	0.1859	-0.09868	0.01417	
	115	-0.4	1.5	0.1167	-0.1049	0.1893	0.6731	0.08108	-0.07873	0.1422	0.4906	0.05607	-0.04329	
	116	-0.4	2.1	0.8191	0.2459	-0.1287	0.08362	0.5856	0.1563	-0.1035	0.0431	0.3934	0.06632	
	117	-0.4	2.7	0.2323	0.729	0.02512	-0.08209	0.1671	0.5239	0.02002	-0.06134	0.1043	0.3091	
	121	0.0	0.9	-0.1442	0.02947	0.7412	0.2654	-0.1167	0.02239	0.6371	0.2471	-0.09196	0.0129	
	122	0.0	1.5	0.1503	-0.1009	0.2634	0.8441	0.1449	-0.07654	0.2273	0.7037	0.133	-0.04302	
	123	0.0	2.1	1	0.2882	-0.121	0.1568	0.8226	0.2427	-0.09775	0.1303	0.6476	0.1318	
	124	0.0	2.7	0.2882	1	0.02421	-0.0847	0.2683	0.7356	0.01923	-0.06469	0.2058	0.5061	
	128	0.4	0.9	-0.121	0.02421	1	0.2838	-0.09874	0.01846	0.7428	0.2953	-0.07783	0.01067	
	129	0.4	1.5	0.1568	-0.0847	0.2838	1	0.193	-0.06498	0.2646	0.85	0.206	-0.03692	
	130	0.4	2.1	0.8226	0.2683	-0.09874	0.193	1	0.2797	-0.08044	0.1923	0.8288	0.1878	
	131	0.4	2.7	0.2427	0.7356	0.01846	-0.06498	0.2797	1	0.01464	-0.05001	0.2527	0.648	
	135	0.8	0.9	-0.09775	0.01923	0.7428	0.2646	-0.08044	0.01464	1	0.3063	-0.06339	0.008507	
	136	0.8	1.5	0.1303	-0.06469	0.2953	0.85	0.1923	-0.05001	0.3063	1	0.231	-0.02822	
	137	0.8	2.1	0.6476	0.2058	-0.07783	0.206	0.8288	0.2527	-0.06339	0.231	1	0.2097	
	138	0.8	2.7	0.1318	0.5061	0.01067	-0.03692	0.1878	0.648	0.008507	-0.02822	0.2097	1	

Table A.2: Correlation matrix from 3D unfolding (cont.).

Bin Central Value				Asym.	0.5									
				bin #	156	157	158	159	163	164	165	166	170	171
				$\cos(\theta)$	-0.8	-0.8	-0.8	-0.8	-0.4	-0.4	-0.4	-0.4	0.0	0.0
Asym.	bin #	$\cos(\theta)$		ΣE_{e^-} (MeV)	0.9	1.5	2.1	2.7	0.9	1.5	2.1	2.7	0.1	1.5
0.3	107	-0.8	0.9	0.395	-0.02824	-0.03806	0.01538	0.3247	-0.03847	-0.04549	0.01753	0.21	-0.04299	
	108	-0.8	1.5	0.2301	0.2947	-0.0583	-0.04533	0.1779	0.2598	-0.08257	-0.05145	0.1253	0.1756	
	109	-0.8	2.1	-0.08865	0.009929	0.1256	0.007266	-0.07417	-0.01105	0.08809	-0.003577	-0.06524	-0.03028	
	110	-0.8	2.7	0.01539	-0.07167	0.017	0.1651	0.0129	-0.07768	0.007278	0.1309	0.0113	-0.07382	
	114	-0.4	0.9	0.3594	-0.0344	-0.03985	0.01576	0.3225	-0.03836	-0.04782	0.01809	0.2456	-0.03928	
	115	-0.4	1.5	0.2313	0.2598	-0.08495	-0.04877	0.1936	0.2729	-0.095	-0.0563	0.1554	0.2357	
	116	-0.4	2.1	-0.1063	0.001348	0.088	-0.008056	-0.08926	-0.0007644	0.09542	-0.004409	-0.07875	-0.002342	
	117	-0.4	2.7	0.01697	-0.08001	-0.003072	0.1743	0.01438	-0.08663	0.0003366	0.1816	0.01277	-0.08229	
	121	0.0	0.9	0.2917	-0.03978	-0.03743	0.01449	0.2922	-0.03858	-0.04511	0.01671	0.2598	-0.03503	
	122	0.0	1.5	0.1959	0.1773	-0.1042	-0.04555	0.1835	0.2399	-0.09896	-0.05341	0.18	0.2599	
	123	0.0	2.1	-0.0998	-0.02414	0.02514	-0.02526	-0.08393	-0.00749	0.07055	-0.01006	-0.07508	0.01278	
	124	0.0	2.7	0.01725	-0.08384	0.006699	0.1024	0.01453	-0.09278	0.02177	0.1442	0.01268	-0.09006	
	128	0.4	0.9	0.2097	-0.03998	-0.03242	0.01216	0.2388	-0.03228	-0.03931	0.01405	0.2541	-0.02463	
	129	0.4	1.5	0.1455	0.08255	-0.1085	-0.03638	0.1493	0.1714	-0.09328	-0.04364	0.1665	0.2376	
	130	0.4	2.1	-0.07861	-0.03781	-0.02638	-0.03401	-0.06722	-0.01286	0.03522	-0.01374	-0.06094	0.02008	
	131	0.4	2.7	0.0127	-0.05933	-0.01821	0.05026	0.01077	-0.06654	-0.004635	0.1059	0.009484	-0.0655	
	135	0.8	0.9	0.1557	-0.03687	-0.02656	0.009883	0.1868	-0.02623	-0.03267	0.01134	0.2193	-0.0191	
	136	0.8	1.5	0.1101	0.01883	-0.09971	-0.02655	0.121	0.1126	-0.08264	-0.03282	0.1574	0.1998	
	137	0.8	2.1	-0.06112	-0.04228	-0.05878	-0.03828	-0.05298	-0.01191	-0.005471	-0.01949	-0.04789	0.02868	
	138	0.8	2.7	0.007282	-0.03151	-0.02658	0.01696	0.006103	-0.03629	-0.01559	0.06001	0.005412	-0.03617	
Bin Central Value				Asym.	0.5									
				bin #	172	173	177	178	179	180	184	185	186	187
				$\cos(\theta)$	0.0	0.0	0.4	0.4	0.4	0.4	0.8	0.8	0.8	0.8
Asym.	bin #	$\cos(\theta)$		ΣE_{e^-} (MeV)	2.1	2.7	0.9	1.5	2.1	2.7	0.9	1.5	2.1	2.7
0.3	107	-0.8	0.9	-0.04281	0.01688	0.1346	-0.04902	-0.03122	0.01287	0.09641	-0.04745	-0.02346	0.009979	
	108	-0.8	1.5	-0.1034	-0.05076	0.09402	0.08358	-0.1049	-0.03852	0.07288	0.02275	-0.0909	-0.02593	
	109	-0.8	2.1	0.02025	-0.01725	-0.05702	-0.04442	-0.02542	-0.03435	-0.05217	-0.04813	-0.04861	-0.03216	
	110	-0.8	2.7	-0.004896	0.06689	0.009259	-0.0617	-0.008649	0.0464	0.008035	-0.04767	-0.01349	0.03025	
	114	-0.4	0.9	-0.04478	0.01739	0.169	-0.04286	-0.03254	0.01343	0.1452	-0.03918	-0.02453	0.01016	
	115	-0.4	1.5	-0.09554	-0.05615	0.1321	0.1747	-0.08902	-0.04324	0.1142	0.1175	-0.07181	-0.03039	
	116	-0.4	2.1	0.07206	-0.007108	-0.06836	-0.006284	0.0432	-0.01831	-0.06225	-0.011	0.01402	-0.01775	
	117	-0.4	2.7	0.003513	0.1252	0.01043	-0.06923	0.002957	0.09994	0.009037	-0.05394	-0.004645	0.04917	
	121	0.0	0.9	-0.04223	0.01604	0.1929	-0.03481	-0.0308	0.01258	0.1826	-0.02921	-0.02331	0.009291	
	122	0.0	1.5	-0.0785	-0.05356	0.1671	0.2469	-0.063	-0.04182	0.1728	0.2056	-0.04316	-0.03071	
	123	0.0	2.1	0.09974	0.003286	-0.06489	0.02719	0.1037	0.001511	-0.05919	0.02711	0.08236	0.0006396	
	124	0.0	2.7	0.033	0.1602	0.0103	-0.07722	0.03763	0.1888	0.008867	-0.06148	0.0275	0.1494	
	128	0.4	0.9	-0.0363	0.01339	0.2061	-0.0193	-0.02672	0.01068	0.2247	-0.01019	-0.02019	0.007633	
	129	0.4	1.5	-0.05726	-0.04346	0.1651	0.2724	-0.03356	-0.03458	0.1836	0.2618	-0.0101	-0.02663	
	130	0.4	2.1	0.1041	0.009626	-0.05292	0.04635	0.1489	0.01873	-0.04821	0.05756	0.1488	0.02092	
	131	0.4	2.7	0.01689	0.1468	0.007692	-0.05707	0.04158	0.2184	0.006639	-0.04516	0.04629	0.1825	
	135	0.8	0.9	-0.02972	0.01071	0.1891	-0.0115	-0.02217	0.008593	0.246	0.002703	-0.01671	0.006088	
	136	0.8	1.5	-0.04183	-0.03256	0.1592	0.2614	-0.01598	-0.0268	0.2113	0.2918	0.01328	-0.02172	
	137	0.8	2.1	0.06783	0.003627	-0.041	0.06208	0.1372	0.01796	-0.03691	0.08057	0.1691	0.02609	
	138	0.8	2.7	0.00256	0.09943	0.004404	-0.0318	0.02525	0.1632	0.003808	-0.02465	0.0317	0.1605	

Table A.2: Correlation matrix from 3D unfolding (cont.).

Bin Central Value			Asym.	0.7									
			bin #	205	206	207	208	212	213	214	215	219	220
			cos(θ)	-0.8	-0.8	-0.8	-0.8	-0.4	-0.4	-0.4	-0.4	0.0	0.0
Asym.	bin #	cos(θ)	ΣE_{e^-} (MeV)	0.9	1.5	2.1	2.7	0.9	1.5	2.1	2.7	0.1	1.5
0.3	107	-0.8	0.9	0.008909	0.05007	0.004806	0.004045	-0.02265	-0.0006965	-0.04337	-0.005662	0.00262	0.05895
	108	-0.8	1.5	-0.02289	-0.001593	-0.04028	-0.003734	-0.006095	-0.04263	-0.1037	-0.04025	-0.01517	-0.0125
	109	-0.8	2.1	-0.008409	-0.04951	-0.09225	-0.03643	0.002473	-0.01981	-0.04722	-0.03094	-0.005526	-0.04873
	110	-0.8	2.7	0.003057	-0.02218	-0.04287	-0.028	0.01644	0.07603	0.005542	0.004205	0.00194	-0.01458
	114	-0.4	0.9	0.03132	0.07976	0.004794	0.003969	-0.01909	0.01477	-0.04607	-0.006232	0.01478	0.08073
	115	-0.4	1.5	-0.01709	0.0129	-0.04434	-0.004335	-0.007961	-0.04424	-0.1171	-0.04611	-0.011	0.00456
	116	-0.4	2.1	-0.01087	-0.05352	-0.1054	-0.0412	0.002956	-0.02185	-0.05035	-0.03333	-0.007137	-0.04996
	117	-0.4	2.7	0.003668	-0.02446	-0.04272	-0.02998	0.02938	0.09834	0.005098	0.003913	0.002315	-0.01654
	121	0.0	0.9	0.04625	0.1057	0.004277	0.003547	-0.01289	0.03293	-0.04129	-0.005596	0.02673	0.09453
	122	0.0	1.5	-0.008443	0.03162	-0.04138	-0.00411	-0.007477	-0.03905	-0.1064	-0.04214	-0.004581	0.02578
	123	0.0	2.1	-0.01018	-0.0497	-0.09809	-0.03684	0.002067	-0.0158	-0.03964	-0.02646	-0.006658	-0.04183
	124	0.0	2.7	0.003362	-0.02729	-0.04846	-0.03222	0.03412	0.1082	0.004295	0.003405	0.002129	-0.01883
	128	0.4	0.9	0.05449	0.1204	0.003325	0.002891	-0.006996	0.04137	-0.03181	-0.004241	0.03533	0.09943
	129	0.4	1.5	-0.003755	0.04141	-0.03339	-0.003137	-0.006346	-0.0299	-0.0864	-0.03465	0.00007989	0.03832
	130	0.4	2.1	-0.008569	-0.03752	-0.07893	-0.02895	0.002067	-0.0158	-0.03964	-0.02646	-0.005712	-0.03097
	131	0.4	2.7	0.002548	-0.01706	-0.03441	-0.02497	0.02746	0.09228	0.003682	0.002883	0.00162	-0.01234
	135	0.8	0.9	0.03974	0.1118	0.002584	0.002257	-0.004627	0.0418	-0.02233	-0.002644	0.03339	0.08924
	136	0.8	1.5	-0.0004863	0.04358	-0.02478	-0.00199	-0.004614	-0.0228	-0.06164	-0.02364	0.00296	0.04463
	137	0.8	2.1	-0.006562	-0.03151	-0.05653	-0.01873	0.001299	-0.009613	-0.02084	-0.01453	-0.004263	-0.02105
	138	0.8	2.7	0.00166	-0.01047	-0.0206	-0.01464	-0.007034	0.01294	0.00298	0.003293	0.001019	-0.007955
Bin Central Value			Asym.	0.7									
			bin #	221	222	226	227	228	229	233	234	235	236
			cos(θ)	0.0	0.0	0.4	0.4	0.4	0.4	0.8	0.8	0.8	0.8
Asym.	bin #	cos(θ)	ΣE_{e^-} (MeV)	2.1	2.7	0.9	1.5	2.1	2.7	0.9	1.5	2.1	2.7
0.3	107	-0.8	0.9	0.005888	0.003469	0.0007866	0.0539	0.004335	0.00256	-0.01656	-0.01657	-0.02344	0.001471
	108	-0.8	1.5	-0.04349	-0.001803	-0.01828	-0.01415	-0.0375	-0.00147	-0.003226	-0.03047	-0.06329	-0.0005596
	109	-0.8	2.1	-0.1012	-0.03083	-0.003843	-0.04203	-0.08538	-0.02515	0.001539	-0.008244	-0.02913	-0.02112
	110	-0.8	2.7	-0.04568	-0.02387	0.001759	-0.009693	-0.03788	-0.02144	0.003481	0.05147	0.003982	-0.01756
	114	-0.4	0.9	0.006458	0.003692	0.005724	0.07283	0.004553	0.002782	-0.01561	-0.005084	-0.02492	0.001517
	115	-0.4	1.5	-0.04721	-0.002774	-0.01725	0.00106	-0.04012	-0.001829	-0.00432	-0.03268	-0.07375	-0.000963
	116	-0.4	2.1	-0.1147	-0.03632	-0.005097	-0.04421	-0.0968	-0.02942	0.001824	-0.01054	-0.03081	-0.02541
	117	-0.4	2.7	-0.04944	-0.02968	0.002076	-0.0119	-0.04079	-0.02685	0.007187	0.06061	0.003466	-0.02095
	121	0.0	0.9	0.006288	0.003493	0.01101	0.08815	0.004327	0.00259	-0.01334	0.009908	-0.02256	0.001394
	122	0.0	1.5	-0.04324	-0.003232	-0.01436	0.02037	-0.03609	-0.001856	-0.003951	-0.02728	-0.06508	-0.001076
	123	0.0	2.1	-0.1034	-0.03337	-0.004715	-0.03791	-0.08492	-0.02714	0.001672	-0.0132	-0.03794	-0.0234
	124	0.0	2.7	-0.05374	-0.02964	0.001912	-0.01457	-0.04646	-0.02696	0.009172	0.06563	0.002746	-0.02102
	128	0.4	0.9	0.005594	0.002892	0.01403	0.09504	0.003949	0.002068	-0.01092	0.01916	-0.0176	0.001165
	129	0.4	1.5	-0.03405	-0.003018	-0.0104	0.03261	-0.0276	-0.001512	-0.003418	-0.02181	-0.05365	-0.0008591
	130	0.4	2.1	-0.08494	-0.02768	-0.00406	-0.02923	-0.06933	-0.023	0.001263	-0.008974	-0.02478	-0.01958
	131	0.4	2.7	-0.04097	-0.02388	0.001451	-0.009666	-0.03276	-0.02226	0.00612	0.06482	0.002246	-0.01497
	135	0.8	0.9	0.004721	0.002188	0.01424	0.08394	0.00346	0.001544	-0.008498	0.02476	-0.0121	0.0009626
	136	0.8	1.5	-0.02416	-0.002529	-0.007132	0.0381	-0.01941	-0.0008662	-0.002472	-0.01582	-0.03831	-0.0005466
	137	0.8	2.1	-0.06172	-0.01899	-0.002875	-0.02045	-0.05016	-0.01606	0.0008287	-0.005478	-0.01413	-0.01411
	138	0.8	2.7	-0.01762	-0.01303	0.0009251	-0.006024	-0.01593	-0.01108	-0.005634	0.01081	0.003534	-0.008129

Table A.2: Correlation matrix from 3D unfolding (cont.).

Bin Central Value				Asym.	0.5									
				bin #	156	157	158	159	163	164	165	166	170	171
				$\cos(\theta)$	-0.8	-0.8	-0.8	-0.8	-0.4	-0.4	-0.4	-0.4	0.0	0.0
Asym.	bin #	$\cos(\theta)$		ΣE_{e^-} (MeV)	0.9	1.5	2.1	2.7	0.9	1.5	2.1	2.7	0.1	1.5
0.5	156	-0.8	0.9	1	0.06974	-0.04283	0.009974	0.2624	0.07339	-0.04979	0.01141	0.2031	0.05139	
	157	-0.8	1.5	0.06974	1	0.2112	-0.1098	0.03942	0.8918	0.1902	-0.1246	0.02916	0.7016	
	158	-0.8	2.1	-0.04283	0.2112	1	0.2143	-0.03495	0.1674	0.8675	0.1798	-0.03311	0.09797	
	159	-0.8	2.7	0.009974	-0.1098	0.2143	1	0.008326	-0.1193	0.174	0.748	0.0076	-0.1127	
	163	-0.4	0.9	0.2624	0.03942	-0.03495	0.008326	1	0.0497	-0.04047	0.009567	0.1876	0.04244	
	164	-0.4	1.5	0.07339	0.8918	0.1674	-0.1193	0.0497	1	0.1918	-0.1365	0.05376	0.8578	
	165	-0.4	2.1	-0.04979	0.1902	0.8675	0.174	-0.04047	0.1918	1	0.192	-0.03892	0.1645	
	166	-0.4	2.7	0.01141	-0.1246	0.1798	0.748	0.009567	-0.1365	0.192	1	0.008788	-0.1299	
	170	0.0	0.9	0.2031	0.02916	-0.03311	0.0076	0.1876	0.05376	-0.03892	0.008788	1	0.05799	
	171	0.0	1.5	0.05139	0.7016	0.09797	-0.1127	0.04244	0.8578	0.1645	-0.1299	0.05799	1	
	172	0.0	2.1	-0.0465	0.1027	0.5903	0.1082	-0.03706	0.1576	0.8226	0.1752	-0.03531	0.1964	
	173	0.0	2.7	0.01104	-0.1258	0.1271	0.485	0.009236	-0.1377	0.1858	0.6821	0.008454	-0.1304	
	177	0.4	0.9	0.1585	0.02881	-0.0315	0.006314	0.1641	0.06025	-0.03653	0.007207	0.2103	0.07898	
	178	0.4	1.5	0.03195	0.4526	0.02855	-0.09416	0.03109	0.6771	0.116	-0.1096	0.05599	0.8487	
	179	0.4	2.1	-0.03474	0.03633	0.3043	0.03668	-0.02804	0.1097	0.5854	0.1222	-0.02756	0.1837	
	180	0.4	2.7	0.008121	-0.09521	0.03237	0.2399	0.00688	-0.107	0.1208	0.4681	0.006291	-0.1039	
	184	0.8	0.9	0.1495	0.01297	-0.03018	0.005768	0.1483	0.0566	-0.03567	0.006542	0.2405	0.0803	
	185	0.8	1.5	0.01789	0.2748	-0.01426	-0.07286	0.01777	0.4975	0.06771	-0.08547	0.04992	0.7148	
	186	0.8	2.1	-0.02637	-0.002607	0.1329	-0.007292	-0.02189	0.0704	0.3807	0.07707	-0.02053	0.1607	
	187	0.8	2.7	0.006341	-0.06485	-0.01863	0.1736	0.005342	-0.07399	0.05446	0.3368	0.004709	-0.07281	
Bin Central Value				Asym.	0.5									
				bin #	172	173	177	178	179	180	184	185	186	187
				$\cos(\theta)$	0.0	0.0	0.4	0.4	0.4	0.4	0.8	0.8	0.8	0.8
Asym.	bin #	$\cos(\theta)$		ΣE_{e^-} (MeV)	2.1	2.7	0.9	1.5	2.1	2.7	0.9	1.5	2.1	2.7
0.5	156	-0.8	0.9	-0.0465	0.01104	0.1585	0.03195	-0.03474	0.008121	0.1495	0.01789	-0.02637	0.006341	
	157	-0.8	1.5	0.1027	-0.1258	0.02881	0.4526	0.03633	-0.09521	0.01297	0.2748	-0.002607	-0.06485	
	158	-0.8	2.1	0.5903	0.1271	-0.0315	0.02855	0.3043	0.03237	-0.03018	-0.01426	0.1329	-0.01863	
	159	-0.8	2.7	0.1082	0.485	0.006314	-0.09416	0.03668	0.2399	0.005768	-0.07286	-0.007292	0.1736	
	163	-0.4	0.9	-0.03706	0.009236	0.1641	0.03109	-0.02804	0.00688	0.1483	0.01777	-0.02189	0.005342	
	164	-0.4	1.5	0.1576	-0.1377	0.06025	0.6771	0.1097	-0.107	0.0566	0.4975	0.0704	-0.07399	
	165	-0.4	2.1	0.8226	0.1858	-0.03653	0.116	0.5854	0.1208	-0.03567	0.06771	0.3807	0.05446	
	166	-0.4	2.7	0.1752	0.6821	0.007207	-0.1096	0.1222	0.4681	0.006542	-0.08547	0.07707	0.3368	
	170	0.0	0.9	-0.03531	0.008454	0.2103	0.05599	-0.02756	0.006291	0.2405	0.04992	-0.02053	0.004709	
	171	0.0	1.5	0.1964	-0.1304	0.07898	0.8487	0.1837	-0.1039	0.0803	0.7148	0.1607	-0.07281	
	172	0.0	2.1	1	0.2236	-0.03199	0.1986	0.8273	0.2137	-0.03082	0.169	0.6483	0.145	
	173	0.0	2.7	0.2236	1	0.006897	-0.1103	0.2032	0.6702	0.006184	-0.08584	0.1584	0.5644	
	177	0.4	0.9	-0.03199	0.006897	1	0.08203	-0.02473	0.005092	0.2782	0.07238	-0.01882	0.003844	
	178	0.4	1.5	0.1986	-0.1103	0.08203	1	0.2253	-0.0905	0.09634	0.8456	0.2236	-0.06443	
	179	0.4	2.1	0.8273	0.2032	-0.02473	0.2253	1	0.2724	-0.02487	0.2232	0.8506	0.2239	
	180	0.4	2.7	0.2137	0.6702	0.005092	-0.0905	0.2724	1	0.004576	-0.07181	0.2687	0.7152	
	184	0.8	0.9	-0.03082	0.006184	0.2782	0.09634	-0.02487	0.004576	1	0.1011	-0.01783	0.003454	
	185	0.8	1.5	0.169	-0.08584	0.07238	0.8456	0.2232	-0.07181	0.1011	1	0.259	-0.0518	
	186	0.8	2.1	0.6483	0.1584	-0.01882	0.2236	0.8506	0.2687	-0.01783	0.259	1	0.2464	
	187	0.8	2.7	0.145	0.5644	0.003844	-0.06443	0.2239	0.7152	0.003454	-0.0518	0.2464	1	

Table A.2: Correlation matrix from 3D unfolding (cont.).

Bin Central Value			Asym.	0.7									
			bin #	205	206	207	208	212	213	214	215	219	220
			cos(θ)	-0.8	-0.8	-0.8	-0.8	-0.4	-0.4	-0.4	-0.4	0.0	0.0
Asym.	bin #	cos(θ)	ΣE_{e^-} (MeV)	0.9	1.5	2.1	2.7	0.9	1.5	2.1	2.7	0.1	1.5
0.5	156	-0.8	0.9	-0.005211	0.01859	0.00256	0.002694	-0.007034	0.01294	0.00298	0.003293	-0.004054	0.02798
	157	-0.8	1.5	-0.007263	0.181	-0.05909	-0.03355	-0.005736	0.1476	-0.07724	-0.04082	-0.002907	0.06461
	158	-0.8	2.1	0.0003545	0.1646	0.3066	0.004681	0.0008245	0.1159	0.2552	-0.008552	-0.0001434	0.07342
	159	-0.8	2.7	0.001153	-0.06937	0.0636	0.2378	0.0008686	-0.05671	0.0556	0.2021	0.0007644	-0.05097
	163	-0.4	0.9	-0.002486	0.02543	0.001971	0.002196	-0.006896	0.01877	0.002656	0.002777	-0.0008048	0.02784
	164	-0.4	1.5	-0.007353	0.1679	-0.07031	-0.03658	-0.006173	0.1554	-0.08219	-0.04424	-0.002799	0.1016
	165	-0.4	2.1	0.0004198	0.1561	0.266	-0.01312	0.001057	0.1287	0.2823	-0.005633	-0.0001859	0.113
	166	-0.4	2.7	0.001314	-0.07548	0.05691	0.1821	0.0009481	-0.06193	0.06771	0.1844	0.0008461	-0.05596
	170	0.0	0.9	0.004787	0.03697	0.001441	0.001936	-0.00142	0.02784	0.002748	0.003028	0.003902	0.03532
	171	0.0	1.5	-0.006478	0.1174	-0.08172	-0.03433	-0.005247	0.1311	-0.08483	-0.04173	-0.001873	0.1185
	172	0.0	2.1	0.0005805	0.1078	0.1559	-0.03872	0.0008705	0.1073	0.225	-0.01566	-0.0001548	0.13
	173	0.0	2.7	0.001239	-0.08082	0.04951	0.1555	0.001031	-0.06627	0.07967	0.1967	0.0008817	-0.05892
	177	0.4	0.9	0.001393	0.03744	0.000773	0.001577	0.0009557	0.02766	0.002393	0.002576	0.006482	0.03026
	178	0.4	1.5	-0.004972	0.07669	-0.07356	-0.02904	-0.004029	0.1014	-0.07117	-0.03562	-0.001104	0.123
	179	0.4	2.1	0.0006035	0.05767	0.04152	-0.05386	0.0009034	0.07305	0.1346	-0.02602	0.00002133	0.1152
	180	0.4	2.7	0.00117	-0.06103	0.01261	0.0678	0.0009054	-0.05132	0.05309	0.1211	0.0007858	-0.04838
	184	0.8	0.9	-0.003875	0.03787	0.0005194	0.001401	-0.004911	0.02289	0.002607	0.003285	0.0006236	0.0335
	185	0.8	1.5	-0.00388	0.04135	-0.0633	-0.02236	-0.003159	0.06158	-0.05808	-0.02762	-0.00108	0.1021
	186	0.8	2.1	0.0005851	0.03346	-0.005073	-0.05547	0.000991	0.04615	0.08296	-0.03086	0.0001244	0.09248
	187	0.8	2.7	0.0007364	-0.04135	-0.02303	0.03589	0.0006754	-0.03501	0.01229	0.07699	0.0005546	-0.03332
Bin Central Value			Asym.	0.7									
			bin #	221	222	226	227	228	229	233	234	235	236
			cos(θ)	0.0	0.0	0.4	0.4	0.4	0.4	0.8	0.8	0.8	0.8
Asym.	bin #	cos(θ)	ΣE_{e^-} (MeV)	2.1	2.7	0.9	1.5	2.1	2.7	0.9	1.5	2.1	2.7
0.5	156	-0.8	0.9	0.002869	0.002334	-0.006032	0.02637	0.002641	0.001755	-0.005634	0.01081	0.003534	0.001737
	157	-0.8	1.5	-0.07738	-0.02897	-0.003934	0.02949	-0.07498	-0.02196	-0.003374	0.004596	-0.06034	-0.01628
	158	-0.8	2.1	0.1569	-0.0445	0.0009823	0.03504	0.06409	-0.04874	0.0009315	0.02236	0.003731	-0.05797
	159	-0.8	2.7	0.03592	0.09746	0.0005826	-0.03639	0.007958	0.06686	0.0004965	-0.0322	-0.001953	0.04332
	163	-0.4	0.9	0.002791	0.001994	-0.008002	0.02688	0.002247	0.0015	-0.0073	0.01314	0.002676	0.001336
	164	-0.4	1.5	-0.07606	-0.03291	-0.004362	0.06151	-0.07207	-0.02511	-0.003719	0.03304	-0.05943	-0.0184
	165	-0.4	2.1	0.253	-0.0235	0.0012	0.07473	0.1838	-0.03361	0.001138	0.05803	0.112	-0.04056
	166	-0.4	2.7	0.0593	0.1739	0.0006068	-0.03845	0.04543	0.1451	0.0005203	-0.03396	0.04282	0.1083
	170	0.0	0.9	0.002421	0.001815	-0.005395	0.03541	0.001598	0.001393	-0.005158	0.01734	0.003185	0.001266
	171	0.0	1.5	-0.06897	-0.03215	-0.003775	0.0922	-0.05932	-0.02443	-0.003228	0.07026	-0.04805	-0.01748
	172	0.0	2.1	0.283	-0.006607	0.001066	0.111	0.2712	-0.01624	0.001104	0.1016	0.2161	-0.009019
	173	0.0	2.7	0.1013	0.2337	0.0006924	-0.03916	0.1016	0.2296	0.0005752	-0.03741	0.1008	0.2141
	177	0.4	0.9	0.001406	0.00146	-0.002145	0.03775	0.0007686	0.001178	-0.002569	0.01664	0.003447	0.000838
	178	0.4	1.5	-0.05187	-0.02834	-0.002933	0.1118	-0.03909	-0.02165	-0.002509	0.09635	-0.03271	-0.01516
	179	0.4	2.1	0.2486	0.003943	0.0009812	0.1164	0.2953	-0.002581	0.0009747	0.1138	0.2708	0.01865
	180	0.4	2.7	0.09389	0.2053	0.0006162	-0.0317	0.1189	0.2069	0.0005486	-0.02988	0.1291	0.2072
	184	0.8	0.9	0.0004008	0.00127	-0.004317	0.03538	0.0001904	0.001046	-0.005842	0.02064	0.003458	0.0006422
	185	0.8	1.5	-0.0395	-0.02257	-0.002024	0.1092	-0.02624	-0.01669	-0.001863	0.1064	-0.01919	-0.01165
	186	0.8	2.1	0.2069	0.004893	0.0009286	0.1214	0.3053	0.000652	0.0008515	0.1402	0.3178	0.04006
	187	0.8	2.7	0.06063	0.174	0.0004688	-0.02163	0.09873	0.1994	0.0003838	-0.02049	0.1229	0.2367

Table A.2: Correlation matrix from 3D unfolding (cont.).

Bin Central Value				Asym.	0.7									
				bin #	205	206	207	208	212	213	214	215	219	220
				cos(θ)	-0.8	-0.8	-0.8	-0.8	-0.4	-0.4	-0.4	-0.4	0.0	0.0
Asym.	bin #	cos(θ)		ΣE_{e^-} (MeV)	0.9	1.5	2.1	2.7	0.9	1.5	2.1	2.7	0.1	1.5
0.7	205	-0.8	0.9	1	0.02741	0.0001868	0.0001589	0.03088	0.03357	0.0003141	0.0002042	0.0233	0.02438	
	206	-0.8	1.5	0.02741	1	0.1326	-0.0348	0.02263	0.2876	0.1086	-0.03859	0.01714	0.1946	
	207	-0.8	2.1	0.0001868	0.1326	1	0.262	0.0002424	0.0804	0.8828	0.2344	0.0001688	0.04446	
	208	-0.8	2.7	0.0001589	-0.0348	0.262	1	0.00009829	-0.02583	0.241	0.7646	0.0001191	-0.02625	
	212	-0.4	0.9	0.03088	0.02263	0.0002424	0.00009829	1	0.02315	0.0003306	0.0001349	0.02408	0.01585	
	213	-0.4	1.5	0.03357	0.2876	0.0804	-0.02583	0.02315	1	0.07797	-0.02914	0.01725	0.1933	
	214	-0.4	2.1	0.0003141	0.1086	0.8828	0.241	0.0003306	0.07797	1	0.2618	0.0002138	0.06748	
	215	-0.4	2.7	0.0002042	-0.03859	0.2344	0.7646	0.0001349	-0.02914	0.2618	1	0.0001354	-0.0297	
	219	0.0	0.9	0.0233	0.01714	0.0001688	0.0001191	0.02408	0.01725	0.0002138	0.0001354	1	0.01211	
	220	0.0	1.5	0.02438	0.1946	0.04446	-0.02625	0.01585	0.1933	0.06748	-0.0297	0.01211	1	
	221	0.0	2.1	0.0001852	0.07342	0.6222	0.1689	0.00004373	0.0696	0.8081	0.2333	0.0002165	0.09655	
	222	0.0	2.7	0.0001577	-0.03336	0.09301	0.4607	0.00009832	-0.02511	0.1659	0.5601	0.00009324	-0.02372	
	226	0.4	0.9	0.0211	0.01475	0.0001816	0.00005595	0.02726	0.0152	0.0002174	0.000073	0.0182	0.009915	
	227	0.4	1.5	0.02362	0.1202	0.02244	-0.01805	0.01903	0.1328	0.05015	-0.0195	0.01431	0.1921	
	228	0.4	2.1	0.0006245	0.04033	0.3599	0.07285	0.0002703	0.04639	0.5956	0.1572	0.0003645	0.08434	
	229	0.4	2.7	0.0001068	-0.02659	0.05471	0.3776	0.00005551	-0.02034	0.1112	0.4851	0.00005104	-0.01925	
	233	0.8	0.9	0.02698	0.01354	0.0001279	0.00003973	0.02333	0.01599	0.0001438	0.00005697	0.01642	0.01017	
	234	0.8	1.5	0.017	0.09137	0.007637	-0.01598	0.01462	0.09782	0.032	-0.0189	0.01102	0.1583	
	235	0.8	2.1	-0.00007133	-0.001847	0.1706	0.01917	-0.00003377	0.008883	0.3927	0.1031	0.0000033	0.04036	
	236	0.8	2.7	0.00007903	-0.01911	0.001107	0.2721	0.0001723	-0.01583	0.0435	0.3519	0.0001232	-0.0135	
Bin Central Value				Asym.	0.7									
				bin #	221	222	226	227	228	229	233	234	235	236
				cos(θ)	0.0	0.0	0.4	0.4	0.4	0.4	0.8	0.8	0.8	0.8
Asym.	bin #	cos(θ)		ΣE_{e^-} (MeV)	2.1	2.7	0.9	1.5	2.1	2.7	0.9	1.5	2.1	2.7
0.7	205	-0.8	0.9	0.0001852	0.0001577	0.0211	0.02362	0.0006245	0.0001068	0.02698	0.017	-0.00007133	0.00007903	
	206	-0.8	1.5	0.07342	-0.03336	0.01475	0.1202	0.04033	-0.02659	0.01354	0.09137	-0.001847	-0.01911	
	207	-0.8	2.1	0.6222	0.09301	0.0001816	0.02244	0.3599	0.05471	0.0001279	0.007637	0.1706	0.001107	
	208	-0.8	2.7	0.1689	0.4607	0.00005595	-0.01805	0.07285	0.3776	0.00003973	-0.01598	0.01917	0.2721	
	212	-0.4	0.9	0.00004373	0.00009832	0.02726	0.01903	0.0002703	0.00005551	0.02333	0.01462	-0.00003377	0.0001723	
	213	-0.4	1.5	0.0696	-0.02511	0.0152	0.1328	0.04639	-0.02034	0.01599	0.09782	0.008883	-0.01583	
	214	-0.4	2.1	0.8081	0.1659	0.0002174	0.05015	0.5956	0.1112	0.0001438	0.032	0.3927	0.0435	
	215	-0.4	2.7	0.2333	0.5601	0.000073	-0.0195	0.1572	0.4851	0.00005697	-0.0189	0.1031	0.3519	
	219	0.0	0.9	0.0002165	0.00009324	0.0182	0.01431	0.0003645	0.00005104	0.01642	0.01102	0.00000335	0.0001232	
	220	0.0	1.5	0.09655	-0.02372	0.009915	0.1921	0.08434	-0.01925	0.01017	0.1583	0.04036	-0.0135	
	221	0.0	2.1	1	0.2186	-0.00001682	0.09998	0.8283	0.1736	-0.00004943	0.08008	0.6777	0.1449	
	222	0.0	2.7	0.2186	1	0.00003135	-0.01269	0.2036	0.7554	0.00002614	-0.01493	0.1814	0.5367	
	226	0.4	0.9	-0.00001682	0.00003135	1	0.01268	0.000159	0.0000262	0.01831	0.01031	0.00004411	0.0001222	
	227	0.4	1.5	0.09998	-0.01269	0.01268	1	0.1072	-0.01022	0.01186	0.2191	0.0747	-0.00384	
	228	0.4	2.1	0.8283	0.2036	0.000159	0.1072	1	0.1689	0.0001859	0.0953	0.8581	0.1982	
	229	0.4	2.7	0.1736	0.7554	0.0000262	-0.01022	0.1689	1	0.00003319	-0.01214	0.1626	0.6264	
	233	0.8	0.9	-0.00004943	0.00002614	0.01831	0.01186	0.0001859	0.00003319	1	0.009211	0.00001925	0.00007749	
	234	0.8	1.5	0.08008	-0.01493	0.01031	0.2191	0.0953	-0.01214	0.009211	1	0.0659	-0.005673	
	235	0.8	2.1	0.6777	0.1814	0.00004411	0.0747	0.8581	0.1626	0.00001925	0.0659	1	0.2535	
	236	0.8	2.7	0.1449	0.5367	0.0001222	-0.00384	0.1982	0.6264	0.00007749	-0.005673	0.2535	1	

Table A.2: Correlation matrix from 3D unfolding (cont.).

Appendix B

Colophon

This document was set in the Times Roman typeface using $\text{\LaTeX}$ and $\text{\BibTeX}$.

The Feynman diagrams in figures 2.13, 2.15, and 2.16, were created with a Javascript application and $\text{\LaTeX}$ package written by Alec Aivazis¹.

Data analysis and histogram generation was carried out using ROOT v.5.34/36 and RooUnfold v.1.1.1².

¹<http://github.com/AlecAivazis/feynman>

²<http://hepunx.rl.ac.uk/~adye/software/unfold/RooUnfold>

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