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Nuclear Physics B 923 (2017) 378–393

NUCLEAR PHYSICS B

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Hawking radiation power equations for black holes

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Received 3 July 2017; accepted 14 August 2017

Editor: Hubert Saleur

Abstract

We derive the Hawking radiation power equations for black holes in asymptotically flat, asymptotically Anti-de Sitter (AdS) and asymptotically de Sitter (dS) black holes. This is done by using the greybody factor for these black holes. We observe that the radiation power equation for asymptotically flat black holes, corresponding to greybody factor at low frequency, depends on both the Hawking temperature and the horizon radius. However, for the greybody factors at asymptotic frequency, it only depends on the Hawking temperature. We also obtain the power equation for asymptotically AdS black holes both below and above the critical frequency. The radiation power equation for at asymptotic frequency is same for both Schwarzschild AdS and Reissner–Nordström AdS solutions and only depends on the Hawking temperature. We also discuss the power equation for asymptotically dS black holes at low frequency, for both even or odd dimensions.

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http://dx.doi.org/10.1016/j.nuclphysb.2017.08.010

1. Introduction

The evaporation of a black hole can be understood in terms of the black body factor and the greybody factor. The black body factor is calculated from the probability of a particle being created in the vicinity of a horizon, and the greybody factor is calculated from the probability that this particle penetrates the potential barrier and escapes to infinity. The analysis of the black hole greybody factors shows that Hawking emission from a highly rotating black hole is strongly spin dependent, with particles of highest spin (gravitons) dominating the energy spectrum [1]. Gravitational greybody factors are analytically computed for static, spherically symmetric black holes, including black holes with charge and in the presence of a cosmological constant [2]. In this context, the greybody factors for both asymptotically dS and AdS spacetimes can be obtained. There are many distinct models with exact black hole solutions in literature, which in turn implies the need for concrete calculations of the corresponding greybody factors (some recent developments can be found in [3–14]).

These greybody factors were studied long ago in [15,16]. It is very simple to understand the basic set-up of the calculations. It might be of some difficulty to extract exact results by such set-up. Black holes scattering is described via linearized wave equations, which governs the particle perturbation in the black hole geometry [17]. The equation describing particle perturbations in black hole geometries, in this spherically symmetric context, can always be written as a one-dimensional Schrödinger-like equation. The quantum Hawking evaporation of near-extremal Reissner–Nordström black holes is studied recently, where the effective curvature potential causes distortion in the familiar radiation spectrum of genuine $(3 + 1)$-dimensional perfect black-body emitters [18]. The Hawking radiations of black holes in asymptotically flat, AdS and dS spacetimes though greybody factors are subject of interesting investigations. With the help of radiation power expression, one can compute total energy emitted as Hawking radiation by multiplying the power with black hole evaporation time scale and this must be equal to the total mass of the black hole by virtue of energy conservation. This may provide an insight to the process of black hole evaporation.

It may be noted that we use a formalism that depends on the frequency, and it can be consistently applied as the greybody factors are functions of all frequencies [2]. So, we use a form of the greybody factor which is defined for all the frequencies. Here we note that the low energy expression for the greybody factor for scalar fields in the background of a higher-dimensional Schwarzschild black hole have been obtained derived in [19]. The low-energy greybody factor for a higher-dimensional dS black hole have also been studied [20]. The emission of Hawking radiation of higher-dimensional black holes in the bulk (greybody factors and radiation spectra) are studied in details for the emission of scalar modes, and the ratio of the missing energy over the visible one is calculated for different values of the number of extra dimensions [21]. The process we consider here is the absorption of a scalar wave by a black hole. Since the scalar wave propagates from infinity throughout spacetime, is partly reflected by the potential barrier of the black hole, and close to horizon the transmitted wave seems as the incoming radiation into the black hole. In fact, the greybody factor for low frequency scattering is identical to the absorption probability of the black hole as the scattering and absorption processes are reverse to each other and therefore we consider real frequencies at low energy. We shall mainly work with the results of [2] for our paper, where it is shown that the leading contribution to the greybody factor, in the low frequency limit, comes from the $l = 0$ mode.

With this motivation, in this paper, we calculate the Hawking radiation power equation for black holes in $d+1$-dimensional asymptotically flat, AdS and dS spacetimes with the help of following equation[1] [22,23]

$$P^{d+1} = \frac{T_H}{2\pi} \int_0^\infty d\omega \frac{\gamma(\omega) x}{e^x - 1}, \tag{1}$$

where $x \equiv \frac{\omega}{T_H}$, T_H is the Hawking temperature and $\gamma(\omega)$ represents greybody factor (the probability for an outgoing wave, in the ω-mode, to reach infinity). First, we consider the greybody factor at the low frequency limit for black holes in asymptotically flat spacetime and compute the respective Hawking radiation power equation which depends on both, the Hawking temperature and horizon radius R_H under different power law. Then, we derive the Hawking radiation power equation corresponding to the greybody factor at asymptotic frequency for Schwarzschild solution and find that it depends on the Hawking temperature only and does not depend on horizon radius. Interestingly, we observe that the radiation power equations corresponding to greybody factors at asymptotic frequency are same for all spacetime dimensions as it does not depend on the dimension d. In fact, it depends on Hawking temperature only with power law. Further, we compute the Hawking radiation power equation along greybody factor at the low frequency limit for asymptotically AdS black holes. We consider here both cases for which the frequencies are much lower and higher than the critical frequency (at which the black hole absorbs all of the radiation which is sent towards it). The radiation power equations for asymptotically AdS black holes at asymptotic frequency are also derived for both the Schwarzschild and Reissner–Nordström solutions which are found same and depend on the Hawking temperature only. Corresponding to greybody factor at the low frequency, we demonstrate the Hawking radiation power equations for asymptotically dS black holes for both even and odd spacetimes. Here, we find that the radiation power equation in five-dimensional dS spacetimes depends on both the horizon radius and Hawking temperature. In case of even spacetime dimensions, we get simpler form of it, however it has an infinite sum series of Hurwitz Zeta function for the case of odd spacetime dimensions. It may be noted that as the greybody factors depend on all frequencies [2], our formalism can be applied for any given greybody factors. We have plotted diagrams also to understand the behavior of the radiation power equation with respect to both the Hawking temperature and horizon radius.

This paper is organized as following. In section 2, we compute the radiation power equation for asymptotically flat spacetimes. Specifically, we derive radiation power equation for the black holes with the help of greybody factors at both the low frequency and asymptotic frequency for Schwarzschild solution. In section 3, we evaluate the Hawking radiation power equations for the asymptotically AdS black holes. Here, the radiation power equations corresponding to greybody factors at both low and asymptotic frequencies for Schwarzschild solution and Reissner–Nordström solution are discussed. Further, in section 4, we derive the radiation power equation for greybody factors at low frequency only for asymptotically dS black holes in even and odd spacetimes. In the last section, we draw conclusion with final remarks.

[1] We use the unit system in which $G = \hbar = c = k_B = 1$.

2. Asymptotically flat spacetimes

In this section, we analyse the radiation power equation for asymptotically flat black holes. First, we shall derive the radiation power equation for greybody factor at low frequency limit $\omega \ll T_H$, $\omega R_H \ll 1$ for Schwarzschild solution. Here, the low frequency limit precisely means that the wavelength of the scalar wave is much larger than any of the characteristic scales associated with the black hole. Then, we discuss the radiation power equation for greybody factor at asymptotic frequency for the case of Schwarzschild solution.

2.1. Power equation for greybody factors at low frequency

In this subsection, we compute the radiation power equation for greybody factor at low frequency. Since the greybody factor in the low frequency limit for asymptotically flat black holes is given by [2]

$$\gamma(\omega) = \frac{4\pi\omega^{d-2}R_H^{d-2}}{2^{d-2}[\Gamma(\frac{d-1}{2})]^2},$$

where R_H is the horizon radius. Here we clarify that to the above greybody factor is calculated by assuming that the scalar wave propagates from infinity throughout spacetime, is partly reflected by the potential barrier of the black hole, and near the horizon the transmitted radiation appears as purely incoming radiation into the black hole. We note that, as the scattering and absorption processes are reverse to each other, the greybody factor for low frequency scattering is identical to the absorption probability of a black hole.

For a convenience, we change dimension $d \to (d+1)$[2] and, hence, the greybody factor for $d+1$ dimensional black hole takes following form:

$$\gamma(\omega) = \frac{4\pi\omega^{d-1}R_H^{d-1}}{2^{d-1}[\Gamma(\frac{d}{2})]^2}. \tag{2}$$

For greybody factor in the low frequency limit as given in (2), the Hawking radiation power equation for asymptotically flat black holes is given by

$$P_{low}^{(d+1)} = \frac{T_H}{2\pi}\int_0^\infty d\omega \frac{4\pi\omega^{d-1}R_H^{d-1}}{2^{d-1}[\Gamma(\frac{d}{2})]^2}\frac{\omega}{T_H(e^{\frac{\omega}{T_H}}-1)},$$

where we have utilized relation (1). After further simplification, this reduces to the following expression:

$$P_{low}^{(d+1)} = \frac{R_H^{d-1}}{2^{d-2}[\Gamma(\frac{d}{2})]^2}\int_0^\infty d\omega \frac{\omega^d}{e^{\frac{\omega}{T_H}}-1}. \tag{3}$$

In order to solve above equation, we recall Riemann Zeta function, which is given by,

$$\zeta(s) = \frac{1}{\Gamma(s)}\int_0^\infty dy \frac{y^{s-1}}{e^y-1} \quad \text{for } Re(s) > 1. \tag{4}$$

[2] Throughout the paper $d \to (d+1)$ represents that d is being replaced by $(d+1)$, where d stands for an arbitrary number of dimensions.

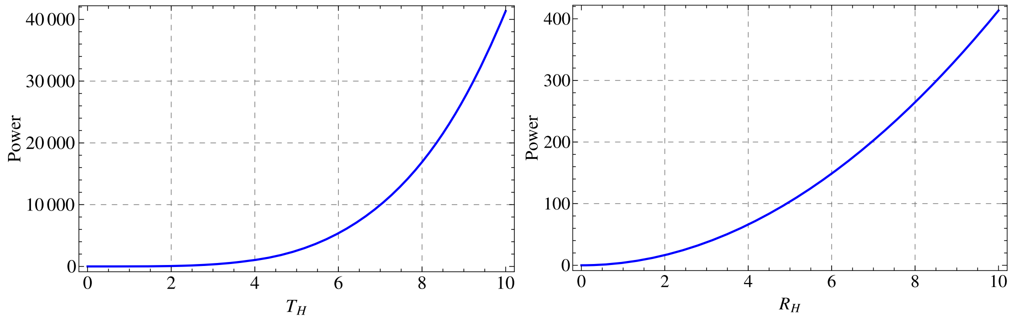

Fig. 1. Left: Hawking radiation power versus Hawking temperature for $d = 3$ and $R_H = 1$. Right: Hawking radiation power versus horizon radius for $d = 3$ and $T_H = 1$.

To match this Zeta function with the desired integral, we replace $y = \frac{\omega}{T_H}$ in above equation to get,

$$\zeta(s) = \frac{1}{T_H^s \Gamma(s)} \int_0^\infty d\omega \frac{\omega^{s-1}}{e^{\frac{\omega}{T_H}} - 1} \quad \text{for} \;\; Re(s) > 1. \tag{5}$$

Now, exploiting (3) and (5), the Hawking radiation power equation for asymptotically flat black holes is given by

$$P_{low}^{(d+1)} = C_{low}^{(d+1)} T_H^{d+1} R_H^{d-1}, \tag{6}$$

where explicit form of $C_{low}^{(d+1)}$ is $C_{low}^{(d+1)} = \frac{\zeta(d+1)\Gamma(d+1)}{2^{d-2}[\Gamma(\frac{d}{2})]^2}$. Here we see that the Hawking radiation power equation depends on both the Hawking temperature and horizon radius with different power law. For example, for a black hole in four dimensional spacetime, the Hawking radiation power equation is proportional to T_H^4 and R_H^2. The behavior of Hawking radiation power with respect to Hawking temperature and horizon radius can been seen from Fig. 1.

2.2. *Power equation for greybody factors at asymptotic frequency*

In this subsection, we determine power equation for greybody factors at asymptotic frequency for Schwarzschild solution. In order to determine the radiation power equation, we first write the Schwarzschild greybody factor at asymptotic frequency by [2],

$$\gamma(\omega) = \frac{e^{\frac{\omega}{T_H}} - 1}{e^{\frac{\omega}{T_H}} + 3}. \tag{7}$$

The poles of above greybody factor precisely correspond to the asymptotic quasinormal frequencies. Now, using binomial theorem, we can write the greybody factor at asymptotic frequency as

$$\left(e^{\frac{\omega}{T_H}} - 1\right)\left(3 + e^{\frac{\omega}{T_H}}\right)^{-1} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{3^n} \left(e^{\frac{n\omega}{T_H}} - e^{\frac{(n-1)\omega}{T_H}}\right). \tag{8}$$

With the help of expression (1) and greybody factor (8), the radiation power equation corresponding to the greybody factors at asymptotic frequency for Schwarzschild solution takes the following form:

$$P^{(d+1)}_{asym} = \frac{1}{2\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{3^n}\left(\int_0^{\infty} d\omega \frac{\omega e^{\frac{n\omega}{T_H}}}{e^{\frac{\omega}{T_H}}-1} - \int_0^{\infty} d\omega \frac{\omega e^{\frac{(n-1)\omega}{T_H}}}{e^{\frac{\omega}{T_H}}-1}\right). \tag{9}$$

In order to simplify the above integral, we can use the Hurwitz (generalized Riemann) Zeta function[3] as it has the following definition:

$$\zeta(s,q) = \frac{1}{\Gamma(s)}\int_0^{\infty} dy \frac{y^{s-1}e^{y(1-q)}}{e^y-1} \quad \text{for } Re(s) > 1.$$

In order to match the form of above Hurwitz Zeta function integral with the radiation power equation (9), we make the following identification: $y = \frac{\omega}{T_H}$. With such identification, the expression of Hurwitz Zeta function results to

$$\zeta(s,q) = \frac{1}{T_H^s\Gamma(s)}\int_0^{\infty} d\omega \frac{\omega^{s-1}e^{\frac{\omega}{T_H}(1-q)}}{e^{\frac{\omega}{T_H}}-1} \quad \text{for } Re(s) > 1. \tag{10}$$

Comparing expressions (9) and (10), one can easily derive the radiation power equation corresponding to greybody factors at asymptotic frequency for Schwarzschild solution as

$$\begin{aligned} P^{(d+1)}_{asym} &= \frac{1}{2\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{3^n}\left[\Gamma(2)\zeta\left(2,1-n\right)T_H^2 - \Gamma(2)\zeta\left(2,1-(n-1)\right)T_H^2\right] \\ &= C^{(d+1)}_{asym}T_H^2, \end{aligned} \tag{11}$$

where, coefficient $C^{(d+1)}_{asym} = \frac{1}{2\pi}\sum_{n=1}^{\infty}\left[\frac{(-1)^{n+1}}{3^n}\left(\zeta\left(2,1-n\right)-\zeta\left(2,2-n\right)\right)\right]$. Here, we have used $\Gamma(n) = (n-1)!$ for $\Gamma(2) = 1! = 1$. Remarkably, we find that the radiation power equations corresponding to greybody factors at asymptotic frequency in any arbitrary dimensions are same as it does not depend on the dimension d. Also, we notice that the radiation power equation corresponding to greybody factors at asymptotic frequency depends on Hawking temperature only with power law T_H^2 and does not depend on horizon radius. However, for greybody factor at low frequency, it depends on both the Hawking temperature and horizon radius.

3. Asymptotically AdS spacetimes

In this section, we derive the radiation power equation of black hole in asymptotically AdS spacetimes corresponding to the greybody factors at both the low frequency and asymptotic frequency.

[3] Here we note that for this type of Zeta function there are two different forms exists 1) $\zeta(s,q) = \sum_{a=0}^{\infty}(a+q)^{-s}$ for $q > 0$ and 2) $\zeta(s,q) = \sum_{a=0}^{\infty}[(a+q)^2]^{-\frac{s}{2}}$ for $q < 0$. We also note that both are identical for $Re(q) > 0$.

3.1. Power equation for greybody factors at low frequency

The greybody factor for asymptotically AdS black holes, in the low frequency regime $\hat{\omega} \ll 1$ is given by [2],

$$\gamma(\hat{\omega}) = 1 - \left| \frac{1 - z(\hat{\omega})}{1 + z(\hat{\omega})} \right|^2, \tag{12}$$

where $\hat{\omega} = \frac{\omega}{k}$ is a dimensionless variable for the frequency and quantity

$$z(\hat{\omega}) = \frac{\pi}{2^{d-1}[\Gamma(\frac{d}{2})]^2} \frac{\omega^{d-1}}{k^{2d-2} R_H^{d-1}}. \tag{13}$$

Now, we define $\frac{\pi}{2^{d-1}[\Gamma(\frac{d}{2})]^2 k^{2d-2} R_H^{d-1}} := \alpha$, so that we can write $z(\hat{\omega}) = \alpha \omega^{d-1}$. Here note that we have used $d \to (d+1)$ in original form (which can be derived back by using $d \to (d-1)$ in above expressions). For this greybody factor for asymptotically AdS black holes in the low frequency regime (12), the radiation power equation (1) takes the following form:

$$P_{low}^{(d+1)} = \frac{1}{2\pi} \left[\int_0^\infty d\omega \frac{\omega}{e^{\frac{\omega}{T_H}} - 1} - \int_0^\infty d\omega \left(\frac{1-z}{1+z} \right)^2 \frac{\omega}{e^{\frac{\omega}{T_H}} - 1} \right]. \tag{14}$$

Here we see that the first integral of above expression can be solved by using (5), as a result we get $\int_0^\infty d\omega \frac{\omega}{e^{\frac{\omega}{T_H}} - 1} = \zeta(2)\Gamma(2) T_H^2 = \frac{\pi^2 T_H^2}{6}$. With this simplification, the expression for radiation power (14) reads,

$$P_{low}^{(d+1)} = \frac{\pi}{12} T_H^2 - \frac{1}{2\pi} \int_0^\infty d\omega \left(\frac{1 - \alpha\omega^{d-1}}{1 + \alpha\omega^{d-1}} \right)^2 \frac{\omega}{e^{\frac{\omega}{T_H}} - 1},$$

here we have utilized $z = \alpha\omega^{d-1}$. Now, taking $\alpha\omega^{d-1} = \delta$, the above expression can further be simplified as follows,

$$P_{low}^{(d+1)} = \frac{\pi}{12} T_H^2 - \frac{1}{2\pi} \frac{1}{(d-1)\alpha^{\frac{2}{d-1}}} \times \int_0^\infty d\delta \left(1 - \frac{4}{1+\delta} + \frac{4}{(1+\delta)^2} \right) \frac{\delta^{\frac{3-d}{d-1}}}{\exp\left[\frac{1}{T_H} \left(\frac{\delta}{\alpha} \right)^{\frac{1}{d-1}} \right] - 1}.$$

Here, exploiting binomial expansion for $(1+\delta)^{-1}$ and $(1+\delta)^{-2}$, we find that $(1+\delta)^{-1} - (1+\delta)^{-2} = \sum_{n=1}^\infty (-1)^{n+1} n \delta^n$. With this result, the above expression for the radiation power equation reduces to,

$$P_{low}^{(d+1)} = \frac{\pi}{12} T_H^2 - \frac{1}{2\pi} \frac{1}{(d-1)\alpha^{\frac{2}{d-1}}} \times \int_0^\infty d\delta \left(1 - 4 \sum_{n=1}^\infty (-1)^{n+1} n \delta^n \right) \frac{\delta^{\frac{3-d}{d-1}}}{\exp\left[\frac{1}{T_H} \left(\frac{\delta}{\alpha} \right)^{\frac{1}{d-1}} \right] - 1}. \tag{15}$$

In order to solve above integral, we plug $y = \frac{1}{T_H}\left(\frac{\delta}{\alpha}\right)^{\frac{1}{d-1}}$ in the expression of Zeta function (4), and we have

$$\zeta(s) = \frac{1}{T_H^s \Gamma(s)} \frac{1}{\alpha^{\frac{s}{d-1}}(d-1)} \int_0^\infty d\delta \frac{\delta^{\frac{s-d+1}{d-1}}}{\exp\left[\frac{1}{T_H}\left(\frac{\delta}{\alpha}\right)^{\frac{1}{d-1}}\right]-1} \quad \text{for } Re(s) > 1. \tag{16}$$

Finally, using (15) and (16), we get simplified expression for the radiation power equation corresponding to greybody factor in low frequency limit as

$$P_{low}^{(d+1)} = \sum_{n=1}^{\infty} C_{low}^{(d+1)} \frac{T_H^{nd-n+2}}{k^{2n(d-1)} R_H^{n(d-1)}}, \tag{17}$$

where, $C_{low}^{(d+1)} = \frac{2}{\pi}\left[(-1)^{n+1}\frac{n\pi^n}{2^{n(d-1)}\left[\Gamma\left(\frac{d}{2}\right)\right]^{2n}}\zeta(nd-n+2)\Gamma(nd-n+2)\right]$. Here, the radiation power equation has an infinite sum series with terms depending on Hawking temperature and horizon radius differently. Also, we notice that, contrary to flat spacetime case, the radiation power equation depends on horizon radius with inverse power law.

Here, we note that there exists a critical frequency $\hat{\omega}_c$ for which there is no reflection of radiation for black hole i.e., the black hole absorbs all of the radiation which is sent towards it. Alternatively, in the reverse process, i.e., for emission of radiation from the black hole, all of the emitted radiation at critical frequency will reach the asymptotic region.

In this condition, $z(\hat{\omega}_c) = 1$ and critical frequency $\hat{\omega}_c$ simplifies from (13) as [2]

$$\hat{\omega}_c = \frac{2\left[\Gamma\left(\frac{d-1}{2}\right)\right]^{\frac{2}{d-2}}}{\pi^{\frac{1}{d-2}}} k R_H.$$

Here we note that only for small AdS black holes one can achieve the critical frequency. Now, there are two cases possible in calculation of power radiation equation for AdS black hole. Firstly, if we now consider frequencies much lower than the critical frequency ($\hat{\omega} << \hat{\omega}_c$) and secondly, if we now consider instead frequencies much higher than the critical frequency ($\hat{\omega} >> \hat{\omega}_c$).

3.1.1. *Case I: when $\hat{\omega} << \hat{\omega}_c$*

In this case, the greybody factor for $\hat{\omega} << \hat{\omega}_c$ is given as follows,

$$\gamma(\hat{\omega}) = 4z(\hat{\omega}) = \frac{\pi}{2^{d-2}\left[\Gamma\left(\frac{d-1}{2}\right)\right]^2} \frac{\hat{\omega}^{d-2}}{(kR_H)^{d-2}}.$$

Here we notice that the greybody factor is inversely proportional to the area of the black hole, whereas it is proportional to ω^{d-2}. Therefore, the frequencies much lower than the critical frequency is identical to $\hat{\omega} << kR_H$. This means that large AdS black holes (with $kR_H >> 1$) are always in a frequency regime much lower than the critical frequency. For convenience (without loss of generality), we rewrite the above expression of the greybody factor for $d \to (d+1)$ as,

$$\gamma(\hat{\omega}) = \frac{\pi}{2^{d-1}\left[\Gamma\left(\frac{d}{2}\right)\right]^2} \frac{\omega^{d-1}}{k^{2d-2} R_H^{d-1}}. \tag{18}$$

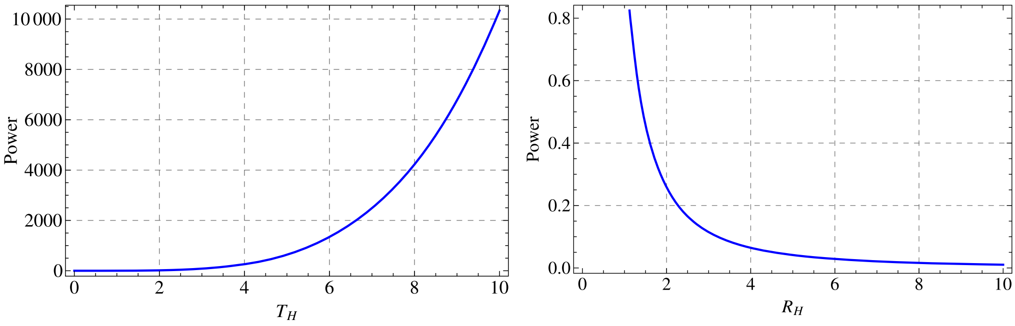

Fig. 2. Left: Hawking radiation power versus Hawking temperature for $d = 3$ and $k = R_H = 1$. Right: Hawking radiation power versus horizon radius for $d = 3$ and $k = T_H = 1$.

For this greybody factor, the power radiation equation (1) is given by

$$P^{(d+1)}_{low-I} = \frac{1}{2^d[\Gamma\left(\frac{d}{2}\right)]^2}\frac{1}{k^{2d-2}R_H^{d-1}}\int_0^\infty d\omega \frac{\omega^d}{e^{\frac{\omega}{T_H}}-1},$$

here, subscript $low - I$ stands for low frequency in region I (i.e. $\hat{\omega} << \hat{\omega}_c$). In order to obtain explicit expression for the power radiation, we need to solve the integral, which can be done easily with the help of Zeta function (5). Hence, we get

$$P^{(d+1)}_{low-I} = C^{(d+1)}_{low-I}\frac{T_H^{d+1}}{k^{2(d-1)}R_H^{d-1}}, \tag{19}$$

where the constant $C^{(d+1)}_{low-I} = \frac{\zeta(d+1)\Gamma(d+1)}{2^d\left[\Gamma\left(\frac{d}{2}\right)\right]^2}$. Here we observe that the radiation power corresponding to greybody factor for frequencies much lower than the critical frequency depends on both the Hawking temperature with power law $\sim T_H^{d+1}$ and horizon radius with power law $\sim R_H^{d-1}$. In four-dimensional spacetimes, the radiation power equation reduces to $P^{(d+1)}_{low-I} = \frac{\pi^3}{30}\frac{T_H^4}{k^4R_H^2}$. The behavior of Hawking radiation power with respect to Hawking temperature and horizon radius can be seen in Fig. 2.

3.1.2. Case II: when $\hat{\omega} >> \hat{\omega}_c$

In this case of frequencies much higher than the critical frequency, the greybody factor for $d \to (d+1)$ is given by,

$$\gamma\left(\hat{\omega}\right) = \frac{2^{d-1}\left[\Gamma\left(\frac{d}{2}\right)\right]^2}{\pi}\frac{R_H^{d-1}k^{2d-2}}{\omega^{d-1}}. \tag{20}$$

Here, greybody factor is proportional to the area of the black hole, whereas it is instead inversely proportional to ω^{d-2}. Here we note that this frequency regime is possible for small AdS black holes with $kR_H << 1$ only. Now, we follow the same procedure as discussed in case I and get the following expression for radiation power equation:

$$P^{(d+1)}_{low-II} = C^{(d+1)}_{low-II}T^{-d+3}k^{2(d-1)}R_H^{d-1}, \tag{21}$$

where constant $C^{(d+1)}_{low-II} = \frac{2^{d-2}\left[\Gamma\left(\frac{d}{2}\right)\right]^2}{\pi^2}\zeta(-d+3)\Gamma(-d+3)$. Here subscript $low-II$ stands for low frequency in region II (i.e. $\hat{\omega} >> \hat{\omega}_c$). Remarkably, we observe that the radiation power depends both on Hawking temperature with power law $\sim T^{-d+3}$ and horizon radius with power law $\sim R_H^{d-1}$. This clearly means that for four-dimensional spacetimes the radiation power depends only on horizon radius and does not depend on Hawking temperature. However, for space dimensions $d \neq 3$, the radiation power depends on temperature with different nature. For $d > 3$, it depends on temperature with inverse power law and, for $d < 3$, it depends on temperature with direct power law.

3.2. Power equation for greybody factors at asymptotic frequency

Remarkably, the greybody factor at asymptotic frequency for the Schwarzschild solution and the Reissner–Nordström solution is same and given by [2],

$$\gamma(\omega) = 1. \tag{22}$$

For this greybody factor, the Hawking radiation power equation reads,

$$P^{(d+1)}_{asym} = \frac{1}{2\pi}\int_0^\infty d\omega \frac{\omega}{e^{\frac{\omega}{T_H}} - 1}.$$

The above integration can be performed with the help of (5). Thus, we find the value of radiation power equation in simplified form as

$$P^{(d+1)}_{asym} = C^{d+1}_{asym} T_H^2, \tag{23}$$

where the coefficient $C^{(d+1)}_{asym} = \frac{\zeta(2)\Gamma(2)}{2\pi} = \frac{\pi}{12}$. Here we conclude that the radiation power equation corresponding to greybody factor at asymptotic frequency depends on Hawking temperature only.

4. Asymptotically de Sitter spacetimes

In this section we shall find the Hawking radiation power equation corresponding to the greybody factor, at low frequencies, for black holes in asymptotically dS spacetimes. The greybody factor (after considering $d \to (d+1)$) in this case is given by [2]

$$\gamma(\omega) = 4h(\hat{\omega})(kR_H)^{(d+1)-2} = 4h(\hat{\omega})(kR_H)^{d-1}, \tag{24}$$

where function $h(\hat{\omega})$ for even $(d+1) \geq (3+1)$ is expressed by

$$h(\hat{\omega}) = \prod_{n=1}^{\frac{(d+1)-2}{2}} \left(1 + \frac{\hat{\omega}^2}{(2n-1)^2}\right) = \prod_{n=1}^{\frac{d-1}{2}} \left(1 + \frac{\hat{\omega}^2}{(2n-1)^2}\right), \tag{25}$$

however, for odd $(d+1) \geq (4+1)$ we have,

$$h(\hat{\omega}) = \frac{\pi\hat{\omega}}{2}\coth\left(\frac{\pi\hat{\omega}}{2}\right)\prod_{n=1}^{\frac{(d+1)-3}{2}}\left(1 + \frac{\hat{\omega}^2}{(2n)^2}\right) = \frac{\pi\hat{\omega}}{2}\coth\left(\frac{\pi\hat{\omega}}{2}\right)\prod_{n=1}^{\frac{d-2}{2}}\left(1 + \frac{\hat{\omega}^2}{(2n)^2}\right). \tag{26}$$

Here we see that $h(\hat{\omega}) \to 1$ and $h(\hat{\omega}) \to 0$ as $\hat{\omega} \to 0$, for even and odd spacetime dimension respectively. Now, we shall calculate the power radiation equation for the given greybody factor by considering specific cases. For example, we shall study (a) $(d+1)=4$ and (b) $(d+1)=6$ for the even spacetimes, and (c) $(d+1)=5$ and (d) $(d+1)=7$ for the odd spacetimes.

4.1. *Even Spacetimes case I: $(d+1)=4$*

In this case, the function $h(\hat{\omega})$ (25) leads to

$$h(\hat{\omega}) = 1 + \hat{\omega}^2 = 1 + \frac{\omega^2}{k^2}.$$

With this value of $h(\hat{\omega})$, the greybody factor, at low frequencies, for asymptotic dS black holes (24) reads,

$$\gamma(\omega) = 4k^2 R_H^2 + 4\omega^2 R_H^2. \tag{27}$$

Now, exploiting relation (1), we write the power radiation equation for four-dimensional dS black hole as follows:

$$P^{(3+1)}_{low-even} = \frac{2}{\pi} R_H^2 \left[k^2 \int_0^\infty d\omega \frac{\omega}{e^{\frac{\omega}{T_H}} - 1} + \int_0^\infty d\omega \frac{\omega^3}{e^{\frac{\omega}{T_H}} - 1} \right]. \tag{28}$$

The integrals of above expression can be solved very easily with the help of Riemann Zeta function (5). After simplification the expression (28) reduces to

$$\begin{aligned} P^{(3+1)}_{low-even} &= \frac{2}{\pi} R_H^2 \left(\frac{1}{6}\pi^2 k^2 T_H^2 + \frac{6}{90}\pi^4 T_H^4 \right), \\ &= \frac{1}{3}\pi k^2 R_H^2 T_H^2 + \frac{2}{15}\pi^3 R_H^2 T_H^4. \end{aligned} \tag{29}$$

It is evident that the radiation power for four-dimensional asymptotically dS black hole corresponding to greybody factor at low frequency depends on both the Hawking temperature and horizon radius. The behaviors of radiation power for four-dimensional asymptotically dS black hole corresponding to greybody factor at low frequency with respect to Hawking temperature and horizon radius can be seen in Fig. 3.

4.2. *Even Spacetimes case II: $(d+1)=6$*

In case $(d+1)=6$, the expression for $h(\hat{\omega})$ given in (25) has the following form:

$$h(\hat{\omega}) = 1 + \frac{10\omega^2}{9k^2} + \frac{\omega^4}{9k^4}.$$

With this value of $h(\hat{\omega})$, the greybody factor, at low frequencies, for asymptotically dS black holes (24) is given by

$$\gamma(\omega) = 4k^4 R_H^4 + \frac{40}{9}\omega^2 k^2 R_H^2 + \frac{4}{9}\omega^4 R_H^4. \tag{30}$$

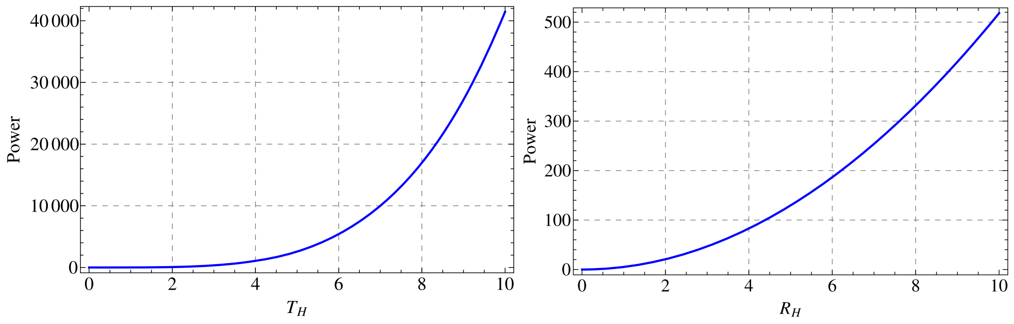

Fig. 3. Left: Hawking radiation power versus Hawking temperature for $d = 3$ and $k = R_H = 1$. Right: Hawking radiation power versus horizon radius for $d = 3$ and $k = T_H = 1$.

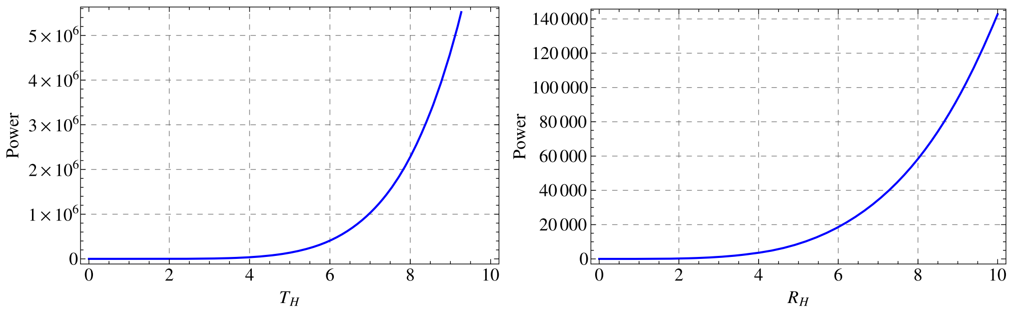

Fig. 4. Left: Hawking radiation power versus Hawking temperature for $d = 5$ and $k = R_H = 1$. Right: Hawking radiation power versus horizon radius for $d = 3$ and $k = T_H = 1$.

Once the expression for the greybody factor is known, it is matter of calculation to obtain the radiation power equation which utilizes relation (1). For the greybody factor (30), the expression for the radiation power equation is given by

$$P^{(5+1)}_{low-even} = \frac{2}{\pi} R_H^4 \left[k^4 \int_0^\infty d\omega \frac{\omega}{e^{\frac{\omega}{T_H}} - 1} + \frac{10}{9} k^2 \int_0^\infty d\omega \frac{\omega^3}{e^{\frac{\omega}{T_H}} - 1} + \frac{1}{9} \int_0^\infty d\omega \frac{\omega^5}{e^{\frac{\omega}{T_H}} - 1} \right]. \quad (31)$$

In order to solve the integrals in above expression, we utilize the Zeta function (5). After doing so, the expression for the radiation power equation simplified to

$$\begin{aligned} P^{(5+1)}_{low-even} &= \frac{2}{\pi} R_H^4 \left[k^4 \zeta(2)\Gamma(2) T_H^2 + \frac{10k^2}{9} \zeta(4)\Gamma(4) T_H^4 + \frac{1}{9} \zeta(6)\Gamma(6) T_H^6 \right], \\ &= \frac{1}{3} \pi k^4 R_H^4 T_H^2 + \frac{4}{27} \pi^3 k^2 R_H^4 T_H^4 + \frac{16}{567} \pi^5 R_H^4 T_H^6. \end{aligned} \quad (32)$$

Here, we see that the expression for the radiation power equation depends on both the horizon radius and Hawking temperature. In particular, it depends on the sum of different powers of Hawking radiation. In order to see behavior of the radiation power equation with respect to horizon radius and Hawking temperature, we plot Fig. 4.

4.3. Odd Spacetimes case I: $(d+1)=5$

The expression for $h(\hat{\omega})$ given in (26) for Odd Spacetimes $(d+1)=5$ takes following value:

$$h(\hat{\omega}) = \frac{\pi\omega}{2k}\coth\left(\frac{\pi\omega}{2k}\right)\left(1+\frac{\omega^2}{4k^2}\right). \tag{33}$$

Now, in order to simplify above expression we utilize following definition: $\coth(x) = \frac{1+e^{-2x}}{1-e^{-2x}}$. With this definition, we have

$$\coth\left(\frac{\pi\omega}{2k}\right) = \left(1+e^{-\pi\frac{\omega}{k}}\right)\left(1-e^{-\pi\frac{\omega}{k}}\right)^{-1}. \tag{34}$$

With the help of binomial expansion, we can write $\left(1-e^{-\pi\frac{\omega}{k}}\right)^{-1} = \sum_{m=1}^{\infty} e^{(1-m)\pi\frac{\omega}{k}}$. As a result, the expression (34) reduces to $\coth\left(\frac{\pi\omega}{2k}\right) = \sum_{m=1}^{\infty}\left(e^{-m\pi\frac{\omega}{k}} + e^{(1-m)\pi\frac{\omega}{k}}\right)$. By inserting this value of $\coth\left(\frac{\pi\omega}{2k}\right)$ into (33), the function $h\left(\frac{\omega}{k}\right)$ takes the following expression:

$$h\left(\frac{\omega}{k}\right) = \frac{\pi\omega}{2k}\left(1+\frac{\omega^2}{4k^2}\right)\sum_{m=1}^{\infty}\left(e^{-m\pi\frac{\omega}{k}} + e^{(1-m)\pi\frac{\omega}{k}}\right).$$

Plugging this value of $h\left(\frac{\omega}{k}\right)$ in (24), the greybody factor for five-dimensional asymptotic dS black holes at low frequencies reduces to

$$\gamma(\omega) = \frac{2\pi\omega}{k}(kR_H)^3\left(1+\frac{\omega^2}{4k^2}\right)\sum_{m=1}^{\infty}\left(e^{-m\pi\frac{\omega}{k}} + e^{(1-m)\pi\frac{\omega}{k}}\right). \tag{35}$$

Now, it is matter of calculation to evaluate the radiation power equation for a given greybody factor. So, utilizing relations (1) and (35), we write the radiation power equation for black hole in five-dimensional dS spacetimes as

$$\begin{aligned} P^{(4+1)}_{low-odd} &= \frac{T_H}{2\pi}\int_0^{\infty} d\omega \frac{\left(2\pi\omega^2 k^2 R_H^3\right)}{T_H\left(e^{\frac{\omega}{T_H}}-1\right)}\left(1+\frac{\omega^2}{4k^2}\right)\sum_{m=1}^{\infty}\left(e^{-m\pi\frac{\omega}{k}} + e^{(1-m)\pi\frac{\omega}{k}}\right), \\ &= R_H^3\sum_{m=1}^{\infty}\left[\int_0^{\infty} d\omega\left(e^{-m\pi\frac{\omega}{k}} + e^{(1-m)\pi\frac{\omega}{k}}\right)\left(\frac{k^2\omega^2}{e^{\frac{\omega}{T_H}}-1} + \frac{1}{4}\frac{\omega^4}{e^{\frac{\omega}{T_H}}-1}\right)\right]. \end{aligned} \tag{36}$$

In order to simplify the integrals, we utilize the Hurwitz Zeta function (10) and by doing so, we get the following explicit expression for the radiation power equation:

$$\begin{aligned} P^{(4+1)}_{low-odd} &= 2k^2 R_H^3 T_H^3\sum_{m=1}^{\infty}\left[\zeta\left(3, 1+\frac{m\pi}{k}T_H\right) + \zeta\left(3, 1-\frac{(1-m)\pi}{k}T_H\right)\right] \\ &+ 6R_H^3 T_H^5\sum_{m=1}^{\infty}\left[\zeta\left(5, 1+\frac{m\pi}{k}T_H\right) + \zeta\left(5, 1-\frac{(1-m)\pi}{k}T_H\right)\right]. \end{aligned} \tag{37}$$

Here, it is evident that although the radiation power equation in five-dimensional dS spacetimes depends on both the horizon radius and Hawking temperature but has an infinite sum series of Hurwitz Zeta function as well.

Next, we shall derive the radiation power equation for the $(d+1)=7$.

4.4. Odd Spacetimes case II: $(d+1)=7$

The expression for function $h(\hat{\omega})$ (26) for Odd Spacetimes $(d+1)=7$ is given by:

$$h(\hat{\omega}) = \frac{\pi\omega}{2k}\coth\left(\frac{\pi\omega}{2k}\right)\left(1+\frac{5\omega^2}{16k^2}+\frac{\omega^4}{64k^4}\right). \tag{38}$$

Now, plugging the value of $\coth\left(\frac{\pi\omega}{2k}\right)$ calculated in the above last subsection, the above function $h(\hat{\omega})$ reduces to the following form:

$$h\left(\frac{\omega}{k}\right) = \frac{\pi\omega}{2k}\left(1+\frac{5\omega^2}{16k^2}+\frac{\omega^4}{64k^4}\right)\sum_{m=1}^{\infty}\left(e^{-m\pi\frac{\omega}{k}}+e^{(1-m)\pi\frac{\omega}{k}}\right). \tag{39}$$

With this value of $h\left(\frac{\omega}{k}\right)$ (39), the greybody factor for seven-dimensional asymptotically dS black holes at low frequencies (24) has following value:

$$\gamma(\omega) = 2\pi\frac{\omega}{k}(kR_H)^5\left(1+\frac{5\omega^2}{16k^2}+\frac{\omega^4}{64k^4}\right)\sum_{m=1}^{\infty}\left(e^{-m\pi\frac{\omega}{k}}+e^{(1-m)\pi\frac{\omega}{k}}\right). \tag{40}$$

Once the expression for greybody factor of black hole is known, the Hawking radiation power equation can easily be calculated from relation (1). For a given greybody factor (40) at low frequency in $(d+1)=7$ dimensions, the Hawking radiation power equation reads,

$$\begin{aligned}
P^{(6+1)}_{low-odd} &= \frac{1}{2\pi}\int_0^{\infty} d\omega\frac{(2\pi\omega^2k^4R_H^5)}{e^{\frac{\omega}{T_H}}-1}\left(1+\frac{5\omega^2}{16k^2}+\frac{\omega^4}{64k^4}\right)\sum_{m=1}^{\infty}\left(e^{-m\pi\frac{\omega}{k}}+e^{(1-m)\pi\frac{\omega}{k}}\right), \\
&= R_H^5\sum_{m=1}^{\infty}\left[\int_0^{\infty} d\omega\frac{\left(e^{-m\pi\frac{\omega}{k}}+e^{(1-m)\pi\frac{\omega}{k}}\right)}{e^{\frac{\omega}{T_H}}-1}\left(k^4\omega^2+\frac{5}{16}k^2\omega^4+\frac{1}{64}\omega^6\right)\right].
\end{aligned} \tag{41}$$

In order to simplify the above expression, we can use Hurwitz Zeta function (10). By doing so, we get following expression for the Hawking radiation power equation:

$$\begin{aligned}
P^{(6+1)}_{low-odd} &= 2k^4R_H^5T_H^3\sum_{m=1}^{\infty}\left[\zeta\left(3,1+\frac{m\pi}{k}T_H\right)+\zeta\left(3,1-\frac{(1-m)\pi}{k}T_H\right)\right] \\
&\quad+\frac{15k^2}{2}R_H^5T_H^5\sum_{m=1}^{\infty}\left[\zeta\left(5,1+\frac{m\pi}{k}T_H\right)+\zeta\left(5,1-\frac{(1-m)\pi}{k}T_H\right)\right] \\
&\quad+\frac{45}{4}R_H^5T_H^7\sum_{m=1}^{\infty}\left[\zeta\left(7,1+\frac{m\pi}{k}T_H\right)+\zeta\left(7,1-\frac{(1-m)\pi}{k}T_H\right)\right].
\end{aligned} \tag{42}$$

Here, we can see that the Hawking radiation power equation in $(d+1)=7$ dimensions also depends on both the horizon radius and Hawking temperature but with different power law. Similar to the previous case, here also, Hawking radiation power has an infinite sum series of Hurwitz Zeta function.

5. Conclusions

In this paper, we have evaluated the Hawking radiation power equations for given greybody factors in asymptotically flat, AdS and dS black holes in $(d+1)$ dimensions. First of all, we have derived the Hawking radiation power equation for asymptotically flat Schwarzschild solution of black hole corresponding to the greybody factor at low frequency. We have solved the Hawking radiation power equation with the help of Zeta function and found that it depends on both the Hawking temperature and horizon radius with different power law. Moreover, we compute the same for the greybody factor at asymptotic frequency and found that in this region of frequency the Hawking radiation power equation depends on the Hawking temperature only with square power law and remains same for all spacetime dimensions.

Furthermore, we have evaluated the radiation power equations for black hole in asymptotically AdS spacetimes. Here, also we have considered the greybody factors in different regime of frequency in order to calculate power equations. For the low energy regime, we note that the critical frequency exists only for small AdS black holes at which there are no reflection of radiation for black hole and/or no emission of radiation from the black hole. Here, we have discussed the cases: 1) if one considers frequencies much lower than the critical frequency, and 2) if one considers instead frequencies much higher than the critical frequency. For former case, we have observed that the radiation power depends on both the Hawking temperature with power law and horizon radius. However, for later case, remarkably, we observed that though it depends on both Hawking temperature and horizon radius but with different power law. In fact, for four-dimensional spacetimes the radiation power depends on horizon radius only but not on the Hawking temperature. The radiation power for greybody factors at asymptotic frequency is independent of horizon radius and dimensionality of spacetime, however, depends on Hawking temperature only.

For asymptotically dS spacetime, the Hawking radiation power corresponding to greybody factor at low frequency highly depends on dimensionality of spacetime. We have computed this for even and odd spacetimes. In case of even dimensions, we have obtained simpler form of the radiation power equation which depends on both the horizon radius and Hawking temperature. However, the radiation power equation in odd dS spacetimes although depends on both the horizon radius and Hawking temperature but has an infinite sum series of Hurwitz Zeta function.

It is known that by multiplying the radiation power expression of black hole with black hole evaporation time, one can estimate the total energy emitted as Hawking radiation which must be equal to the total mass of the black hole by virtue of energy conservation. This might give an insight to understand the process of black hole evaporation. Such analysis a subject of future work.

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