ROLE OF SPIN EFFECTS IN THE NUCLEON CHARGE-EXCHANGE PROCESS $n + p \to p + n$ AT ZERO ANGLE

V.L.Lyuboshitz and V.V.Lyuboshitz [†]

*Joint Institute for Nuclear Research,*
*141980 Dubna, Moscow region, Russia*
[†] *E-mail: Valery.Lyuboshitz@jinr.ru*

## 1. Isotopic structure of $NN$-scattering

Taking into account the isotopic invariance, the nucleon-nucleon scattering is described by the following operator:

$$\hat{f}(\mathbf{p},\mathbf{p}') = \hat{a}(\mathbf{p},\mathbf{p}') + \hat{b}(\mathbf{p},\mathbf{p}')\hat{\boldsymbol{\tau}}^{(1)}\hat{\boldsymbol{\tau}}^{(2)}\,. \tag{1}$$

Here $\hat{\boldsymbol{\tau}}^{(1)}$ and $\hat{\boldsymbol{\tau}}^{(2)}$ are vector Pauli operators in the isotopic space, $\hat{a}(\mathbf{p},\mathbf{p}')$ and $\hat{b}(\mathbf{p},\mathbf{p}')$ are 4-row matrices in the spin space of two nucleons; $\mathbf{p}$ and $\mathbf{p}'$ are the initial and final momenta in the c.m. frame, the directions of $\mathbf{p}'$ are defined within the solid angle in the c.m. frame, corresponding to the front hemisphere.

One should note that the process of elastic neutron-proton scattering into the back hemisphere is interpreted as the charge-exchange process $n + p \to p + n$ .

According to (1), the matrices of amplitudes of proton-proton, neutron-neutron and neutron-proton scattering take the form:

$$\hat{f}_{pp\to pp}(\mathbf{p},\mathbf{p}') = \hat{f}_{nn\to nn}(\mathbf{p},\mathbf{p}') = \hat{a}(\mathbf{p},\mathbf{p}') + \hat{b}(\mathbf{p},\mathbf{p}')\,;$$

$$\hat{f}_{np\to np}(\mathbf{p},\mathbf{p}') = \hat{a}(\mathbf{p},\mathbf{p}') - \hat{b}(\mathbf{p},\mathbf{p}')\,; \tag{2}$$

meantime, the matrix of amplitudes of the charge transfer process is as follows:

$$\hat{f}_{np\to pn}(\mathbf{p},\mathbf{p}') = 2\hat{b}(\mathbf{p},\mathbf{p}') = \hat{f}_{pp\to pp}(\mathbf{p},\mathbf{p}') - \hat{f}_{np\to np}(\mathbf{p},\mathbf{p}')\,. \tag{3}$$

It should be stressed that the differential cross-section of the charge-exchange reaction $\hat{f}_{np\to pn}(\mathbf{p},\mathbf{p}')$, defined in the front hemisphere $0 \le \theta \le \frac{\pi}{2}$, $0 \le \phi \le 2\pi$ (here $\theta$ is the angle between the momenta of initial neutron and final proton, $\phi$ is the azimuthal angle), should coincide with the differential cross-section of the elastic neutron-proton scattering $\hat{f}_{np\to np}(\mathbf{p},-\mathbf{p}')$ into the back hemisphere by the angle $\widetilde{\theta} = \pi - \theta$ at the azimuthal angle $\widetilde{\phi} = \pi + \phi$ in the c.m. frame. Due to the antisymmetry of the state of two fermions with respect to the total permutation, including the permutation of momenta ( $\mathbf{p}' \to -\mathbf{p}'$), permutation of spin projections and permutation of isotopic projections ($p \leftrightarrows n$), the following relation between the amplitudes $\hat{f}_{np\to pn}(\mathbf{p},\mathbf{p}')$ and $\hat{f}_{np\to np}(\mathbf{p},-\mathbf{p}')$ holds [1] :

$$\hat{f}_{np\to pn}(\mathbf{p},\mathbf{p}') = -\hat{P}^{(1,2)}\hat{f}_{np\to np}(\mathbf{p},-\mathbf{p}')\,, \tag{4}$$

where $\hat{P}^{(1,2)}$ is the operator of permutation of spin projections of two particles with equal spins; the matrix elements of this operator are [2]: $\langle m_1' m_2' \mid \hat{P}^{(1,2)} \mid m_1 m_2\rangle = \delta_{m_1' m_2}\delta_{m_2' m_1}$. For particles with spin 1/2 [1,2]

$$\hat{P}^{(1,2)} = \frac{1}{2}(\hat{I}^{(1,2)} + \hat{\boldsymbol{\sigma}}^{(1)}\hat{\boldsymbol{\sigma}}^{(2)}), \tag{5}$$

where $\hat{I}^{(1,2)}$ is the four-row unit matrix, $\hat{\boldsymbol{\sigma}}^{(1)}$, $\hat{\boldsymbol{\sigma}}^{(2)}$ – vector Pauli operators. It is evident that $\hat{P}^{(1,2)}$ is the unitary and Hermitian operator:

$$\hat{P}^{(1,2)} = \hat{P}^{(1,2)+}, \quad \hat{P}^{(1,2)}\hat{P}^{(1,2)+} = \hat{I}^{(1,2)}. \tag{6}$$

Taking into account the relations (5) and (6), the following matrix equality holds:

$$\hat{f}^{+}_{np\to pn}(\mathbf{p},\mathbf{p}')\hat{f}_{np\to pn}(\mathbf{p},\mathbf{p}') = \hat{f}^{+}_{np\to np}(\mathbf{p},-\mathbf{p}')\hat{f}_{np\to np}(\mathbf{p},-\mathbf{p}'). \tag{7}$$

As a result, the differential cross-sections of the charge-exchange process $n+p\to p+n$ and the elastic $np$-scattering in the corresponding back hemisphere coincide at any polarizations of initial nucleons:

$$\frac{d\sigma_{np\to pn}}{d\Omega}(\mathbf{p},\mathbf{p}') = \frac{d\sigma_{np\to np}}{d\Omega}(\mathbf{p},-\mathbf{p}'). \tag{8}$$

However, the separation into the spin-dependent and spin-independent parts is different for the amplitudes $\hat{f}_{np\to pn}(\mathbf{p},\mathbf{p}')$ and $\hat{f}_{np\to np}(\mathbf{p},-\mathbf{p}')$!

## 2. Nucleon charge-exchange process at zero angle

Now let us investigate in detail the nucleon charge transfer reaction $n+p\to p+n$ at zero angle. In the c.m. frame of the $(np)$-system, the amplitude of the nucleon charge transfer in the "forward" direction $\hat{f}_{np\to pn}(0)$ has the following spin structure:

$$\hat{f}_{np\to pn}(0) = c_1\hat{I}^{(1,2)} + c_2\,[\hat{\boldsymbol{\sigma}}^{(1)}\hat{\boldsymbol{\sigma}}^{(2)} - (\hat{\boldsymbol{\sigma}}^{(1)}\mathbf{l})(\hat{\boldsymbol{\sigma}}^{(2)}\mathbf{l})] + c_3\,(\hat{\boldsymbol{\sigma}}^{(1)}\mathbf{l})(\hat{\boldsymbol{\sigma}}^{(2)}\mathbf{l}), \tag{9}$$

where $\mathbf{l}$ is the unit vector directed along the incident neutron momentum. In so doing, the second term in Eq. (9) describes the spin-flip effect, and the third term characterizes the difference between the amplitudes with the parallel and antiparallel orientations of the neutron and proton spins.

The spin structure of the amplitude of the elastic neutron-proton scattering in the "backward" direction $\hat{f}_{np\to np}(\pi)$ is analogous:

$$\hat{f}_{np\to np}(\pi) = \widetilde{c}_1\hat{I}^{(1,2)} + \widetilde{c}_2[\hat{\boldsymbol{\sigma}}^{(1)}\hat{\boldsymbol{\sigma}}^{(2)} - (\hat{\boldsymbol{\sigma}}^{(1)}\mathbf{l})(\hat{\boldsymbol{\sigma}}^{(2)}\mathbf{l})] + \widetilde{c}_3(\hat{\boldsymbol{\sigma}}^{(1)}\mathbf{l})(\hat{\boldsymbol{\sigma}}^{(2)}\mathbf{l}). \tag{10}$$

However, the coefficients $\widetilde{c}$ in Eq.(10) do not coincide with the coefficients $c$ in Eq.(9). According to Eq.(4), the connection between the amplitudes $\hat{f}_{np\to pn}(0)$ and $\hat{f}_{np\to np}(\pi)$ is the following:

$$\hat{f}_{np\to pn}(0) = -\hat{P}^{(1,2)}\hat{f}_{np\to np}(\pi), \tag{11}$$

where the unitary operator $\hat{P}^{(1,2)}$ is determined by Eq.(5).

As a result of calculations with Pauli matrices, we obtain:

$$c_1 = -\frac{1}{2}(\widetilde{c}_1 + 2\widetilde{c}_2 + \widetilde{c}_3); \quad c_2 = -\frac{1}{2}(\widetilde{c}_1 - \widetilde{c}_3); \quad c_3 = -\frac{1}{2}(\widetilde{c}_1 - 2\widetilde{c}_2 + \widetilde{c}_3). \tag{12}$$

Hence, it follows from here that the "forward" differential cross-section of the nucleon charge-exchange reaction $n+p\to p+n$ for unpolarized initial nucleons is described by the expression:

$$\frac{d\sigma_{np\to pn}}{d\Omega}(0) = |c_1|^2 + 2|c_2|^2 + |c_3|^2 =$$

$$= \frac{1}{4}\,|\widetilde{c}_1 + 2\widetilde{c}_2 + \widetilde{c}_3|^2 + \frac{1}{2}\,|\widetilde{c}_1 - \widetilde{c}_3|^2 + \frac{1}{4}\,|\widetilde{c}_1 - 2\widetilde{c}_2 + \widetilde{c}_3|^2 = |\widetilde{c}_1|^2 + 2|\widetilde{c}_2|^2 + |\widetilde{c}_3|^2. \qquad (13)$$

Thus,

$$\frac{d\sigma_{np\to pn}}{d\Omega}(0) = \frac{d\sigma_{np\to np}}{d\Omega}(\pi),$$

just as it must be in accordance with the relation (8).

## 3. Spin-independent and spin-dependent parts of the cross-section of the reaction $n + p \to p + n$ at zero angle

It is clear that the amplitudes of the proton-proton and neutron-proton elastic scattering at zero angle have the structure (9) with the replacements $c_1, c_2, c_3 \to c_1^{(pp)}, c_2^{(pp)}, c_3^{(pp)}$, $c_1, c_2, c_3 \to c_1^{(np)}, c_2^{(np)}, c_3^{(np)}$, respectively. It follows from the isotopic invariance (see Eq. (3)) that

$$c_1 = c_1^{(pp)} - c_1^{(np)}, \quad c_2 = c_2^{(pp)} - c_2^{(np)}, \quad c_3 = c_3^{(pp)} - c_3^{(np)}. \qquad (14)$$

In accordance with the optical theorem, the following relation holds, taking into account Eq.(14):

$$\frac{4\pi}{k}\text{Im } c_1 = \frac{4\pi}{k}(\text{Im } c_1^{(pp)} - \text{Im } c_1^{(np)}) = \sigma_{pp} - \sigma_{np}\,, \qquad (15)$$

where $\sigma_{pp}$ and $\sigma_{np}$ are the total cross-sections of interaction of two unpolarized protons and of an unpolarized neutron with unpolarized proton, respectively (due to the isotopic invariance, $\sigma_{pp} = \sigma_{nn}$); $k = |\mathbf{p}| = |\mathbf{p}'|$ is the modulus of neutron momentum in the c.m. frame of the colliding nucleons [1)].

Taking into account Eqs. (9), (13) and (15), the differential cross-section of the process $n + p \to p + n$ in the "forward" direction for unpolarized nucleons can be presented in the following form, distinguishing the spin-independent and spin-dependent parts:

$$\frac{d\sigma_{np\to pn}}{d\Omega}(0) = |c_1|^2 + 2|c_2|^2 + |c_3|^2 = \frac{d\sigma^{(si)}_{np\to pn}}{d\Omega}(0) + \frac{d\sigma^{(sd)}_{np\to pn}}{d\Omega}(0)\,. \qquad (16)$$

In doing so, the spin-independent part $\frac{d\sigma^{(si)}_{np\to pn}}{d\Omega}(0)$ in Eq.(16) is determined by the difference of total cross-sections of the unpolarized proton-proton and neutron-proton interaction:

$$\frac{d\sigma^{(si)}_{np\to pn}}{d\Omega}(0) = |c_1|^2 = \frac{k^2}{16\pi^2}(\sigma_{pp} - \sigma_{np})^2(1 + \alpha^2)\,, \qquad (17)$$

where $\alpha = \text{Re } c_1/\text{Im } c_1$. The spin-dependent part of the cross-section of the "forward" charge-exchange process is

$$\frac{d\sigma^{(sd)}_{np\to pn}}{d\Omega}(0) = 2|c_2|^2 + |c_3|^2. \qquad (18)$$

Meantime, according to Eqs. (10), (12) and (13), the spin-dependent part of the cross-section of the "backward" elastic $np$-scattering is

$$\frac{d\sigma^{(sd)}_{np\to np}}{d\Omega}(\pi) = 2|\widetilde{c}_2|^2 + |\widetilde{c}_3|^2. \qquad (19)$$

[1)] We use the unit system with $\hbar = c = 1$.

We see that $\frac{d\sigma^{(sd)}_{np\to pn}}{d\Omega}(0) \neq \frac{d\sigma^{(sd)}_{np\to np}}{d\Omega}(\pi)$.

Further it is advisable to deal with the differential cross-section $\left.\frac{d\sigma}{dt}\right|_{t=0}$, being a relativistic invariant ($t = -(p_1-p_2)^2 = (\mathbf{p}-\mathbf{p}')^2-(E-E')^2$ is the square of the 4-dimensional transferred momentum). In the c.m. frame we have: $t = 2k^2(1-\cos\theta)$ and $\frac{d\sigma}{dt} = (\pi/k^2)\frac{d\sigma}{d\Omega}$ . So, in this representation, the spin-independent and spin-dependent parts of the differential cross-section of the "forward" charge transfer process $\left.\frac{d\sigma_{np\to pn}}{dt}\right|_{t=0}$ are as follows: $\left.\frac{d\sigma^{(si)}_{np\to pn}}{dt}\right|_{t=0} = (\pi/k^2)\,|c_1|^2$, $\left.\frac{d\sigma^{(sd)}_{np\to pn}}{dt}\right|_{t=0} = (\pi/k^2)\,(2|c_2|^2+|c_3|^2)$ , and we may write, instead of Eq.(16):

$$\left.\frac{d\sigma_{np\to pn}}{dt}\right|_{t=0} = \left.\frac{d\sigma^{(sd)}_{np\to pn}}{dt}\right|_{t=0} + \frac{1}{16\pi}(\sigma_{pp}-\sigma_{np})^2(1+\alpha^2)\,. \tag{20}$$

Now it should be noted that, in the framework of the impulse approach, there exists a simple connection between the spin-dependent part of the differential cross-section of the charge-exchange reaction $n+p\to p+n$ at zero angle $\left.\frac{d\sigma^{(sd)}_{np\to pn}}{dt}\right|_{t=0}$
( not the "backward" elastic neutron-proton scattering, see Section 2) and the differential cross-section of the deuteron charge-exchange breakup $d+p\to(pp)+n$ in the "forward" direction $\left.\frac{d\sigma_{dp\to(pp)n}}{dt}\right|_{t=0}$ at the deuteron momentum $\mathbf{k}_d = 2\mathbf{k}_n$ ($\mathbf{k}_n$ is the the initial neutron momentum) . In the case of unpolarized particles we have [3,4,5]:

$$\left.\frac{d\sigma_{dp\to(pp)n}}{dt}\right|_{t=0} = \frac{2}{3}\left.\frac{d\sigma^{(sd)}_{np\to pn}}{dt}\right|_{t=0}. \tag{21}$$

In doing so, this formula remains still valid if one takes into account the deuteron $D$-wave state [5].

It is easy to understand also that, due to the isotopic invariance, the same relation (like Eq. (21)) takes place for the process $p+d\to n+(pp)$ at the proton laboratory momentum $\mathbf{k}_p = \mathbf{k}_n$ and for the process $n+d\to p+(nn)$ at the neutron laboratory momentum $\mathbf{k}_n$.

Thus, in principle, taking into account Eqs. (20) and (21),the modulus of the ratio of the real and imaginary parts of the spin-independent charge transfer amplitude at zero angle ($|\alpha|$) may be determined using the experimental data on the total cross-sections of interaction of unpolarized nucleons and on the differential cross-sections of the "forward" nucleon charge transfer process and the charge-exchange breakup of an unpolarized deuteron $d+p\to(pp)+n$ in the "forward" direction.

At present there are few *reliable* experimental data on the differential cross-section of the deuteron charge-exchange breakup on a proton. However, the analysis shows: if we suppose that the real part of the spin-independent amplitude of charge transfer $n+p\to p+n$ at zero angle is smaller or of the same order as compared with the imaginary part ($\alpha^2 \le 1$), then it follows from the available experimental data on the differential cross-section of charge transfer $\left.\frac{d\sigma_{np\to pn}}{dt}\right|_{t=0}$ and the data on the total cross-sections $\sigma_{pp}$ and $\sigma_{np}$ that the main contribution into $\left.\frac{d\sigma_{np\to pn}}{dt}\right|_{t=0}$ is provided namely by the spin-dependent part $\left.\frac{d\sigma^{(sd)}_{np\to pn}}{dt}\right|_{t=0}$ .

If the differential cross-section $\frac{d\sigma}{dt}$ is given in the units of $mbn/(\frac{GeV}{c})^2$ and the total cross-sections are given in $mbn$, then the spin-independent part of the "forward" charge transfer cross-section may be expressed in the form :

$$\left.\frac{d\sigma^{(si)}_{np\to pn}}{dt}\right|_{t=0} \approx 0.0512\,(\sigma_{pp}-\sigma_{np})^2(1+\alpha^2). \tag{22}$$

Using (22) and the data from the works [6,7,8], we obtain the estimates of the ratio $\frac{d\sigma^{(si)}_{np\to pn}}{dt}\Big|_{t=0} / \frac{d\sigma_{np\to pn}}{dt}\Big|_{t=0}$ at different values of the neutron laboratory momentum $k_n$:

1) $k_n = 0.7\ \frac{GeV}{c}$; $\frac{d\sigma_{np\to pn}}{dt}\Big|_{t=0} = 268\ mbn/(\frac{GeV}{c})^2$; $\sigma_{pp} - \sigma_{np} = -22.6\ mbn$;

$$\frac{d\sigma^{(si)}_{np\to pn}}{dt}\Big|_{t=0} / \frac{d\sigma_{np\to pn}}{dt}\Big|_{t=0} \approx 0.1\,(1+\alpha^2).$$

2) $k_n = 1.7\ \frac{GeV}{c}$; $\frac{d\sigma_{np\to pn}}{dt}\Big|_{t=0} = 37.6\ mbn/(\frac{GeV}{c})^2$; $\sigma_{pp} - \sigma_{np} = 10\ mbn$;

$$\frac{d\sigma^{(si)}_{np\to pn}}{dt}\Big|_{t=0} / \frac{d\sigma_{np\to pn}}{dt}\Big|_{t=0} \approx 0.136\,(1+\alpha^2).$$

3) $k_n = 2.5\ \frac{GeV}{c}$; $\frac{d\sigma_{np\to pn}}{dt}\Big|_{t=0} = 17.85\ mbn/(\frac{GeV}{c})^2$ ; $\sigma_{pp} - \sigma_{np} = 5.5\ mbn$;

$$\frac{d\sigma^{(si)}_{np\to pn}}{dt}\Big|_{t=0} / \frac{d\sigma_{np\to pn}}{dt}\Big|_{t=0} \approx 0.085\,(1+\alpha^2).$$

So, it is well seen that, assuming $\alpha^2 \lesssim 1$, the spin-dependent part $\frac{d\sigma^{(sd)}_{np\to pn}}{dt}\Big|_{t=0}$ provides at least $(70 \div 90)\%$ of the total magnitude of the "forward" charge transfer cross-section.

The preliminary experimental data on the differential cross-section of "forward" deuteron charge-exchange breakup $d+p \to (pp)+n$, obtained recently in Dubna (JINR, Laboratory of High Energies), also confirm the conclusion about the predominant role of the spin-dependent part of the differential cross-section of the nucleon charge-exchange reaction $n+p \to p+n$ in the "forward" direction.

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