WHAT IS THE DARK MATTER? IMPLICATIONS FOR GALAXY FORMATION AND PARTICLE PHYSICS

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ABSTRACT We discuss three arguments that the dark matter which dominates the present universe is not baryonic - based on excluding specific baryonic models, deuterium abundance, and the absence of small-angle fluctuations in the microwave background radiation. If the dark matter consists of elementary particles, it may be classified as hot (free streaming erases all but supercluster-scale fluctuations), warm (free streaming erases fluctuations smaller than galaxies), or cold (free streaming is unimportant). We consider scenarios for galaxy formation in all three cases. We discuss several potential problems with the hot (neutrino) case: making galaxies early enough, with enough baryons, and without too much increase in $M_{tot}/M_{\ell um}$ from galaxy to rich cluster scales. The reported existence of dwarf spheroidal galaxies with relatively heavy halos is a serious problem for both hot and warm scenarios. Zeldovich (n = 1) adiabatic initial fluctuations in cold dark matter (axions, or a heavy stable "ino") appear to be lead to observed sizes and other properties of galaxies, and may also yield large scale structure such as voids and filaments.

I. INTRODUCTION

There is abundant observational evidence that dark matter (DM) is responsible for most of the mass in the universe (1). Dark matter is detected through its gravitational attraction in the massive extended halos of disk galaxies and in groups and clusters of galaxies of all sizes. It is appropriate to call this matter "dark" because it is detected in no other way; it is not observed to emit or absorb electromagnetic radiation of any wavelength. Matter observed in these latter ways we will call "luminous". Here we consider the nature of the dark matter.

II. THE DM IS PROBABLY NOT BARYONIC

There are three arguments that the DM is not "baryonic", that is, that it is not made of protons, neutrons, and electrons as all ordinary matter is. As Richard Feynman has said in other contexts, one argument would suffice if it were convincing. All three arguments have loopholes. The arguments that DM $\neq$ baryons are as follows:

A. Excluding Baryonic Models (2)

The dark matter in galaxy halos cannot be gas (it would have to be hot to be pressure supported, and would radiate); nor frozen hydrogen "snowballs" (they would sublimate); nor dust grains (their "metals", elements of atomic number ≥ 3, would have prevented formation of the observed low-metallicity Population II stars); nor "jupiters" (how to make so many hydrogen balls too small to initiate nuclear burning without making a few large enough to do so?); nor collapsed stars (where is the matter they must have ejected in collapsing?).

The weakest argument is probably that which attempts to exclude "jupiters": arguments of the form "how could it be that way?" are rarely entirely convincing.

B. Deuterium Abundance (3)

In the early universe, almost all the neutrons which "freeze out" are synthesized into ^{4}He. The fraction remaining in D and ^{3}He is a rapidly decreasing function of η, the ratio of baryon to photon number densities. The presently observed D abundance (compared, by number, to H) is $(1-4)\times 10^{-5}$. Since D is readily consumed but not produced in stars, 10^{-5} is also a lower limit on the primordial D abundance. This, in turn, implies an upper limit $\eta \leq 10^{-9}$, or

$$\Omega_b \leq 0.035\, h^{-2}\, (T_o/2.7)^3, \tag{1}$$

where Ω_b is the ratio of the present average baryon density ρ_b to the critical density

$$\rho_c = 3H_o^2/8\pi G = 1.9\times 10^{-29}\, h^2\ \mathrm{g\,cm^{-3}} = 11\, h^2\ \mathrm{keV\,cm^{-3}}, \tag{2}$$

H_o is the Hubble parameter (the subscript o denotes the present epoch), and observationally $h \equiv H_o\,(100\,\mathrm{km s^{-1} Mpc^{-1}})^{-1}$ lies in the range $\frac{1}{2} \leq h \leq 1$. The total cosmological density $\Omega \equiv \rho_{tot}/\rho_c$ is very difficult to determine observationally, but it appears to lie in the range $0.1 \leq \Omega \lesssim 2$. Cosmological models in which the universe passes through an early de Sitter "inflationary" stage, predict Ω very close to unity.

In a baryon dominated universe ($\Omega \simeq \Omega_b$), the deuterium bound, Eq. (1), is consistent only with the lower limit on Ω, and then only for the Hubble parameter at its lower limit. An Einstein-de Sitter or inflationary ($\Omega = 1$) or closed ($\Omega > 1$) universe cannot be baryonic.

C. Galaxy Formation

In the standard cosmological model, which we will adopt, large scale structure forms when perturbations $\delta \equiv \delta\rho/\rho$ grow to $\delta \gtrsim 1$, after which they cease to expand with the Hubble flow. Let us further assume that perturbations in matter and radiation density are correlated (these are called adiabatic perturbations, since the entropy per baryon is constant; these are the sort of perturbations predicted in grand unified models). Then photon diffusion ("Silk damping") erases perturbations of baryonic mass smaller than (4)

$$M_{Silk,b} \approx 3 \times 10^{13}\, \Omega_b^{-1/2}\, \Omega^{-3/4}\, h^{-5/2}\, M_\odot \,. \qquad (3)$$

Thus galaxies ($M_b \lesssim 10^{11-12} M_\odot$) can form only after the "pancake" collapse of larger-scale perturbations (5). Perturbations δ in a matter dominated universe grow linearly with the scale factor

$$\delta \propto a = (1+z)^{-1} = T_o/T \qquad (4)$$

where $z = (\lambda_o - \lambda)/\lambda$ is the redshift and T is the radiation temperature. In a baryonic universe, δ grows only between the epoch of hydrogen recombination, $z_r \simeq 1300$, and $z \simeq \Omega^{-1}$. It follows that at recombination $\delta T/T \approx \delta\rho/3\rho \geq 3 \times 10^{-3}$ for $M \gtrsim M_{Silk}$, which corresponds to fluctuations on observable angular scales $\theta > 4'$ today. Such temperature fluctuations are an order of magnitude larger than present observational upper limits (6).

The main loophole in this argument is the assumption of adiabatic perturbations. It is true that the orthogonal mode, perturbations in baryonic density which are uncorrelated with radiation (called isothermal perturbations), do not arise naturally in currently fashionable particle physics theories where baryon number is generated in the decay of massive grand unified theory (GUT) bosons, since in such theories $\eta = n_b/n_\gamma$ is determined by the underlying particle physics and should not vary from point to point in space. But galaxies originating as isothermal perturbations do avoid both Silk damping and contradiction with present $\delta T/T$ limits.

A second loophole is the possibility that matter was reionized at some $z \gtrsim$, by hypothetical very uv sources. Then the fluctuations in $\delta T/T$ at recombination associated with baryonic protopancakes could be washed out by rescattering.

Despite the loopholes in each argument, we find the three arguments together to be rather persuasive, even if not entirely compelling. If it is indeed true that the bulk of the mass in the universe is not baryonic, that is yet another blow to anthropocentricity: not only is man not the center of the universe physically (Copernicus) or biologically (Darwin), we and all that we see are not even made of the predominant variety of matter in the universe!

III. THREE TYPES OF DM PARTICLES: HOT, WARM & COLD

If the dark matter is not baryonic, what is it? We will consider here the physical and astrophysical implications of three classes of elementary particle DM candidates, which we will call hot, warm, and cold. (We are grateful to Dick Bond for proposing this apt terminology.)

Hot DM refers to particles, such as neutrinos, which were still in thermal equilibrium after the most recent phase transition in the hot early universe, the QCD deconfinement transition, which presumably took place at $T_{QCD} \sim 10^2$ MeV. Hot DM particles have a cosmological number density roughly comparable to that of the microwave background photons, which implies an upper bound to their mass of a few tens of eV. As we shall discuss shortly, free streaming destroys any perturbations smaller than supercluster size, $\sim 10^{15} M_\odot$.

Warm DM particles interact much more weakly than neutrinos. They decouple (i.e., their mean free path first exceeds the horizon size) at $T > T_{QCD}$, and consequently their number density is roughly an order of magnitude lower, and their mass an order of magnitude higher, than hot DM particles. Perturbations as small as large galaxy halos, $\sim 10^{12} M_\odot$, could then survive free streaming. It was initially suggested that, in theories of local supersymmetry broken at $\sim 10^6$ GeV, gravitinos could be DM of the warm variety (7). Other candidates are also possible, as we will discuss.

Cold DM consists of particles for which free streaming is of no cosmological importance. Two different sorts have been proposed, a cold Bose condensate such as axions, and heavy remnants of annihilation or decay such as heavy stable neutrinos. As we will see, a universe dominated by cold DM looks remarkably like the one astronomers actually observe.

It is of course also possible that the dark matter is NOTA -

none of the above! A perennial candidate, primordial black holes, is becoming increasingly implausible (8-10). Another possibility which, for simplicity, we will not discuss, is that the dark matter is a mixture, for example "jupiters" in galaxy halos plus neutrinos on large scales (3).

IV. GALAXY FORMATION WITH HOT DM

The standard hot DM candidate is massive neutrinos (3-5), although other, more exotic, theoretical possibilities have been suggested, such as a "majoron" of nonzero mass which is lighter than the lightest neutrino species, and into which all neutrinos decay (11). For definiteness, we will discuss neutrinos.

A. Mass Constraints

Left-handed neutrinos of mass $\lesssim 1\,\text{MeV}$ will remain in thermal equilibrium until the temperature drops to $T_{\nu d}$, at which point their mean free path first exceeds the horizon size and they essentially cease interacting thereafter, except gravitationally (12). Their mean free path is, in natural units ($\hbar = c = 1$), $\lambda_\nu \sim [\sigma_\nu n_{e^\pm}]^{-1} \sim [(G_{wk}^2 T^2)(T^3)]^{-1}$, and the horizon size is $\lambda_h \sim (G\rho)^{-\frac{1}{2}} \sim M_{P\ell} T^{-2}$, where the Planck mass $M_{P\ell} \equiv G^{-\frac{1}{2}} = 1.22 \times 10^{19}\,\text{GeV} = 2.18 \times 10^{-5}\,\text{g}$. Thus $\lambda_h/\lambda_\nu \sim (T/T_{\nu d})^3$, with the neutrino decoupling temperature

$$T_{\nu d} \sim M_{P\ell}^{-1/3}\, G_{wk}^{-2/3} \sim 1\,\text{MeV}. \tag{5}$$

After T drops below 1 MeV, e^+e^- annihilation ceases to be balanced by pair creation, and the entropy of the e^+e^- pairs heats the photons. Above 1 MeV, the number density $n_{\nu i}$ of each left-handed neutrino species (counting both ν_i and $\bar{\nu}_i$) is equal to that of the photons, n_γ, times the factor 3/4 from Fermi vs. Bose statistics; but e^+e^- annihilation increases the photon number density relative to that of the neutrinos by a factor of 11/4.[1] Thus today, for each species,

$$n_\nu^o = \frac{3}{4} \cdot \frac{4}{11}\, n_\gamma^o = 109\, \left(\frac{T_\gamma}{2.7K}\right)^3 \text{cm}^{-3}\,. \tag{6}$$

Since the cosmological density

$$\rho = \Omega\rho_c = 11\ \Omega h^2\,\text{keV cm}^{-3}, \tag{7}$$

it follows that

$$\sum_i m_{\nu i} < \rho/n_\nu^o \leq 100\ \Omega h^2\ \text{eV}, \tag{8}$$

where the sum runs over all neutrino species with $m_{\nu_i} \lesssim 1\,\mathrm{MeV}$.[2] Observational data imply that Ωh^2 is less than unity (3). Thus if one species of neutrino is substantially more massive than the others and dominates the cosmological mass density, as for definiteness we will assume for the rest of this section, then a reasonable estimate for its mass is $m_\nu \sim 30\,\mathrm{eV}$.

At present there is apparently no reliable experimental evidence for nonzero neutrino mass. Although one group reported (15) that $14\,\mathrm{eV} < m_{\nu_e} < 40\,\mathrm{eV}$ from tritium β end point data, according to Boehm (16) their data are consistent with $m_{\nu_e} = 0$ with the resolution corrections pointed out by Simpson. The so far unsuccessful attempts to detect neutrino oscillations also give only upper limits on neutrino masses times mixing parameters (16).

B. Free Streaming

The most salient feature of hot DM is the erasure of small fluctuations by free streaming. It is easy to see that the minimum mass of a surviving fluctuation is of order $M_{P\ell}^3/m_\nu^2$ (17,4).

Let us suppose that some process in the very early universe - for example, thermal fluctuations subsequently vastly inflated, in the inflationary scenario (18) - gave rise to adiabatic fluctuations on all scales. Neutrinos of nonzero mass m_ν stream relativistically from decoupling until the temperature drops to m_ν, during which time they will traverse a distance $d_\nu \approx \lambda_h(T = m_\nu) \sim M_{P\ell}\, m_\nu^{-2}$. In order to survive this free streaming, a neutrino fluctuation must be larger in linear dimension than d_ν. Correspondingly, the minimum mass in neutrinos of a surviving fluctuation is $M_{J,\nu} \sim d_\nu^3 m_\nu n_\nu(T = m_\nu) \sim d_\nu^3 m_\nu^4 \sim M_{P\ell}^3 m_\nu^{-2}$. By analogy with Jeans' calculation of the minimum mass of an ordinary fluid perturbation for which gravity can overcome pressure, this is referred to as the (free-streaming) Jeans mass. A more careful calculation (4,19) gives

$$d_\nu = 41\,(m_\nu/30\,\mathrm{eV})^{-1}\,(1+z)^{-1}\,\mathrm{Mpc}, \qquad (9)$$

and

$$M_{J,\nu} = 1.77\,M_{P\ell}^3\,m_\nu^{-2} = 3.2 \times 10^{15}\,(m_\nu/30\,\mathrm{eV})^{-2}\,M_\odot, \qquad (10)$$

which is the mass scale of superclusters. Objects of this size are the first to form in a ν-dominated universe, and smaller scale structures such as galaxies can form only after the initial collapse of supercluster-size fluctuations.

C. Growth of Fluctuations

The absence of small angle $\delta T/T$ fluctuations is compatible with this picture. When a fluctuation of total mass $\sim 10^{15} M_\odot$ enters the horizon at $z \sim 10^4$, the density contrast of the radiation plus baryons δ_{RB} ceases growing and instead starts oscillating as an acoustic wave, while that of the neutrinos δ_ν continues to grow linearly with the scale factor $a = (1+z)^{-1}$. Thus by recombination, at $z_r \approx 1300$, $\delta_{RB}/\delta_\nu < 10^{-1}$, with possible additional suppression of δ_{RB} by Silk damping (depending on the parameters in Eq. (3)). This picture, as well as the warm and cold DM schemes, predicts small angle fluctuations in the microwave background radiation just slightly below current observational upper limits (6).

In numerical simulations of dissipationless gravitational clustering starting with a fluctuation spectrum appropriately peaked at $\lambda \approx d_\nu$, the regions of high density form a network of filaments, with the highest densities occurring at the intersections and with voids in between (5,20-22). The similarity of these features to those seen in observations (23,24) is certainly evidence in favor of this model.

D. Potential Problems with ν DM

A number of potential problems with the neutrino dominated universe have emerged in recent studies, however. (1) From studies both of nonlinear (22) clustering ($\lambda \lesssim 10\,\mathrm{Mpc}$) and of streaming velocities (25) in the linear regime ($\lambda > 10\,\mathrm{Mpc}$), it follows that supercluster collapse must have occurred recently: $z_{sc} \lesssim 0.5$ is indicated (25), and in any case $z_{sc} < 2$ (22). But then, if QSOs are associated with galaxies, their abundance at $z > 2$ is inconsistent with the "top-down" neutrino dominated scheme in which superclusters form first: $z_{sc} > z_{galaxies}$. (2) Numerical simulations of the nonlinear "pancake" collapse taking into account dissipation of the baryonic matter show that at least 85% of the baryons are so heated by the associated shock that they remain ionized and unable to condense, attract neutrino halos, and eventually form galaxies (25a). (3) The neutrino picture predicts (26) that there should be a factor of ~5 increase in M_{tot}/M_{lum} between large galaxies ($M_{tot} \sim 10^{12} M_\odot$) and large clusters ($M_{tot} \gtrsim 10^{14} M_\odot$), since the larger clusters, with their higher escape velocities, are able to trap a considerably larger fraction of the neutrinos. Although there is indeed evidence for a trend of increasing M_{tot}/L with M_{tot} (1,27), when one takes into account the large amount of x-ray emitting gas in rich clusters (28) one finds comparable $M_{tot}/M_{lum} \approx 14$ for galaxies with large halos and for rich clusters (29,30). (M_{lum} here includes matter luminous in x-ray as well as optical wavelengths, in contrast to luminosity L that includes only the latter.) (4) Both theoretical arguments (31) and data on Draco (32,33) imply that dark matter dominates the gravitational potential of dwarf spheroidal galaxies. The phase-space constraint (34) then sets a lower limit (33) $m_\nu > 500\,eV$, which is completely incompatible with the cosmological constraint Eq. (8). (Note that for neutrinos as the DM in spiral galaxies, the phase space constraint implies $m_\nu > 30\,eV$.)

These problems, while serious, may not be fatal for the hypothesis that neutrinos are the dark matter. It is possible that galaxy density does not closely correlate with the density of dark matter, for example because the first generation of luminous objects heats nearby matter, thereby increasing the baryon Jeans mass and suppressing galaxy formation. This could complicate the comparison of nonlinear simulations (22) with the data. Also, if dark matter halos of large clusters are much larger in extent than those of individual galaxies and small groups, then virial estimates would underestimate M_{tot} on large scales and the data could be consistent with M_{tot}/M_{lum} increasing with M_{lum}. But it is hard to avoid the constraint on z_{sc} from streaming velocities in the linear regime (25) except by assuming that the local group velocity is abnormally low. And the only explanation for the high M_{tot}/L of dwarf spheroidal galaxies in a neutrino-dominated universe is the rather ad hoc assumption that the dark matter in such objects is baryons rather than neutrinos. Of course, the evidence for massive halos around dwarf spheroidals is not yet solid.

V. GALAXY FORMATION WITH WARM DM

Suppose the dark matter consists of an elementary particle species X that interacts much more weakly than neutrinos. The Xs decouple thermally at a temperature $T_{Xd} >> T_{\nu d}$ and their number density is not thereafter increased by particle annihilation at temperatures below T_{Xd}. With the standard assumption of conservation of entropy per comoving volume, the X number density today n_X^o and mass m_X can be calculated in terms of the effective number of helicity states of interacting bosons (B) and fermions (F), $g = g_B + (7/8)g_F$, evaluated at T_{Xd} (35). These are plotted in Fig. 1, assuming the "standard model" of particle physics. The simplest grand unified theories predict $g(T) \approx 100$ for T between 10^2 GeV and $T_{GUT} \sim 10^{14}$ GeV, with possibly a factor of two increase in g beginning near 10^2 GeV due to N = 1 supersymmetry partner particles. Then for T_{Xd} in the enormous range from ~1 GeV to $\sim T_{GUT}$, $n_X^o \sim 5 g_X cm^{-3}$ and correspondingly $m_X \simeq 2\Omega h^2 g_X^{-1}$ keV (36), where g_X is the number of X helicity states. Such "warm" DM particles of mass $m_X \sim 1\,keV$ will cluster on a scale $\sim M_{P\ell}^3 m_X^{-2} \sim 10^{12} M_\odot$, the scale of large galaxies such as our own (7,37,38).

What might be the identity of the warm DM particles X? It was initially (7) suggested that they might be the $\pm\frac{1}{2}$ helicity states of the gravitino $\tilde{G}$, the spin 3/2 supersymmetric partner of the graviton G. The gravitino mass is related to the scale of supersymmetry breaking by $m_{\tilde{G}} = (4\pi/3)^{1/2} m_{SUSY}{}^2 m_{P\ell}^{-1}$, so $m_{\tilde{G}} \sim 1$ keV corresponds to $m_{SUSY} \sim 10^6$GeV. This now appears to be phenomenologically dubious, and supersymmetry models with $m_{SUSY} \sim 10^{11}$ GeV and $m_{\tilde{G}} \sim 10^2$ GeV are currently popular (39). In such models, the photino $\tilde{\gamma}$, the spin ½ supersymmetric partner of the photon, is probably the lightest R-odd particle, and hence stable. But in supersymmetric GUT models $m_{\tilde{\gamma}} \sim 10\, m_{\tilde{g}}$, and there is a phenomenological lower bound on the mass of the gluino $m_{\tilde{g}} > 2$ GeV (40). The requirement that the photinos almost all annihilate, so that they do not contribute too much mass density, implies that $m_{\tilde{\gamma}} \gtrsim 2$ GeV (14,41), and they become a candidate for cold rather than warm dark matter.

A hypothetical right-handed neutrino ν_R could be the warm DM particle (42), since if right-handed weak interactions exist they must be much weaker than the ordinary left-handed weak interactions, so $T_{\nu_R d} >> T_{\nu d}$ as required. But particle physics provides no good reason why any ν_R should be light.

Thus there is at present no obvious warm DM candidate elementary particle, in contrast to the hot and cold DM cases. But our ignorance about the physics above the ordinary weak interaction scale hardly allows us to preclude the existence of very weakly interacting light particles, so we will consider the warm DM case, mindful of Hamlet's prophetic admonition

> There are more things in heaven and earth, Horatio,
> Than are dreamt of in your philosophy.

A. Fluctuation Spectrum

The spectrum of fluctuations δ_ν at late times in the hot DM model is controlled mainly by free streaming; $\delta_\nu(M)$ is peaked at $\sim M_{J,\nu}$, Eq. (10), for any reasonable primordial fluctuation spectrum. This is <u>not</u> the case for warm or cold DM.

The primordial fluctuation spectrum can be characterized by the magnitude of fluctuations as they just enter the horizon. It is expected that no mass scale is singled out, so the spectrum is just a power law

$$\delta_{DM,H} = \left(\frac{\delta\rho_{DM}}{\rho_{DM}}\right)_H = \kappa\left(\frac{M_{DM}}{M_o}\right)^{-\alpha} \tag{11}$$

Furthermore, to avoid too much power on large or small mass scales requires $\alpha \approx 0$ (43), and to form galaxies and large scale structure by the present epoch without violating the upper limits on both small (6) and large (44) scale (quadrupole) angular variations in the microwave background radiation requires $\kappa \sim 10^{-4}$. Eq. (11)

corresponds to $|\delta_k|^2 \propto k^n$ with $n = 6\alpha + 1$. The case $\alpha = 0$ $(n = 1)$ is commonly referred to as the Zeldovich spectrum.

Inflationary models predict adiabatic fluctuations with the Zeldovich spectrum (18). In the simplest models κ is several orders of magnitude too large, but it is hoped that this will be remedied in more realistic - possible supersymmetric - models (45).

The important difference between the fluctuation spectra δ_{DM} at late times in the hot and warm DM cases is that $\delta_{DM,warm}$ has power over an increased range of masses, roughly from 10^{11} to $10^{15} M_\odot$. As for the hot case, the lower limit, $M_X \sim M_{P\ell}^3 m_X^{-2}$, arises from the damping of smaller-scale fluctuations by free streaming. In the hot case, the DM particles become nonrelativistic at essentially the same time as they become gravitationally dominant, because their number density is nearly the same as that of the photons. But in the warm case, the X particles become nonrelativistic and thus essentially stop free streaming at $T \sim m_X$, well before they begin to dominate gravitationally at $T_{eq} \approx 6\Omega h^2$ eV. The subscript "eq" refers to the epoch when the energy density of massless particles equals that of massive ones:

$$z_{eq} = \frac{\Omega \rho_c c}{4\sigma T_0^4(1+\gamma)} = 2.47 \times 10^4 \, \Omega h^2 \left(\frac{1.681}{1+\gamma}\right) \theta^{-4} . \tag{12}$$

We assume here that there are n_ν species of very light or massless neutrinos, and $\gamma \equiv \rho_\nu^0/\rho_\gamma^0 = (7/8)(4/11)^{4/3} n_\nu$ $(= 0.681$ for $n_\nu = 3)$, $\theta \equiv T_0/2.7\mathrm{K}$, and σ is the Stefan-Boltzmann constant. During the interval between $T \sim m_X$ and $T \sim T_{eq}$, growth of δ_{DM} is inhibited by the "stagspansion"[3] phenomenon (also known as the Meszaros (46) effect), which we will discuss in detail in the section on cold DM. Thus the spectrum δ_{DM} is relatively flat between M_X and

$$M_{eq} = \frac{4\pi}{3} \left(\frac{ct_{eq}}{1+z_{eq}}\right)^3 \rho_c \Omega = 2.2 \times 10^{15} (\Omega h^2)^{-2} M_\odot . \tag{13}$$

Fluctuations with masses larger than M_{eq} enter the horizon at $z < z_{eq}$, and thereafter δ_{DM} grows linearly with $a = (1+z)^{-1}$ until nonlinear gravitational effects become important when $\delta_{DM} \sim 1$. Since for $\alpha = 0$ all fluctuations enter the horizon with the same magnitude, and those with larger M enter the horizon later in the matter-dominated era and subsequently have less time to grow, the fluctuation spectrum falls with M for $M > M_{eq}$: $\delta_{DM} \propto M^{-2/3}$. For a power-law primordial spectrum of arbitrary index,

$$\delta_{DM} \propto M^{-\alpha - 2/3} = M^{-(n+3)/6}, \quad M > M_{eq} . \tag{14}$$

This is true for hot, warm, or cold DM. In each case, after recombination at $z_r \approx 1300$ the baryons "fall in" to the dominating DM fluctuations on all scales larger than the baryon Jeans mass, and by $z \approx 100$, $\delta_b \approx \delta_{DM}$ (47).

In the simplest approximation, neglecting all growth during the "stagspansion" era, the fluctuation spectrum for $M_X < M < M_{eq}$ is just $\delta_{DM} \propto M^{-\alpha} = M^{-(n-1)/6} = M^{-(n_{eff}+3)/6}$, where $n_{eff} = n - 4$; i.e., the spectrum is flattened by a factor of $M^{2/3}$ compared to the primordial spectrum. The small amount of growth that does occur during the "stagspansion" era slightly increases the fluctuation strength on smaller mass scales: $n_{eff} \approx n - 3$. Detailed calculations of these spectra are now available (19,37).

B. Which Formed First, Galaxies or Superclusters?

For $\alpha \gtrsim 0$, $\delta_X(M)$ has a fairly broad peak at $M \sim M_X$. Consequently, objects of this mass - galaxies and small groups - are the first to form, and larger-scale structures - clusters and superclusters - form later as $\delta_X(M)$ grows toward unity on successively larger mass scales. For a particular primordial spectral index α, one can follow Pebbles (48,49) and use the fact that the galaxy autocovariance function $\xi(R) \approx 1$ for $R = 5h^{-1}$, together with the (uncertain) assumption that the DM is distributed on such scales roughly like the galaxies, to estimate when galaxies form in this scenario. For $\alpha = 0$, $z_{galaxies} \sim 4$, which is consistent with the observed existence of quasars at such redshifts. But superclusters do not begin to collapse until $z < 2$, so one would not expect to find similar Lyman α absorption line redshifts for quasars separated by $\sim 1\,h^{-1}$ Mpc perpendicular to the line of sight (50). Indeed, Sargent et al. (51) found no such correlations. This is additional evidence against hot DM.

C. Potential Problems with Warm DM

The warm DM hypothesis is probably consistent with the observed features of typical large galaxies, whose formation would probably follow roughly the "core condensation in heavy halos" scenario (52,29,53). The potentially serious problems with warm DM are on scales both larger and smaller than M_X. On large scales, the question is whether the model can account for the observed network of filamentary superclusters enclosing large voids (23,24). A productive approach to this question may require sophisticated N-body simulations with $N \sim 10^6$ in order to model the large mass range that is relevant (54). We will discuss this further in the next section in connection with cold DM, for which the same question arises.

On small scales, the preliminary indications that dwarf spheroidal galaxies have large DM halos (31-33) pose problems nearly as serious for warm as for hot DM. Unlike hot DM, warm DM is (barely) consistent with the phase space constraint (32-34). But since free streaming of warm DM washes out fluctuations δ_X for $M \lesssim M_X \sim 10^{11} M_\odot$, dwarf galaxies with $M \sim 10^7 M_\odot$ can form in this picture only via fragmentation following the collapse of structures of mass $\sim M_X$, much as ordinary galaxies form from superslusters fragmentation in the hot DM picture. The problem here is that dwarf galaxies, with their small escape velocities $\sim 10\,km\,s^{-1}$, would not be expected to bind more than a small fraction of the X particles, whose typical velocity must be $\sim 10^2\,km\,s^{-1}$ (~ rotation velocity of spirals). Thus we expect M_{tot}/M_{lum} for dwarf galaxies to be much smaller than for large galaxies - but the indications are that they are comparable (31-33). Understanding dwarf galaxies may well be crucial for unravelling the mystery of the identity of the DM. Fortunately, data on Carina, another dwarf spheroidal companion of the Milky Way, is presently being analyzed (55).

VI. GALAXY FORMATION WITH COLD DM

Damping of fluctuations by free streaming occurs only on scales too small to be cosmologically relevant for DM which either is not characterized by a thermal spectrum, or is much more massive than 1 keV. We refer to this as cold DM.

A. Cold DM Candidates

Quantum chromodynamics (QCD) with quarks of nonzero mass violates CP and T due to instantons. This leads to a neutron electric dipole moment that is many orders of magnitude larger than the experimental upper limit, unless an otherwise undetermined complex phase θ_{QCD} is arbitrarily chosen to be extremely small. Peccei and Quinn (56) have proposed the simplest and probably the most appealing way to avoid this problem, by postulating an otherwise unsuspected symmetry that is spontaneously broken when an associated pseudoscalar field - the axion (57) - gets a nonzero vacuum expectation value $\langle\phi_a\rangle \sim f_a e^{i\theta}$. This occurs when $T \sim f_a$. Later, when the QCD interactions become strong at $T \sim \Lambda_{QCD} \sim 10^2$ MeV, instanton effects generate a mass for the axion $m_a = m_\pi f_\pi / f_a \approx 10^{-5}\,\text{eV}(10^{12}\,\text{GeV}/f_a)$. Thereafter, the axion contribution to the energy density is (58) $\rho_a = 3m_a T^3 f_a^2 (M_{P\ell}\Lambda_{QCD})^{-1}$. The requirement $\rho_a^o < \rho_c\Omega$ implies that $f_a \lesssim 10^{12}$ GeV, and $m_a \gtrsim 10^{-5}$ eV.[4] The longevity of helium-burning stars implies (59) that $m_a < 10^{-2}$ eV, $f_a > 10^9$ GeV. Thus if the hypothetical axion exists, it is probably important cosmologically, and for $m_a \sim 10^{-5}$ eV gravitationally dominant. (The mass range 10^{9-12} GeV, in which f_a must lie, is also currently popular with particle theorists as the scale of supersymmetry (39) or family symmetry breaking, the later possibility connected with the axion (60).)

Two quite different sorts of cold DM particles are also possible. One is a heavy stable "ino", such as a photino (41) of mass $m_{\tilde{\gamma}} > 2$ GeV as discussed above. By a delicate adjustment of the theoretical parameters controlling the $\tilde{\gamma}$ mass and interactions, the $\tilde{\gamma}$s can be made to almost all annihilate at high temperatures, leaving behind a small remnant that, because $m_{\tilde{\gamma}}$ is large, can contribute a critical density today (14).

The second possibility may seem even more contrived: a particle, such as a ν_R, that decouples while still relativistic but whose number density relative to the photons is subsequently diluted by entropy generated in a first-order phase transition such as the Weinberg-Salam symmetry breaking (36). (Recall that the m_X bound in Fig. 1 assumes no generation of entropy.) More than a factor $\lesssim 10^3$ entropy increase would over dilute $\eta = n_b/n_\gamma$, if we assume η was initially generated by GUT baryosynthesis; correspondingly, $m_X \lesssim 1$ MeV, and $M_X \gtrsim 10^6\,M_\odot$.

Actually, it is not clear that we have a good basis to judge the plausibility of any of these DM candidates, since in no case is there a fundamental explanation - or, even better, a prediction - for the ratio $\omega \equiv \rho^o_{DM}/\rho^o_{lum}$, which is known to lie in the range $10 \lesssim \omega \lesssim 10^2$. Two fundamental questions about the universe which the fruitful marriage of particle physics and cosmology has yet to address are the value of ω and of the cosmological constant Λ. (We have here assumed $\Lambda = 0$, as usual.)

B. "Stagspansion"

Peebles (49) has calculated the fluctuation spectrum for cold DM, with results that are well approximated by the expression

$$|\delta_k|^2 = k^n(1+\alpha k+\beta k^2)^{-2},$$
$$\alpha = 6\,\theta^2 h^{-2}\,\text{Mpc},\ \beta = 2.65\,\theta^4 h^{-4}\,\text{Mpc}^2,\ \theta = T_o/2.7\text{K}. \qquad (15)$$

This calculation neglects the massless neutrinos; we find qualitatively similar results with their inclusion (61). For an adiabatic Zeldovich ($n=1$) primordial fluctuation spectrum, the spectrum of rms fluctuations in the mass found within a randomly placed sphere,[5] $\delta M/M$, is relatively flat for $M < 10^9 M_\odot$, $\propto M^{-1/6}$ ($n_{eff} \approx -2$) for $10^9 M_\odot \leq M \leq 10^{12} M_\odot$, $\propto M^{-1/3}$ ($n_{eff} \approx -1$) for $10^{12} M_\odot \leq M \leq M_{eq}$, and $\propto M^{-2/3}$ ($n=1$, reflecting the primordial spectrum) for $M \geq M_{eq}$.

The flattening of the spectrum for $M < M_{eq}$ is a consequence of "stagspansion",[3] the inihibition of the growth of δ_{DM} for fluctuations which enter the horizon when $z > z_{eq}$, before the era of matter domination. In the conventional formalism (12,48,62) - synchronous gauge, time-orthogonal coordinates - the fastest growing adiabatic fluctuations grow $\propto a^2$ when they are larger than the horizon. When they enter the horizon, however, the radiation and charged particles begin to oscillate as an acoustic wave with constant amplitude (later damped by photon diffusion for $M < M_{Silk}$), and the neutrinos free stream away. As a result, the main source term for the growth of δ_{DM} disappears, and once the fluctuation is well inside the horizon δ_{DM} grows only as (46), (48, pp. 56-59)

$$\delta_{DM} \propto 1 + \frac{3a}{2a_{eq}} \qquad (16)$$

until matter dominance ($a = a_{eq}$); thereafter, $\delta_{DM} \propto a$. Based on Eq. (16), it has sometimes been erroneously remarked [also by the present authors (38), alas] that there is only a factor of 2.5 growth in δ_{DM} during the entire stagspansion regime, from horizon crossing until matter dominance. There is actually a considerable amount of growth in δ_{DM} just after the fluctuation enters the horizon, since $d\delta_{DM}/da$ is initially large and the photon and neutrino source terms for the growth of dark matter fluctuations do not disappear instantaneously. (See reference 61 for details.) This explains how $(\delta M/M)_{DM}$ can grow by a factor ~30 between M_{eq} and $10^9 M_\odot$.

C. Galaxy Formation

When δ reaches unity, nonlinear gravitational effects become important. The fluctuation separates from the Hubble expansion, reaches a maximum radius, and then contracts to about half that radius (for spherically symmetric fluctuations), at which point the rapidly changing gravitational field has converted enough energy from potential to kinetic for the virial relation $\langle PE \rangle = -2\langle KE \rangle$ to be satisfied. (For reviews see (63) and (48).)

Although small-mass fluctuations will be the first to go nonlinear in the cold DM picture, baryons will be inhibited by pressure from falling into them if $M < M_{J,b}$. What is more important is that even for $M > M_{J,b}$, the baryons will not be able to contract further unless they can lose kinetic energy by radiation. Without such mass segregation between baryons and DM, the resulting structures will be disrupted by virialization as fluctuations that contain them go nonlinear (52). Moreover, successively larger fluctuations will collapse relatively soon after one another if they have masses in the flattest part of the $\delta M/M$ spectrum, i.e., (total) mass $\lesssim 10^9 M_\odot$.

Gas of primordial composition (about 75% atomic hydrogen and 25% helium, by mass) cannot cool significantly unless it is first heated to $\sim 10^4$K, when it begins to ionize (65). Assuming a primordial Zeldovich spectrum and normalizing (49) so that

$$\frac{\delta M}{M}(R = 8h^{-1}) = 1, \tag{17}$$

the smallest protogalaxies for which the gas is sufficiently heated by virialization to radiate rapidly and contract have $M_{tot} \sim 10^9 M_\odot$ (65). One can also deduce an <u>upper</u> bound on galaxy masses from the requirement that the cooling time be shorter than the dynamical time (64); with the same assumptions as before, this upper bound is $M_{tot} \lesssim 10^{12} M_\odot$ (65). It may be significant that this is indeed the range of masses of ordinary galaxies. The collapse of fluctuations of larger mass is expected in this picture to lead to clusters of galaxies. Only the outer parts of the member galaxy halos are stripped off; and the inner baryon cores continue to contract, presumably until star formation halts dissipation (29).[6]

D. Potential Problems with Cold DM

Dwarf galaxies with heavy DM halos are less of a problem in the cold than in the hot or warm DM pictures. There is certainly plenty of power in the cold DM fluctuation spectrum at small masses; the problem is to get sufficient baryon cooling and avoid disruption. Perhaps dwarf spheroidals are relatively rare because most suffered disruption.

The potentially serious difficulties for the cold and warm DM pictures arise on very large scales, where galaxies are observed to form filamentary superclusters with large voids between them (23,24). These features have seemed to some authors to favor the hot DM model, apparently for two main reasons: (1) it is thought that formation of caustics of supercluster size by gravitational collapse requires a fluctuation power spectrum sharply peaked at the corresponding wavelength, and (2) the relatively low peculiar velocities of galaxies in superclusters are seen as evidence for the sort of dissipation expected in the baryonic shock in the "pancake" model. Recent work by Dekel (67) suggests, however, that nondissipative collapse fits the observed features of superclusters. Results from N-body simulations with $N \sim 10^6$ (54) will soon show whether broad fluctuation spectra lead to filaments.

VII. SUMMARY AND REFLECTIONS

The hot, warm, and cold DM pictures are compared schematically in Fig. 2. Although only very tentative conclusions can be drawn on the basis of present information, it is our impression that the hot DM model is in fairly serious trouble. Maybe that is mainly because it has been the most intensively studied of the three possibilities considered here.

Probably the greatest theoretical uncertainly in all three DM pictures concerns the relative roles of heredity vs. environment. For example, are elliptical galaxies found primarily in regions of high galaxy density, and disk galaxies in lower density regions, because such galaxies form after the regions have undergone a large-scale dissipative collapse which provides the appropriate initial conditions, as in the hot DM picture? Or is it because disks form relatively late from infall of baryons in an extended DM halo, which is disrupted or stripped in regions of high galaxy density? An exciting aspect of the study of large scale structure and DM is the remarkable recent increase in the quality and quantity of relevant observational data, and the promise of much more to come.

Perhaps even more remarkable is the fact that this data may shed important light on the interactions of elementary particles on very small scales. Fig. 3 is redrawn from a sketch by Shelley Glashow which recently was reproduced in *The New York Times Magazine* (68). Glashow uses the snake eating its tail - the uroboros, an ancient symbol associated with creation myths (69) - to represent the idea that gravity may determine the structure of the universe on both the largest and smallest scales. But there is another fascinating aspect to this picture. There are left-right connections across it: medium-small-to-medium-large, very-small-to-very-large, etc. Not only does electromagnetism determine structure from atoms to mountains (70), and the strong and weak interactions control properties and compositions of stars and solar systems. The dark matter, which is gravitationally dominant on all scales larger than galaxy cores, may reflect fundamental physics on still smaller scales. And if cosmic inflation is to be believed, cosmological structure on scales even larger than the present horizon arose from interactions on the seemingly infinitesimal grand unification scale.

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FOOTNOTES

1. This discussion is approximate. Since neutrino decoupling and e^+e^- annihilation so nearly coincide, there is actually a little heating of the neutrinos too (13).
2. It is also possible that the DM is heavy stable neutrinos with mass $\gtrsim 2$ GeV, almost all of which would have annihilated (14). This is a possible form of cold DM, discussed below.
3. In economic "stagflation", the economy stagnates but the economic yardstick inflates. The behavior of δ_{DM} during the "stagspansion" era is analogous: $\delta_{DM} \approx$ constant but a expanding. We suggest here the term stagspansion rather than stagflation for this phenomenon since it occurs during the ordinary expansion (Friedmann) era rather than during a possible very early "inflationary" (de Sitter) era.
4. One might worry that such a light particle could give rise to a force that at short distances $(10^{-5}\,eV)^{-1} \sim 2\,cm$ would be much stronger than gravity. But because the axion is pseudoscalar, its nonrelativistic couplings to fermions are $\sim \vec{\sigma} \cdot \vec{p}$.
5. One calculates δ_k initially. In order to discuss mass fluctuations it is more convenient to use $\delta M/M$ than $\delta\rho/\rho$, the Fourier transform of δ_k (49). Note that there is a simple relationship between $|\delta\rho/\rho|^2$ and $|\delta_k|^2$ only for a power law fluctuation spectrum $|\delta_k|^2 \propto k^n$.
6. The model presented by Peebles at the Moriond conference differs from that sketched here mainly in Peebles' assumption that there is sufficient cooling from molecular hydrogen for baryon condensation to occur rapidly even on globular cluster mass scales.

REFERENCES

1. Faber,S.M. and Gallagher,J.S.1979 Ann.Rev.Astron. and Astrophys.17,pp 135-187.
2. Hegyi,D.J. and Olive,K.A.1982 U.Mich.preprint; Hegyi,D.J.1983 Rencontre de Moriond,Astrophysics.
3. Steigman,G.1981 in *ν81* and in Heusch,C.ed.,*Particles and Fields 1981: Testing the Standard Model* (A.I.P.Conf.Proc.,No.81) pp 548-571; Schramm,D.N. and Steigman,G.1981 Astrophys.J.,241, pp 1-7.
4. Bond,J.R.,Efstatiou,G. and Silk,J.1980 Phys.Rev.Lett.45,pp 1980-1984.
5. Doroshkevich,A.G.,Khlopov,M.Yu.,Sunyaev,R.A.,Szalay,A.S. and Zeldovich,Ya.B.1981 *Proc. Xth Texas Symposium on Relativistic Astrophysics* Ann.N.Y.Acad.Sci.375,pp 32-42; Sato,H.,ibid,pp 43-53; and ref. therein.
6. Partridge,R.B.1980 Astrophys.J.,235,pp 681-687.
7. Pagels,H.R. and Primack,J.R.1982 Phys.Rev.Lett.48,pp 223-226.
8. Carr,B.1978 Comments on Astrophys.7,pp 161-173.
9. Canizares,R.1982 Astrophys.J.,263,pp 508-517.
10. Lacey,C.1983 Rencontre de Moriond Astrophysics.
11. Gelmini,G.B.,Nussinov,S. and Roncadelli,M.1982 Nucl. Phys.B209, pp 157.
12. Weinberg,S.,*Gravitation and Cosmology*(Wiley,1972),pp 534.

13. Dicus, D.A.,Kolb,E.W.,Gleeson,A.M.,Sudarshan,E.C.G.,Teplitz, V.L.and Turner,M.S.1982 Phys.Rev.D26,pp 2694-2706.
14. Lee,B.W. and Weinberg,S.1977 Phys.Rev.Lett.39,pp 165-168.
15. Lyubimov,V.A.,et al.1980 Phys.Lett.94B,pp 266-268.
16. Boehm,F. 1983 *Proc. 4th Workshop on Grand Unification*, in press.
17. Bisnovatyi-Kogan,G.S. and Novikov,I.D.1980 Sov.Astron.24,pp 516-517.
18. Gibbons,G.,Hawking,S., and Siklos,S.,eds.1983 *The Very Early Universe* (Cambridge Univ. Press) and ref. therein.
19. Bond,J.R. and Szalay,A.S.1981 Proc. *ν81*,1,pp 59; Astrophys.J., in press.
20. Melott,A.1983 Mon.Not.R.astr.Soc.202,pp 595-604, and ref. therein.
21. Klypin,A.A. and Shandarin,S.F.1982 preprint.
22. Frenk,C.,White,S.D.M. and Davis,M.1983 Astrophys.J., in press; White,S.D.M.,Frenk,C. and Davis,M.1983 Rencontre de Moriond, Astrophysics.
23. Zeldovich,Ya.B.,Einasto,J. and Shandarin,S.F.1982 Nature,300, 407-413, and refs. therein.
24. Oort,J.1983 Ann.Rev.Astron. and Astrophys. in press and refs. therein.
25. Kaiser,N. Berkeley preprint.
25a. Bond,J.R.1983 Rencontre de Moriond,Astrophysics; Shapiro,P.R., Struck-Marcell,C. and Melott,A.L.1983 preprint.
26. Bond,J.R.,Szalay,A.S. and White,S.D.M.1983 Nature,301,pp 584-585.
27. Press,W.H. and Davis,M.1982 Astrophys.J.,259,pp 449-473.
28. Forman,W. and Jones,C.1982 Ann.Rev.Astron.Astrophys.20,pp 547-585.
29. Faber,S.M.1982 *Astrophysical Cosmology* (eds. Bruck,H.,Coyne,G. and Longair,M.),pp 191-234.
30. Gunn,J.E.1982 ibid.
31. Faber,S.M. and Lin,D.N.C.1983 Astrophys.J.(Lett.)266,pp L17-L20.
32. Aronson,M.1983 Astrophys.J.(Lett.)266,pp L11-L15.
33. Lin,D.N.C. and Faber,S.M.1983 Astrophys.J.(Lett.)266,pp L21-L25.
34. Tremaine,S.D. and Gunn,J.E.1979 Phys.Rev.Lett.42,pp 407-410.
35. Steigman,G.1979 Ann.Rev.Astron.Astrophys.17,pp 135-
36. Primack,J.R.1981 *Proc. 1981 Banff Summer Institute on Particles and Fields*, in press.
37. Bond,J.R., Szalay,A.S. and Turner,M.S.1982 Phys.Rev.Lett.48, pp 1636-1639.
38. Blumenthal,G.R.,Pagels,H. and Primack,J.R.1982 Nature,299,pp 37-38.
39. Savoy,C.A.1983 Rencontre de Moriond, Elementary Particles.
40. Ellis,J.1983 private communication.
41. Goldberg,H.1983 Phys.Rev.Lett.50,pp 1419-1422.
42. Olive,K.A. and Turner,M.S.1982 Phys.Rev.D25,pp 213-216.

43. Harrison,E.R.1970 Phys.Rev.D1,pp 2726-2730; Peebles,P.J.E. and Yu,J.T.1970 Astrophys.J.,162,pp 815-836; Zeldovich,Ya.B. 1972 Mon.Not.R.astr.Soc.160,pp 1P-3P.
44. Lubin,P.M.,Epstein,G.L. and Smoot,G.F.1983 Phys.Rev.Lett.50, pp.616-619; Fixin,D.J.,Cheng,E.S. and Wilkinson,D.T.1983 Phys.Rev.Lett.50,pp 620-622.
45. Olive,K.A.1983 Rencontre de Moriond, Astrophysics.
46. Guyot,M. and Zeldovich,Ya.B.1970 Astron.Astrophys.9,pp 227-231; Meszaros,P.1974 Astron.Astrophys.37,pp 225-228.
47. Doroshkevich,A.G.,Zeldovich,Ya.B.,Sunyaev,R.A. and Khlopov, M.Yu.1980 Sov.Astron.Lett.6,pp 252; Chernin,A.D.1981 Sov. Astron.25,pp 14-16.
48. Peebles,P.J.E.1980 *The Large Scale Structure of the Universe*, Princeton Univ. Press.
49. Peebles,P.J.E.1982 Astrophys.J.(Lett.)263,pp L1-L5; Astrophys. J.,258,pp 415-424.
50. Dekel,A.1982 Astrophys.J.(Lett.)261,pp. L13-L17.
51. Sargent,W.L.W.,Young,P. and Schneider,D.P.1981 Astrophys.J., 256,pp 374-385.
52. White,S.D.M. and Rees,M.1977 Mon.Not.R.astr.Soc.
53. Silk,J.1983 Nature,301,pp 574-578.
54. Davis,M.,Efstatiou,G.,Frenk,C. and White,S.D.M.1983 private communication.
55. Schechter,P.1983 private communication.
56. Peccei,R. and Quinn,H.1977 Phys.Rev.Lett.38,pp 140.
57. Weinberg,S.1978 Phys.Rev.Lett.40,pp 223; Wilczek,F.1978 Phys. Rev.Lett.40,pp 279.
58. Abbott,L. and Sikivie,P.1983 Phys.Lett.120B,pp 133; Dine,M. and Fischler,W.1983 Phys.Lett.120B,pp 137; Preskill,J., Wise,M. and Wilszek,F.1983 Phys.Lett.120B,pp. 127; Ipser,J. and Sikivie,P.1983 Phys.Rev.Lett.50,pp 925.
59. Dicus,D.,Kolb,E.,Teplitz,V. and Wagoner,R.1978 Phys.Rev.D18, pp 1829; Fukugita,M.,Watamura,S. and Yoshimura,M.1982 Phys. Rev.Lett.48,pp.1522.
60. Wilczek,F.1982 preprint NSF-ITP-82-100.
61. Blumenthal,G.R. and Primack,J.R., in preparation.
62. Press,W.H. and Vishniac,E.T.1980 Astrophys.J.,239,pp 1-11.
63. Gott,J.R.1977 Ann.Rev.Astron.Astrophys.15,pp 235-266.
64. Rees,M.J. and Ostriker,J.P.1977 Mon.Not.R.astr.Soc.179,pp 541-559.
65. Blumenthal,G.R.,Faber,S.M.,Primack,J.R. and Rees,M., in preparation.
66. Peebles,P.J.E.1983 Rencontre de Moriond, Astrophysics.
67. Dekel,A.1983 Astrophys.J.,264,pp 373-391.
68. Ferris,T.1982 *The New York Times Magazine*, Sept.26,1982,pp 38.
69. Neumann,E.1954 *Origins and History of Consciousness*, Princeton Univ. Press.
70. Weisskopf,V.F.1962 *Knowledge and Wonder*, Hinemann.

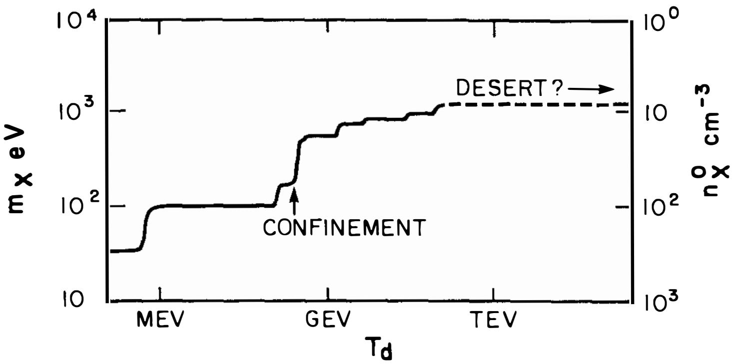

Figure 1. The mass m_X and present number density n_X^0 of warm dark matter particles X, calculated assuming the standard particle physics model and no entropy generation. The mass scale should be multiplied by the factor $h^2\Omega$.

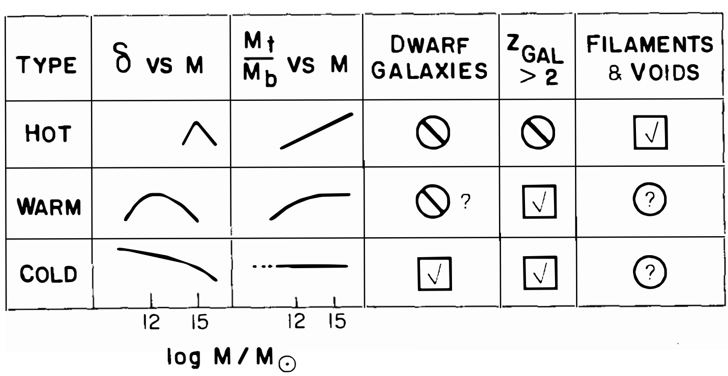

Figure 2. Consumers' Guide to dark matter. The circle with a bar means trouble and the box with a check means consistency.

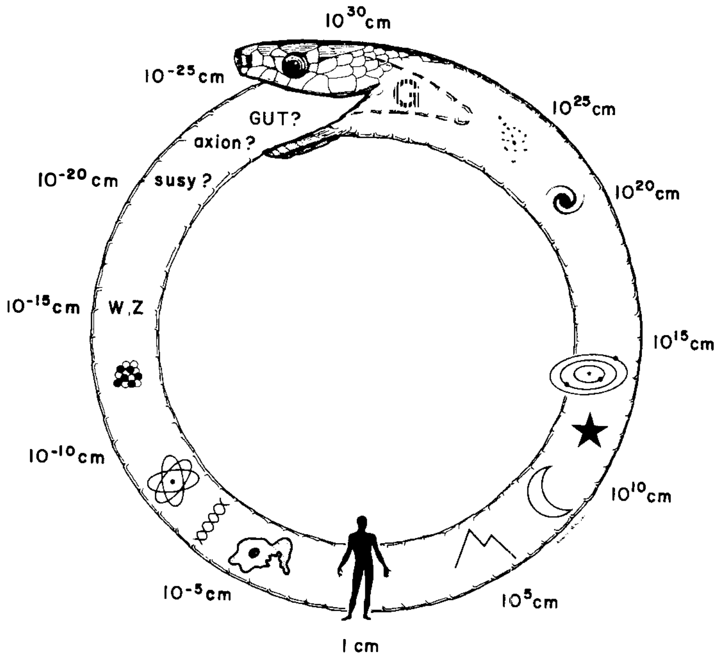

Figure 3. Physics Uroboros (after Glashow (68)).