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Evolution of the gauge couplings and Weinberg angle in 5-dimensions for a G_2 gauge group

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Abstract. We explicitly test, in a simplified 5-dimensional model with G_2 gauge symmetry, the evolution of the gauge couplings. We assume that all the matter fields are propagating in the bulk, and consider orbifolds based on Abelian discrete groups which lead to 5-dimensional gauge theories compactified on an S^1/Z_2 . The gauge couplings evolution is derived at one-loop level and used to test the impact on lower energy observables, in particular the Weinberg angle.

1. Introduction

Note that in coming years a new era of unexpected and exciting discoveries in high energy physics. For example, the CERN Large Hadron Collider (LHC) has been operating for some time and much experimental data has already been collected [1]. So far, the greatest achievement of the LHC has been the discovery of the missing building block of the SM, the Higgs particle (or, at least, a particle which most likely is the SM Higgs particle). On the other hand, no direct evidence of new physics beyond the SM has been found, yet. However, there are many reasons to believe that new physics should in fact show-up at, or about, the TeV scale [2, 3].

Theories which contain light elementary scalars can look unnatural because their masses receive quadratically divergent loop corrections, this problem is known as the hierarchy problem of the Standard Model (SM), as in this case the masses are pushed up to a cutoff scale [4]. In order to solve this problem we need to lower the cutoff scale (as was done in the case of large extra dimensional models), or to embed the Higgs field in a multiplet of a symmetry group larger than the 4-dimensional Poincare group (such as supersymmetry (SUSY)) [5]. As we know SUSY is a space-time symmetry mapping particles and fields of integer spin (bosons) into particles and fields of half

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integer spin (fermions), and vice-versa. Aside from SUSY there are other extensions of the 4-dimensional Poincare group, the most natural such choice would be to use the Poincare group of a higher dimensional gauge theory [6, 7]. A gauge theory defined in more than four dimensions can have many attractive features, one of these features is that the interactions at low energies may be truly unified and some of the distinct fields in four dimensions can be integrated in a single multiplet in higher dimensions, Higgs fields could be a part of the gauge fields. Another feature is the topology and structure of the extra-dimension may provide us with new ways of breaking symmetries, accounting for, at the same time, the hierarchy problem [8]. As such the SM Higgs field may originate from extra components of a higher dimensional gauge field. We therefore plan to discuss the evolution equations of the gauge couplings and Weinberg angle for a G_2 gauge group.

The structure of this paper is as follows: In section 2 we outline the G_2 model construction, in section 3 we explore the evolution of the gauge couplings and Weinberg angle for a G_2 gauge group. Our results and discussions will be given in section 4, In section 5 we conclude.

2. The G_2 Model Construction

In order to build a successful model we are first required to find a gauge group that contains $SU(2)_L \times U(1)_Y$ (and optionally the strong $SU(3)_c$), where the adjoint representation contains a doublet of SU(2) to be identified with the Higgs doublet. The second step is to normalise the U(1) gauge coupling so that the candidate Higgs has the correct hypercharge.

In this section we will explore a simplest five-dimensional gauge Higgs unification scenarion, where we use the gauge symmetry to be G_2 . The extra dimension is compactified on a circle of radius R with a Z_2 orbifolding [9]. This orbifold is given as $Z_2: y \to -y$, so our physical space is in the interval $y \in [0, \pi R]$ and has two fixed points at y = 0 and $y = \pi R$ [10, 11]. We assume that all matter fields are propagating in the bulk. The gauge bosons arise from the 4-dimensional components of the 5-dimensional gauge fields, whilst the Higgs field arises from the internal components of the gauge group G_2 compactified on an S^1/Z_2 orbifold [13, 14, 3].

The G_2 gauge group contains SU(3) as his maximal subgroup, and the decomposition under SU(3) is given by:

$$14 = 8 + 3 + \bar{3},\tag{1}$$

form this we can see that there are two possible doublets, one contained in the adjoint of SU(3), and the other in the triplets, where in the first case $g_1 = \sqrt{3} g_2$ while in the other case $g_1 = g_2/\sqrt{3}$. The decomposition under $SU(2) \times U(1)$ is given by:

$$14 = \left(3_0 + (2 + \bar{2})_{\sqrt{3}/2} + 1_0\right) + (2 + \bar{2})_{1/2\sqrt{3}} + (1 + \bar{1})_{-1/\sqrt{3}}$$
(2)

The other maximal subgroup is $SU(2) \times SU(2)$ under which:

$$14 = (1, 3) + (3, 1) + (2, 4)$$
(3)

decompose as:

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where in this case the first SU(2) has to be identified with the one contained in the SU(3) gauge group, and then in this case, we can perform two possible way for breaking the group, either it is the rotation, which is now breaks $G_2 \rightarrow SU(3)$, and hence the glide to $SU(2) \times SU(2)$ or the viceversa. The fundamental representations under SU(3)

$$7 = 3 + \bar{3} + 1$$
 (4)

and the fundamental representations under $SU(2) \times U(1)$ decompose as:

$$7 = (2 + \bar{2})_{1/2\sqrt{3}} + (1 + \bar{1})_{-1/\sqrt{3}} + 1_0$$
(5)

We need the renormalization group equations (RGEs) to fill in the space between the predictions of the model at $\mu \gg M_Z$ and the experimental ones at $\mu \leq M_Z$. We can describe the contributions from the SM and Kaluza-Klein modes (KK) to the betafunctions in two separate terms, they are different and independent [12, 15].

3. The evolution of the gauge couplings and Weinberg angle for a G_2

The evolution of the gauge couplings in 4-dimensions for the G_2 gauge group at one-loop is given by:

$$16\pi^2 \frac{dg_i}{dt} = b_i^G g_i^3, (6)$$

where the numerical coefficients in equation (6) are given by:

$$b_i^G = \left[\frac{53}{6}, -\frac{21}{6}, -\frac{63}{6}\right].$$
(7)

We can then rewrite equation (6) in terms of α_i^{-1} as follows

$$\frac{1}{\alpha_i(\mu)} \frac{d\ln\alpha_i(\mu)}{d\ln\mu} = \frac{b_i}{2\pi}.$$
(8)

The one-loop beta functions for the gauge couplings in 5-dimensions for the G_2 gauge group are given by:

$$16\pi^2 g_3^{-3} \frac{d g_3}{d t} = -(S(t) - 1) \left(\frac{14}{6}\right), \tag{9}$$

$$16\pi^2 g_2^{-3} \frac{dg_2}{dt} = (S(t) - 1) \left(\frac{7}{24}\right),\tag{10}$$

$$16\pi^2 g_1^{-3} \frac{dg_1}{dt} = (S(t) - 1) \left(\frac{35}{8}\right),\tag{11}$$

where $S(t) = M_Z R e^t$ is the number of KK states, $t = \ln\left(\frac{\mu}{M_Z}\right)$ is the energy scale parameter, for $M_Z < \mu < 1/R$. We have chosen the Z boson mass as the renormalization point, that is when the energy $\mu = 1/R$ or S(t) = 1, in this case the whole beta-function reduces to the normal beta-functions [16, 17]. IOP Conf. Series: Journal of Physics: Conf. Series 889 (2017) 012012



Figure 1. Running of α_i^{-1} in 4-dimensions for one-loop beta-functions, where α_3^{-1} is the (green) dotted line, α_2^{-1} is the (blue) dashed line and α_1^{-1} is the (red) solid line.



Figure 2. The evolution of theof α_i^{-1} in 5-dimensions for one-loop beta-functions, for the G_2 gauge group, where α_3^{-1} is the (green) dotted line, α_2^{-1} is the (blue) dashed line and α_1^{-1} is the (red) solid line.

4. Results and discussions

For our numerical calculation we choose the initial values for the gauge couplings based on the renormalisation point M_Z scale as: $g_1(M_Z) = 0.46597$, $g_2(M_Z) = 0.63337$ and $g_3(M_Z) = 1.02739$. In Figure 1, we present the evolution of α_i^{-1} in 4-dimensions for the one-loop beta-functions, in this figure we see that α_1^{-1} and α_3^{-1} are approximately unified at some energy scale $t \sim 8.0$. Whilst Figure 2, shows the evolution of the α_i^{-1} in 5-dimension for the one-loop beta-function for the G_2 gauge group. From this one can see that $\alpha_1^{-1}, \alpha_2^{-1}$ and α_3^{-1} are approximately unified at $t \sim 4.0$.

In Figure 3, we present the evolution of the Weinberg angle for the one-loop betafunctions, for different values of compactification scale, as example, for $R^{-1} = 10 \text{ TeV}$, IOP Conf. Series: Journal of Physics: Conf. Series 889 (2017) 012012 doi:10.10

 $\sin^2 \theta_W \sim 0.42$ at $t \sim 5.56$. When the fifth dimension KK modes become kinematically accessible, there are large changes in the Weinberg angle up until we reach the cut-off scale. We have chosen the cut-off for our effective theory, as $g_1 = g_2$, as shown in Table 1.

Scenario	$t(R_1)$	$t(R_2)$	$t(R_3)$
$5D G_2$	4.529	5.045	5.245

Table 1. The cut-offs in 5 dimensions for the G_2 gauge group for three different compactification radii $R^{-1} = 1, 5$ and 10 TeV, where $t = \ln(\mu/M_Z)$.



Figure 3. The evolution of the Weinberg angle $\sin^2 \theta_W$ for the one-loop beta-function, for different values of compactification scales, $R^{-1} = 1$ TeV (red), $R^{-1} = 5$ TeV (blue) and $R^{-1} = 10$ TeV (green) as a function of t.

5. Conclusion

In conclusion, in this paper we explicitly tested, in a simplified 5-dimensional model with a G_2 gauge symmetry, the evolution of the gauge couplings and Weinberg angle. We observed that when the fifth dimension KK-modes switch on all the previous physical observables evolution change rapidly.

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