THE a_1 MASS FROM $\tau \to a_1\nu$ DECAY

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The three recent experiments of the MARK II, DELCO, and ARGUS groups are reanalysed using an analytically correct Breit-Wigner form including apart from the $\rho\pi$ also the $K^*\bar{K} + \bar{K}^*K$ thresholds, and an energy dependent real part (a mass shift function involving Chew-Mandelstam functions) to the a_1 mass parameter. It is found that all three experiments can be fitted with an a_1 mass of (1250 ± 40) MeV in agreement with the Particle Data Group value (1275 ± 28) MeV, but in contrast to results presented by the experimental groups.

However, the width of a_1 determined from the $\tau \to a_1\nu \to 3\pi\nu$ decay data is considerably larger than the conventional value.

1. Introduction.

The determination of the true experimental mass of the a_1 (previously denoted A_1) meson is an important question for several reasons: (i) The a_1 mass is crucial for the determination of the 1^{++} nonet mixing structure, (in particular the OZI rule violating mixing angle)[**1**]. (ii) For tests of the Weinberg relation $m_{a_1} = \sqrt{2} \cdot m_\rho$ and related sum rules[**2**]. (iii) Finally being a very broad resonance (> 300MeV) the a_1 is also interesting from the point of view of theoretical models of resonance shapes and mass determinations. One can test the validity of different approximations of Breit-Wigner forms. For a very narrow resonance the latter question is clearly not very interesting, since the naivest approximation gives in practice the same resonance parameters as a more detailed analytic treatment.

The production of the a_1 through the τ lepton decay ($e^+e^- \to \tau\bar{\tau}$, and $\tau \to a_1\nu \to \pi^+\pi^-\pi^0\nu$) has been considered as a promising way for studying the properties of the a_1 meson, since it is expected to be essentially free from background from other processes. In the hadronic production experiments[**3-4**] of a_1 one is plagued by the diffractive "Deck effect", which through interference distorts the a_1 resonance shape, especially at low momentum transfers. The latter data must therefore be analysed in a model dependent way, which makes a reliable extraction of a_1 parameters difficult and model dependent.

The $\tau \to a_1\nu \to 3\pi\nu$ data are, however, not either completely free from background. There can be a background of in particular $\tau \to \pi(1300)\nu \to 3\pi$, which is also expected to go predominantly via the $\rho\pi$ intermediate state, although in a P-wave. The experimental groups[**5-7**] (in particular ARGUS[**7**]) find that this is less than 10%. Also PCAC predicts that this channel should be suppressed, as it would be generated by the derivative of the axial vector current, whereas a_1 production can go directly via the W (Fig.1).

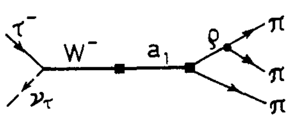

Fig.1. The $\tau \to a_1\nu \to 3\pi\nu$ diagram.

Another contribution to the background can come from higher axial vector resonances (as $2\ ^3P_1$), which would affect the upper end of the mass spectrum. However, since these are expected only near 2GeV, and above, their contribution is presumably small.

Soon after the first data on $\tau \to a_1\nu \to 3\pi\nu$ decay appeared in 1978 from SPEAR [**8**] and DORIS [**9**], Basdevant and Berger [**10**] realized that the a_1 shape is expected to be distorted due to phase space and due to the energy dependence in the production by the weak interaction matrix element. They could fit these first low statistics experiments with an a_1 mass compatible with one of their previous solutions for the diffractively produced a_1. Their preferred solution (denoted E) had $m_{a_1} = (1180\pm50)$MeV and $\Gamma_{a_1} = (400\pm50)$MeV, defined as the position of the second sheet pole. Another solution (denoted C) of the hadronic production data with $m_{a_1} = 1385$ MeV and $\Gamma_{a_1} = 425$ MeV did not fit the early $\tau \to a_1\nu \to 3\pi\nu$ data. Their analysis thus seemed to favour a rather light a_1. The low statistics data then available did not motivate the authors to include all constraints from analyticity, coupled channels, finite rho widths etc., which as we shall see at least partly explain why their analysis, as well as the analyses of the experimental groups discussed below give a too low a_1 mass.

Recently three new large statistics experiments have been reported on the process $\tau \to a_1\nu \to 3\pi\nu$ by the DELCO, MARKII and ARGUS groups[**5-7**], and a fourth (MAC) is in progress. These report masses and widths very different from the present Particle Data Group avarage:

Experiment	mass/MeV	Width/MeV
DELCO[**5**]	$1056 \pm 20 \pm 5$	$476^{+132}_{-120} \pm 54$
MARKII[**6**]	$1194 \pm 14 \pm 10$	$462 \pm 56 \pm 30$
ARGUS[**7**]	1046 ± 11	521 ± 27
Particle Data Group [**11**]	1275 ± 28	316 ± 45

2. Which is the correct Breit-Wigner form?

As discussed by the previous analyses[**5-7**],[**12**] the 3π mass spectrum can be described by the following form:

$$\frac{dN}{dq^2} \propto \left(\frac{m_{a_1}}{m}\right)^x \left[1 + 2\frac{q^2}{m_\tau^2}\right]\left[1 - \frac{q^2}{m_\tau^2}\right]^2 \cdot BW(q^2). \tag{1}$$

Here BW stands for the Breit-Wigner function, the analytic form of which form is the main point of this note. It will be discussed it in more detail below.

The first factor in Eq.(1) is a weak interaction form factor (for the $W - a_1$ transition (cf. Fig.1) parametrized by the power x following Bowler[**12**] and the experimental groups. (Ref.[**5**] has x=-2, Ref.[**6**] x=+1 corresponding to no form factor, while Ref.[**7**] has x=0.) As discussed by Bowler by comparing with form factors which give best fits to $e^+e^- \to \rho \to \pi\pi$ as well as to the present a_1 data the best χ^2 is obtained for x near +1, which corresponds to no form factor. We also find very similar result, although adding or subtracting one power of x does not change the χ^2 very much. Thus this theoretical ambiguity remains, and consequently will give a theoretical uncertainty to the mass determination of the order of 30 MeV.

The second and third factor in Eq.(1) comes from the $\tau \to a_1\nu$ matrix element and the phase space factor.

The last factor BW(s) is the Breit-Wigner function which contains the the a_1 resonance pole and the final state interactions, but not factors coming from the resonance production.

The most naive approximation of the BW function used in Ref.[**7**] puts both the width and the "running mass" $m(s)$ defined below equal to constants.

$$BW_{naive}(s) = \frac{M\Gamma_{\rho\pi}}{(s - M^2)^2 + (M\Gamma_{tot})^2}, \quad s = q^2. \tag{2}$$

This is certainly a much too crude approximation for a broad resonance like the a_1. Much better is the "usual" approximation where the partial width $\Gamma_{\rho\pi}(s)$ and the total width $\Gamma_{tot}(s)$ are s dependent, including phase space and angular momentum barrier factors, but the mass parameter M still without an s dependent behaviour:

$$BW_{usual}(s) = \frac{M\Gamma_{\rho\pi}(s)}{(s - M^2)^2 + (M\Gamma_{tot}(s))^2}. \tag{3}$$

In full generality the analytically "correct" BW should be written:

$$BW_{correct}(s) = \frac{M\Gamma_{\rho\pi}(s)}{(s - m^2(s))^2 + (M\Gamma_{tot}(s))^2}. \tag{4}$$

This result can most directly be seen within a field (or S-matrix) theory framework as arising from the propagator $(M_0^2+\Pi(s)-s)^{-1}$, where the renormalization term $\Pi(s)$ comes from the sum of all loop diagrams shown in Fig.2.

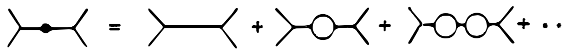

Fig.2. The resonance propagator including loops.

The imaginary part $Im\Pi$ is by unitarity given by the width as discussed below.

In Eq.(4) the mass parameter M is replaced by a function $m(s)$ which we call the "running mass". It's square $m^2(s)$ can be written as a sum of a constant term M_0^2, the "bare mass" squared, and a "mass shift function" $\delta^2(s) = Re\Pi(s)$:

$$m^2(s) = M_0^2 + \delta^2(s), \tag{5}$$

where

$$\delta^2(s) = Re\Pi(s) = \frac{1}{\pi}\int_{s^{th}}^{\infty} \frac{-Im\Pi(s')}{s-s'}ds', \qquad s^{th} = (m_a+m_b)^2, \tag{6}$$

and in which $Im\Pi(s)$ determines the total width:

$$Im\Pi(s) = -M\cdot\Gamma_{tot}(s) = -M\cdot\sum_i \Gamma_i(s). \tag{7}$$

By subtracting a term $\Pi(s_0)$ from the dispersion relation Eq.(6), the mass shift function $\delta^2(s)$ is defined to vanish at $s = s_0$. The constant $Re\Pi(s_0)$ (which is logaritmically divergent if there is no cut off in the form factor $F_i(s)$ below) is absorbed into M_0^2 by renormalization. If s_0 is chosen equal to M_0^2 the "bare mass" parameter M_0 will be equal to the resonance mass, defined as below at the point where the phase shift passes 90^o. This choice of s_0 is of course quite arbitrary, and does not remove the energy dependence of $\delta^2(s)$ which modifies the resonance shape. (Below in Eq.(12) we chose s_0 to be at the threshold s^{th}.)

We define the mass and width of the resonance by the zero of the real part the inverse propagator i.e.,

$$M = m(s)|_{s=M^2}, \qquad \Gamma = \Gamma(M^2). \tag{8}$$

At this point a few remarks are in order: This definition obviously differs from the position of the second sheet pole (which is at the position of the zero of the whole inverse propagator $(M_0^2 + \Pi(s) - s)$. Although the latter has, in theory, simple factorization properties, the definition of Eq.(8) has the practical virtue of coinciding with the usual definition of the energy where the phase shift passes 90^o, and resembles the definition of M^2 in the "usual" BW of Eq.(3). This definition is closely related to the data, which lie at the real s axis, and is not sensitive to analytic continuation, which depend on details of theoretical assumptions of asymtotic behavior and on distant singularities. Of course, with a given analytic form there is a one to one correspondence with our definition and the second sheet pole position. For a broad resonance the two definitions, independently of details, differ considerably numerically. We feel our definition should be preferred, for the above reasons, and because we believe that historically papers quoting second sheet pole positions have led to more confusion than insight.

For S-wave 2-body thresholds each threshold contributes a term to $-Im\Pi(s)$ which is proportional to the 2-body relativistic phase space $\pi k_i^{cm}/\sqrt{s}$ multiplied by a hadronic form factor $F_i^2(s)$. In fact, in the present case since ρ is unstable the situation is slightly more complicated, as one must integrate over the 3π Dalitz plot in calculating the a_1 to $\rho\pi$ width. This is however, a rather straightforward complication and has been taken into account in all fits, including our own. Essentially it means one must "smooth" (c.f. [**13**]) the formulas below over the ρ mass distribution, taking into account the interference of the overlapping ρ bands. Disregarding these complications in our formulas below and summing over the thresholds i we have:

$$Im\Pi(s) = -\sum_i g_i^2 \frac{k_i^{cm}}{\sqrt{s}} F_i^2(s)\Theta(s - s_i^{th}), \qquad ((9))$$

where the g_i's are defined as coupling constants at the thresholds. As long as one is mainly interested in a region reasonably near the threshold the form of the form factor F_i should not be very important, since one expects it to be a smoothly decreasing function. Putting $F_i(s) = 1$, as for pointlike couplings, the dispersion relation defines $\Pi(s)$ as a sum of "Chew-Mandelstam functions" (so denoted for historical reasons in Ref.[**13-14**]) $C(s, m_{a,i}^2, m_{b,i}^2)$, where $m_{a,i}, m_{b,i}$ are the masses of the threshold particles (cf. Figs. 3a,b):

$$\Pi(s) = \frac{1}{\pi}\sum_i g_i^2 C(s, m_{a,i}^2, m_{b,i}^2). \qquad (10)$$

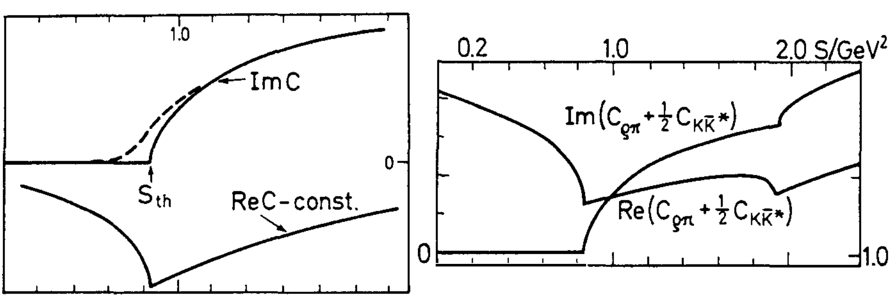

Fig.3. (a) The Chew-Mandelstam function for $\rho\pi$. (b) The sum of the $\rho\pi$ and $K^*\bar{K}$ + $\bar{K}^*K$ C-M functions, c.f. Eqs.(11,12). Taking into account the finite ρ width (as is done in the fits) the functions are "smoothed" as shown in (a) for ImC.

With a subtraction at the threshold C has the following analytic form for real s. Let us define $E_\pm = (|s - (m_a \pm m_b)^2|)^{\frac{1}{2}}$ and $\rho(s, m_a^2, m_b^2) = E_+E_-/(2s)$

The imaginary part $-Im\ C$ is defined as the 2-body relativistic phase space

$$Im\ C(s, m_a^2, m_b^2) = -\pi\rho(s, m_a^2, m_b^2)\cdot\Theta(s - s^{th}). \qquad (11a)$$

The real part is given by

$$Re\,C(s,m_a^2,m_b^2) = \frac{s-(m_a+m_b)^2}{2s}\frac{m_a-m_b}{(m_a+m_b)}\ln\frac{m_a}{m_b} + \rho(s,m_a^2,m_b^2)\cdot G(s,m_a^2,m_b^2), \quad (11b)$$

where the function G for $s > (m_a+m_b)^2$ or $s < (m_a-m_b)^2$ is:

$$G(s,m_a^2,m_b^2) = Sign\cdot\ln|\frac{E_+ - E_-}{E_+ + E_-}|, \quad (11c)$$

where $Sign = +1$ for the first mentioned interval and $Sign = -1$ for the second. On the other hand for the interval $(m_a-m_b)^2 < s < (m_a+m_b)^2$ the function G is given by:

$$G(s,m_a^2,m_b^2) = 2\arctan\frac{E_+}{E_-} - \pi. \quad (11d)$$

One may ask why is one led to such a complicated form, although the imaginary part is just simply phase space? Why could one not just take the phase space function i.e. $\Pi(s) \propto -i\rho(s,m_a^2,m_b^2)$ and continue it below threshold, whereby $Re\Pi$ would vanish above threshold and be proportional to this same function, ρ, evaluated below the threshold. Below a threshold one would then have a similar square root behaviour as also displayed by the Chew-Mandelstam function (Fig.3). Unitarity would be intact, but there would be a spurious unphysical pole at s=0 in this amplitude! This need not always be a very bad approximation, since s=0 is often a "comparatively distant spurious singularity". In fact, this approximation is often (implicitely) done by many authors, e.g., within most K-matrix analyses. But, even in this approximation the $K^*\bar{K} + \bar{K}^*K$ thresholds would contribute an s-dependent contribution to the mass shift function $\delta^2(s)$ in the a_1 region.

In summary, including the $\rho\pi$ and $K^*\bar{K} + \bar{K}^*K$ thresholds into the function Π and assuming no form factor one gets a contribution which is shown in Fig.3b, and which has the functional form:

$$\Pi(s) \propto C(s,m_\rho^2,m_\pi^2) + \frac{1}{2}C(s,m_K^2,m_{K^*}^2), \quad (12)$$

where the factor $\frac{1}{2}$ comes from the relative flavour Clebsh Gordan coefficients. If instead of $F_i(s) = 1$, one assumes a falling form factor, e.g., $F_i = exp[-(k_i^{cm}/k_{cutoff})^2]$, which is reasonable for hadrons of finite size, the function C is modified, but as long as the cutoff is reasonable large the change is not very important for our purpose.

3. Results from fits to experiment.

In the fits I have not taken into account uncertainties in the data due to mass resolution nor details of background subtraction. Including these could change our numbers somewhat, but not the qualitative conclusions. The main observation is that the value of the a_1 mass is increased by 30 to 60 MeV (depending on which experiment) compared to a situation where one uses the "usual" form for the BW Eq.(3). In particular for the ARGUS data (Fig.4) which have the largest statistics (approx. 1400 events) we get (for x=1) M_{a_1} = 1224MeV, Γ_{a_1} = 592MeV, wheras the usual BW under similar conditions give: M_{a_1}=1196MeV, Γ_{a_1} = 446MeV.

The χ^2 is improved compared to a similar fit using instead a "usual" BW from 41.9 to 29.9. A side remark worth noting is that if the $K^*\bar{K} + \bar{K}^*K$ relative Clebsch-Gordan coefficient is left free in the fit we get =0.65 compared to the 0.5 expected from exact $SU3_f$. More results for other data are given in Table 1.

x	DELCO[5] mass width	MARKII[6] mass width	ARGUS[7] mass width χ^2
2	1303 619	1311 468	1259 555 35.6
2	(1245 469)	(1271 383)	(1224 437 41.2)
1	1299 775	1301 561	1224 592 29.9
1	(1222 523)	(1247 419)	(1196 425 41.9)
0	1271 790	1277 642	1182 582 31.2
0	(1192 537)	(1220 439)	(1165 425 48.8)

Table 1. The results of our fits for the a_1 mass and width in MeV for the three experiments for different values of the power x. With x=1 (no $W - a_1$ form factor) the best fits are obtained. See Eqs.(1),(4). In parenthesis are the masses and widths when the usual approximate BW of Eq.(3) is used instead.

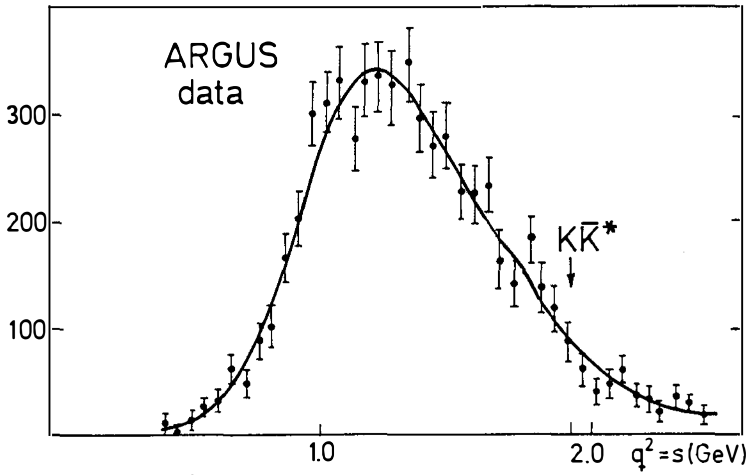

Fig.4. The ARGUS[7] data with our fit.

In conclusion comparing the fits for the three experiments we find that a considerable improvement (about 30%) in χ^2 is obtained if the "correct" BW is used and that the mass of the a_1 is increased such that an a_1 mass

$$M_{a_1} = 1250 \pm 40 MeV, \tag{12}$$

gives a good overall description of the three experiments. Our mass is larger than that of Bowler, who also fitted the three experiments using the BW of Eq.(3), mainly because of our inclusion of the $K^*\bar{K} + \bar{K}^*K$ thresholds with the correct associated mass shift function.

However, the a_1 width comes out to be considerably larger than usual:

$$\Gamma = 600 \pm 100 MeV, \tag{13}$$

which is to be compared with the Particle Data Group value of 316 ± 45MeV. Part of this larger width comes from our inclusion of the Real part $Re\Pi(s)$ into the BW, whereas part of it is also present when using the usual form[**12**]. Whether this is due to background in the τ data or a reflection of the fact that that usual fits to the hadronically produced a_1 data tend to be biased towards too narrow widths remains to be seen.

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