## Radiative Scaling Neutrino Mass with $A_4$ Symmetry and Warm Dark Matter

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## Abstract

In scotogenic model, neutrino mass comes from a one-loop mechanism with dark matter candidate in the loop. An assumption of producing small neutrino mass proportional to small dark matter mass in the model consisting of singlet Majorana fermions  $N_i$  with masses of order 10 keV was proposed recently. Neutrino masses are scaled down from them by factors of about  $10^{-5}$ . We study the consequences in the phenomenology of  $N_i$  as warm dark matter of implementing the non-Abelian discrete symmetry  $A_4$  for neutrino mixing in the model.

In the scotogenic model [1] the radiative neutrino mass connects with dark matter with considering three neutral fermion singlets  $N_i$  and a new scalar doublet  $(\eta^+, \eta^0)$  are odd under a new  $Z_2$  symmetry. Hence Majorana neutrino masses are generated in one loop through the interaction  $h_{ij}(\nu_i\eta^0 - l_i\eta^+)N_j$  as shown in Fig. 1. This mechanism has been called "scotogenic", from the Greek "scotos" meaning darkness. Because of the allowed  $(\lambda_5/2)(\Phi^{\dagger}\eta)^2 + H.c.$  interaction,  $\eta^0 = (\eta_R + i\eta_I)/\sqrt{2}$  is split so that  $m_R \neq m_I$ . The diagram of Fig. 1 can be computed exactly [1], i.e.

$$(\mathcal{M}_{\nu})_{ij} = \sum_{k} \frac{h_{ik} h_{jk} M_k}{16\pi^2} \left[ \frac{m_R^2}{m_R^2 - M_k^2} \ln \frac{m_R^2}{M_k^2} - \frac{m_I^2}{m_I^2 - M_k^2} \ln \frac{m_I^2}{M_k^2} \right].$$
(1)



Figure 1: One-loop generation of scotogenic Majorana neutrino mass.

The usual assumption for neutrino mass in Eq. (1) is

$$m_I^2 - m_R^2 << m_I^2 + m_R^2 << M_k^2, (2)$$

in which case

$$(\mathcal{M}_{\nu})_{ij} = \frac{\lambda_5 v^2}{8\pi^2} \sum_k \frac{h_{ik} h_{jk}}{M_k} \left[ \ln \frac{M_k^2}{m_0^2} - 1 \right], \tag{3}$$

where  $m_0^2 = (m_I^2 + m_R^2)/2$  and  $m_R^2 - m_I^2 = 2\lambda_5 v^2$  ( $v = \langle \phi^0 \rangle$ ). This scenario is often referred to as the radiative seesaw. There is another interesting scenario, i.e.

$$M_k^2 << m_R^2, \ m_I^2.$$
 (4)

Neutrino masses are then given by [2]

$$(\mathcal{M}_{\nu})_{ij} = \frac{\ln(m_R^2/m_I^2)}{16\pi^2} \sum_k h_{ik} h_{jk} M_k.$$
 (5)

In the above expression neutrino mass is now not inversely proportional to some large scale. In that case, how do we understand the smallness of  $m_{\nu}$ ? The answer is lepton number. In this model,  $(\nu, l)_i$  have lepton number L = 1 and  $N_k$  have L = -1, and L is conserved in all interactions except for the Majorana mass terms  $M_k$  which break L to  $(-1)^L$ . We may thus argue that  $M_k$  should be small compared to all other mass terms which conserve L. For instance, if  $M_k \sim 10$  keV then  $m_{\nu} \sim 0.1$  eV is obtained if  $h^2 \sim 10^{-3}$  in Eq. (5).

The scotogenic model [1] with large  $M_k$ , i.e. Eq. (3), has been extended recently [3] to include the well-known non-Abelian discrete symmetry  $A_4$  [4, 5, 6]. Simpler discrete symmetries, such as  $Z_2$  [7, 8], have been used in the literature to describe the neutrino mixing. In Eq. (5) we let  $(\eta^+, \eta^0)$  be a singlet under  $A_4$  and both  $(\nu_i, l_i)$  and  $N_k$  to be triplets [9]. In that case,

$$h_{ik} = h\delta_{ik},\tag{6}$$

and

$$\mathcal{M}_{\nu} = \zeta \mathcal{M}_N,\tag{7}$$

where  $\zeta = h^2 \ln(m_R^2/m_I^2)/16\pi^2$  is the scale factor. The soft breaking of  $A_4$  which shapes  $\mathcal{M}_N$  is then directly transmitted to  $\mathcal{M}_{\nu}$ .

There are two consequences of the scaling mechanism for neutrino mass. First, quasidegenerate scenario of neutrino mass can be obtained if  $M_{1,2,3}$  are in the same order of magnitude. Second, the interactions of  $N_{1,2,3}$  with the charged leptons through  $\eta^+$  depend only on h and the mismatch between the charged-lepton mass matrix and the neutrino mass matrix, i.e. the experimentally determined neutrino mixing matrix  $U_{l\nu}$ . Hence  $\mu \to e\gamma$  is highly suppressed because the leading term of its amplitude is proportional to  $\sum_k h_{\mu k} h_{ek}^* = |h|^2 \sum_k U_{\mu k} U_{ek}^* = 0$ . Note that  $A_4$  may be replaced by any other flavor symmetry as long as it is possible to have Eq. (8) using the singlet and triplet representations of that symmetry.

 $N_k$  becomes good warm dark-matter candidates [10, 11] if the interactions of  $N_k$  with the neutrinos and charged leptons are weaker than the usual weak interaction when  $\eta^{\pm}$ ,  $\eta_R$ ,  $\eta_I$  are of order 10<sup>2</sup> GeV, i.e.  $N_k$  may be considered "sterile". Since  $N_k$  are assumed light in the range of keV, muon decay proceeds at tree level through  $\eta^+$  exchange, i.e.  $\mu \to N_{\mu} e \bar{N}_e$ , with the rate

$$\Gamma(\mu \to N_{\mu} e \bar{N}_{e}) = \frac{|h|^{4} m_{\mu}^{5}}{6144 \pi^{3} m_{n^{+}}^{4}}.$$
(8)

Since  $N_{\mu}$  and  $\bar{N}_{e}$  are invisible just as  $\nu_{\mu}$  and  $\bar{\nu}_{e}$  are invisible in the dominant decay  $\mu \rightarrow \nu_{\mu}e\bar{\nu}_{e}$  (with rate  $G_{F}^{2}m_{\mu}^{5}/192\pi^{3}$ ), this would change the experimental value of  $G_{F}$ . Using the experimental uncertainty of  $10^{-5}$  in the determination of  $G_{F}$ , we find

$$m_{\eta^+} > 70 \text{ GeV} \tag{9}$$

for  $|h|^2 = 10^{-3}$ . Whereas the lightest scotino, say  $N_1$ , is absolutely stable,  $N_{2,3}$  will decay into  $N_1$  through  $\eta_R$  and  $\eta_I$ . The decay rate of  $N_3 \to N_1 \bar{\nu}_1 \nu_3$  is given by

$$\Gamma(N_3 \to N_1 \bar{\nu}_1 \nu_3) = \frac{|h|^4}{256\pi^3 M_3} \left(\frac{1}{m_R^2} + \frac{1}{m_I^2}\right)^2 \times \left(\frac{M_3^6}{96} - \frac{M_1^2 M_3^4}{12} + \frac{M_1^6}{12} - \frac{M_1^8}{96M_3^2} + \frac{M_1^4 M_3^2}{8} \ln \frac{M_3^2}{M_1^2}\right).$$
(10)

Let  $M_1 = 10$  keV,  $M_3 = 14.85$  keV,  $|h|^2 = 10^{-3}$ ,  $m_R = 240$  GeV,  $m_I = 150$  GeV, then this rate is  $1.0 \times 10^{-46}$  GeV, corresponding to a lifetime of  $2.1 \times 10^{14}$  y, which is much longer than the age of the Universe of  $13.75 \pm 0.11 \times 10^9$  y. The lifetime of  $N_2$  is even longer because  $\Delta m_{21}^2 << \Delta m_{31}^2$ . Hence both  $N_2$  and  $N_3$  are stable enough to be components of warm dark matter.

Since  $\eta^+$  may be as light as 70 GeV, it may be observable at the LHC. The inclusive decay of  $\eta^{\pm} \rightarrow l^{\pm}N_{1,2,3}$ , which is of universal strength. At the LHC, the pair production of  $\eta^+\eta^$ will then lead to  $l_i^+l_j^-$  final states with equal probability for each flavor combination. This signature together with the large missing energy of  $N_{1,2,3}$  may allow it to be observed at the LHC. However, these events also come from  $W^+W^-$  production and their subsequent leptonic decays. The cross sections for the signal events are smaller than the dominant  $W^+W^$ background even after a large  $E_T$  cut. This is because both the signal and background events have similar missing energy distributions, but the  $W^+W^-$  production is much larger.

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