

COMPOSITENESS OF QUARKS AND LEPTONS

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Introduction

The theoretical talks we have heard so far at this symposium have been full of precision and technical detail in explaining the consequences of the gauge theories of the strong, weak, and electromagnetic interactions. The level of confusion and far-reaching speculation has been remarkably less than at previous conferences in this series, in great part because our current theories seem to be both fundamental and in good accord with experimental data. One might wish to believe that the study of fundamental physics is almost complete. Is it true, at least, that the quarks, leptons, and gauge bosons that we now observe are among the most basic building-blocks of Nature? Despite the temptation to agree, a stream of theoretical papers – a stream which has, in the past year, become a torrent – has argued that they are not. I have been asked to review this set of theoretical ideas. My review will necessarily contain more than its proper share of speculation and guesswork. But I hope to make clear both the current status and the consequences of the idea that the quarks and leptons are composite, that another level of fundamental physics lies waiting to be discovered.

There is, at the moment, no experimental evidence which would require the compositeness of quarks and leptons. Indeed, the simplicity of their gauge interactions makes quarks and leptons look convincingly elementary. What, then, encourages physicists to take the notion of their compositeness seriously?

Three reasons come to mind. The first is the observed proliferation of quarks and leptons. We still do not understand the reason for the muon. Now that we have identified the tau lepton, and a variety of heavy quarks, the question is even more puzzling. The second problem, theoretically even more pressing, is the proliferation of parameters. It remains a major unsolved problem to compute the masses of the quarks and leptons from some deeper theory. In the standard models of weak interactions, such a computation is impossible as a matter of principle: The quark and lepton masses are proportional to the coupling constants which describe the coupling fermions to Higgs scalar fields. These coupling constants are renormalized parameters, like α in QED, which must be specified from outside to define the theory consistently.¹ One cannot ask why the τ or b is heavy. The Higgs field is often viewed as the least appealing aspect of our current models; numerous authors, beginning with Wilson,² have denounced fundamental Higgs fields as objects unsatisfactory to be components of a fundamental theory. But if we wish to remove fundamental Higgs fields from our theories, we need some dynamical construct to take their place in producing gauge boson and fermion masses. The third reason might be termed the proliferation of speculation. What self-respecting theorist could insist that nothing novel happens between here and 10^{15} GeV, without inventing something to fill the gap?

This talk will be, essentially, a survey of these inventions; I will review along grand lines the theoretical ideas associated with the notion that quarks and leptons are composite. I will first discuss various constituent pictures which have been proposed to account for the quantum numbers of the observed quarks and leptons, a study I will call the Quantum Numerology. I will then discuss some new theoretical developments of the past two years which bear on the subject of composite fermions and which make plausible (or rule out) some of the

major dynamical assumptions of these constituent models. Finally, I will discuss the consequences of the compositeness of quarks and leptons by setting up a series of scenarios for this compositeness and exploring, for each scenario, its experimental implications.

To complement this list of topics, let me offer a list of disclaimers. First, I will, throughout this talk, assume the validity of the "standard" gauge theory of color $SU(3) \times SU(2) \times U(1)$. This assumption is not necessary, and does not imply that I believe the case for this theory is settled; it does allow us a fixed base for our developments. Secondly, I will often assume that the interactions which bind the constituents of quarks and leptons are also gauge interactions. This assumption will exclude some models which depend on very unusual dynamics. But I believe it is important to know what restrictions on the dynamics of composite fermions may be deduced in the one class of field theories that we do understand well. Third, I have, at several points in this review, oversimplified the arguments of papers under discussion in an attempt to avoid digressions from the current of my analysis; I apologize to the offended authors and to their admirers. Fourth, I will not actually present a solution to the problem of determining the masses of quarks and leptons; this promise of composite theories has not yet been realized in any sensible model. Finally, I will not attempt to improve the existing terminology for fermionic constituents. Preons, quincks, haplons, gleeks, and other such demons run rampant through the literature; it is only good taste not to arbitrate their disputation.

The Quantum Numerology

The compositeness of atoms, of nuclei, and of hadrons was, historically, realized when it became apparent that there were many varieties of each, characterized by different discrete values of quantum numbers. The quantum numbers of the constituents were deduced as those of a minimal set of objects which could generate the appropriate pattern of varieties. In searching for a composite description of quarks and leptons, it is natural to begin also by attempting to understand and decompose the pattern of their quantum numbers. Our phenomenological resources are, however, limited: the pattern that we observe is the simplistic repetition of generations, sets of quarks and leptons with the quantum numbers of (u, d, e, ν_e) . This pattern does not single out a unique assignment of composites; in fact, models of very different structure can be found in the literature. I will review here the three simplest schemes which have been proposed. (Some more complex proposals, which I will not have time to discuss, are listed as ref. 3).

The most straightforward scheme for constituent quantum numbers imagines that separate constituents within the quarks and leptons carry color, flavor, and, perhaps also, generation number. A quark or lepton might then be a bound state of a fermion and a boson or of three fermions (Fig. 1). The earliest

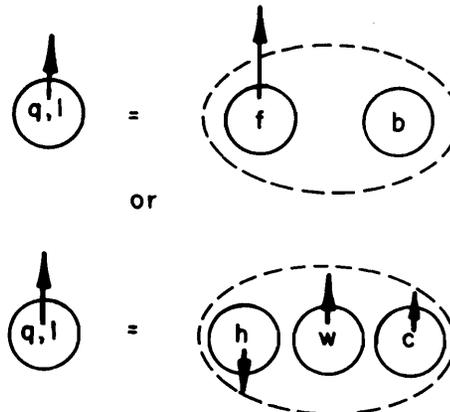


Fig. 1: Elementary schemes of quark and lepton composition

composite models of quarks and leptons, proposed by Pati, Salam, and Strathdee⁴ and by Matumoto⁵, were of the first type. Indeed, Pati, Salam, and Strathdee created their model as a direct outgrowth of their proposal that lepton-ness should be considered as a fourth color: They assigned to one set of constituents (the "valency quartet" or "flavons") the flavor quantum numbers of (u,d,s,c) and to the other (the "color quartet" or "chromons") one of the four colors. More recently, models in which quarks and leptons are fermion-boson composites have been proposed by Greenberg and Sucher⁶ Ne'eman,⁷ Fritzsche and Mandelbaum,⁸ and Višnjić-Triantafillou,⁹ Veltman¹⁰ and Derman¹¹ have made the interesting suggestion that the quarks and leptons are bound states of a (four-colored) fermion with a Higgs boson; the generations are assumed to arise as radial excitations of this bound-state system. The first attempt to study the detailed dynamics of a composite model of quarks and leptons was made by Akama, Chikashige, and Terazawa¹² in a model containing three-fermion bound states; they called the three constituents the "hakem", "wakem", and "chrom", and assigned them respectively the quantum numbers of generation number (e,μ,τ), isospin (u,d), and color or leptonicity. The gauge bosons of color and SU(2) x U(1) arise as fermion-antifermion bound states of these constituents. Models of this type have also been presented by Taylor¹³ and by Sharatchandra;¹⁴ a related model based on a grand unification of the constituents in SU(5) has been offered by Ansel'm.¹⁵

A second scheme for constituent quantum numbers is slightly more intricate, and relies more closely on the eventual grand unification of the various gauge interactions of the constituents. This scheme is, however, quite elegant. One may observe that the state comprising the spinor representation of O(2N), and their charge conjugates, can be constructed as the states of an assembly of N two-level systems. One can, for example, build the fermions of the O(10) grand unified theory - which fill a 16-dimension representation of O(10) and its conjugate - as the 32 composites of five objects with two-valued quantum numbers. (Fig. 2). Mansouri,¹⁶ Yasuè,¹⁷ Casalbuoni and Gatto,¹⁸ and Macrae¹⁹ have taken this notion seriously as a model of quarks and leptons built of five constituents. All of these authors regard the underlying symmetry group as being much smaller than O(10); they consider only a discrete or, at most, Abelian, subgroup of O(10) to be fundamental. The gauge symmetries of color SU(3) and weak SU(2) - and their associated gauge bosons - are postulated to appear dynamically.

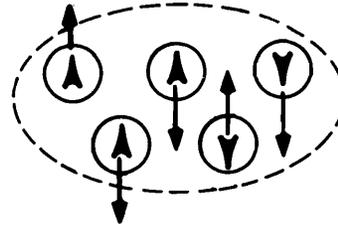


Fig. 2: Possible composite structure for fermions of the O(10) grand unified theory.

A third scheme for the quantum numbers of constituents seeks economy in just the opposite way, renouncing a simple embedding in a grand unified theory. Harari²⁰ and Shupe²¹ have independently proposed a model in which quarks and leptons are built of triples of two objects T,V, with electric charges 1/3 and 0, respectively, according to the pattern:

$$\begin{aligned} \nu &= VVV & u &= VTT \\ \bar{d} &= VVT & e^+ &= TTT \end{aligned} \quad (1)$$

The correct color assignments are reproduced if the T is a 3 and the V is a $\bar{3}$ of color, and the 3 fermions are placed in a totally antisymmetric state. Shupe called these objects "quips"; Harari proposed the name "rishons", and I will refer to them in this way also. More recently, Squires²² and Raito²³ have proposed variants of this scheme and Nelson²⁴ has offered an interesting extension of the model.

The rishon model, in the version I have just given, contains color as a fundamental symmetry. The weak interaction $SU(2)$, however, has no relation to the symmetries of rishons; this symmetry, and the associated gauge bosons W^\pm and Z^0 , must somehow arise from the dynamics. In terms of rishons, the ordinary process of β -decay ($d \rightarrow ue^{-}\bar{\nu}$) becomes the complex reaction shown in Figure 3.

The rishon model occupies a special position in the study of composite models of quarks and leptons, because Harari, in collaboration with Seiberg, has clothed this schema in a complete dynamical theory and has explored this theory in great detail, uncovering unexpected phenomenological features and difficulties.^{25,26} The status of this model has recently been described by Harari and Seiberg in a provocative paper²⁷ which I recommend to all serious students of composite models. They should be warned, however, that many of the conclusions which Harari and Seiberg reach are, if not demonstrably incorrect, at least extremely implausible. I will discuss some of these points as I proceed; a more specific and complete critique of this work may be found in a paper of Okun.²⁸

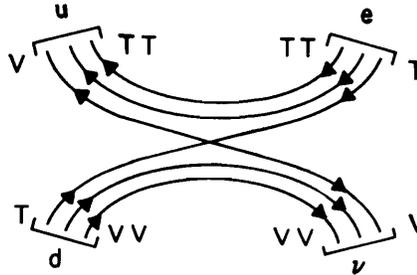


Fig. 3: Beta-decay in the rishon model

Evidence Against Compositeness

We have seen, then, that there are several plausible schemes for assigning quantum numbers to constituents in order to reproduce the quantum numbers of quarks and leptons. Our arguments about these schemes imagined that the quarks and leptons were simple bound states of these constituents, in the same way that the structure of nuclei or of hadrons is given correctly by the simplest picture of the constituent binding. Unfortunately, there are specific things which we know about the physics of quarks and leptons which strongly oppose this intuitive picture. Let me now review these more uncomfortable matters.

While composite systems at rest may look relatively simple, we expect that a serious jolt should bring out such obvious signs of its compositeness as the appearance of form factors or even dissociation into constituents. How serious a jolt is necessary? For nuclei, a projectile with a few MeV of energy can already cause a major disruption. For hadrons, the energy required is of the order 1 GeV, that is, of the order of the mass of the composite state. Quarks and leptons, however, show no form factor effects and certainly no dissociation up to the highest energies we have so far explored — energies four to five orders of magnitude larger than the masses of e , u , or d . The new bounds from PETRA on deviations from the electroweak predictions for $e^+e^- \rightarrow q\bar{q}$ or $l\bar{l}$, presented in Branson's talk at the symposium,²⁹ are, indeed, striking; as an example, I have reproduced in Table 1 the limits on the QED cutoff parameters Λ_+ and Λ_- reported by the MARK-J group.

A second place that a form factor due to the compositeness of leptons could have appeared is in the value of $(g-2)$. The Landé g factors for the nucleons are nowhere near 2, a phenomenon well explained by the quark model of nucleon structure. One should, then, expect composite structure inside the electron and muon to affect their g values, perhaps even by a large factor.³⁰ But no

TABLE 1

Limits from the MARK-J experiment on deviations from electroweak predictions for $e^+e^- \rightarrow f\bar{f}$, expressed as limits on the parameters Λ_+ , Λ_- ²⁹

$e^+e^- \rightarrow$	95% conf. Λ_+	Lower limits on Λ_-
for deviation from SU(2) x U(1):		
e^+e^-	128	161
$q\bar{q}$	190	285
for deviation from QED:		
$\mu^+\mu^-$	194	153
$\tau^+\tau^-$	126	116

such effect has been observed: If one defines

$$\delta a = \frac{(g-2) - (\text{QED value})}{2}$$

then δa is known to be extremely close to zero.³¹

$$\begin{aligned} \text{For } e: & \quad |\delta a| < 3.2 \times 10^{-10} \\ \text{For } \mu: & \quad |\delta a| < 1.5 \times 10^{-8} \end{aligned} \quad (2)$$

Composite models which are constructed economically, to explain the multiplicity of generations and to unify quarks and leptons, often contain another uncomfortable feature: Exotic decay processes may proceed readily at the level of quark and lepton constituents by simple constituent exchange. Two processes whose appearance is especially easy, and especially dangerous, are $\mu \rightarrow e\gamma$ and proton decay. The rishon scheme, in the simple form in which I have just presented it, allows proton decay via the baryon-number violating process shown in Figure 4.^{28,32} This process is, indeed, simpler than the one shown in Figure 3; it is possible that proton decay is easier than β -decay in this model.

Bounds on such rare decays and on the muon and electron $(g-2)$ certainly place some stringent limits on possible composite structure of quarks and leptons. It is important to determine more precisely what these limits are, what bounds these processes give on the physical size of the quark or lepton. In order to make this determination, I would like to introduce a small amount of formalism. This formalism will prove useful throughout the rest of the investigation, providing a language convenient for our analysis.

Let me, then, introduce the notion of the effective action which represents the low-energy effects of higher-energy dynamics. We would like to describe the

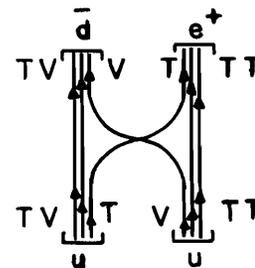
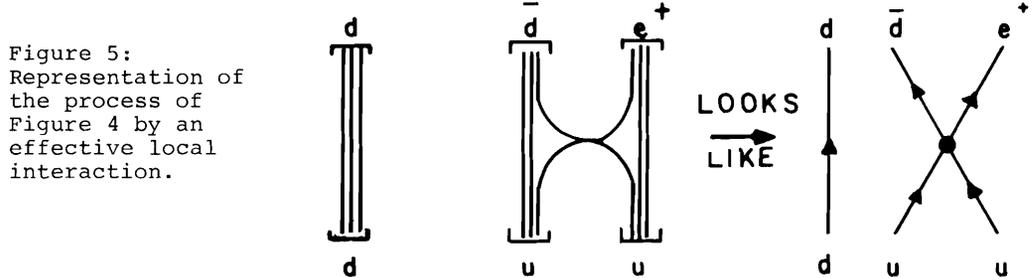


Fig. 4: A baryon-number violating process ($uu \rightarrow \bar{d}e^+$) in the rishon model

influence on accessible processes of the dynamics which bind the constituents of quarks and leptons; this dynamics necessarily operates on very large mass scales, that is, on very small distance scales. To a proton being bodily converted by a reaction of constituent interchange between quarks, the perturbation of its structure looks like a contact interaction between the quarks, occurring at a point (Figure 5). We may represent this perturbation by writing an interaction



term in the form of a local operator.

$$\delta\mathcal{L} = G \cdot (\bar{e} \cdot u \cdot \bar{d} \cdot u), \quad (3)$$

where e , u , and d are the field operators associated with the electron, up quark, and down quark, respectively, with Dirac indices contracted in a Lorentz-scalar combination, and G is a fixed coefficient, which must have the dimensions of $(\text{mass})^{-2}$. $\delta\mathcal{L}$ is an additional term to be added to the Lagrangian describing the motion of quarks and leptons. Let us refer to this term, or, more generally, to the sum of such local operators which represent the effects of constituent interactions, as the effective action produced by these interactions. The theory of constituent binding should tell us what operators appear in the effective action and what coefficients they have.

If we do not have, or do not wish to commit ourselves to, a particular theory of constituent structure, we cannot compute the effective action explicitly, but we can still use it to organize our thinking about constituent interactions, if we supplement the idea of a local operator description with two simple assumptions: first, that any operator may appear in the effective action if it is invariant to Lorentz transformations and to all other unbroken symmetries of the composite model; second, that the coefficient of each operator may be estimated by dimensional analysis. If $\delta\mathcal{L}$ is to have the dimensions of a Lagrangian, the coefficient of any operator of dimension d has the dimensions of

$$(\text{mass})^{-(d-4)} .$$

I will estimate this coefficient by using the mass scale M which corresponds to the physical size of a quark or lepton or the strength of the forces which bind their constituents. Clearly M is well-defined only in its order of magnitude. I will refer to M henceforth as the "compositeness scale". If any further new interactions arise between 1 GeV and the scale M , these interactions do not alter the power of M which appears in the effective action coefficients as long as the quarks and leptons have no new strong interactions; even these are permissible if they are asymptotically free gauge interactions. This estimation procedure sets G in (3) equal simply to $1/M^2$.

We can now rephrase our worry about dangerously allowed rare decays as the possibility that unusual operators of low dimension will appear in the effective action with coefficients which are unacceptably large. Some dangerous operators which may appear, listed with their estimated coefficients, are the following:

$$\begin{aligned} \frac{e}{M} \bar{e} \sigma^{\mu\nu} F_{\mu\nu} & \quad (\text{mediates } \mu \rightarrow e\gamma) \\ \frac{e}{M} \bar{e} \sigma^{\mu\nu} F_{\mu\nu} e & \quad (\text{produces an electron } (g-2)) . \end{aligned} \tag{4}$$

Before making use of these terms, however, we should recall one more operator on our list:

$$M \bar{e} e \quad (\text{gives the electron a mass of order } M!) . \tag{5}$$

This operator is allowed by all obvious symmetries. We must confront its appearance and find a way to eliminate it. But this simply returns us to the basic problem which I had indicated at the beginning of this section: How could it be reasonable that the quarks and leptons have masses far smaller than the mass scale corresponding to their physical size or to the binding energy of their composite structure?

Massless Composite Fermions

It is remarkable that this question has an answer; it is not hard to understand why that answer is one of the most intriguing theoretical developments of the past two years. To understand why composite fermions may be very light with respect to their compositeness scale, it is useful to carry this phenomenon to its extreme, to imagine circumstances in which zero mass composites arise from dynamics at the fixed scale M . In this extreme case, one can make powerful use of symmetries and their constraints. Indeed, 't Hooft³³ and Dimopoulos, Raby, and Susskind³⁴ have shown a mechanism by which a symmetry can forbid any mass acquisition by composite fermions! I would now like to devote some considerable space to a discussion of their idea and its ramifications. This discussion will necessarily be rather abstract and theoretical. Readers who wish to avoid such a discussion should skip ahead several sections; let me warn them, however, that they will miss the most interesting aspects of this subject.

To explain their idea, let me recall one familiar feature of quantum electrodynamics and, indeed, of any gauge interaction: The coupling of gauge field quanta to massless fermions conserves helicity. If, for example, we separate a fermion field ψ into the components ψ_L, ψ_R which create and destroy left- and right-handed massless fermions, the Hamiltonian governing the electrodynamics of this field takes the form:

$$H = \int d^3x \{ \psi_L^\dagger \vec{\sigma} \cdot (-i\vec{\nabla} - e\vec{A}) \psi_L + \psi_R^\dagger \vec{\sigma} \cdot (i\vec{\nabla} + e\vec{A}) \psi_R + m(\psi_L^\dagger \psi_R + \psi_R^\dagger \psi_L) \} \tag{6}$$

The kinetic energy terms involve the left- and right-handed fields separately. The mass term is essentially a helicity flip operator. One can understand this decomposition in the following way: A massive fermion must have both helicities, since a left-handed massive particle can be converted by a boost into a right-handed one, as shown in Figure 6. However, a massless particle moves at the



Fig 6: A massive particle cannot have definite helicity

speed of light; a boost cannot catch up to it. Thus, a massless fermion need have only one helicity. It is, still, a surprising property of gauge field couplings that this picture, based on the dynamics of free particles, survives the inclusion of interactions.

The mass term in equation (6) violates a conservation law which is respected by the remainder of that Hamiltonian: the separate conservation of the numbers of left- and right-handed fermions. We could forbid the appearance of this mass term by insisting on this conservation law, called chirality conservation, or on a (chiral) symmetry which implies it.

An argument for zero-mass fermions which makes use of such a conservation law or symmetry principle can clearly be generalized from this single system to other systems with fermions, even to composite fermions. Indeed, we can force a set of composite fermions to have zero mass by making the following assumptions about the interactions which form their structure:

- (1) These interactions should respect a chiral symmetry which forbids the mixing of left- and right-handed composite states.
- (2) These interactions, though they are strong, do not cause this chiral symmetry to be broken spontaneously.

The first assumption is, in my opinion, very mild; it is satisfied if the interactions which bind constituents are any gauge interactions. The second assumption, however, is unexpectedly strong. To understand its status, let us review briefly the chiral symmetries of one very familiar gauge theory, QCD with 3 quark flavors.

Let us imagine altering the usual strong interactions slightly to make the u, d, and s quarks massless. Assuming that the theory of the strong interactions is QCD, a gauge theory, the left- and right-handed components q_L, q_R of the quark fields are not connected, and the theory thus has a separate SU(3) flavor symmetry for each helicity. These two symmetry groups form the famous SU(3) x SU(3) chiral symmetry which, treated as an approximate symmetry of the strong interactions, leads to the results of current algebra. In the strong interactions, however, only the overall flavor SU(3) symmetry is manifest and useful in classifying particles; the additional symmetry, which would involve performing an SU(3) rotation on q_L and the opposite rotation on q_R , is apparently spontaneously broken. The pattern of its breaking is explained by the following simple mechanism: The strong attraction between massless quarks and antiquarks causes these particles to bind into pairs. The light pairs can condense, as electron pairs do in a superconductor, so that the vacuum of space contains an indefinite number of them. A pair with zero momentum and angular momentum has net helicity (Figure 7), and therefore net chiral charge.

Figure 7: A quark-antiquark pair with vacuum quantum numbers.



The vacuum has indefinite chiral charge and, so, it can absorb chiral charge. One then would expect

$$\langle 0 | \bar{\psi}\psi | 0 \rangle = \langle 0 | \psi_L^\dagger \psi_R + \psi_R^\dagger \psi_L | 0 \rangle \neq 0 \quad (7)$$

This mechanism breaks SU(3)_L x SU(3)_R spontaneously to SU(3), breaking 8

continuous symmetries and generating, thereby, 8 massless Goldstone bosons. These two phenomena, and the identification of these 8 bosons with the light mesons π, K, η are, in fact, the basic assumptions of current algebra.³⁵

This whole picture hinges, of course, on the assumption of quark-antiquark pair binding and condensation. Is this assumption necessary? Simple dynamical arguments do confirm this picture for QCD,³⁶ but none of these calculations are really definitive. It is certainly possible that in some variant of QCD, perhaps in a theory with more massless fermion flavors, the pair condensation would not occur, and the whole chiral symmetry group would be realized as a classification symmetry. If we had n massless flavors, there would be an exact $SU(n) \times SU(n)$ flavor symmetry. In Figure 8, I have shown some typical composite

Fig. 8: An example of states which may be kept massless by chirality conservation. The small arrows represent the spin orientations.



fermion states one might form. Since the left-handed fermions have the quantum numbers of only the first $SU(n)$ and the right-handed fermions have only the quantum numbers of the second, the left- and right-handed composite fermions in Figure 8 will generally have different quantum numbers under this $SU(n) \times SU(n)$. The constituent binding interactions therefore may not mix these states; the new interactions at the compositeness scale M are forbidden by symmetry from producing for them a mass term $\psi_L^\dagger \psi_R$.

The notion of manifest chirality conservation will be a central ingredient in our explication of quark and lepton composite structure. By adopting this notion, we automatically solve the problem of the previous section, of obtaining quarks and leptons light compared to the mass scale of their internal structure. But we pay a price for this solution: Chirality conservation places unusual constraints on the structure of the composite states, making their dynamics different in kind from that of ordinary nonrelativistic bound states. I will discuss some of those constraints in detail in a moment. First, however, I would like to use this idea to finish the argument of the previous section, to estimate the bounds on the compositeness scale associated with bounds on $(g-2)$ and on the rate for $\mu \rightarrow e\gamma$. This will already give a hint of the novel character of these bound states.

In order to use the principle of chirality conservation to make realistic estimates, I must bend this principle to make allowance for the fact that the quarks and leptons do have nonzero masses, masses much smaller, however, than M . It is natural to assume that these finite mass terms arise from a small symmetry-breaking perturbation of the chirality-conserving constituent-binding dynamics. If this perturbation allows the appearance of a mass term (5) in the effective action, it must be a very weak effect; its strength must be given by the dimensionless parameter (m/M) , where m is the lepton or quark mass, to suppress (5) to the required level. This perturbation will allow other chirality-nonconserving operators to appear in the effective action, but these must also have coefficients proportional to this small dimensionless parameter. But note that both of the dangerous operators (4) are operators which flip helicity and therefore change chiral quantum numbers. If they appear in the effective action, they must carry in their coefficients this same suppression factor.³⁷ The

operator contributing to the muon ($g-2$) then appears in the form:

$$\delta\mathcal{L} \sim e \frac{m_\mu}{M^2} (\bar{\mu}_L \sigma^{\mu\nu} \mu_R + \text{h.c.})$$

which gives

$$\delta a_\mu \sim \left(\frac{m_\mu}{M}\right)^2 \quad (8)$$

and places a bound $M > 900 \text{ GeV}$.³⁸ The bound derived from the electron ($g-2$) is weaker still. Simple nonrelativistic bound state models give $\delta a \sim (m/M)$,^{9,39} a result qualitatively different and, I would think, irrelevant to our study. The operator contributing to $\mu \rightarrow e\gamma$ should appear as

$$\delta\mathcal{L} \sim e \frac{m_\mu}{M^2} [\bar{\mu}_L \sigma^{\mu\nu} F_{\mu\nu} e_R + \text{h.c.}] ;$$

this yields

$$\Gamma(\mu \rightarrow e\gamma) \sim m_\mu^5 / M^4 \quad . \quad (9)$$

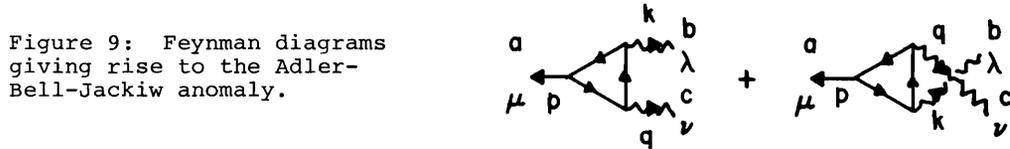
Then the current bound $\text{BR}(\mu \rightarrow e\gamma) < 2 \times 10^{-10}$, gives a limit $M > 200 \text{ TeV}$.³⁷ This limit on M is more stringent, but it assumes that the muon and the electron are built of the same constituents.

Before concluding this discussion, I should note that manifest chirality conservation is not the only mechanism which could insure the masslessness of composite fermions. In an intriguing paper, Bardeen and Višnjić⁴⁰ have argued that one may also obtain massless composite fermions as the result of spontaneously broken supersymmetry. Supersymmetry transformations convert bosons to fermions, and the charges which generate supersymmetry transformations have fermionic quantum numbers; thus, spontaneous breaking of supersymmetry gives rise to a massless Goldstone fermion for each broken symmetry direction. Bardeen and Višnjić wish to interpret all the quarks and leptons as Goldstone fermions of an almost-exact supersymmetry. This interpretation, unfortunately, has serious difficulties: It requires a supersymmetry with N distinct generators, where N is the number of handed quarks and leptons. Counting colors separately, and not including the top quark, $N > 39$. But supersymmetry theories with N generators necessarily contain particles of spin $N/4$;⁴¹ one must, then, confront the difficulties of quantizing fields of extremely high spin. Perhaps additional, more attractive possibilities for insuring the masslessness of composite fermions remain to be discovered. For the remainder of my review, however, I will assume that it is manifest chirality conservation which determines the structure of quarks and leptons and concern myself with analyzing the consequences of this assumption.

The Axial-Vector Anomaly and its Restrictions

In the previous section, I emphasized that the gauge interactions of massless fermions automatically respect chiral symmetry. In this section, I would like to recant at least a part of that conclusion. It seems obvious from a glimpse of the Hamiltonian (6) that this system conserves chirality, but in quantum field theory, any conclusion apparently obvious from the equations of motion may be spoiled by effects of renormalization. The separate conservation of the chiral symmetry currents $J_R^{\mu a}$, $J_L^{\mu b}$ is in fact spoiled in this way. One would be tempted to consider this an affliction, were it not for the fact that the terms which spoil chirality conservation are of a very special form which, remarkably, can be put to good use in our analysis. In this section, I will review the nature of this effect, known as the Adler-Bell-Jackiw axial vector anomaly,⁴² and explain an amazing observation of 't Hooft by which this effect sheds light on systems with massless composite fermions.

The origin of this anomaly may be seen by considering the process in which a current $J^{\mu a}$ creates a virtual fermion-antifermion pair, which then converts to two gauge bosons. The simplest diagrams for this process are shown in Figure 9.



In this figure, p, k, q represent momenta, μ, λ, ν , the Lorentz indices of the current and the gauge bosons, and a, b, c , the color or flavor indices of those objects. The amplitude shown in Figure 9 must have overall Bose symmetry with respect to the interchange of (k, λ, b) and (q, ν, c) . What does this amplitude look like in low momentum limit ($k, p, q \rightarrow 0$)? There are two possible Lorentz structures proportional to one power of momentum:

- (1) $[g^{\mu\nu}(p-q)^\lambda + g^{\nu\lambda}(q-k)^\mu + g^{\lambda\mu}(k-p)^\nu] \cdot (\text{antisymmetric in } a, b, c)$
- (2) $\epsilon_{\mu\nu\lambda\sigma}(k-q)^\sigma \cdot (\text{symmetric in } a, b, c)$.

The first of these is the form of the usual 3-gluon vertex of a gauge theory; it is consistent with $\partial_\mu J^{\mu a} = 0$. The second form is not consistent with $\partial_\mu J^{\mu a} = 0$. Since it contains the ϵ tensor, it can arise only if Figure 9 contains currents of indefinite parity, but we are interested precisely in the case where $J^{\mu a}$ is a handed current. In this case, the contribution to the loop integral from very large momenta always produces a term of this second form; one cannot remove this term by renormalization without spoiling the conservation of one of the currents to which the gauge bosons in Figure 9 couple. This term is the Adler-Bell-Jackiw anomaly. The term appears, quite generally, in the vertex function of three currents (of which at least one is handed), spoiling the conservation of one of the three.

It is a classic result of current algebra that the anomaly allows one to predict the rate for the decay processes $\pi^0 \rightarrow 2\gamma$.⁴² It will be useful to recall, if not the precise arguments which connect these phenomena, at least the underlying logic of this prediction. The anomaly has two very different aspects to its behavior: First, it arises as a renormalization effect; it comes from the short distance structure of the theory. In QCD, or in any asymptotically free gauge theory, this means that we can compute the coefficient of the anomaly term using perturbation theory. Remarkably, Figure 9 is the only contribution in any order of perturbation theory. Secondly, the anomaly predicts the low-energy behavior of a current matrix element. It therefore dictates the couplings of massless particles which can be created by the current. If this current is a chiral symmetry current, and chiral symmetry is spontaneously broken, as in QCD, these massless states are the Goldstone bosons π, K, η . Combining these observations, we see that certain appropriate strong-interaction couplings at low energy, of which the coupling of the π^0 - γ - γ coupling is an example, are predicted by QCD perturbation theory through the assistance of the anomaly.

This is the situation when the chiral symmetry is spontaneously broken. What happens if we assume that the chiral symmetry remains exact? In that case, the chiral currents no longer create massless bosons, but these currents may create pairs of massless composite fermions. The couplings of these composite fermions are then constrained by the anomaly. But, if each of the three currents involved in the vertex with the anomaly is the current of a classification symmetry, the amplitude for each current to create a pair of fermions is dictated by the quantum numbers of the fermions with respect to that symmetry.

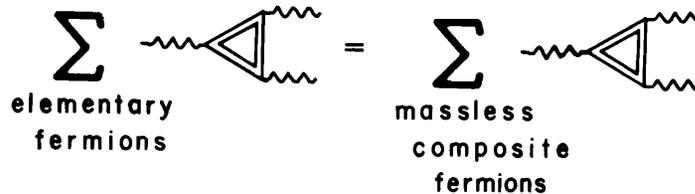
The only parameter which one has left to adjust is the number of massless states with given symmetry quantum numbers. This is the striking observation of 't Hooft, that the value of the coefficient of the anomaly in a general strongly-interacting gauge theory, a quantity computable simply from the triangle diagram of Figure 9, places a restriction on the spectrum of massless composite fermionic bound states. Let us denote the anomalous part of the 3-current vertex function as indicated in Figure 10. Now imagine that we have

Figure 10: A notation for the anomaly contribution from Figure 9.



a strong interaction theory whose unbroken chiral symmetries protect the masslessness of a set of composite fermions. Choose any three currents of the flavor group $SU(n) \times SU(n)$, or, more generally, of the group of chiral symmetries which are not spontaneously broken. Then these currents give rise to a restriction, the 't Hooft Anomaly Condition, shown in Figure 11: the short-distance computation of the anomaly of these three currents, which should be done

Figure 11: The 't Hooft Anomaly Condition.



using the elementary fermions, should agree with the computation of the anomaly from the low-energy structure of the theory, which can be done using the massless composite states with the couplings dictated by their quantum numbers.^{33, 43}

For a given symmetry group, but for fermions of arbitrary quantum numbers in loop, Figure 10 may be evaluated in the form

$$(Fig. 10) \frac{1}{8\pi^2} \epsilon_{\mu\nu\lambda\sigma} (k-q)^\sigma d^{abc} A \quad (10)$$

where d^{abc} is a group-theoretic invariant which depends only on the symmetry group. The overall coefficient A , the "anomaly coefficient", contains all the dependence on the quantum number of the fermions.⁴⁴ The computation of this coefficient is discussed in ref. 45. According to 't Hooft, a strong interaction theory can produce a set of massless composite fermions only if the sum of the anomaly coefficients of these composites matches that of the original fermions. This is a bizarre constraint, since it is cubic in symmetry charges, but it is elementary, exact, and, as we will see, exceedingly restrictive.

As a first example of the application of 't Hooft's condition, let us consider the case of QCD with three massless flavors. I will assume that color

remains confined. Let us ask whether the flavor symmetry $SU(3) \times SU(3)$ might, however, remain exact, so that the composite states of confined quarks may include massless composite fermions. The simplest choice for these massless states is the multiplet shown in Figure 8. If we assume that the quarks occupy a spatial wavefunction with no orbital or radial excitation, and that they are in a color singlet state, the two quarks with the same helicity must appear in a symmetric combination. (Quarks with different helicity are distinguished by that quantum number, and may have arbitrary symmetry.) The original left- and right-handed quarks each transformed under only one of the two handed flavor $SU(3)$ groups; we may assign them to the $(3,1)$ and $(1,3)$ representation, respectively, of $SU(3) \times SU(3)$. Then the left- and right-handed composite states of Figure 8, with the symmetry restriction just mentioned, belong, respectively, to the $(6,3)$ and $(3,6)$ representations. These representations contain states with the quantum numbers of the baryon octet and the helicity $\pm\frac{1}{2}$ states of the decuplet; they are the obvious candidates for the lightest fermions of this theory. Let us check whether 't Hooft would permit these states as massless composites by computing the anomaly coefficient of three currents of the left-handed $SU(3)$ from elementary and composite fermions. In $SU(3)$, the anomaly coefficients for the 3 and 6 representations are 1 and 7 respectively. The elementary fermions in the $(3,1)_L$ and $(1,3)_R$ give

$$A = (1 \cdot (3 \text{ colors})) = 3 \ .$$

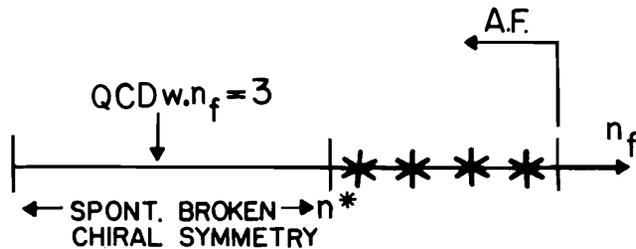
Only the left-handed fermions contribute. The composites in the $(6,3)_L$ and $(3,6)_R$ give

$$A = (7 \cdot 3 - 1 \cdot 6) = 15 \ .$$

(The right-handed fermions contribute to A with the opposite sign.) Clearly, it doesn't work.

't Hooft's original paper contained a more general analysis which extended this result to strong interaction gauge theories of the form of QCD but with any number of colors n_c and flavors n_f . This argument has since been refined by Frishman, Schwimmer, Banks and Yankielowicz⁴³ and by Farrar.⁴⁶ Under the assumption we used above, that the composite states have no orbital excitation, 't Hooft's condition has no solution for any value of n_c and for any $n_f > 2$.⁴⁷ If one allows the massless composite states to be arbitrarily exotic but assumes that the form of these states is independent of the number of flavors, then still one can find no solutions to the anomaly conditions for any value of n_c . Further, Coleman and Witten⁴⁸ have demonstrated that the chiral symmetry will always be spontaneously broken in the limit $n_c \gg n_f$, at least for fermions in the fundamental representation of the color group. Some rather complex solutions have been found for specific choices of n_c, n_f .^{49,50} But if these QCD-like theories have massless composite fermions at all, their behavior as a function of the number of flavors should look like that represented in Figure 12. At some value of n_f , one loses asymptotic freedom. Between this value and a value

Figure 12: The possible region of manifest chiral symmetry in generalizations of QCD with n_f massless flavors.



n^* , one must have a new and different pattern of massless composites for each value of n_f .

Let us now broaden our perspective and consider strong-interaction gauge theories of more general types. One situation which is suggested by the first class of quantum numerical models is to consider gauge theories of strongly interacting fermions and scalar bosons, and to assume that the composite states are fermion-boson bound states. If the chiral symmetry of the fermions is $SU(n_f) \times SU(n_f)$, the original fermions transform under this symmetry as $(n_f, 1)_L$ and $(1, n_f)_R$. The composites belong to these same representations, but they are color singlet states. Hence, we find for the anomaly coefficient of the elementary fermions

$$A = 1 \cdot n_c ,$$

while, for one multiplet of the composite fermions we have

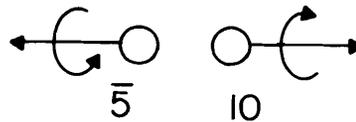
$$A = 1 .$$

For $n_c = 1$ (an Abelian gluon) we are allowed only this one composite state. For a large number of colors, the anomaly conditions require n_c multiplets of composites. By setting $n_c = 3$ we might explain the multiplicity of generations.

The mechanism for producing generations was first proposed by Barbieri, Maiani, and Petronzio³⁷ in the context of a slightly more involved model; it has been pursued by Casalbuoni and Gatto⁵¹. The mechanism is an attractive one, but an important question associated with it is, to my knowledge, unanswered: These authors have not given us any idea of the physical origin of this multiplicity of states. If their idea is correct, one ought to be able to construct a set of n_c states, with identical quantum numbers but differing in their internal structure, which could be massless fermions. Some bits of light have been shed on the physics of this situation by Banks and Kaplunovsky:⁵² They have studied a theory of the type considered by Barbieri, Maiani, and Petronzio in the limit of strong gauge coupling, using the techniques of lattice gauge theories. They find evidence for massless composite states, but they do not find evidence for a multiplicity of states with the same quantum numbers.

Dimopoulos, Raby, and Susskind³⁴ have considered a different generalization of the QCD-like models: They have examined chiral gauge theories, theories in which the gauge couplings themselves are handed. An example of such a theory is the Georgi-Glashow $SU(5)$ model of grand unification: There the $SU(5)$ bosons couple to left-handed fermions only, these fermions being assigned to the 10 and $\bar{5}$ representations of $SU(5)$. These models are interesting for the following reason: The physical argument for the spontaneous breaking of chiral symmetry which we gave earlier for QCD does not generalize to chiral gauge theories. In QCD, we identified a mode in which quark-antiquark pairs were likely to condense and fill the vacuum. But each pair (Figure 7) was composed of a left-handed quark and the antiparticle of a right-handed quark, or vice versa. Without the right-handed fermions, we can make no fermion-antifermion pairs of zero momentum and angular momentum. We might try to pair two left-handed fermions, but in the $SU(5)$ model, the closest thing we can make to an object with vacuum quantum numbers is shown in Figure 13: This is an object which, though a Lorentz scalar,

Figure 13: A pair of fermions from the $SU(5)$ gauge theory with zero total momentum and angular momentum.



is not a color singlet. It is a combination of fermions with the quantum numbers of the operator

$$\epsilon^{\alpha\beta} \psi_{\alpha}^{(10)ab} \bar{\psi}_{\beta a}^{(5)} , \quad (11)$$

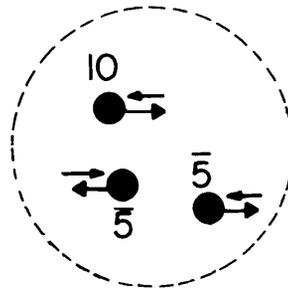
where the ψ fields are the SU(5) fermion fields, $a,b=1,-,5$, and $\alpha,\beta=1,2$ are the spinor indices of a left handed fermion. If this pair is not a color singlet, should confinement prevent it from condensing? Dimopoulos, Raby and Susskind find it likely that this object will not condense but, rather, will attach itself through the confining gauge forces to other fermions. We can make a color singlet composite fermion by attaching this colored system to a $\bar{5}$ fermion, to produce a state with the quantum numbers of

$$\epsilon^{\alpha\beta} \psi_{\alpha}^{ab} \psi_{\beta a} \psi_{\gamma b} . \quad (12)$$

Remarkably, the assumption that the SU(5) model produces precisely one fermion with these quantum numbers satisfies the 't Hooft anomaly condition for this model. The same construction produces sets of composite massless fermions consistent with 't Hooft's condition for any theory of left-handed fermions coupled to an SU(N) gauge field in the \bar{N} and antisymmetric tensor, or \bar{N} and symmetric tensor, representations.⁵³

What do these composite states look like?⁵⁴ A state with the quantum numbers of (12), consisting of three massless fermions almost at rest, is shown in Figure 14. The notion of massless fermions held almost at rest, to appear as

Fig. 14: A composite fermion with the quantum numbers of the operator (12).



simple constituents of bound states, is an ill-defined one; I have assigned infinitesimal momenta to these fermions in such a way that the helicities and the overall angular momentum J^2 of the state work out properly. Figure 14 is, however, itself a picture of massless fermions at rest. To find out what this state would look like when viewed more realistically, it is necessary to boost it to some finite momentum. Let us recall that in an interacting quantum field theory a boost operator may create particle-antiparticle pairs from the vacuum, forming a parton sea. This pair creation process must, however, conserve the basic quantum numbers of the theory, including the handed flavor quantum numbers. The boosted state thus has the form shown in Figure 15. Note that the sea, which in a QCD composite state is considered neutral, here carries a chiral flavor quantum number.

It is a question of detailed dynamics whether the valence part of the wave function should be considered to contain two fermions, leaving the sea with a net fermion number, or a single fermion, making the sea spin-zero state. In the latter case, butnot the former, the chiral quantum numbers of the sea can slip away to $x = 0$ and disappear, returning us to the case of an elementary fermion

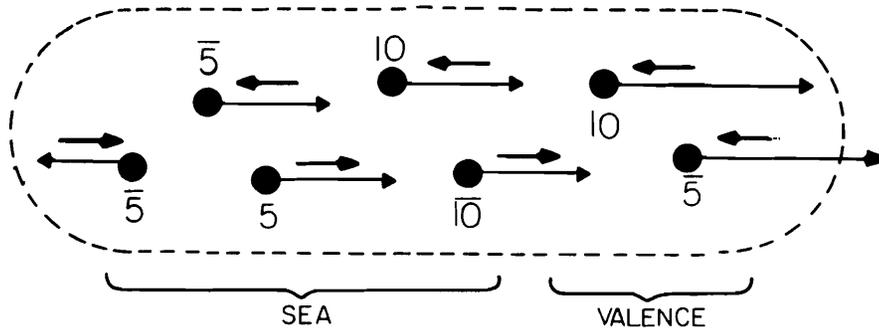


Figure 15: The state of Figure 14, boosted to finite momentum

(and its neutral sea) on a background vacuum of indefinite chiral charge. The fact that one can slip continuously, in any given quantum state, from the situation of massless composite fermions to that of spontaneously broken chiral symmetry is a remarkable property of the whole class of scenarios we have been discussing; Dimopoulos, Raby, and Susskind,³⁴ who first noted this property and clarified its theoretical significance, refer to it as "complementarity".

Following this work, many authors investigated further the anomaly conditions in chiral gauge theories. Banks, Yankielowicz, and Schwimmer,⁵⁵ in a beautiful paper, clarified the structure of 't Hooft's constraints and found some new solutions to them. Bars and Yankielowicz⁵⁶ extended their technique to find solutions with large numbers of massless composite fermions, enough, in fact, to accommodate 8 generations of quarks and leptons. Albright, Schrempp, and Schrempp⁵⁷ have produced a set of composite states for QCD, with $n_c = 3$, $n_f = 6$, which are similar in form to the states of Dimopoulos, Raby and Susskind, and which solve the 't Hooft anomaly conditions for the $SU(6) \times SU(6)$ chiral symmetry; however one must assume that the overall conservation of quark number is spontaneously broken. These states come in right- and left-handed pairs, of which the left handed composites are shown in Figure 16; the two right-handed quarks are antisymmetrized in flavor. This model is interesting because one may identify the 6 flavors with the two color triplets of the rishon model. The multiplet shown in Figure 16 in fact contains the composite states of the rishon model and, as we will see, a bit more. I should note that the spontaneous breaking of rishon number presents some serious difficulties for this interpretation. Albright, Schrempp, and Schrempp have, however, found a variant of the rishon model which seems to evade these difficulties.

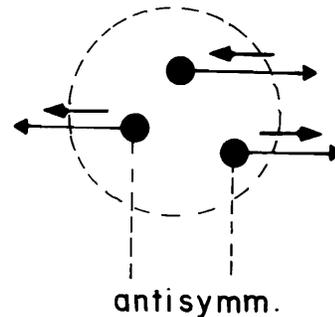


Figure 16: Left-handed composites suggested by Albright, Schrempp, and Schrempp.

Massless Composite Vector Bosons?

We have seen in the previous two sections that massless composite fermions are rapidly becoming theoretically comprehensible and, indeed, respectable objects. Many of the quantum numerical models, however, require a larger class of massless composites: We saw earlier that the most economical of these models also require that

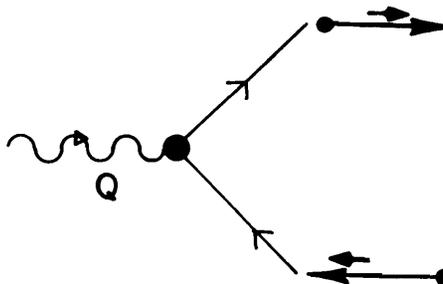
gauge bosons be composites of the underlying constituents. It is thus important to determine whether it is possible to form massless composite states of spin 1, and perhaps also of higher spin. In contrast to the case of spin $\frac{1}{2}$ composites, however, the most important result here is a negative one: A theorem proved twenty years ago by Case and Gasirowicz,⁵⁸ and recently rediscovered and extended by Weinberg and Witten,⁵⁹ provides a most stringent limitation on massless composite states of high spin. In this section, I will review that theorem and discuss the difficulty of evading its constraints.

The theorem of Case, Gasirowicz, Weinberg and Witten is the following:

- (1) A theory with a Lorentz-covariant conserved 4-vector current J^μ can have no massless particle of spin $> \frac{1}{2}$ with a nonvanishing value of the associated conserved charge.
- (2) A theory with a Lorentz-covariant energy-momentum tensor $\Theta^{\mu\nu}$ can have no massless particle at all with spin > 1 .

The proof of these results is so simple that I have room to give it here: Under the assumptions of Lorentz covariance, the scattering of a massless particle of momentum p from J^μ or $\Theta^{\mu\nu}$ conserves helicity: $\langle p', \lambda' | J^\mu | p, \lambda \rangle$ and $\langle p', \lambda' | \Theta^{\mu\nu} | p, \lambda \rangle$ are nonzero only if the helicities λ' and λ are equal. But consider the process shown in Figure 17, the backward scattering of a massless

Figure 17: The scattering process used in the proof of the Case-Gasirowicz-Weinberg-Witten theorem.



particle from J^μ or $\Theta^{\mu\nu}$. This scattering cannot vanish under assumptions given, since the limit as $Q \rightarrow 0$ of this scattering process measures the charge or the energy, respectively, of the particle. Helicity conservation insists that the angular momentum of the particle changes by $\Delta J^z = 2 \cdot (\text{spin})$. This angular momentum must be supplied by the operator inside the scattering matrix element; thus,

$$(\text{spin of particle}) \leq \frac{1}{2}(\text{spin of operator}).$$

Can one evade this theorem? Surely, there must be some escape, because it would seem to forbid the existence of non-Abelian gauge bosons and gravitons. In fact, these two cases do not satisfy the hypotheses of the theorem: In the former case, the gauge currents J_μ^a are Lorentz-invariant only up to a gauge transformation; in the latter case, $\Theta^{\mu\nu}$ is Lorentz-invariant only up to a local coordinate transformation. In each case, the theorem is evaded only by insisting that the underlying theory has an exact local (non-Abelian) gauge invariance. This is a serious price to pay for the privilege of omitting gauge fields from the fundamental equations of motion, but this course has been accepted by several authors.⁶⁰ In practice, one needs to impose gauge invariance at some state in a calculation which seeks to generate gauge fields: Gauge

field couplings obey relations such as those shown in Figure 18, which must hold even when one includes effects of radiative corrections. Imposing

Figure 18: Relations among coupling constants required for gauge-invariant theory.

$$\text{Tree-level vertex} = \text{Tree-level vertex with fermion loop} = \left[\text{Tree-level vertex with fermion loop} \right]^{1/2}$$

constraints on the renormalization constants to force such relations to hold is essentially equivalent to assuming gauge invariance from the beginning.

Generating gauge fields from systems with only local symmetries, is, then, impossible in Lorentz-invariant theories. However, several authors have shown explicitly that gauge symmetries may arise in the continuum limit of a lattice field theory with only global symmetries.⁶¹ In the Lorentz-invariant continuum limit theory, the gauge bosons appear as elementary fields and gauge invariance appears exact; the fact that the underlying system has a lower symmetry is seen only at momenta sufficiently high that the effects of the lattice, which break Lorentz invariance, become visible. It is tempting to speculate that the fluctuations of space-time associated with quantum gravity might have a similar effect, allowing a theory of low symmetry at the Planck mass scale to appear as a theory with local gauge invariance at all smaller energy scales. I will pursue and embellish this speculation later in my discussion.

The Implications of Compositeness

The previous few sections have been concerned with the theoretical aspects of composite quarks and leptons. I have reviewed the theoretical developments of the past two years which show how composite fermions may be produced with masses much smaller than the mass scale M characterizing the strength of binding of their constituents or the size of their composite structure. It is time now to put these theories to work, to ask how one can build from them realistic models of composite quarks and leptons, and to ask, more importantly, what the distinguishing characteristics of such models might be. Can we learn, experimentally, whether quarks and leptons are composite? The answer, I am afraid, is complicated, for two reasons: First, the notion of composite quarks and leptons encompasses several different classes of models: Each class may be associated with a different order of magnitude assumed for the compositeness scale M ; each class is characterized by a different set of novel phenomena. We will need to examine each case in detail. Secondly, the answer we will find, case by case, is the response of Sarastro's ghostly chorus: "Bald, oder nie!"⁶² We will need grace, as well as skill, to find evidence for the internal structure of quarks and leptons.

Where should one look for such evidence? Let us consider a range of scenarios for the composite structure:

1) Scenarios with nearby Compositeness ($M < 1$ TeV)

In our study of the experimental lower bounds on the compositeness scale M , we found (in Eq. (8)) that, if we assume the constituent-binding interactions conserve baryon number and separate electron and muon numbers, this scale could be smaller than 1 TeV in order of magnitude. Indeed, our crude estimation procedure could easily allow $M \sim 300$ GeV; I will use that value in numerical estimates of this section. If M really were this small, interactions with $Q^2 \sim (100 \text{ GeV})^2$ which will be produced at the next generation of accelerators would probe distance scales approaching the physical size of the composite states. For such values of M , then, the compositeness of quarks and leptons

would become manifest, producing small but noticeable deviations from the standard gauge model predictions as effects of nontrivial quark and lepton form factors. It is important to be aware of the possibility of such form factor effects and to search for them, even at the few-percent level. Compositeness may, in addition, manifest itself in a number of less direct but more interesting ways, which I now wish to review:

We saw in the previous section that the massless gauge bosons could not arise as composite states in a theory with only global symmetries. The same result applies to massive vector particles with masses m_V much smaller than the compositeness scale M : It is well known that massive vector particles have scattering amplitudes which rise too rapidly to be consistent with unitarity unless their couplings obey the relations, such as those of Figure 18, which characterize a gauge theory.⁶³ Such bad high-energy behavior lasts, however, only up to the compositeness scale; it is not a problem if this scale is sufficiently close to m_V , that is, if

$$m_V^2 \gtrsim g^2 M^2,$$

where g is the vector boson coupling constant. Thus, if M were of order 1 TeV, the W and Z bosons could be composite states, produced as the result of some underlying strong interaction dynamics. Models of composite, strongly interacting W and Z bosons have been proposed by Terazawa⁶⁴ and Fritzsch and Mandelbaum;⁸ a particularly elegant model has been constructed by Abbott and Farhi.⁶⁵ These models provide a realization of a more general phenomenology of strongly interacting W and Z bosons constructed some time ago by Terazawa,⁶⁶ Bjorken,⁶⁷ and Hung and Sakurai;⁶⁸ the last paper, in particular, provides a detailed picture of the structure expected for the Z^0 resonance as a function of its mass. The composite models require a larger value of the coupling constant g than that expected in the standard $SU(2) \times U(1)$ model. They thus predict higher values for the W and Z masses: m_Z should be in the range of 125–200 GeV, still accessible to the later stages of LEP. They also predict one notable correction to the theory of Hung and Sakurai: The cross section for fermion pair production on the Z^0 peak should be enhanced by 10% as the result of form factor effects.

Two additional phenomena to be expected for such small values of M are, properly, mere form factor effects, but they deserve special mention. The first is that a truly anomalous contribution to the muon magnetic moment should appear, with

$$\delta a_\mu \sim 10^{-8}.$$

This effect would be of the order of the current bound, and, could well be larger than the weak-interaction contribution to δa_μ . I should note, though that improvement of the current limits on δa_μ will require improvement of the theory of the hadronic corrections to δa_μ as well as a more accurate experimental determination of the muon $(g-2)$.³¹ The second phenomenon is the possibility that the Cabbibo angle, which is, after all, the $Q^2 \rightarrow 0$ limit of a form factor, may acquire a Q^2 dependence:⁶⁹ $\sin^2 \theta_c$ may change by 10% between $Q^2 = 0$ and $Q^2 = (100 \text{ GeV})^2$. This effect can and should be searched for at an ep collider.

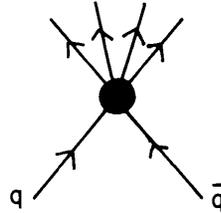
Finally, if quarks and leptons have strong interactions on the energy scale of 1 TeV, one would expect to see, in very high energy reactions, events exhibiting multiple production of quarks and leptons. De Rujula⁷⁰ has labelled such events "glints" and has paused from his essentially linguistic analysis to describe their characteristics: They may be distinguished as wide angle multilepton events or as hadronic events exceptional in both transverse momentum and multiplicity. The cross section for such events is, however, not very large: Thinking simply of the geometrical cross section for breakup of a

quark or lepton of size M^{-1} , one would estimate roughly

$$\sigma(p\bar{p} \rightarrow \text{glint}) \sim 10^{-5} \cdot \sigma_{\text{tot}}(p\bar{p}) \quad . \quad (13)$$

However, even the result (13) is valid only for center of mass energies $\sqrt{s} \sim M$. If $\sqrt{s} \ll M$, glints are produced by a local interaction and may be analyzed using the effective action. The relevant vertices are of the form of Figure 19; the

Figure 19: The local interaction which gives rise to a glint.



simplest helicity-conserving vertex of this type contains a derivative in addition to the six fermions and thus has a coefficient of order M^{-6} . Thus, for $s \ll M$

$$\sigma(ff' \rightarrow \text{glint}) \sim \frac{1}{M^2} \left(\frac{s}{M^2} \right)^5 \quad , \quad (14)$$

a severe suppression. Glints may, however, be visible on the Z^0 : The vertex coupling the Z^0 to four fermions has a coefficient $\sim M^{-4}$, giving

$$\text{B.R.}(Z^0 \rightarrow \text{glint}) \sim \left(\frac{m_Z}{M} \right)^8 \sim 10^{-4} \quad .$$

This small branching ratio might be compensated, however, by the large event rate on the Z^0 resonance, which should allow selection of rare but unusual events.

2) Scenarios with $M \sim 100$ TeV

A model of composite quarks and leptons in which the muon and electron are built of the same constituents, and which therefore allows $\mu \rightarrow e\gamma$ by interactions at the constituent level, is restricted by our earlier analysis (eq. (9)) to have a compositeness scale M of order 100 TeV. If M is this large, the various direct effects of composite structure discussed in the previous section will not be visible at any presently conceived - perhaps, at any conceivable - accelerator. In such a case, we can find evidence for compositeness only through indirect effects which might be visible at energy scales much lower than the scale M . Some of the possible effects are quite remarkable, but the possibility of these effects depends as much on the relation between the constituent-binding interactions and the other fundamental interactions as on the simple fact of quark and lepton compositeness.

In order to work out what effects should appear, I will adopt the specific picture of this relation which I find the most plausible. This picture is closely related to models of weak-interaction symmetry breaking incorporating a certain dynamical mechanism ("extended technicolor") to generate quark and lepton masses;^{71,72} many of my conclusions are, in fact borrowed directly from the literature on those models. Basically, though, the picture entails three simple assumptions: The first is that quarks and leptons are built up, at a scale of M of order 100 TeV, as a multiplet of massless composite fermions, corresponding to a representation of an unbroken chiral symmetry group. This

group must include the exact (gauge) symmetries of quarks and leptons, $SU(3) \times SU(2) \times U(1)$; it is probably larger. The second is that a different interaction, acting at energies of 300 GeV, is responsible for the breaking of $SU(2) \times U(1)$; this interaction may be another strong interaction (technicolor) or simply a light Higgs boson created at the scale M . The third is that quarks and leptons are coupled into the $SU(2) \times U(1)$ breaking only weakly, perhaps through interactions at the scale M .

The generation of quark and lepton masses requires breaking of their chiral symmetries, and in particular the breaking of $SU(2) \times U(1)$. Therefore, we expect that their masses will appear in the general form

$$m \sim a \cdot (300 \text{ GeV}) ,$$

where a is a small dimensionless number representing the strength of the weak link between these fermions and the $SU(2) \times U(1)$ breaking. It is likely, however, that some fermions will not receive masses in the leading order in a but, rather, in order a^2 or a^3 . Such hierarchies of light fermion masses appear naturally in the relatively simple models for light fermion mass generation which have been studied in literature;^{50,73} they are, indeed, necessary to account for the great range of quark and lepton masses. Putting $a \sim 1/10$ sets the highest level of masses as being of order 30 GeV; it is not unlikely that we have, so far, explored only the lower ranges of this hierarchy. If this is true, we might expect that most of the quarks and leptons belong to this highest rung (since only exceptional states would escape acquiring mass in order a) and thus still await discovery.

This idea, that one should see a dramatic increase in the number of quarks and leptons at center of mass energies of 60-100 GeV, gives rise to another notion, no less plausible: Composite models of quarks and leptons generally allow as composite fermions, not only these observed states, but also exotic states belonging to higher representations of color $SU(3)$. As an example, consider the multiplet of massless fermions indicated in Figure 16. The constituents may each appear in one of six flavors; let us identify these with the six states $T_a, V_a, a=1,2,3$ of Harari's rishon model. Then one can readily work out the color quantum numbers of the composites: The multiplet contains 2 color singlets (leptons) and 6 color triplets (quarks), but also 2 color 6's (quixes), 2 color 8's (quaits), and 2 color 15's (quifs). To obtain a reasonable spectrum, we must assume that most of these states (all but 2 leptons and 2 quarks) receive mass in leading order in a , but that is a reasonable consequence of first-order perturbation theory. Color 6, 8, and 15 fermions produce remarkable signatures in e^+e^- annihilation: They give large steps in R , proportional to the dimension of their color representation, and they appear in visible heavy hadrons, since they are stable with respect to the strong, weak, and electromagnetic interactions. These fermions might decay by some additional interactions, the most plausible ones being 4-fermion interactions from the effective action of the constituent binding forces. Such interactions would, however, yield a decay rate proportional to M^{-4} , giving a lifetime $\tau \sim 1$ meter. In this scenario for compositeness, a major fraction of Z^0 decays will involve pair-production of these unmistakable objects. The complete multiplet of originally-massless composites should be visible at the highest energies available to LEP.

A second consequence of the composite-binding interaction is connected more closely to my assumption that these interactions link all flavors: These interactions should produce, at some level, flavor-mixing interactions and rare flavor-changing decay processes. The occurrence of such processes in technicolor theories was noted by Eichten, Lane, and Preskill^{72,74} and has been worked out in some detail by Dimopoulos and Ellis.⁷⁵ The effects noted by these authors appear in basically the same way in composite models of the type we now consider. We have already noted that $M \sim 100$ TeV should lead to

$$B.R.(\mu \rightarrow e\gamma) \sim 10^{-10}$$

of the order of the current bound. Other processes sensitive to such flavor

mixing have been listed by Cahn and Harari⁷⁶ and by Kane and Thun;⁷⁷ for two particularly interesting reactions (and $M \sim 100$ TeV) we should expect

$$\begin{aligned} \text{BR}(K_L \rightarrow \mu e) &\sim 10^{-9} \\ \text{BR}(\Sigma^+ \rightarrow p \mu e) &\sim 10^{-12} . \end{aligned}$$

The first of these is of order the current bound, the second might perhaps be reached with the recently improved CERN hyperon beams.

A warning should, however, accompany this prediction. A much more sensitive probe of flavor mixing than any of these rare decays is the magnitude of the $K_L - K_S$ mass difference: Even as innocuous a coupling at the scale M as a neutral-current interaction with a different strength for the d and s quarks would give an unacceptably large contribution to this mass difference. One must, then, arrange that the d and s quarks enter equivalently in the effective action not only in their coupling to $SU(2) \times U(1)$ but also in their 4-fermion interaction self-couplings. A major difficulty of technicolor models is that this requirement seems to be inconsistent with that of giving different masses to the d and s .^{74,75,78} I suspect, though, that this difficulty stems from the particularly simple form (1-gauge boson exchange) which these models give for the interactions which couple light fermions to the $SU(2) \times U(1)$ breaking mechanism. In a composite model one may have enough freedom to satisfy both constraints. It is much more difficult, however, to reduce CP violation to the observed level if it is produced by these interactions. I believe that the scenario which I have been describing requires a superweak mechanism for CP violation.⁷⁹

Having discussed this difficulty with composite structure at 100 TeV, we must now confront a greater one: We must impose some constraint on the constituent binding interactions to insure that they may not mediate proton decay. One strategy is to insist that these interactions conserve baryon number exactly. Casalbuoni and Gatto⁵¹ have explored systematically the solutions to the 't Hooft conditions in gauge theories with vector couplings; they have shown that only a few exceptional cases allow a conserved baryon number, which takes the same value for all quarks, to be defined. Chiral gauge theories, however, may provide more possibilities. An alternative strategy, suggested by Domokos and Kövesi-Domokos,⁸⁰ is to use constraints from chiral symmetry to reduce the rate for baryon decay to an acceptable level. Harari, Mohapatra, and Seiberg²⁶ have found that the rishon model of ref. 27 contains an amusing illustration of this strategy, which I would like to review: For reasons best left unexplained, these authors favor the following assignments of handed rishons ($T_L, T_R; V_L, V_R$) to handed quarks and leptons:

$$\begin{aligned} u_L &= T_L T_L V_L; & (\bar{d})_L &= T_L V_L V_L \\ \text{but} & & (e^+)_L &= T_R T_R T_L . \end{aligned}$$

This last assignment makes sense if the right-handed T 's are imagined to occupy the two lowest positions in the state shown in Figure 16. With these assignments, the baryon-number-violating process shown in Figure 4 is forbidden by chirality conservation. The simplest operator which respects chirality conservation but violates baryon number contains 6 fermion operators; if it is to be Lorentz-invariant, it must also contain a derivative; thus, it appears in the effective action with a coefficient of order M^{-6} . This would give the proton lifetime

$$\tau_{\text{proton}} \sim \frac{M_{12}}{m_p^{13}} \sim 10^{30} \text{ yr.}$$

for $M \sim 100$ TeV. (For completeness, I should note that Harari, Mohapatra, and Seiberg have also identified another mechanism of proton decay in their model which, though less generally interesting, is numerically dominant. The curious reader should consult ref. 26 for details).

3) Scenarios with $M \sim 10^{15}$ GeV

One suggestion which has been favored by many authors^{4,15,16-19,81,82} is to place the compositeness scale at the location of the grand unification of strong, weak, and electromagnetic interactions. The idea is, in principle, an interesting one, but I know of no observable phenomena which would distinguish this class of models from more ordinary grand unified theories. Such models might produce the multiplicity of quark and lepton generations, and, indeed, many published models require a number of generations equal to 3 or larger⁸¹. But this conclusion is not sufficiently remarkable to keep us from the next level of our discussion.

4) Scenarios with $M \sim 10^{19}$ GeV

In this final class of models of compositeness, we have moved the compositeness scale to a region sensitive to the effects of quantum gravity. For such a large value of M , we can imagine that gauge bosons, as well as quarks and leptons, arise as bound states of a set of constituents. But we can also imagine another development, more unusual, more profound: the identification of the constituents and their binding interactions with the elements of a theory of supergravity, a theory which unifies the graviton with particles of lower spin via a supersymmetry which mixes bosons with fermions.⁸³ This latter idea has led to some very interesting speculations, which I would now like to describe.

The largest of the class of supergravity theories is constructed around a manifest $O(8)$ symmetry: the theory contains 8 spin 3/2 fermionic partners of the graviton. However, when Cremmer and Julia⁸⁴ first constructed this theory explicitly, they found that it contained, magically, a much larger symmetry group, including a gauge invariance under the group $SU(8)$. Unlike $O(8)$, this group is large enough to accommodate $SU(3) \times SU(2) \times U(1)$ as a subgroup; the discovery of Cremmer and Julia thus fueled a hope that supergravity suffices as a theory of all the fundamental interactions. This $SU(8)$ symmetry was not without its problems: Cremmer and Julia identified it only as a formal symmetry of the Lagrangian of supergravity; the simultaneous conservation of all the $SU(8)$ currents was apparently spoiled by an axial-vector anomaly. Further, the supergravity theory which they constructed did not contain the $SU(8)$ gauge bosons as elementary fields. Cremmer and Julia answered this latter problem with the speculation that these bosons could appear as composites of the basic fields of the theory; later, Curtwright and Freund⁸⁵ imagined that spin $\frac{1}{2}$ fermions sufficient to cancel the $SU(8)$ anomalies could also be generated as composites.

Ellis, Gaillard, Maiani and Zumino⁸⁶ tried to make these speculations more concrete by assuming that if the $SU(8)$ gauge bosons were formed as composites, the rest of the supersymmetry multiplet to which they belong would be formed as well. They constructed and analyzed this multiplet, but they found that its fermions still caused axial-vector anomalies which ruined the $SU(8)$ symmetry. Undaunted (for the most part), Ellis, Gaillard, and Zumino⁸⁷ proposed the following solution: One must assume that the theory has a sensible low-energy limit which includes a measure of gauge symmetry, and one must throw away (unbind) composites until the remaining composite fermions and gauge bosons form a consistent, anomaly-free system. This procedure is not necessarily sensible: It is, first of all, the reverse of the effective action philosophy which I have been following up to this point; instead of trying to compute in the high-energy theory and observing the (perhaps trivial) consequences at low energies, one here insists on obtaining a certain type of low-energy result. It also turns out to force the breaking of supersymmetry. But the procedure does produce an intriguing result: The largest consistent subset of their original multiplet is the $SU(5)$ grand unified theory with precisely 3 generations of quarks and leptons!

The scenario of Ellis, Gaillard, and Zumino, is, then, a most tempting one. They begin with a fundamental theory of unmatched beauty and intricacy; they end with a theorist's conception of the real world. One might be tempted to ignore the *ad hoc* assumptions necessary to link these endpoints; better still, one should be tempted to understand this linkage better, to find a way to derive it from a precise mechanical analysis. Achiman⁸² and Derendinger, Ferrara, and Savoy⁸⁸ have attempted, along different routes, to simplify those assumptions; their efforts, however, seem only the first steps of a long and difficult road.

CONCLUSIONS

This last remarkable speculation brings my review to an end. I have surveyed a variety of theoretical ideas associated with the notion that quarks and leptons are composite, following each strand of thought to the boundary of that tangle where our current understanding guides us no further. Each strand ends in a different place. Can we assemble them and unite them? The best conclusions I can give are a list of requests, a list of problems whose solution might give insight into how these various ideas do lock together into a picture of quark and lepton structure.

For experimentalists, I have three rather modest requests: The first is to explore the Z^0 resonance, looking for more flavors and for fermions in exotic color representations. The second is to improve the bounds on rare weak decays, quark and lepton form factors, and the muon ($g-2$). The last is simply to keep your eyes (and minds) open to the possibility of small effects which lie outside the standard gauge theories.

For theorists, my requests are more serious: The first is to find a way to study quantitatively the realization of chiral symmetry in confining gauge theories. It is likely that the techniques of lattice gauge theories will be essential in the solution of this problem, despite the difficulties of implementing chiral symmetry on the lattice. The second is to find a model which satisfies in a simple and compelling way both the constraints of the quantum numerology and the dynamical constraint of 't Hooft. The third, and most pressing, is to learn how to compute the mass spectrum of quarks and leptons in composite models, even in models too simple to be realistic. It is, after all, only through the computation of the quark and lepton masses that the idea of compositeness can really fulfill its promise.

Are quarks and leptons composite? We are far from knowing the answer. But this situation is not unfortunate: It means that the deepest explorations into this subject – and the most entrancing surprises – lie ahead.

ACKNOWLEDGEMENTS

I am grateful to many of the originators of the ideas I have reviewed for discussions of those ideas. Some of those discussions have been brief but enlightening; others have been lengthy arguments stretching over months or years. Among those who have helped me in this way are Tom Banks, Itzhak Bars, Eugene Cremmer, Savas Dimopoulos, Estia Eichten, John Ellis, Ed Farhi, Glennys Farrar, Haim Harari, Gerard 't Hooft, Gordy Kane, Ken Lane, John Preskill, Stuart Raby, Adam Schwimmer, Lenny Susskind, Hidezumi Terazawa, Steven Weinberg, Ed Witten, and Shimon Yankielowicz. I am grateful, as well, to my colleagues at Cornell, particularly to Orlando Alvarez, Paul Ginsparg, and Henry Tye for their patient criticism of my work. Finally, I thank Lev Okun for some intense sessions of debate, at the Bonn Symposium itself, which helped me to see clearly, at the last moment, some of the most confusing aspects of this subject.

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DISCUSSION

A. ALI, DESY :

You have discussed the limits on the scale of lepton substructure due to the experimental constraints from $(g-2)$ measurements. There is, however, another powerful constraint on model building which comes from the limits on the flavor changing neutral currents. For example, the compatibility of non-zero Cabibbo angles with the small K_L-K_S mass difference is a serious problem for the theories of the technicolor type. Do you think this would also severely restrict the composite models, since you generate similar couplings among the known fermions there?

PESKIN:

I think the point is a very serious one. Consider first the case of compositeness at one TeV. Then flavor-changing couplings have to be forbidden by symmetries of some sort. This is accomplished in the model by Abbott and Farhi [ref. 65]. In the models at 100 TeV where you want to have, in principle, all the currents allowed by quantum numbers, it's a much more touchy point. The only process I know which imposes a real restriction at this level is the non CP-violating part of the K_L-K_S -mass difference. To get rid of the CP-violating part you can always insist that CP is superweak; however, that gives a prediction that ϵ'/ϵ is truly zero and not the value of roughly 0.01 predicted in the Kobayashi-Maskawa theory [ref. 89]. If you assume that the effects at 100 TeV are not CP-violating, then one has only the constraint I mentioned on the 4-fermion couplings of d and s. But you have to insure that somehow. In technicolor models, nobody knows how to do that. In the composite models I think it's easier, because there are fewer fundamental exchanges relative to the number of composites, but no one has tried in a serious way. The solution to the problem should not suppress the rare decays which I mentioned if the last two proceed via (Pati-Salam-type) quark-lepton transitions.

R. JAFFE, MIT :

You explained how it is possible to produce zero-mass fermions as composites. But are you sure that when you give these fermions small masses, and, particularly, when you make inter-generational mass differences, that you do not generate form factors or other effects containing this low mass scale?

PESKIN:

There exist some toy models in which this can be verified; I cited some of these models [as refs. 50,73]. But there exist no realistic models at all in which one generates small masses for light fermions. [Note added: Actually, the model of Abbott and Farhi, ref. 65, is a realistic scheme in which Jaffe's problem can be seen explicitly to be absent.]

H. TERAZAWA, Tokyo :

I would like to point out that the recent work of T. Matsumoto on confinement in QCD based on the conjecture of Banks-Rabinovici-Fradkin-Shenker may answer your first request for theorists. You did not mention his work. You must be aware of it.

I. MONTVAY, Hamburg University :

Is there a good place for CP-violation in composite models of quarks and leptons?

PESKIN:

In my answer to Ali, I proposed tossing CP-violation up to some higher scale. If you are going to do that you have to believe that there are many dynamical scales between here and the Planck mass. But I think in fact that assumption is required in composite models, if just for philosophical consistency. If you demand a Grand Unification, there might well be nothing between here and there. That's one viewpoint. But in the models with compositeness at 100 TeV, one must already introduce two new interactions. Is that disturbing? Certainly not. So far in physics we have discovered something new every few orders of magnitude in scale. Why shouldn't this continue for ever? If so, then the next level up is perhaps the right place to put CP-violation.