PION EXCHANGE BETWEEN NUCLEONS IN STATIC LATTICE QCD

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ABSTRACT

Baryon-baryon interactions are studied within the framework of QCD with static quarks. Gluon exchange is treated by simulating an SU(3) Lagrangian on a space-time lattice. Meson exchange is described by putting a static quark-antiquark pair distribution between the baryons.

INTRODUCTION

Quark theory provides a new set of degrees of freedom at the subnuclear level which were priviously provided by meson theory. The vacuum of QCD allows for the creation of gluons and quarks. Thus the nucleon-nucleon forces are mediated by gluon exchange between the constituent quarks for short distances whereas for longer distances the production of quark-antiquark pairs is the dominating mechanism. The quark-antiquark exchange can be treated as an effective meson exchange which lead to the construction of the Bonn and Paris potentials [1]. The meson theoretical potentials give a satisfying description of the nucleon-nucleon scattering data which are mainly sensitive to long range distances. The gluon exchange is studied by phenomenological potential and bag models allowing for a first insight into the interaction mechanism of the six-quark system [2].

Both quark and meson potentials contain parameters and are based on phenomenology. Today the aim should be to calculate the nucleon-nucleon forces from the equations of QCD itself. Although we are at present far from a treatment of QCD from first principles some attempts can be performed in the framework of a restricted QCD. In the last few years the simula-

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tion of quantum field theories on computers made fast progress and reached a high standard. Thus one should start now with the investigation of nucleon-nucleon interactions on the basis of a simple (and may be unrealistic) QCD. With the development of the next computer generations the codes can be extended in order to take finally full QCD into account.

During the last years lattice calculations have demonstrated that the potential between a static quark and a static antiquark of a meson is confining [3]. The same result turns out for the three quarks of a baryon [4]. All these outcomes of the static approximation make us believe QCD to be the proper theory of strong interactions. Thus an application of static QCD to the nucleon-nucleon system seems interesting. First investigations of the gluon exchange between two three-quark clusters yielded an attractive potential limited to the overlap region of the baryons [5]. The extension of the pure gluon Lagrangian to the full QCD Lagrangian would provide for the creation of virtual quark-antiquark pairs out of the vacuum. Calculations with the total QCD Lagrangian are in a very preliminary stage dealing with many technical problems. An alternative to simulate the meson exchange between two three-quark clusters is to set a static quark-antiquark pair between them and to calculate the potential energy with varying distance of the two nucleons. Although a static quark-antiquark pair is not a virtual meson this process may bring some insight into QCD based meson exchange and is technically feasible.

THEORY

At the present state in our approach quarks are restricted to fulfill the static Dirac equation whereas gluons are treated as dynamical SU(3)-Maxwell fields [6]. We introduce creation and annihilation operators $\Psi_a^+(\vec{r}_i,t)$ and $\Psi_a(\vec{r}_i,t)$ for the static quarks with color a at position $\vec{r}_i$ and time t as well as charge conjugate operators Ψ_a^{c+} and Ψ_a^c for the antiquarks. The static fields Ψ_a, Ψ_b^+ satisfy equal-time anticommutation relations with respect to space and color and obey the static time-evolution equation

$$\left(\frac{\partial}{\partial t} - i\vec{\lambda}A_o(\vec{r}_i t)\right)\Psi(\vec{r}_i t) = 0 \tag{1}$$

where $\vec{A}_o$ is the time component of the gluon field and $\vec{\lambda}$ are the generators of SU(3) in the fundamental representation. This equation can be integrated and yields the quark propagator

$$\Psi(\vec{r}_i,t) = \hat{T}\exp(i\int_o^t dt'\vec{\lambda}\vec{A}_o(\vec{r}_i,t'))\Psi(\vec{r}_i,0) \tag{2}$$

with the time ordering operator $\hat{T}$.

The free energy $F_N(\vec{r}_1,\ldots,\vec{r}_N)$ of a system of N quarks and antiquarks in a gluonic field is defined by the thermodynamical expectation value

$$\exp(-\frac{1}{T}F_N(\vec{r}_1,\ldots,\vec{r}_N)) := \frac{1}{3^N}\sum_{|s_q s_g>} <s_{q_1}\ldots s_{q_N}s_g|e^{-\frac{1}{T}H}|s_{q_1}\ldots s_{q_N}s_g> \tag{3}$$

The Hamiltonian H contains the static Dirac operator and the temperature T is equivalent to inverse Euclidean time t [6]. The expectation value has to be taken over all quark states $|s_{q_1}\ldots s_{q_N}>$ and over all states $|s_g>$ of the gluon field. Inserting the creation and annihilation operators $\Psi_a^+(\vec{r}_i,0)$, $\Psi_a(\vec{r}_i,0)$ for the quark states $|s_{q_i}>$ and writing the time evolution e^{-Ht} of these operators in terms of the quark propagator (2) leads to the expression

$$\exp(-\frac{1}{T}F_N(\vec{r}_1,\ldots,\vec{r}_N)) = \sum_{|s_g>} <s_g|e^{-\frac{1}{T}H}L(\vec{r}_1)\ldots L(\vec{r}_N)|s_g> \tag{4}$$

where the Wilson-Polyakov loops $L(\vec{r}_i)$ are an abbreviation for the trace over the static propagator

$$L(\vec{r}_i) = \frac{1}{3}\mathrm{tr}\hat{T}\exp(i\int_o^t dt'\vec{\lambda}\vec{A}_o(\vec{r}_i,t)) \tag{5}$$

The above expectation value (4) can be interpreted as a path integral in the Euclidean Hamiltonian formulation which can be recasted into the Lagrange form. Normalization with respect to the quark-vacuum energy F_o yields the Feynman path integral representation for the expectation value of a product of N Wilson-Polyakov loops

$$\exp(-\frac{1}{T}F(\vec{r}_1,\ldots,\vec{r}_N)) = \frac{\int DA^{\mu}(\vec{r},t)L(\vec{r}_1)\ldots L(\vec{r}_N)\exp(-\int_0^{\frac{1}{T}}dt\int_V d^3x\mathcal{L}(A))}{\int DA^{\mu}(\vec{r},t)\exp(-\int_0^{\frac{1}{T}}dt\int_V d^3x\mathcal{L}(A))}$$

$$= <L(\vec{r}_1)\ldots L(\vec{r}_N)>, \tag{6}$$

where $F(\vec{r}_1,\ldots,\vec{r}_N):=F_N(\vec{r}_1,\ldots,\vec{r}_N) - F_o$ is the net free energy of the N-quark system and v the spatial volume. The Euclidean Lagrange density $\mathcal{L}(A)$ for the pure gluon field is given by

$$\mathcal{L}(A) = \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} \text{ with } F^a_{\mu\nu} := \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g_o f^{abc}A^b_\mu A^c_\nu, \qquad (7)$$

g_o the bare coupling constant, and f^{abc} the structure constants of SU(3).

In order to compute the path integral (6) we succed by its formulation on a space-time lattice and performing Monte Carlo simulations. We take a lattice of size 16x16x16x6 with hot boundary conditions in space and periodic boundary conditions in time. The coupling $\beta=6/g_o^2=5.85$ corresponds via the renormalization group equation to a lattice distance of $a \sim 0.23$fm. Thus the whole cube has a linear spatial extension of about 3.66fm.

We construct a quark wave function $\Psi(\vec{r})$ by distributing the static quark charges in Gaussians

$$\Psi(\vec{r}) = \frac{1}{K}\exp(-\vec{r}^2/2R^2)L(\vec{r}) \qquad (8)$$

over a sphere with radius R on the lattice where K is a normalization constant. For the description of baryons we take the product of three quark wave functions choosing R=3a and for mesons correlated wave functions of quark and antiquark with R=2a. Because the static quarks carry no spin and flavor all baryons and mesons are degenerate.

To study the pure gluon exchange between baryons the total wave function of the baryon-baryon system is constructed to be a product of Gauss functions with two centers separated by some distance d. This framework is extended for a simulation of meson exchange by placing a quark-antiquark distribution between the two baryons.

RESULTS

First we calculate the total energy of the baryon-baryon system according to (6) and subtract twice the energy of a single baryon. In the figure the broken line demonstrates the baryon-baryon potential caused by creating gluons out of the vacuum between the six-quark system. The interaction between the two baryons is attractive. Its range is restricted to the overlap region of the baryons which is about 6a. Adding quark-

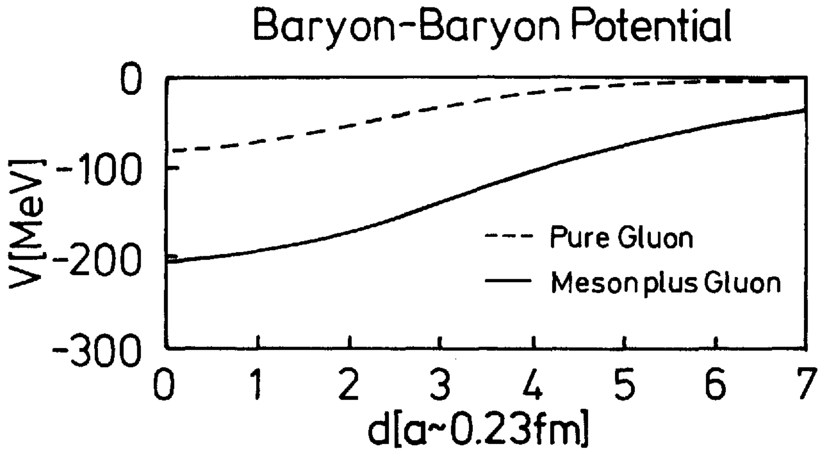

Figure: Contribution of pure gluon exchange (dashed line) and gluon plus meson exchange (full line) to the baryon-baryon potential as a function of the distance d in units of the lattice constant a.

antiquark pairs between the two baryons yields the potential outlined by the full curve in the figure. Here the energy of an isolated meson was subtracted additionally. The potential is about twice as attractive as in the pure gluon exchange case. The range of interaction is given by the overlap of the three-hadron system covering a region of about 10a. It must be noticed that the exchanged meson is not a virtual object but is put in by hand as a real quark-antiquark pair.

The origin of the attractive forces between the two baryons lies in the Coulomb plus confining potential between quarks which tries to attract the two three-quark clusters. When the two baryons become separated their colors are saturated and they build isolated color singlets which do not interact. At least on the lattice no long range forces can be resolved. In the presence of the meson the same mechanism holds true. When the two baryons have no overlap with the meson in between the three hadrons build noninteracting singlets.

Let us discuss the main deficiencies of our approach and propose some improvements for future work.

Recent accurate investigations of the string constant suggest a lattice constant being only half as big [7] which means a $\sim$ 0.12 fm for our case. This would imply that the resulting potentials are twice as deep. The range of the interactions is always restricted to the overlap region of the interacting hadrons.

To compute the energy expectation value F (6) of a quark distribution (8) we had to interchange logarithm and summation for technical reasons. An exact treatment should not effect the general shape of the potentials.

We work with a truncated Dirac equation (1) restricting the quarks to static color charges. The calculated energy of the considered quark wave functions has thus the meaning of the energy content of quark charge distributions in a gluon field. This is the equivalent of the potential of an electric charge distribution in electrostatics. The only difference is that the Coulomb potential mediated by photon exchange is known analytically and thus no quantum field theory has to be simulated. It must be emphasized that the construction of the quark wave functions allows for baryons of reliable size but takes no kinetic energy into account.

Although the exchanged gluons are produced out of the quantum sea the meson exchange is simply treated by a real quark-antiquark pair. However, with the extension of the pure gluon Lagrangian to full QCD Lagrangian the contribution of virtual mesons could be investigated. Our next aim is to compute both virtual gluon and meson exchange between static baryons.

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