Numerical relativity in higher dimensions

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Abstract. We give a status report on our project targeted at performing numerical simulations of a head-on collision of non-spinning black holes in higher dimensional non-compact space-times. These simulations should help us understand black objects in higher dimensions and their stability properties. They are also relevant for the problem of black hole formation and evaporation in particle accelerators and cosmic rays. We use the symmetries of the system to reduce the problem to an effective 3+1 problem, allowing the use of existing numerical codes. As a simple application of the formalism, we present the results for the evolution of a five dimensional single black hole space-time.

1. Introduction

Modern theories of supergravity and strings naturally encompass extra dimensions, and as such, the study and understanding of higher dimensional space-times plays here an important role. To study highly dynamical and non-linear phenomena in gravity, numerical relativity is the only tool available. For four dimensional general relativity, the past decade has seen remarkable achievements since the 2005 breakthroughs [1–3]. The higher dimensional case, however, is mostly unexplored, and only more recently some pioneering works started to appear [4–7]. There are a lot of interesting problems to explore in this field, and we give a few examples in the following.

If the scale of quantum gravity is near the TeV, ultra-relativistic scattering of point particles should be rather insensitive to the structure of the particles, and as such these type of phenomena may be described by black hole scattering [8]. Furthermore, there is now the exciting possibility of seeing black hole production at the LHC [9]. These black holes would quickly evaporate by Hawking radiation, emitting high-energy particles which may be detected. Thus, studies of gravitational radiation are extremely important, and numerical relativity plays here a major role.

Another related and interesting application of black hole physics to high-energy particle systems is suggested by the AdS/CFT conjecture [10], with the possibility of using this duality to understand properties of the quark-gluon plasma at the RHIC by studying black hole collisions in AdS [11]. Numerical relativity in AdS space-times is difficult, and has so far only been studied in very special cases [12, 13] (see also Helvi Witek's contribution).

We here summarise a framework introduced in [14] that allows us to study D dimensional axially symmetric space-times. Symmetry has been a crucial ingredient in solving the Einstein field equations numerically ever since the early attempts of Hahn and Lindquist [15], Eppley [16] and Smarr [17]. Our formalism explores the symmetry of higher dimensional head-on collisions of black holes in order to reduce the space-time dimensionality, in the spirit of the Kaluza-Klein reduction, thereby reducing the problem to an effective four dimensional one with "matter" terms.

2. An effective $3+1$ dimensional system

To make use of the existing numerical codes, we are interested in making a $4+(D-4)$ split of the D dimensional space-time. We start by noting that the most general D dimensional metric compatible with $SO(D-3)$ isometry, in $D \geq 6$, is

$$d\bar{s}^2 = g_{\mu\nu}dx^\mu dx^\nu + \lambda(x^\mu)d\Omega_{D-4}\,, \tag{2.1}$$

where $\mu, \nu = 0, \ldots, 3$ and $d\Omega_{D-4}$ is the line element on a unit S^{D-4} sphere. For such a metric element, the D dimensional Einstein vacuum equations yield the following system

$$R_{\mu\nu} = \frac{D-4}{2\lambda}\left(\nabla_\mu\partial_\nu\lambda - \frac{1}{2\lambda}\partial_\mu\lambda\partial_\nu\lambda\right)\,, \tag{2.2}$$

$$\nabla^\mu\partial_\mu\lambda = 2(D-5) - \frac{D-6}{2\lambda}\partial_\mu\lambda\partial^\mu\lambda\,. \tag{2.3}$$

In these equations, all operators are covariant with respect to the four dimensional metric $g_{\mu\nu}$. The energy-momentum tensor is[1]

$$T_{\mu\nu} = \frac{D-4}{16\pi\lambda}\left[\nabla_\mu\partial_\nu\lambda - \frac{1}{2\lambda}\partial_\mu\lambda\partial_\nu\lambda - (D-5)g_{\mu\nu} + \frac{D-5}{4\lambda}g_{\mu\nu}\partial_\alpha\lambda\partial^\alpha\lambda\right]\,. \tag{2.4}$$

We can thus view our system as an effective four dimensional problem with a source given by the above energy-momentum tensor, a sort of fake "matter"[2]. With this four dimensional perspective, we introduce the metric γ_{ij} and extrinsic curvature K_{ij} of the three dimensional hypersurface Σ, and perform the usual $3+1$ splitting of the space-time (see, e.g [18, 19]).

The resulting system is

$$(\partial_t - \mathcal{L}_\beta)\,\gamma_{ij} = -2\alpha K_{ij}\,, \tag{2.5a}$$

$$\begin{aligned}(\partial_t - \mathcal{L}_\beta)\,K_{ij} = &-D_i\partial_j\alpha + \alpha\left({}^{(3)}R_{ij} + KK_{ij} - 2K_{ik}K^k{}_j\right)\\ &- \alpha\frac{D-4}{2\lambda}\left(D_i\partial_j\lambda - 2K_{ij}K_\lambda - \frac{1}{2\lambda}\partial_i\lambda\partial_j\lambda\right)\,,\end{aligned} \tag{2.5b}$$

$$(\partial_t - \mathcal{L}_\beta)\,\lambda = -2\alpha K_\lambda\,, \tag{2.5c}$$

$$\frac{1}{\alpha}(\partial_t - \mathcal{L}_\beta)\,K_\lambda = -\frac{1}{2\alpha}D^k\lambda D_k\alpha + (D-5) + KK_\lambda + \frac{D-6}{\lambda}K_\lambda^2 - \frac{D-6}{4\lambda}D^k\lambda D_k\lambda - \frac{1}{2}D^k\partial_k\lambda\,, \tag{2.5d}$$

where D_i is the covariant derivative with respect to the three metric γ_{ij}.

[1] We use the standard form of the Einstein equations $G_{\mu\nu} = 8\pi T_{\mu\nu}$ and choose geometrised units throughout.

[2] We dub the source terms of the lower dimensional Einstein equations as matter although its energy-momentum tensor is not that of canonical matter.

3. Initial data

For the initial data, we follow the approach outlined by Yoshino *et al* [20, 21]. For Brill-Linquist initial data of two black holes with mass parameters μ_1 and μ_2 in D dimensions, the $(D-1)$ dimensional spatial metric element has the form

$$\gamma_{ab}dx^a dx^b = \psi^{\frac{4}{D-3}}\left(dz^2 + d\rho^2 + \rho^2 d\theta^2\right) + \psi^{\frac{4}{D-3}}\rho^2 \sin^2\theta^2 d\Omega^2_{D-4}\,, \tag{3.1}$$

$$\psi = 1 + \frac{\mu_1^2}{4\left[(z-z_1)^2+\rho^2\right]^{\frac{D-3}{2}}} + \frac{\mu_2^2}{4\left[(z-z_2)^2+\rho^2\right]^{\frac{D-3}{2}}}\,, \tag{3.2}$$

where $a, b = 1, \ldots, D-1$, $\theta \in [0, \pi]$, $-\infty < z < +\infty$ and $\rho \in [0, \infty[$. Introducing "incomplete" cartesian coordinates as

$$x = \rho\cos\theta, \qquad y = \rho\sin\theta\,, \tag{3.3}$$

where $-\infty < x < +\infty$ and $0 \le y < +\infty$, we can then write the initial data as

$$\gamma_{ij}dx^i dx^j = \psi^{\frac{4}{D-3}}\left[dx^2 + dy^2 + dz^2\right]\,, \tag{3.4}$$

and

$$\lambda = y^2\psi^{\frac{4}{D-3}}\,, \tag{3.5}$$

$$K_{ij} = 0\,, \tag{3.6}$$

$$K_\lambda = 0\,, \tag{3.7}$$

which close the system (2.5).

4. Numerical results in $D = 5$

For the numerical implementation, we write the evolution equations in the Baumgarte, Shapiro, Shibata and Nakamura (BSSN) formulation [22, 23].

We have performed numerical simulations in $D = 5$ for the evolution of a single black hole. The evolution is stable and the constraints are preserved in this time evolution. In Fig. 1 we exhibit the Hamiltonian constraint and the y component of the momentum constraint for evolution time $t = 28\mu$. One can see that there is some noise, but the overall convergence is still very good.

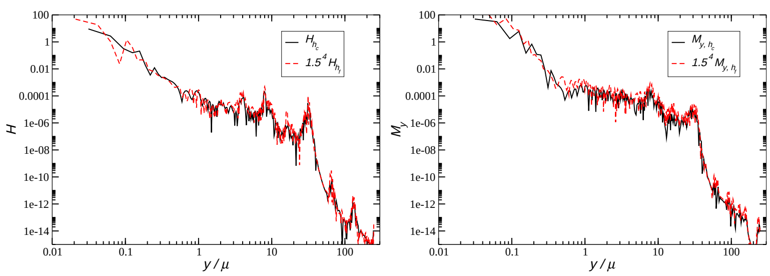

Figure 1. Hamiltonian constraint (left panel) and y-component of the momentum constraint (right panel) at time $t = 28\mu$, for the evolution of a single Tangherlini black hole in five dimensions.

5. Final remarks

The formalism we have introduced allows us to reduce the head-on collision of non-spinning black holes in any dimension to an effective $3+1$ system with source terms. This enables us to study higher dimensional systems (with enough symmetry) using the existing numerical codes. A key advantage of our approach is that the very same code should work for *any* dimension.

We are currently implementing and testing the formalism using the LEAN code [24], developed by one of us (U.S.), and have already found good numerical results for the evolution of a single black hole in $D=5$. For more details see [14].

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