The EMC Effect: Looking at the Quarks in the Nucleus

The European Muon Collaboration's announcement that the deep inelastic structure functions of iron and deuterium were different[1,2] caught the particle and nuclear physics communities by surprise. The difference was not, but should have been, anticipated. In response there has been a flood of papers purporting to explain "the EMC effect," as it is now called. This Comment is an introduction to the subject and a detailed discussion of two attempts at theoretical analysis about which I have strong feelings.

Most attempts to understand the EMC effect take for granted a "convolution model": the nuclear structure function is taken to be a sum of the structure functions of various "packages" (e.g., nucleons, pions, six-quark bags, Δ's, alpha particles, etc.) modulated by their distribution within the nucleus. I will argue that this approach is based on an assumption that is unjustified and in some cases wrong: it is unlikely to give more than a qualitative understanding of the EMC effect. The situation is especially bad if the objects are pions. Another more fundamental approach has recently been advocated by Close, Roberts, and Ross,[3] who employ the QCD language of scale dependent quark, antiquark, and gluon distributions to interpret the data. I will describe it in some detail. Before discussing the theory I must review some essential kinematics and summarize the phenomenological parton analysis of the data.

Comments Nucl. Part. Phys.
1984, Vol. 13, Nos. 1&2, pp. 39–60
0010-2709/84/1302-0039/$18.50/0

1. KINEMATICS

In inelastic leptoproduction a lepton transfers a four momentum q^μ to a nuclear target (atomic number A) with initial momentum P^μ. We are interested in the Bjorken limit: $Q^2 \equiv -q_\mu q^\mu \to \infty$, $q^0 = P \cdot q/M_A \to \infty$ with $x_A \equiv Q^2/2M_A q^0$ fixed, where the cross section is determined by a set of quark probability distribution functions which depend on x_A and weakly on Q^2. Since $(P + q)^2 \geq M_A^2$ we find $0 < x_A \leq 1$. It is more convenient to use a uniform scaling variable independent of the particular target:

$$x_{Bj} \equiv x = \frac{Q^2}{2Mq^0} = \frac{M_A}{M} x_A \tag{1}$$

satisfying

$$0 < x \leq M_A/M, \tag{2}$$

where M is the nucleon mass. Quark distributions in different nuclei are almost identical functions of x (up to an overall factor of A) lying mainly between $x = 0$ and 1. [All of the current excitement is summarized in the "almost."]

Inclusive deep inelastic lepton scattering from any target can be analyzed with QCD, which for this purpose is equivalent to the parton model modulo small corrections varying like log Q^2.[5] The model is summarized in Figure 1: the lepton scatters *elastically* and *incoherently* off of the quarks in the target. This is not assumed in QCD;

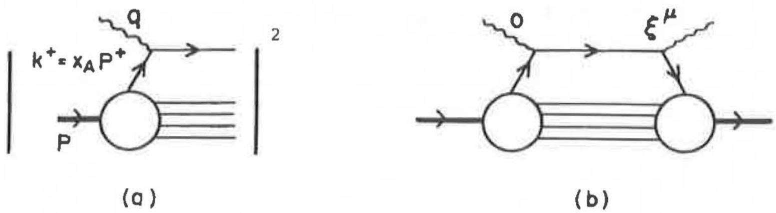


FIGURE 1 (a) The parton model of the inelastic leptoproduction cross section, which is the same as (b), the imaginary part of forward, virtual Compton scattering.

it is proved.[6] The kinematics are simplest if light-cone coordinates are used:

$$a^{\pm} \equiv \frac{1}{\sqrt{2}}(a^0 \pm a^3). \tag{3}$$

Choose the momentum transfer $\mathbf{q}$ to lie in the z direction in the nucleus' rest frame

$$q^\mu \cong [q^0, 0, 0, -q^0 + Mx] \tag{4}$$

so as $q^\mu \to \infty$ with x fixed (the Bjorken limit), $q^2 \cong -2Mxq^0 = -Q^2$ as required. Note that in this limit $q^- \approx \sqrt{2}\, q^0 \to \infty$ but $q^+ = Mx/\sqrt{2}$ remains fixed. It is easy to show that in the Bjorken limit the struck quark carries a fraction x_A of the target's + component of momentum:

$$k^+ = x_A P^+ = xM/\sqrt{2}, \tag{5}$$

where k^μ is the momentum of the struck quark and the second equality is valid in the nucleus' rest frame.

The cross section for leptoproduction is related to the imaginary part of the forward, virtual Compton scattering amplitude (as shown in Figure 1b), which is proportional to the nuclear correlation function for the product of two currents separated by ξ^μ:

$$Q^4 \frac{d\sigma}{dQ^2 dx} \propto \int d^4 \xi e^{iq \cdot \xi} \langle P | J_\mu(\xi)\, J_\nu(0) | P \rangle \tag{6}$$

The distance scale probed is determined by q^μ. Since

$$q \cdot \xi = \xi^+ q^- + \xi^- q^+ - \boldsymbol{\xi}_\perp \cdot \mathbf{q}_\perp,$$

$$\xi^+ \lesssim 1/q^- \approx 1/\sqrt{2} q^0 \tag{7}$$

$$\xi^- \lesssim 1/q^+ = \sqrt{2}/Mx \tag{8}$$

in the nucleus' rest frame. Note that $\xi^+ \to 0$, but ξ^- does not. *Inelastic leptoproduction does not probe short distance structure, rather it probes the light-cone:*

$$0 \leq \xi_\mu \xi^\mu \lesssim 2\xi^+\xi^- \lesssim 4/Q^2 \tag{9}$$

($0 \leq \xi_\mu \xi^\mu$ follows from causality). Combining Eqs. (7) and (8) we find

$$|\boldsymbol{\xi}| \lesssim \frac{1}{Mx} \text{ and } |\xi^0| \lesssim \frac{1}{Mx} \cdot \tag{10}$$

So long range correlations are associated with small x.[7] This simple kinematic analysis has many applications in the study of the EMC effect.

2. DATA AND A PARTON ANALYSIS

Deep inelastic electron or muon scattering measures a single structure function, $F_2^A(x,Q^2)$, which is a linear combination of quark and antiquark distribution functions.[8] At large Q^2 the Q^2 dependence is only logarithmic. At smaller Q^2, of the order 1 GeV, the Q^2 dependence is stronger and more complicated, and the description in terms of quark and antiquark probability distributions breaks down. Ideally one should consider only large Q^2 data, for example, $Q^2 \gtrsim 10$ GeV^2. Practically one is forced to accept lower Q^2 data especially at low x since q^0 is bounded by the beam energy. The data published early this year by the EMC collaboration[2] comparing iron and deuteron targets (see Figure 2) is restricted to $Q^2 \geq 9$ GeV^2.

In the absence of any nuclear effect one expects $F_2^A(x,Q^2) = AF_2^N(x,Q^2)$ where $F_2^N(x,Q^2)$ is the isospin average nucleon structure function. To display deviations from this naive expectation it is conventional to plot $R^A(x,Q^2) = 2/A\, F_2^A(x,Q^2)/F_2^D(x,Q^2)$ where $1/2F_2^D$ has been substituted for the unmeasurable F_2^N. The deviation of $R^{Fe}(x,Q^2)$ from unity shown in Figure 2 caught almost everyone by surprise.[9]

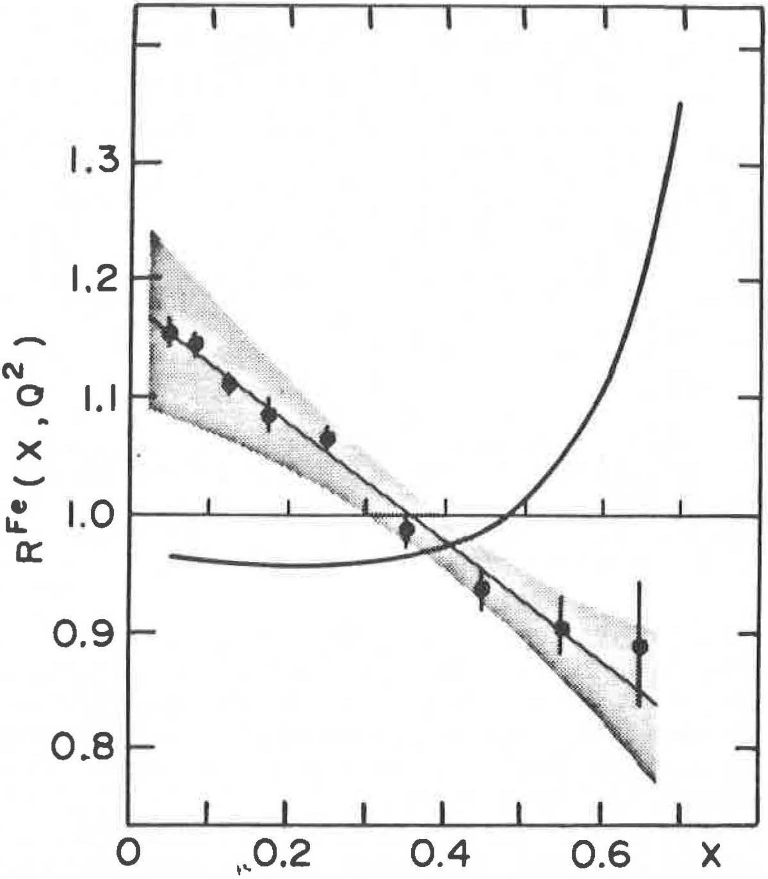


FIGURE 2 The EMC data comparing $F_2(x, Q^2)$ of iron and deuterium. The shaded area denotes the limits of systematic uncertainties.

Confirmation of the EMC effect came soon afterwards from a reanalysis by a Rochester, MIT, SLAC collaboration[10] (RMS) of an old SLAC experiment. The data shown in Figure 3 confirm the large x depletion observed by EMC. RMS do not see an enhancement at small x, but Q^2 values are rather small at low x in their data and nuclear shadowing is expected to deplete the enhancement at low x and low Q^2.

Recently several of the CERN neutrino collaborations have an-

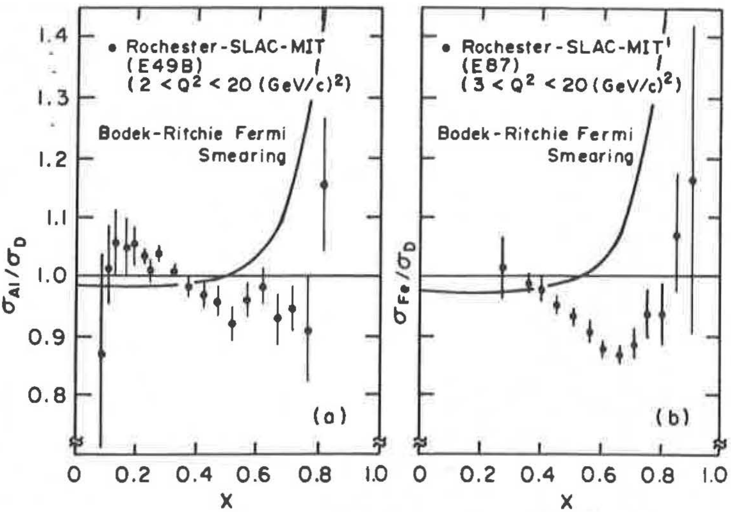


FIGURE 3 The rms data for iron and aluminum targets. The solid curve, taken from Ref. 15, is one estimate of the effect of Fermi motion.

nounced measurements, mostly preliminary, of structure function ratios. For a thorough review see the paper presented by Dydak at the Cornell Conference.[11] So far the neutrino experiments have failed to confirm the low x enhancement seen by EMC. As Dydak points out, the neutrino data have greater systemic uncertainties and lower average Q^2 than the muon data.

Before analyzing the EMC effect as a modification of quark and antiquark distributions in the nucleus we must consider several less interesting possible sources of a deviation of R^{Fe} from unity. Among these are Fermi motion, an isospin correction in iron, higher twist $(O(1/Q^2))$ effects, target mass corrections, shadowing and the possibility that the deuteron structure function is anomalous. Suffice it to say that all these can be excluded as sources of the EMC effect.[12]

The EMC measurements can be converted into quantitative statements about shifts in the valence and ocean quark distributions in iron relative to deuterium.[13] F_2^A may be expressed in terms of valence and ocean quark probability distributions (per nucleon), $q^A(x)$ and $O^A(x)$, respectively: $1/A\ F_2^A(x) = x(5/9\ q^A(x) + 4/3\ O^A(x))$. The weak dependence of F_2^A, q^A and O^A on Q^2 has been suppressed. Here the target nucleus is assumed to be an isoscalar so $u^A(x) = d^A(x) =$

$q^A(x) + O^A(x)$ and the ocean is assumed to be $SU(3)$ symmetric: $\bar{u}^A(x) = \bar{d}^A(x) = s^A(x) = \bar{s}^A(x) = O^A(x)$ while heavy quarks are ignored. The assumption that strange quarks are as important as $\bar{u}$ and $\bar{d}$ quarks in the ocean is suspect, but since the squared charge of the s quark is 1/9 compared to 5/18 for the isospin average u–d quark what we assume about the s quark is relatively unimportant. $q^A(x)$ and $O^A(x)$ obey important sum rules: baryon number $\int_0^A dx\, q^A(x) = 3/2$, "momentum," $\int_0^A x dx\, (2q^A(x) + 6\, O^A(x)) = \epsilon^A$, where ϵ^A is the fraction on the target nucleus' p^+ carried by the quarks and antiquarks (the rest is on gluons). Note the second sum rule is for p^+ in the nucleus' rest frame that boosts into p_3 in an infinite momentum frame. From the nucleus' viewpoint "momentum" is a misnomer: a p^+-distribution requires information about the Hamiltonian (p^0) as well as the momentum space wavefunction for the quarks in the nucleus.

Using the sum rules and the positivity of $O^A(x)$ and $q^A(x)$ it is possible to extract more precise statements about the nature of the EMC effect.[13] This analysis rests on certain plausible assumptions about the behavior of $1/56\, F_2^{\mathrm{Fe}}$ and $1/2\, F_2^D$ in the x regions where they have not been measured ($x < 0.05$ and $0.65 < x < 56$): (1) Both structure functions are very small for $x \gtrsim 0.65$, and (2) R^{Fe} does not fall below unity at very small x. The second assumption has been questioned by Frankfurt and Strickman[14] who believe that shadowing persists at asymptotic Q^2 and very small x. With these assumptions one finds that the valence quarks are degraded in iron relative to deuterium: $\int_0^A dx \delta q^{\mathrm{Fe}}(x) = 0$ but $\delta q^{\mathrm{Fe}} < 0$ for $x \gtrsim 0.35$ and $\delta q^{\mathrm{Fe}} > 0$ for $0.05 \lesssim x \lesssim 0.35$ where $\delta q^{\mathrm{Fe}} = q^{\mathrm{Fe}} - q^D$. Ocean quarks are enhanced at low x: $\int_{0.05}^{0.65} dx \delta O^{\mathrm{Fe}}(x) = 0.72 \pm 0.013$.[15] For comparison $\int_{0.05}^{0.65} dx O^N(x) = 0.125$.[15] Finally, the "momentum" (i.e., p^+) in the quarks and antiquarks is increased $\delta\epsilon^{\mathrm{Fe}} \approx 0.033$. In the nucleon $\epsilon \sim 0.5$.

3. THEORETICAL "EXPLANATIONS" OF THE EMC EFFECT

Most attempts to explain the EMC effect fall into two broad classes: scaling arguments and convolution models. The former are only semiquantitative but theoretically well-founded and (to my mind) extremely interesting. The latter are far more numerous, claim to be quite quantitative but rely on certain assumptions that have not been justified and are probably unjustifiable.

Changes of Scale

The electroproduction structure function is determined by the amplitude to remove a quark from the target at some spacetime point x_μ and replace it at $x_\mu + \xi_\mu$ thereby reconstructing the target as in Figure 1b. ξ_μ is associated with Bjorken's x according to Eq. (10): $|\boldsymbol{\xi}| \lesssim 1/Mx$, $|\xi^0| \lesssim 1/Mx$. Long range correlations in the target ground state are thus associated with the small x behavior of the structure function and vice versa. In Ref. 13 I used this simple kinematic analysis to argue that the degradation of the valence quarks in iron could be understood as an indication of a larger average correlation length. It has long been suspected that quarks in the nucleus are at least partially deconfined, i.e., free to move over longer distances than in an isolated nucleon. To support the qualitative argument I calculated the relation between bag radius and structure function:

$$q'_{R'}(x) = \frac{R'}{R}\, q_R\left(\frac{R'}{R}\, x\right), \tag{11}$$

where q_R ($q_{R'}$) is the quark distribution in a bag of radius $R(R')$. It is easy to see that Eq. (11) has the expected behavior: If $R' > R$ then $q_{R'}(x)$ is shifted to small x relative to $q_R(x)$. To estimate the size of the effect I made a crude model of internucleon correlations—replacing them as six quark bags—and calculated $R^{\mathrm{Fe}}(x)$. Such a calculation falls into the class of convolution models and no longer seems to me very well justified. Whatever its origin a *kinematic* change of scale shifts both valence and ocean distributions to lower x. It does not account for the large enhancement in the ocean quark distribution.

In a recent preprint, Close, Roberts, and Ross[3] put the analysis of length scales on a much firmer footing and in the process achieved a unified interpretation of both the degradation of valence quarks and the enhancement ocean quarks. The key insight is that changes of length or mass scale can be analyzed using the renormalization group in QCD. The idea is extremely simple to anyone familiar with the standard QCD analysis of scaling violations. Close, Roberts, and Ross noticed that the change in F_2 going from deuterium to iron looks very similar to the change in F_2 obtained by evolving any single target upward in Q^2, i.e., valence quarks lose momentum and pairs are created. They look at the flavor non-singlet moments of $F_2^A(x,Q^2)$,

$$M_n^A(Q^2) = \frac{1}{A}\int_0^A dx\, x^{n-2}\, F_2^A(x,Q^2) \tag{12}$$

(flavor singlets are analyzed similarly) which have simple scale dependence

$$M_n^A(Q^2) = (\alpha_s(Q^2)/\alpha_s(\mu^2))^{d_n}\, M_n^A(\mu^2) \tag{13}$$

to leading order in perturbative QCD. $\alpha_s(Q^2)$ is related to $\alpha_s(\mu^2)$ by

$$\alpha_s(Q^2) = \alpha_s(\mu^2)/(1 + \alpha_s(\mu^2)\,(\beta_0/4\pi)\ln Q^2/\mu^2) \tag{14}$$

and the exponents, d_n, are calculable. The moments $M_n(Q^2)$ are related to matrix elements of certain (twist-two) operators $O_n^{\mu_1 \cdots \mu_n}(Q^2)$ renormalized at a scale Q^2.

To compare the structure functions of two different targets we have chosen a common scale and compared $M_n(Q^2)$ with $M_n^A(Q^2)$. [$M_n(Q^2)$ are the moments for a nucleon target.] CRR propose a novel method of comparing two targets: since $M_n^A(Q^2)$ grows monotonically as Q^2 decreases (at least at lowest order in α_s) it is always possible to find two different scales, Q^2 and $\xi_A Q^2$ such that $M_n^A(\xi_A Q^2) = M_n(Q^2)$. This defines a relation, $\xi_A = \xi_A(Q^2,n)$ with the implication that as far as the nth twist-2 operator is concerned, the nucleus observed at a scale $\xi_A Q^2$ looks like a nucleon at a scale Q^2. This mapping always exists and in itself is not surprising.

The surprise comes when CRR use their methods to analyze the EMC data and find that *$\xi A(Q^2,n)$ appears to a good approximation to be independent of n*. Thus the distinction between iron and deuterium, so far as deep inelastic scattering is concerned, is a single, overall change of scale. Since it is n-independent, the scale change can be made directly in the structure function:

$$\frac{1}{A} F_2^A(x,\xi_A Q^2) = \frac{1}{2} F_2^D(x,Q^2). \tag{15}$$

CRR find that over the range of the EMC data $\xi_A = ½$ gives a reasonable fit shown in Figure 4

The scale factor ξ_A is itself scale-dependent. To leading order in α_s, combining Eqs. (13) and (14) we find

$$\xi_A(Q^2) = \xi_A(\mu^2)^{\alpha_s(\mu^2)/\alpha_s(Q^2)}. \tag{16}$$

The smaller μ^2 the closer $\xi_A(\mu^2)$ approaches unity. CRR argue [following Ref. (17)] that the quarks of spectroscopic quark models (like the bag model) are associated with a scale μ_0 typical of hadron masses and transverse momenta. Choosing $\Lambda_{LO} \sim 0.2-0.3$ GeV and μ_0^2 in the range 0.5 – 1.5 GeV^2 they obtain $\xi_A(\mu_0^2) \sim 0.7-0.8$ and interpret $1\sqrt{\xi_A(\mu_0^2)} \sim 1.1-1.2$ as a measure of the increased mean free path of quarks in the nucleus.

Several comments and caveats are in order:

1. This is more than a "naive" change of scale: pairs of quarks are created in the process.
2. CRR predict $1/A F_2^A(x) - F_2^N(x) \approx 0$ at $x \approx 0.2$ independent of A, since this is the value of x at which QCD evolution seems to have no net effect on structure function.
3. If the EMC data are correct in detail then the scale change cannot be the whole story: the data require a slight increase in the "momentum" carried by quarks and antiquarks in iron over deuterium. QCD evolution requires a slight decrease (more "momentum" on gluons). It is clear in Figure 5 that the ocean enhancement (low x) exceeds the prediction of QCD evolution. Alternatively, perhaps scale change is the whole story and the EMC data are too high at low x as electron and neutrino scattering experiments would like.
4. It cannot work for very large moments ($n \rightarrow \infty$). If $\xi_A(Q^2,n)$

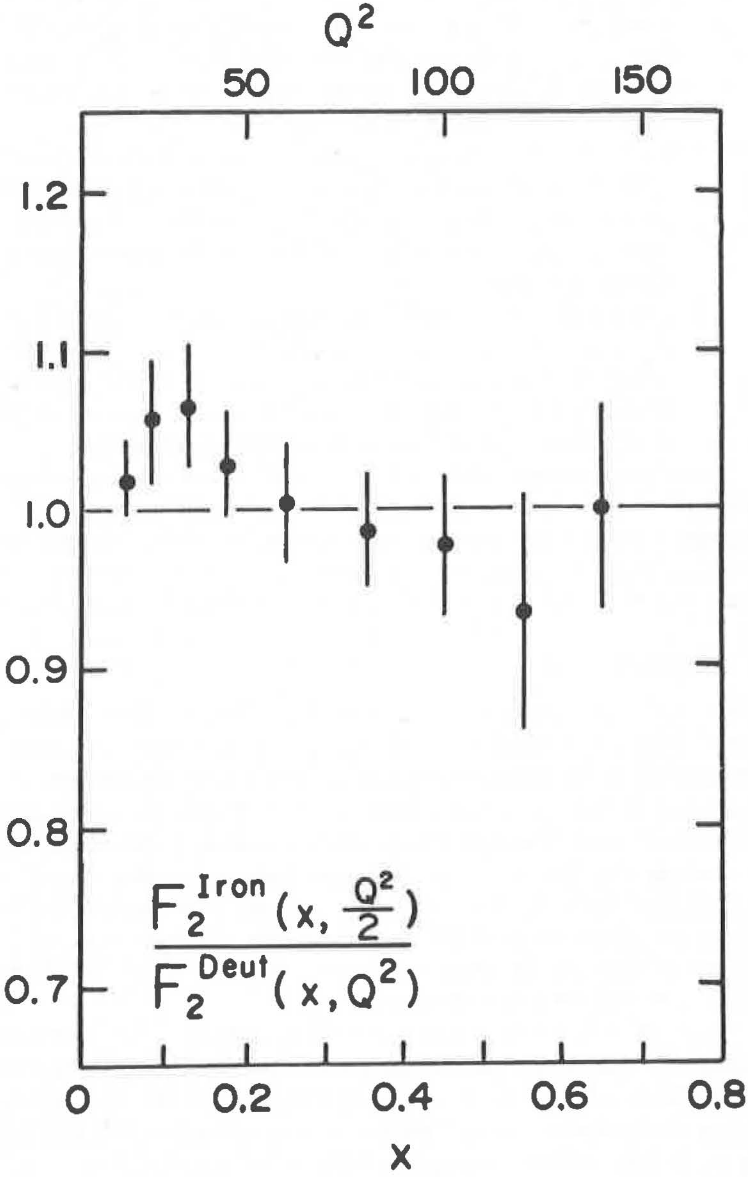


FIGURE 4 The rescaling analysis of CRR with $\xi_{Fe}(Q^2) = ½$.

was independent of n for all n one would predict that the $F_2^A(x,Q^2) = 0$ for $x > 1$, which is not a bad approximation but certainly wrong in principle. Higher order QCD corrections can be shown to limit the range of validity of the CRR analysis to low values of n.[18]

5. The extraction of $\xi_A(\mu^2)$ requires extrapolation of leading order QCD formulas to low mass scales. This is dangerous but higher order effects have been shown to be small in similar application.[17] In any event the scale change analysis can be carried out at any scale.
6. The CRR analysis does not mean that the iron nucleus can be related to deuterium by a simple change of scale. The complete description of a nucleus involves matrix elements of all twists and multiple scales.The gross structure of the EMC effect only involves a few operators of twist-2.

The primary virtues of the CRR analysis are its roots in the well understood phenomenology of asymptotic QCD and its freedom from the flaws in convolution models which I outline below. On the other hand (a critic would quickly add that) it provides no "explanation" for the observed change of scale, only a framework for discussing it.

Convolution Models

The temptation to seek an explanation of the EMC effect by admixing novel hadronic components into the nuclear wavefunction is nearly irresistible, as the flood or preprints with this intention testifies. The basic idea is that the nucleus contains, with some (presumably small) probability, objects different from free nucleons (e.g., pions, off-shell nucleons, Δ's, N^*'s, 6-, 9-, or 12-quark bags, α-particles, etc.). The quark distribution in the nucleus is assumed to be the sum of the quark distributions in all of these "packages" and in some cases is found to resemble the distributions measured by EMC and RMS. There are problems with this approach.

It is probably too weak to be used for quantitative analyses except when the "packages" are the nucleons themselves. I will review the formulation of package models and then analyze the assumptions upon which they are based. Finally, these criticisms notwithstanding, I will briefly review some applications of this approach.

Convolution models are summarized by the diagram in Figure 5. The struck quark carries plus component of momentum $k^+ =$

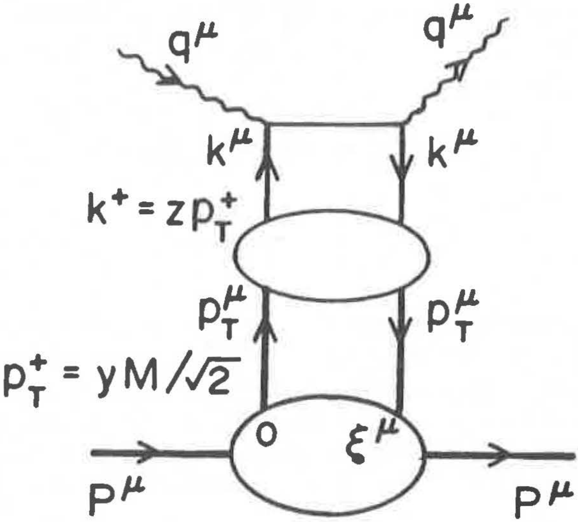


FIGURE 5 The convolution model. T is a nuclear constituent "package" containing quarks of type a.

$xM/\sqrt{2}$, where x is Bjorken's variable [see Eq. (5)]. Suppose the struck quark comes from some "package" T found in the nucleus. Let $f_{i/T}(z)$ be the probability to find a quark with flavor i in "package" T with $k^+ = zp_T^+$ (p_T^μ is the "package's" momentum) and $0 < z < 1$. Similarly, let $g_{T/A}(y)$ be the probability *per nucleon* to find the "package" T in nucleus A with $p_T^+ = yM/\sqrt{2}$. y lies between O and A (because $M/\sqrt{2}$ rather than P_A^+ appears in its definition). Clearly $x = yz$ and the contribution to the distribution of quarks of flavor i in nucleus A from "packages" T is given by a convolution[19]

$$
\begin{aligned}
f_{i/T/A}(x) &= \int_0^A dy \int_0^1 dz\, \delta\,(yz - x)\, f_{i/T}(z) g_{T/A}(y) \\
&= \int_x^A dy\, f_{i/T}(x/y)\, g_{T/A}(y)
\end{aligned}
\tag{17}
$$

$f_{i/T}$ and $g_{T/A}$ are probability distributions in p^+ and are therefore normalized:

$$\int_0^A dy\, g_{T/A}(y) = N_{T/A}/A, \int_0^1 dz\, f_{i/T}(z) = N_{i/T},$$

$$\int_0^A ydy\, g_{T/A}(y) = \epsilon_{T/A}, \int_0^1 zdz\, f_{i/T}(z) = \epsilon_{i/T}.$$

$N_{T/A}$ is the number of "packages" T in nucleus A. Likewise for $N_{i/T}$ though $N_{i/T}$ may be infinite due to ocean pair contributions. $\epsilon_{T/A}$ is the fraction of the nucleus' P^+ carried by target T. Likewise for $\epsilon_{i/T}$. The moment of a convolution is a product of moments, thus $f_{i/T/A}(x)$ obeys the number and momentum sum rules expected on physical grounds. Sum rules constrain $\epsilon_{T/A}$ and $\epsilon_{i/T}$: $\sum_i \epsilon_{i/T} = 1$, $\sum_T \epsilon_{T/A} = 1$.

f and g can be expressed in terms of the imaginary parts of forward virtual scattering amplitudes. Consider $g_{T/A}$. The lowest rung of Figure 5 is the imaginary part of the forward virtual T-nucleus scattering amplitude $A(p,P)$:

$$A(p,P) = \int d^4\xi e^{ip\cdot\xi}\langle P|\phi_T^\dagger(\xi)\phi_T(0)|P\rangle_c \tag{18}$$

In Figure 5 all components of p_T^μ have been integrated over except p_T^+:

$$g_{T/A}(y) = \int d^4p_T\delta(p_T^+ - yM/\sqrt{2})A(p,P) \tag{19}$$

$$= \int d\xi^- e^{-i\xi^- My/\sqrt{2}}\langle P|\phi_T^\dagger(\xi^-)\phi_T(0)|P\rangle_c\Big|_{\xi^+ = \xi_\perp = 0} \tag{20}$$

So $g_{T/A}(y)$ measures a nuclear correlation function along the null plane. It is entirely analogous to the parton distribution function $f_{i/T}(z)$ which measures a null plane correlation of quark fields.

What is wrong with convolution models? In short (for more details see Ref. 17), they assume two levels of incoherence while only one is justified by appeal to QCD. The situation is summarized by Figure 6. Asymptotic freedom allows us to replace the general forward virtual Compton amplitude (Figure 6a) by the parton-model diagram (Figure 6b) at asymptotic Q^2. A requirement for the argument is the "hard momentum, q^μ, which flows along the struck parton leg. Convolution models require a further step: a second level of incoherence, shown in Figure 6c, is extracted from the parton-target forward amplitude.

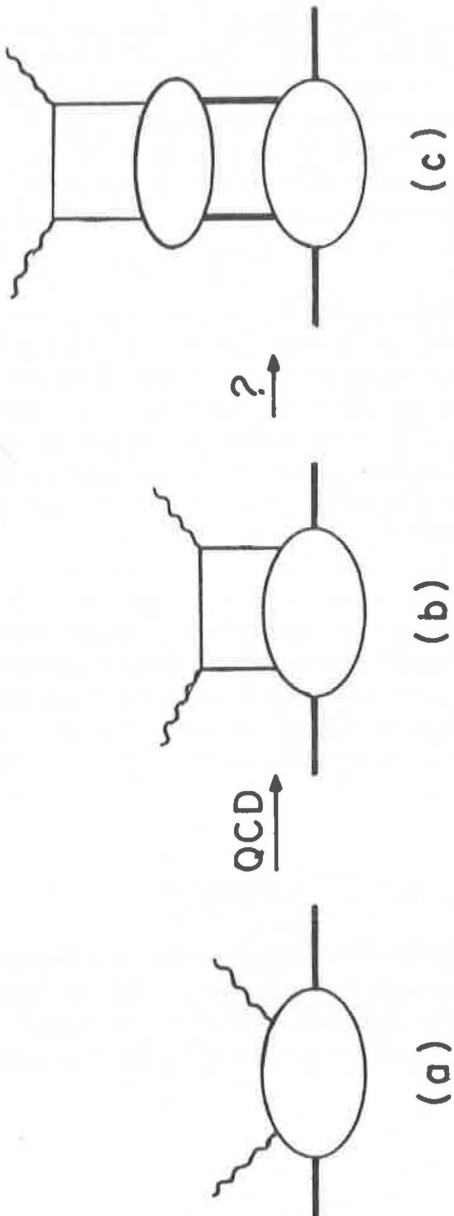
Nothing justifies *a priori* ignoring interactions between fragments of the "package" and spectators in the nucleus which are approximately at rest with respect to one another. Examples are shown in Figures 6d and 6e. From the perspective of leptoproduction these are final state interactions. If these interactions are ignored the amplitude factors into the product of two amplitudes each of which can be expressed as a probability distribution in p^+ and the convolution model emerges.[19]

Without a reason for throwing out final state interactions the convolution model remains at best a qualitative guide to the interpretation of the data. We can get further insight into the importance of the interactions in Figures 6d and 6e by studying the space-time structure of $g_{T/A}(y)$. According to Eq. (20), the space-time separation between emission and absorption of the "package" T is ξ^μ with $\xi^+ = \xi_\perp = 0$ and $\xi^- \lesssim \sqrt{2}/My$ where $y = (p_T^0 + p_T^3)/M$, thus

$$\xi^0 \lesssim \frac{1}{p_T^0 + p_T^3}. \tag{21}$$

If ξ^0 were much less than typical nuclear and hadronic interaction times then one might be justified in ignoring interactions occurring between emission and absorption of the "package." So a sufficient (and probably necessary) condition for ignoring final state interactions is

$$\frac{1}{p_T^0 + p_T^3} << \tau_{\text{int}}. \tag{22}$$


QCD
?
(a)
(b)
(c)

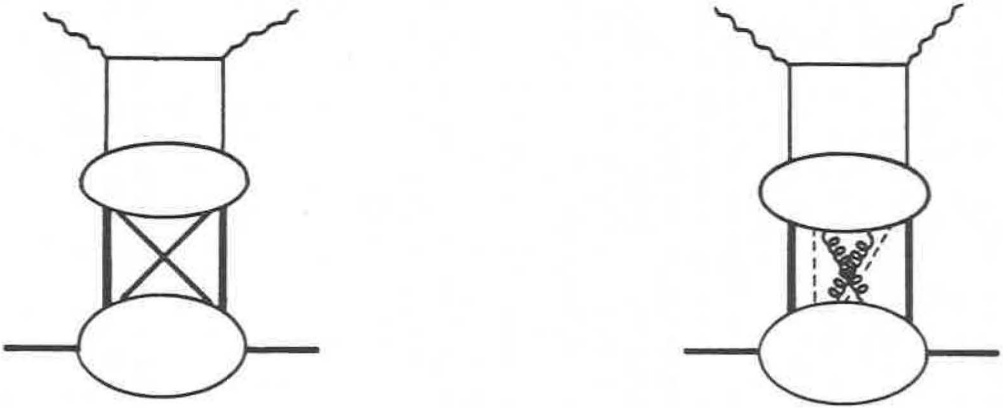


FIGURE 6 The progression from general virtual Compton scattering (a) to the QCD parton model (b) to the convolution model (c) requiring progressively more layers of incoherent scattering. (d) and (e): processes that are ignored in the convolution model.

Let us consider this on a case by case basis. Nucleons are not far off shell on average in the nucleus so $p_N^0 + p_N^3 \sim M$. Typical values for τ_{int} are determined by typical nuclear kinetic and potential energies which are of order 100–300 MeV. So for nucleons Eq. (33) is weakly satisfied and the convolution model may be reasonable. Pions in the nucleus, be they on shell or off, are characterized by $p_\pi^0 + p_\pi^3 \sim 0(m_\pi)$ while τ_{int} remains of order $(100-300)^{-1}$ MeV^{-1}. So for pions, Eq. (22) is not even approximately satisfied and this attempt to justify the convolution model fails. The distinction between nucleons and pions becomes particularly clear in the nuclear weak binding limit: as the binding energy does to zero $\tau_{\text{int}} \to \infty$, $p_N^0 + p_N^3 \to M$ and $p_\pi^0 + p_\pi^3 \sim 1/\tau_{\text{int}} \to 0$ so Eq. (22) is valid for nucleons and not for pions. The situation is less clear for multiquark systems, such as six quark bags. $p^0 + p^3$ is large, of order $2M$, but the effective lifetime of a six quark cluster may be very short.

It appears that convolution models should not be taken very seriously, especially not for pions. Until some justification can be provided for ignoring final state interactions, their predictions should probably be taken "with a grain of salt."

These caveats notwithstanding, I will discuss a couple of applications of the convolution model:

1. Fermi smearing[20]: Suppose the nucleus contains only nucleons distributed according to some $g_{N/A}(y)$. From Eq. (17) we obtain a "smeared" distribution function for the nucleus:

$$f_{i/N/A}(x) = \int_x^A dy = f_{i/N}(x/y)\, g_{N/A}(y).$$

For nucleons $g_{N/A}(y)$ should be sharply peaked about $y = 1$ since $y = p_N^0 + p_N^3/M$ and $p_N^0 \approx M$, $p_N^3 \approx m_\pi$. Thus the convolution smears $f_{i/N}$ over a small interval about x. In this model $N_{T/A} = A$ and $\epsilon_{T/A} = 1$ so

$$\int_0^A dx\, f_{i/N/A}(x) = \int_0^1 dx\, f_{i/N}(x)$$

$$\int_0^A xdx\, f_{i/N/A}(x) = \int_0^1 xdx\, f_{i/N}(x).$$

From these it follows that difference, $f_{i/N/A}(x) - f_{i/N}(x)$, which is the shift due to Fermi smearing must have at least two nodes. Since $f_{i/N}(x) = 0$ for $x > 1$ the difference is positive for $x > 1$. It must go negative for $x < 1$ and then positive for even smaller x. More than two changes of sign are not excluded. Many proposed models of Fermi smearing do not have this property, consequently the nucleon distributions on which they are based do not conserve nucleon number and energy. The "momentum" sum rule for $g_{N/A}(y)$ adds up the energy of the nucleons in the nucleus: $\int_0^1 ydy\, g_{N/A}(y) = M_A/AM \cong 1$. This can only be satisfied if the nucleons are off energy shell. If they were on shell the energy of each nucleon, $\sqrt{M^2 + \mathbf{p}_N^2}$, would exceed M and the sum rule could not be satisfied. Fermi smearing models assume that the structure function of an off-shell nucleon is the same as on-shell, an assumption over which there is no control.

2. Pions: It should be clear that the use of convolution models for pion "packages" should be regarded with skepticism. Recently several detailed studies of pion effects have been made so there is no need to review them here. I have one further caveat to add. It is often remarked that an ocean enhancement at low x due to pions combined with momentum conservation (for nuclear "packages") alone requires a depletion in $F_2^A(x)$ at large x. The argument is a bit oversimplified: one must not confuse momentum which is kinematic, with p^+ which involves p^0 and therefore dynamics. Some of the nucleus' p^+, the argument goes, is carried by pions. Therefore less is carried by nucleons. If a nucleon carries less p^+ so do its valence quarks, so their distribution is degraded. In fact, reducing the mean p^+ of the nucleons drives them further off shell: $p^+ = 1/\sqrt{2}\,\langle p^0\rangle$ since $\langle p^3\rangle = 0$. No one knows which components of the nucleon lose energy as the nucleon goes off shell. It is perfectly conceivable that the energy is extracted entirely from the gluon field in which case the valence quark distribution would be unaffected.

3. Multiquark correlations: Here one is handicapped by not having a good model for the structure function of multiquark hadrons. The scaling law derived from the bag model applies only to spherical

states and is more suited to be input to the CRR QCD analysis than as a detailed model for the structure function.[18] Attempts have been made to use quark counting rules to estimate multiquark structure functions for $x \geq 0.3$.[22] These seem ill-founded. Quark counting rules describe suppression at the edge of phase space: To get to $z = 1$ a quark in a $3n$ quark state must absorb the p^+ carried by all $3n$-1 spectators. The x values measured by current experiments are far from the edge of phase space for a $3n$ quark cluster (I have assumed an effective target mass of $(n/3)M$ so $z = 1$ corresponds to $x = n$). There are other, less suppressed ways to get a large amount of p^+ on a single quark. Consider, for example, a quark in a six-quark state. Standard quark counting ($f_n(x) \sim (1 - 3x/n)^{2n-3}$) yields $f_6(x)$ ($\sim 1 - x/2)^9$ for $x < 2$ when a quark absorbs momentum from five spectators. Suppose, instead, the quark interacts by hard gluon exchange with only two other quarks. This costs $f_3(x) \sim (1 - x)^3$ but can only work for $x < 1$. For $x < 0.75$ $f_3(x) > f_6(x)$ and quark counting rules do not apply. In fact the arguments that lead to the counting rule apply only if $f_{3n}(x) > f_{3n-l}(x)$ in a $3n$ quark state. For $n = 2$, x must be greater than 0.86, but even at $x = 1.3$ the five-quark cluster is suppressed only by a factor 3 compared with the six-quark cluster. So the use of quark counting may not be justified for the x values of interest. Without a detailed model, multiquark packages remain a qualitative notion.

4. SUMMARY

Theorists have sought evidence for quarks and gluons (as distance from nucleons and mesons) in the nucleus for years. Having found it we are at a loss to analyze it: Naive convolution models work (i.e., they undoubtedly can be made to fit the data) but rest on shaky foundations; QCD analyses (à la Close, Roberts and Ross) are intriguing but do not as yet yield a very detailed picture of the phenomena. There is much to do.

Acknowledgment

This work is supported in part through funds provided by the U.S. Department of Energy (DOE) under contract DE-AC02-76ER03069.

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